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DESIGN OF MICROWAVE DEVICES EXPLOITING FIBONACCI AND HYBRID PERIODIC/FIBONACCI ONE DIMENSIONAL PHOTONIC CRYSTALS

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Abstract—We report the transmission response of generalized Fibonacci photonic crystal $F_l(m, n)$ in microwave domain for normal incidence, where l is the generation number, and m and n are parameters of the Fibonacci distribution. The transmission spectra are calculated through the transfer matrix method and studied by varying the Fibonacci parameters. The structure is exploited to design selective optical filters with narrow passbands and polychromatic stop band filters. Therefore, other structure configurations based on the generalized Fibonacci system are proposed. A juxtaposition of p multilayer systems built according to Fibonacci distribution $[F_l(m, n)]^p$ makes possible to have switch-like property (off-on-off-on-off-on-...). Then, a hybrid structure obtained by sandwiching stacks of generalized Fibonacci photonic crystal between two periodic photonic crystals is proposed to enlarge the photonic band gap in microwave domain.

1. INTRODUCTION

Photonic crystals with forbidden photonic bands have attracted much attention due to promising applications in optical devices, such as waveguides, cavities, mirrors, optical switches, channel-drop filters, wave division multiplexers, antireflexion coating [1–5]. The simplest form of a photonic crystal is a one-dimensional periodic structure and known as Bragg Mirror [6, 7]. It consists of a stack of alternating layers having a low and a high refractive indices and a thickness on the order of $\lambda_0/4$ where λ_0 is the wavelength of the light. Photonic

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band gap (PBG) materials may be designed also in two and three dimensions. But one-dimensional PBG structures have been given more importance and investigated extensively because such structures can be fabricated more easily for a large range of wavelength, and also the analytical study and numerical calculations are simpler [8]. To ameliorate the optical functions of the one dimensional photonic crystal, many techniques have been introduced such as defect insertion, combinations of two or more photonic crystals (heterostructures), creating disorder [9–12]. The disorder may be constructed according to a deterministic procedure. So a great deal of effort has been devoted to the study of quasi periodic systems which possess the properties of both periodic and random structures and show interesting properties as the existence of PBGs with some strong resonances, which can localize light very effectively [13, 14]. Some of these works were focused on studying the Fibonacci quasi-periodic multilayer [15–20]. From a structural viewpoint, Fibonacci structures are composed of building blocks exhibiting two incommensurate periods [21].

The photonic heterostructures are combinations between adjacent photonic crystals. This approach has been generally explored to design optical micro cavities [22] and to enlarge the photonic band gap [23–25]. Heterostructures investigated recently are varied. A Photonic crystal formed of two structures of the same period and different layer thicknesses is studied to enhance the omnidirectional reflection [24]. A narrow frequency and sharp angular defect mode is obtained by combining two one-dimensional defective photonic crystals [26]. Some researchers are interested in the combination of two photonic crystals with different periods [27] or two photonic crystals, one of which is dielectric and the other magnetic [28]. The heterostructure revealed in this paper is the combination of periodic and quasi periodic photonic crystals such as the Fibonacci structure. Such a structure has been the subject of some works. It is found for example that a hybrid configuration of the type Bragg mirror-Fibonacci-Bragg mirror is a promising candidate for resonant microcavities with strong mode localization [29].

In this work, we have firstly studied the transmission properties in microwave domain [5 GHz, 50 GHz] of the one-dimensional multilayer system built according to the generalized Fibonacci sequence $F_l(m, n)$ [15, 30, 31] as a function of the Fibonacci parameters. This study reveals interesting properties of the structure since it shows that controlling the Fibonacci parameters permits to obtain selective optical filters with narrow passband and polychromatic stop band filters with varied properties which can be controlled as desired. Secondly, we propose a configuration which is a juxtaposition of

$S_0 = H$
$S_1 = L$
$S_2 = S_1^2 S_0^3$=*LLHHH*
$S_3 = S_2^2 S_1^3$=*LLHHHLLHHHLLL*
$S_4 = S_3^2 S_2^3$=*LLHHHLLHHHLLLLLHHHLLHHHLLLLLHHHLLHHHLLHHH*
......

Figure 1. Scheme of different generations with $m = 2$ and $n = 3$.

several blocks built according to generalized Fibonacci sequence $[F_l(m,n)]^p$. The transmission through this configuration shows the characteristic of switches. This feature could be useful in the design of microwave switches. Finally, we present some hybrid configurations consisting of juxtaposition of periodic and quasi-periodic structures. The quasi-periodic multilayer is built according to the pattern of the Fibonacci sequence. Such configurations enhance the reflection through the system and enlarge the photonic band gap. We calculate transmission spectra through these structures using the transfer Matrix Method [2, 32, 33]. In this paper, we present the mathematical model describing the structures containing the distribution of high and low refractive index layers according to Fibonacci sequence. Next, we give simulation results and finish with a conclusion.

2. MODELS

The first system under consideration is composed of two layers, H and L, stacked alternatively along z direction and following the rules of Fibonacci sequence, i.e., $S_{l+1} = S_l^m S_{l-1}^n$ for $l \geq 1$ with $S_0 = H$ and $S_1 = L$, where l is the generation number, and m and n are parameters characterizing substitution rules generating the sequence. In fact, each transition from a generation to the following one is obtained by doing the substitutions $L \to L^m H^n$, $H \to L$. Figure 1 explains what change in generation number implies when $m = 2$ and $n = 3$. We can build different generations through two procedures, by using the relation between S_{l+1}, S_l and S_{l-1}, or by using the substitution rules between two consecutive generations. Next, we consider a system which is a combination of blocks of Fibonacci structures so having the form $[F_l\,(m,n)]^p$. Finally, a one-dimensional hybrid periodic and generalized quasiperiodic photonic crystal is studied. The structure is built according to the form $(LH)^j\,[F_l\,(m,n)]^p\,(LH)^j$. Here, H and L are defined as two dielectric materials with H the high refractive index material and L the low refractive index one.

We choose the Roger and the Air respectively as the high refractive index material ($n_H = 3.134$) and the low one ($n_L = 1$). Refractive indices of these materials are assumed to be constant in the wavelength region of interest. We assume the individual layers as quarter-wave layers so satisfying the Bragg condition $n_H d_H = n_L d_L = \frac{\lambda_0}{4}$ with λ_0 the reference wavelength chosen to be 12 mm.

3. RESULTS AND DISCUSSIONS

3.1. Generalized Fibonacci Multilayer Structure

3.1.1. Generation Number Effect

In this section, we present in Figure 2 the transmission spectra through the Fibonacci photonic multilayer for different generation numbers. A

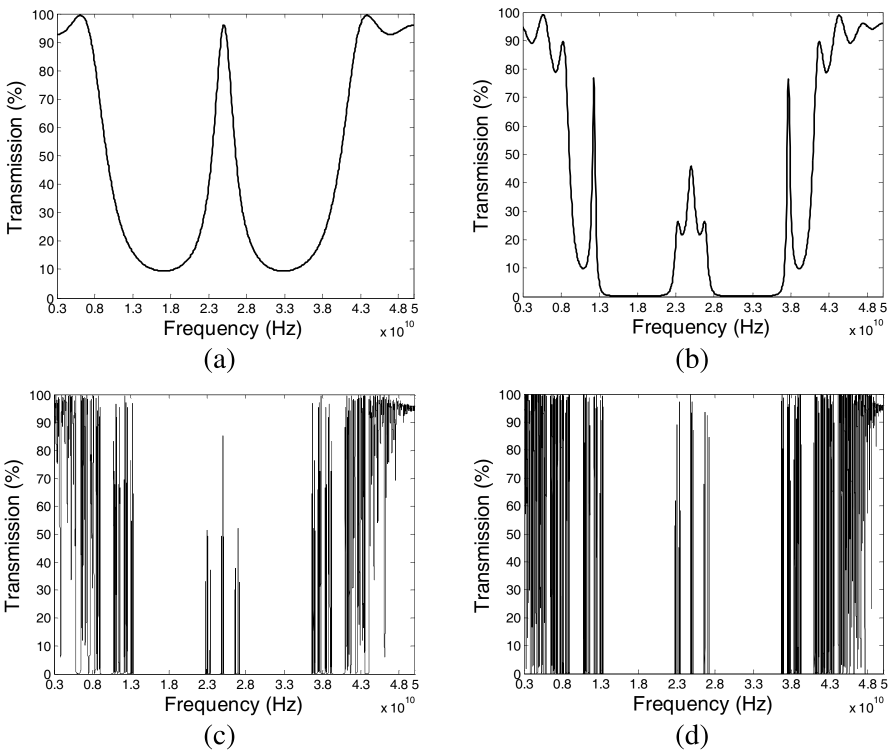


Figure 2. Transmission spectra of a quarter wave Fibonacci structure as function of frequency for different generation numbers with $m = 1$ and $n = 1$: (a) $l = 4$, (b) $l = 6$, (c) $l = 13$, (d) $l = 18$.

symmetric behavior is depicted in the inset of Figures 2(a), (b), (c) and (d). The spectra present a unique mirror symmetrical profile around the band gap center frequency 25 GHz (which is of course the band gap center frequency of a periodic quarter-wavelength one dimensional PC).

With the fourth generation, the transmission falls through two ranges of frequency. The depth of curves varies with the generation number, and two PBGs are formed. Then, with increasing the generation number, the two bands gradually draw together until a flat transmission band emerges with some transmission peaks around the central frequency. We note also the increase of peaks density producing a dark continuum at the edges of the curves. In order to understand the changes in the spectrum shown in Figure 2, we are

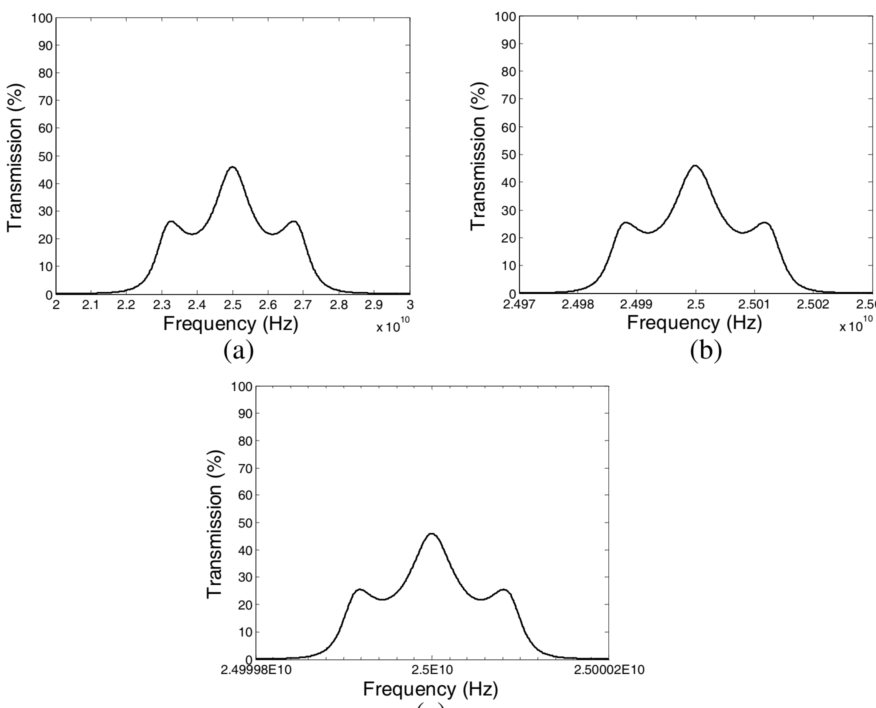


Figure 3. Transmission spectra of a quarter wave Fibonacci structure as function of frequency with $m = 1$ and $n = 1$ (a) for the 6th generation for the reduced range of frequency $20 < f < 30$ (GHz), (b) for the 12th generation for the reduced range of frequency $24.97 < f < 25.03$ (GHz), (c) for the 18th generation for the reduced range of frequency $24.9998 < f < 25.0002$ (GHz).

reminded that the formation of the band gap and the peaks around its center occurs due to firstly the quasiperiodic property of the stack and secondly the auto-similarity that reproduces the structures of the spectrum in smaller scales [15, 34]. To understand this scaling property, we plot in Figure 3(a) the transmission spectrum of Figure 2(b) (which represents the response of the sixth generation quasiperiodic Fibonacci sequence) for the range $20\,\text{GHz} < f < 30\,\text{GHz}$. This spectrum is the same, as shown in Figure 3(b), as the one representing the twelve-generation for the range of frequency reduced by a scale factor approximately equal to 166. When calculating the structure response of the eighteenth generation (4181 layers) and narrowing the frequency range, as shown in Figure 3(c), this spectrum is the same as those depicted in Figure 3(a) and Figure 3(b), in the range of frequency amplified by a scale factor equal to almost 150. It is interesting to notice from Figure 3 and Figure 5 (which represent

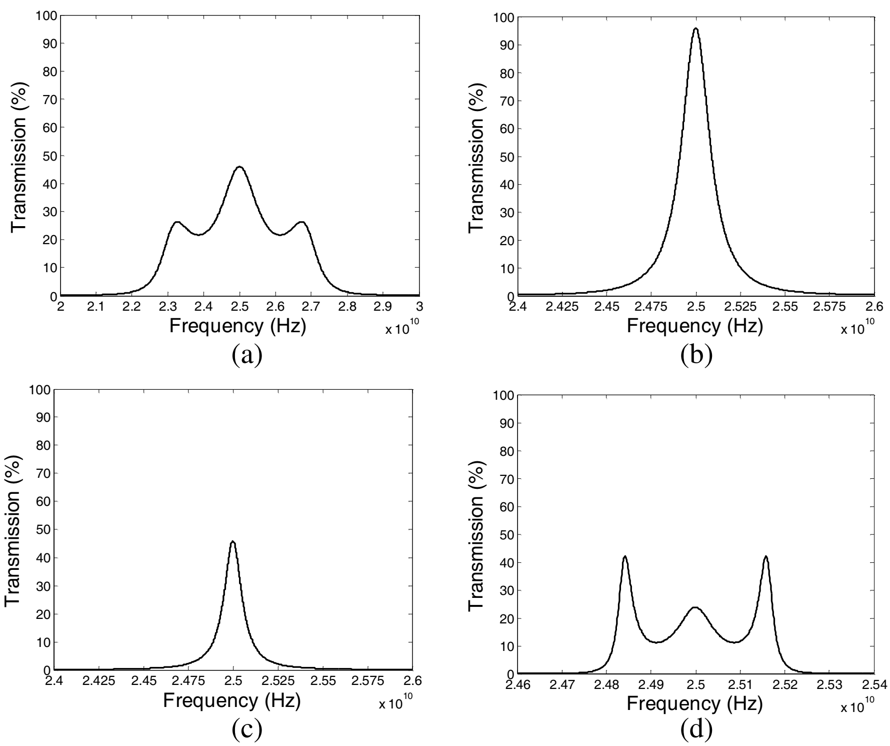


Figure 4. Change in the shape of the peak in band center frequency according to the Fibonacci generation number (a) $l = 6$, (b) $l = 7$, (c) $l = 8$, (d) $l = 9$.

transmission spectra of the seventh and thirteenth generations) that this occurs every time the difference between two generation numbers of the Fibonacci sequence is six [34]. It is worth noting that the auto-similarity is not limited to a narrow central region of the spectra, but it embraces the peaks emerging around of $f = 25\,\text{GHz}$. Concerning the edges of the spectrum, they acquire a fractal shape when the generation number increases. Such a behavior can be also interpreted as a signature of auto-similarity.

We can observe that the bandwidth of the transmittance peaks is much smaller than the width of the bandgap. Thus, it is interesting to make use of the narrow resonate peaks in the bandgap to create selective optical filters with narrow passbands. We have investigated the spectral width of narrow resonate peak in the band center frequency for various Fibonacci iterations. Figure 4 shows the change of the

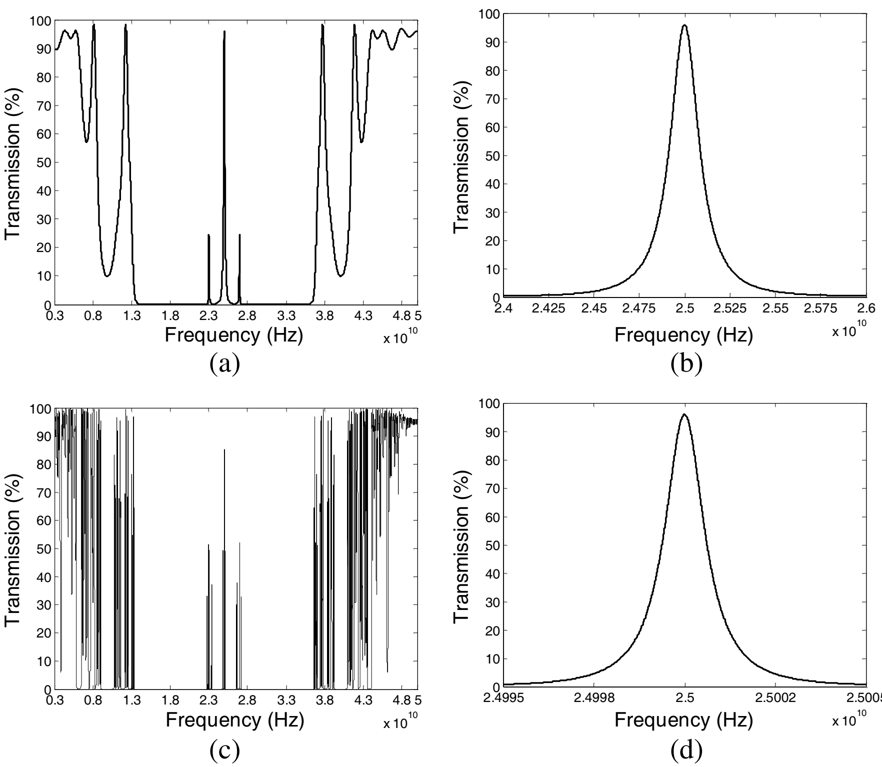


Figure 5. Transmission spectra of a quarter wave Fibonacci structure as function of frequency with $m = 1$ and $n = 1$ (a) for $l = 7$, (b) same as in (a) but for the reduced range of frequency, (c) for $l = 13$, (d) same as in (c), but for the reduced range of frequency.

peak shape at an increase of the Fibonacci generation. We note from Figure 4 and Figure 5 that we can have a peak with almost 96% with the seventh generation and consequently the thirteenth generation. The FWHM (full widths at half maximum) of these peaks are respectively $d7 = 0.18\,\mathrm{GHz}$ and $d13 = 0.00228\,\mathrm{GHz}$.

3.1.2. Effect of the Parameters m and n

To study the effect of parameters m and n on the transmission response of the Fibonacci structure, at first we take the case where $m = n$. This case is interesting since it reveals special properties which will be investigated as follows. Figure 6 shows the transmission spectra corresponding to the third generation and some values of m (or n). It

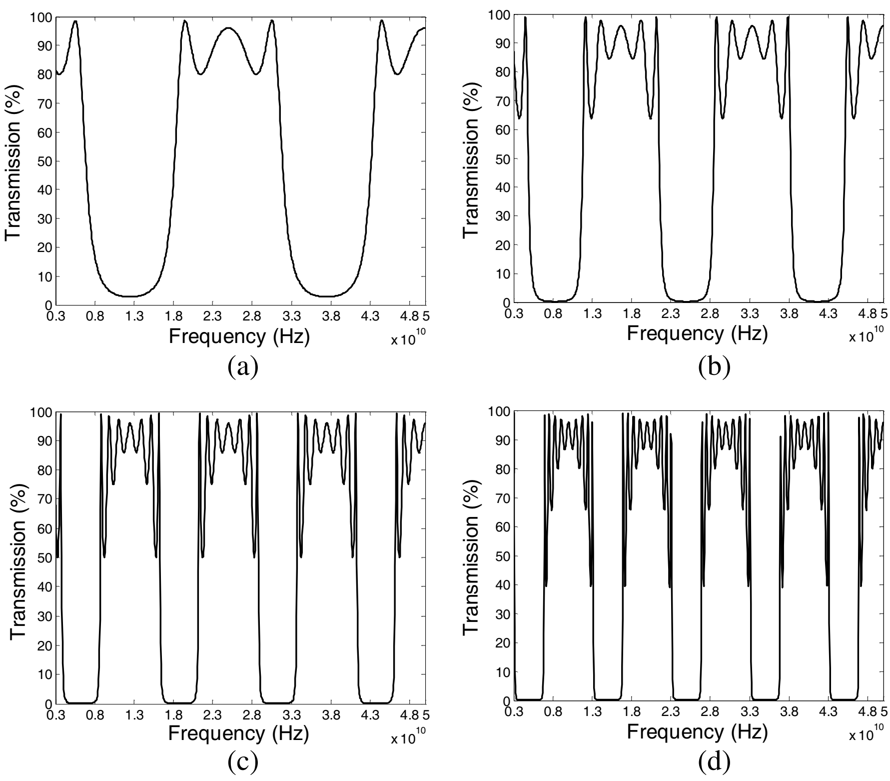


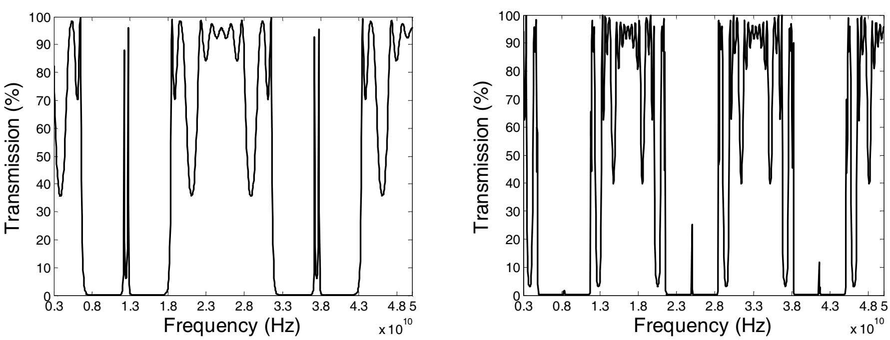
Figure 6. Transmission spectra of a quarter wave Fibonacci structure as function of frequency for the third generation for different values of $m = n$, (a) $m = n = 2$, (b) $m = n = 3$, (c) $m = n = 4$, (d) $m = n = 5$.

is obvious that the number of PBGs is controlled by the variation of m (or n). In the case of $m = n$, the number of PBGs is the same as m (or n) value.

When $m = n = 2$, we obtain 2 PBGs. If $m = n = 5$, the number of PBGs is also 5. But we must note that for each value of m (or n), the PBGs are perfect without transmission peaks for the third generation, but for the fourth generation, transmission peaks appear inside the PBG. For example, with $m = 3$, Figure 6 shows a transmission spectra with 3 PBGs for which the transmission is stopped at all frequencies of these 3 PBGs. From $l = 4$, as shown in Figure 7, some peaks appear inside the PBGs. These phenomenon can be explained if we investigate the multilayer structure for each configuration. For this, we explore the case where $m = n = 3$ as an example.

Figure 8 shows a comparison between the geometry of the third generation and that of the fourth generation. The structure in Figure 8(a) is an alternation of two blocks of layers, a block formed of three layers of low refractive index and a block formed of three layers of high refractive index. So, the multilayer structure behaves as a periodic structure of two elementary layers with refractive indices n_L and n_H and geometric thicknesses with the values $3\frac{\lambda_0}{4n_L}$ and $3\frac{\lambda_0}{4n_H}$, respectively. In other words, the optical length of layers is multiplied by 3.

Figure 9 shows the transmission spectra calculated by the program of a periodic structure with two layers of refractive indices $n_H = 3.134$ and $n_L = 1$ and geometric thicknesses taken three times larger than these of the quarter wave structure. We notice the appearance of three



(a) (b)

Figure 7. Transmission spectra of a quarter wave Fibonacci structure as function of frequency with $m = n$ (a) $m = n = 2$, $l = 4$, (b) $m = n = 3$, $l = 4$.

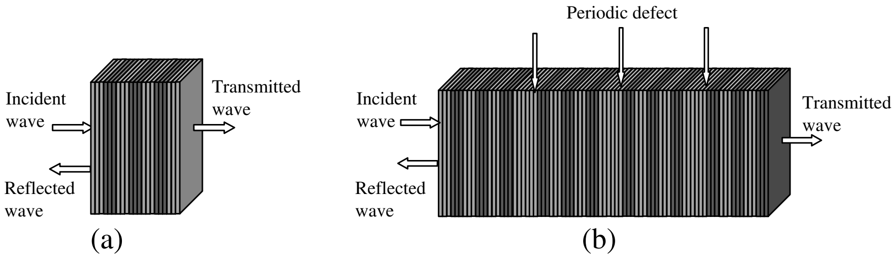


Figure 8. Geometry of the generalized Fibonacci photonic crystal structure with $m = n = 3$, (a) third generation, (b) fourth generation.

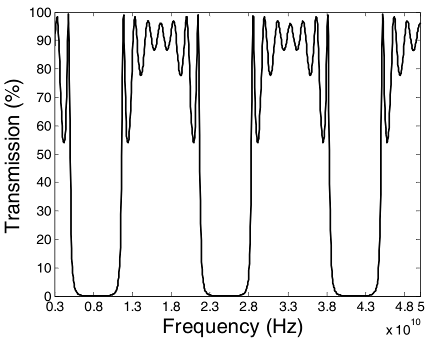


Figure 9. Transmission spectra of a three quarter wave periodic structure as function of frequency.

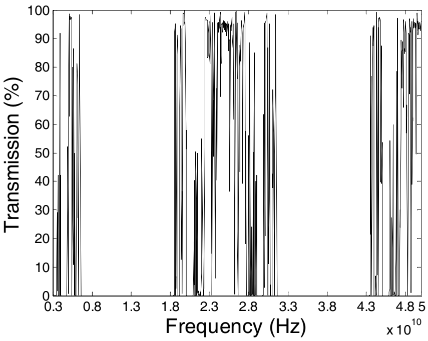


Figure 10. Transmission spectra of a quarter wave Fibonacci structure as function of frequency with $m = n = 2$, $l = 10$.

PBGs in the transmission spectra. If we inspect the geometry of the fourth generation, we note some blocks which contain more than three layers of the same refractive index (6 layers of low refractive index), so the structure behaves as described in the precedent case but with some defects which disturb the periodicity of the structure. The defect can be considered as a layer which has an optical length $6\frac{\lambda_0}{4}$ instead of $3\frac{\lambda_0}{4}$ (the defect itself is periodic, which implies the quasi periodicity of the structure). This enhances the appearance of transmission peaks and explains the fact that for generation more than 3, the PBGs are affected by some peaks.

If we look for the generation number effect on the number of PBGs, we can see from Figure 10 that with the same value of m (or n) and varying the generation number, the number of PBGs remains the same.

Then, we investigate the cases when one of the two parameters m

or n varies and the second set to 1. The transmission is extracted for the fourth generation. The transmission spectrum presents symmetrical profile around the central frequency. It is worth noting from Figure 11 that the number of PBGs through the spectral range increases and contracts by increasing m or n. It is obvious that the number of PBGs is governed by the parameters m and n. The stacking of the PBGs leads to the design of the multi-stop band filters in microwave domain. So we can indicate for each filter the frequency centre and width of the corresponding stop band. Nevertheless, the variation of each m or n has its special effect on the spectral range.

In fact, if parameter m is fixed at 1 and parameter n varies, PBGs are still separated by ripples. Transmission tends to 100% through the spectral ranges of theses ripples. When parameter m varies and parameter n fixed at 1, PBGs are separated by groups of transmission peaks. Increasing m generates an increase density of peaks producing dark narrow bands, and the corresponding ranges may be considered as multi-stop narrow band filters. This behavior can be explained when we know that the fractal aspect is verified for the systems with $n = 1$ and m variable [15]. So the splitting of the transmission peaks when m increases is due to the auto-similarity phenomena which is a manifestation of fractal systems. Whereas, the splitting of the bandgaps when n increases and m fixed at 1, is a distinguishing feature of the quasi-periodic systems (without a fractal aspect).

3.2. Juxtaposition of Generalized Fibonacci Systems

We study the configuration of a multiple of a generalized Fibonacci sequence having the form $[F_l(m,n)]^p$. It is a juxtaposition of p multilayer systems built according to Fibonacci distribution. The transmission spectra of system $[F_l(m,n)]^p$ with $m = n = 1$, $l = 3$ and the repetition number p varying from 1 to 10 are shown in Figure 12. The results show that for $p = 2$, we have 2 PBGs separated by oscillations. The number of oscillations increases as p increases. The proposed structure is similar to a periodic structure with the supercell (LHL). For this type of structure, splitting phenomenon is exhibited in the pass bands but not in the bandgaps [35]. Thus, with increasing the number of periods p, the number of PBG remains 2, whereas the peaks are all split into multiple narrower peaks. Furthermore, the system shows around the central frequency a switch-like property as shown in Figure 13. If we use "on" to represent the high transmission and "off" the low transmission for the central frequency, we note that we have "off" for the odd value of p and "on" for the even value of p. In fact, the transmission through the structure has the following switch-like property: S1(OFF)-S2(ON)-S3(OFF)-S4(ON)-S5(OFF)-S6(ON)-

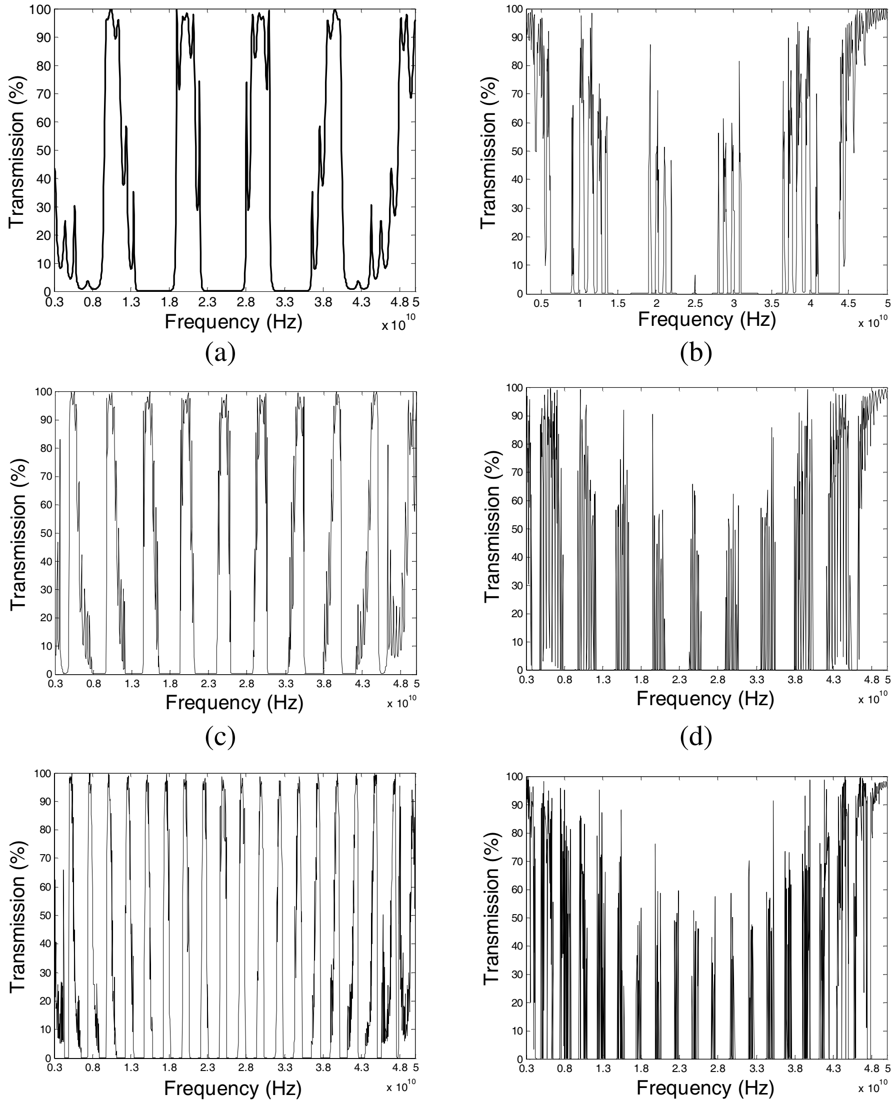


Figure 11. Transmission spectra of a quarter wave Fibonacci structure as function of frequency for the fourth generation for different values of m and n: (a) $m = 1,\ n = 5$, (b) $m = 5,\ n = 1$, (c) $m = 1,\ n = 10$, (d) $m = 10,\ n = 1$, (e) $m = 1,\ n = 20$, (f) $m = 20,\ n = 1$.

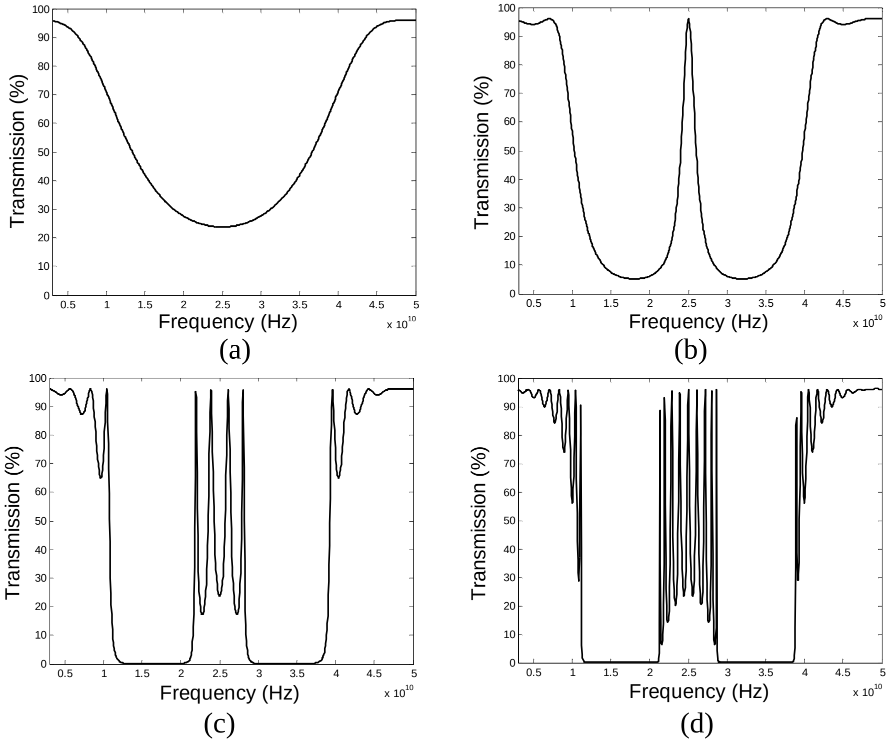


Figure 12. Transmission spectra of the structure $[F_l\,(m,n)]^p$ as function of frequency for the third generation, $m = n = 1$ for different values of p: (a) $p = 1$, (b) $p = 2$, (c) $p = 5$, (d) $p = 10$.

S7(OFF)-S8(ON)-S9(OFF). This reveals interesting results that can be applied to develop new types of nano-switches based on linear photonic crystals. Switching function may be permitted by using a mechanical tuning. It may be realized by directing the incident light to one of two orthogonally orientated stacks, one with an odd value of p and the other with an even value of p. The system may, by controlling polarization, selectively generate and transport the desired wavelength. Light transmission is enabled or disabled by selecting the desired polarization to interact with a geometric aspect. By specifying geometry and orientation, transmission peaks can be modulated to provide multi-state operation.

We can also propose to have a stack with an even value of p. The incident light can propagate through the PC. To inhibit the propagation, a separated block $F_3(1,1)$ can be displaced and stacked with the whole structure. So, it is enough to retire or to add one

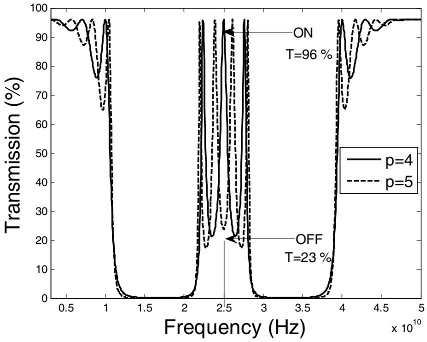


Figure 13. Transmission spectra of the structure $[F_l(m,n)]^p$ as function of frequency for the third generation, $m = n = 1$ for $p = 4$ and $p = 5$.

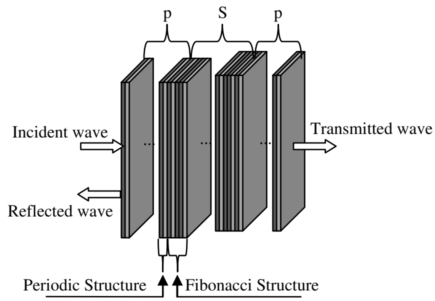


Figure 14. Schematic representation showing the geometry of the structure $(LH)^j[F_l(m,n)]^p(LH)^j$ with $m = n = 1$, $l = 3$.

block $F_3(1,1)$ to realize the on-off states. The mechanical tuning of such device will be in a similar way as in the mechanically switchable photonic crystal filter which is theoretically introduced by Suh and Fan [36]. A detailed study on the proposed switching mechanisms is going to be published.

3.3. Hybrid Structure Periodic/(Fibonacci)^P/Periodic

We now investigate the transmission properties of the hybrid quasi-periodic multilayer structure which is built according to the Fibonacci sequence and sandwiched between two periodic stacks, i.e., $(LH)^j[F_l(m,n)]^p(LH)^j$ (Figure 14). The idea leads to interesting results with the insertion of generalized cantor like multilayer between two identical periodic multilayer structures [37, 38]. Applying the idea for the generalized Fibonacci like multilayer also shows interesting transmission properties.

In fact, if we consider the hybrid configuration $(LH)^j[F_l(m,n)]^p(LH)^j$ with $m = n = 1$, $l = 3$, $p = 7$, $j = 8$, the structure seems almost totally reflective (Transmission is nearly 0%) through a large frequency range with a width of 34 GHz. The total number of layers of this hybrid photonic structure is 53 layers. A second structure exhibits an interesting response in spite of its great number of layers. The configuration is studied with $m = n = 2$, $l = 6$, $p = 8$, $j = 8$. In fact, the structure permits to have a bandwidth equal to 39.2 GHz. The configuration represents a multilayer of 1696 layers.

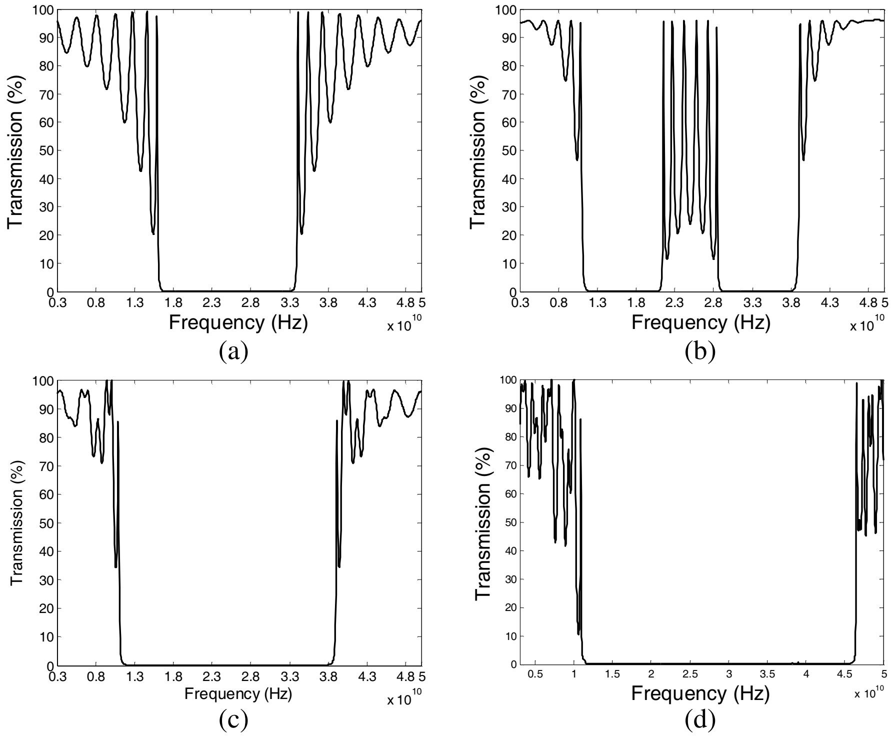


Figure 15. Transmission spectra as a function of frequency for (a) the periodic structure $(LH)^8$, (b) for the structure $[F_3\,(1,1)]^7$, (c) for the structure $(LH)^8\,[F_3\,(1,1)]^7$, (d) for the structure $(LH)^8\,[F_3\,(1,1)]^7\,(LH)^8$.

It is known that the photonic band gap of a photonic crystal can be enlarged by using heterostructures [23–25]. The constituent 1D photonic crystals have to be properly chosen such that photonic band gaps of the adjacent photonic crystals overlap each other. If we plot the transmission spectra of the periodic stack $(LH)^8$ and that of the quasiperiodic stack $[F_3(1,1)]^7$, we understand that the extended photonic band gap obtained through the structure $(LH)^8[F_3(1,1)]^7$ is the result of the overlap of the periodic photonic band with the two bands obtained in the transmission spectrum of the stack $[F_3(1,1)]^7$. So with 37 layers, we can have a photonic band gap with a width of 25.3 GHz. Connecting a second periodic stack at the upper edge of the structure permits, as shown in Figure 15(d), more extended photonic band gap. The structure finishes having the configuration

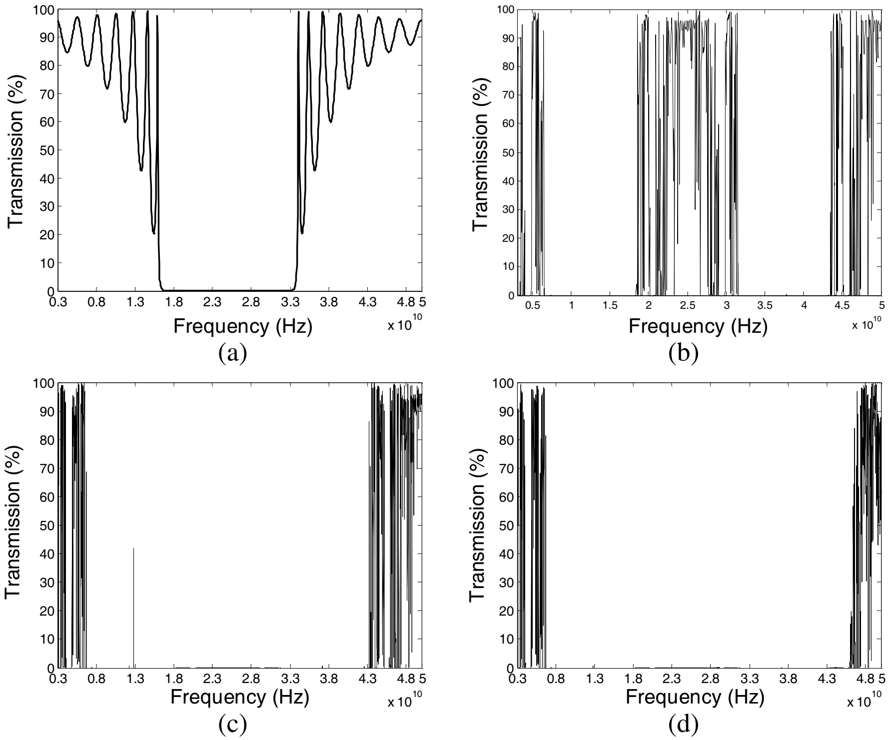


Figure 16. Transmission spectra as a function of frequency for (a) the periodic structure $(LH)^8$, (b) for the structure $[F_6\,(2,2)]^8$, (c) for the structure $(LH)^8\,[F_6\,(2,2)]^8$, (d) for the structure $(LH)^8\,[F_6\,(2,2)]^8\ \ (LH)^8$.

$(LH)^8[F_l(1,1)]^7(LH)^8$ with 53 layers. We show in Figure 16 that the same approach explains the forming of the extended band of the configuration $(LH)^8[F_6(2,2)]^8(LH)^8$. The second sidewall stack permits to eliminate the peak emerging through the extended photonic band gap of the structure $(LH)^8[F_6(2,2)]^8$ and to reduce ripples appearing in the upper photonic band edge. Theses ripples are completely removed by stacking 8 periods in the second side of the multilayer.

4. CONCLUSION

The Fibonacci structure was the subject of many studies. The present paper proposes new configurations revealing new properties which open up a variety of functions. Using Fibonacci structure with some high generation numbers permits to have selective optical filters with narrow passbands. Then a design of stop band filters by using Fibonacci photonic crystal has been studied by varying the Fibonacci parameters. The stacking of several Fibonacci one-dimensional photonic crystals presents a switch-like property which can be used to design nano-switches in microwave domain. Next, sandwiching a several number of Fibonacci structures between two periodic structures has enhanced the zero reflection range. Therefore, a large photonic band gap which covers almost all the microwave domain is obtained with only 53 quarter wave layers. Since different physical phenomena have their own relevant physical scales, we exploited the physical properties of the different proposed structures to obtain different microwave devices.

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