

## UNIAXIAL ANISOTROPIC SUBSTRATE EFFECTS ON THE RESONANCE OF AN EQUITRIANGULAR MICROSTRIP PATCH ANTENNA

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**Abstract**—Using a new combined approach, the effect of the uniaxial anisotropic dielectrics on the resonant frequency and radiation field of an equitriangular patch antenna is presented in this paper. The problem is analysed in the spectral domain using the moment method and an electric field integral equation combined with a mathematical approach. However, the dyadic Green's functions corresponding to the proposed structure are separately developed and the Fourier transform of the basis current components are calculated mathematically using “the reference element” method. Numerical results show that the change in the resonant frequency and the radiation patterns of the antenna is due primarily to a small disturbance of the substrate's nature. Then the effect of the uniaxial anisotropic materials is a significant parameter and most essential on the microstrip antenna characterization.

### 1. INTRODUCTION

During the last three decades, a considerable number of papers have been published on the performance and applications of microstrip patch antennas. These patch antennas possess many desirable features, that make this type of antennas useful for many applications in radar and wireless communication systems. Various patch configurations implemented on different types of substrates have been tested and investigated. In practice, it was found that the choice of the substrate material is of a great importance and plays a significant role in achieving the optimum radiation characteristics of the antenna.

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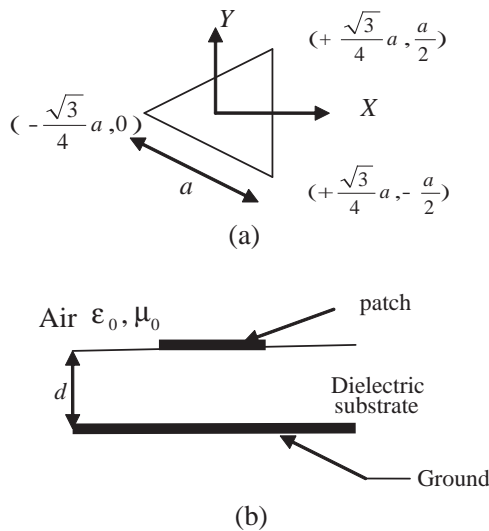
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However, we are interested in this work to study the effects of anisotropic substrate materials on the resonant frequency and radiation field of an equilateral triangular patch antenna. Moreover the study of this type of substrates is of interest, many practical substrates have a significant amount of anisotropy that can affect the performance of printed circuits and antennas, and thus accurate characterization and design must account for this effect [1]. It is found that the use of such materials may have a beneficial effect on circuits or Antennas [1, 2]. For a rigorous solution to the problem of the proposed equitriangular microstrip antenna, Galerkin's method is employed in the spectral domain with an appropriate choice of basis functions and a mathematical method is used to calculate analytically the Fourier transform of these functions corresponding to such shape of patches.

## 2. PROBLEM FORMULATION

The Figure 1 illustrates the proposed antenna under consideration. The structure comprises of an equitriangular patch of side length  $a$ , printed on uniaxial anisotropic dielectric substrate of infinite extent. This substrate has an uniform thickness  $d$ , and assumed to be nonmagnetic material with permeability  $\mu_0$ . The metallization of the patch and ground plane is considered as perfect conductor and had



**Figure 1.** The geometry of the equitriangular microstrip. (a) Top view. (b) Side view.

negligibly thin.

To simplify the analysis, the antenna feed will not be considered in this work. The study of anisotropic substrates is of interest, however, the designers should carefully check for the anisotropic effects in the substrate material with which they will work, and evaluate the effects of anisotropy. The analysis is lead by Galerkin's moment method in the spectral domain using boundary conditions [3, 4] to derive an electric current integral equation.

It is preferable to express the unknown patch current in terms of an appropriate basis functions formed by the set of transverse magnetic modes of a triangular cavity with magnetic side walls and electric top and bottom walls. However an appropriate basis current system was chosen [5], from which we have calculated analytically the Fourier Transform of the current components by applied a mathematical method [6], to overcome the complexity of the geometry. Numerical results were obtained by programming the calculating steps without the use of specific simulation software. Anisotropy is defined as the substrate dielectric constant on the orientation of the applied electric field. Mathematically, the permittivity of an anisotropic substrate can be represented by a tensor or dyadic of this form [1]:

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \quad (1)$$

For a biaxially anisotropic substrate the permittivity is given by:

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \quad (2)$$

For a uniaxially anisotropic substrate the permittivity is:

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \quad (3)$$

$\varepsilon_0$  is the free-space permittivity.

$\varepsilon_z$  is the relative permittivity in the direction of the optical axis  $Z$ .

$\varepsilon_x$  is the relative permittivity in the direction  $X$  perpendicular to  $Z$ .

$\varepsilon_y$  is the relative permittivity in the direction  $Y$  perpendicular to  $Z$ .

Many substrate materials used for printed circuit antenna exhibit anisotropy, especially uniaxial anisotropy [1, 7]. In the following calculations, the substrate material is taken to be isotropic or uniaxially anisotropic with the optical axis normal to the patch. The boundary condition on the patch is given by [2]:

$$E_{inc} + E_{scat} = 0 \quad (4)$$

$E_{inc}$  Tangential components of incident electric field.

$E_{scat}$  Tangential components of scattered electric field.

While it is possible to work with wave equations and the longitudinal components  $\tilde{E}_z$  and  $\tilde{H}_z$  in the Fourier transform domain, it is desired to find the transverse fields in the (TM, TE) representation in terms of the longitudinal components.

Noting that ‘-’ is used for vectorial quantities and ‘ $\sim$ ’ for spectral quantities.

In the spectral domain  $\frac{\partial}{\partial x} = iK_x$ ,  $\frac{\partial}{\partial y} = iK_y$ ,  $\frac{\partial}{\partial z} = iK_z$ ,  $\frac{\partial}{\partial t} = i\omega$ .

Thus, at spectral domain in the representation TE-TM the electric field is represented on matrix form and in terms of longitudinal components as:

$$\bar{E}(K_s, z) = \begin{bmatrix} \tilde{E}^e(K_s, z) \\ \tilde{E}^h(K_s, z) \end{bmatrix} = g(K_s) \begin{bmatrix} \frac{j\varepsilon_z}{K_s \varepsilon_x} \frac{\partial \tilde{E}_z(K_s, z)}{\partial z} \\ \frac{\omega \mu}{K_s} \tilde{H}_z(K_s, z) \end{bmatrix} \quad (5)$$

$k_s = K_x \hat{X} + K_y \hat{Y}$  is the transverse wave vector; with  $K_s = \|k_s\| = \sqrt{K_x^2 + K_y^2}$ , where  $K_x$  and  $K_y$  are the spectral variables corresponding to  $x$  and  $y$  respectively.

$$g(K_s) = \begin{bmatrix} \frac{\omega \varepsilon_0 \varepsilon_x}{K_z} & 0 \\ 0 & \frac{K_z^h}{\omega \mu_0} \end{bmatrix} \quad (6)$$

$K_0 = \omega \sqrt{\varepsilon_0 \mu_0}$  is the propagation constant for free space.

$$K^2 = \omega^2 \mu \varepsilon_0 \text{ and } K_s^2 = K_x^2 + K_y^2$$

$K_z^e = (\varepsilon_x K^2 - \frac{\varepsilon_x}{\varepsilon_z} K_s^2)^{1/2}$  and  $K_z^h = (\varepsilon_x K^2 - K_s^2)^{1/2}$  are respectively the propagation constants for TM and TE waves in the uniaxial dielectric.

After substitutions and some algebraic manipulations the transverse field in the (TM, TE) representation can be written by:

$$\tilde{E}_s(K_s, z) = \begin{bmatrix} \tilde{E}_s^e \\ \tilde{E}_s^h \end{bmatrix} = A(K_s) e^{jK_z^e z} + B(K_s) e^{-jK_z^e z} \quad (7)$$

$$\tilde{H}_s(K_s, z) = \begin{bmatrix} \tilde{H}_s^e \\ \tilde{H}_s^h \end{bmatrix} = \bar{g}(K_s) \left[ A(K_s) e^{jK_z^h z} - B(K_s) e^{-jK_z^h z} \right] \quad (8)$$

In the spectral domain the relationship between the patch current and the electric field on the patch is given by:

$$\tilde{E}_s(K_s) = G(K_s) \cdot \tilde{J}(K_s) \quad (9)$$

$\tilde{J}(K_s)$  is the surface current on the patch.

$G$  is the spectral dyadic Green's function given on matrix form as:

$$\bar{G}(K_s) = \begin{bmatrix} G^e & 0 \\ 0 & G^h \end{bmatrix} \quad (10)$$

where:

$$G^e = \frac{1}{i\omega\varepsilon_0} \frac{-K_z^e K_z \sin(K_{z1}d)}{iK_z^e \sin(K_{z1}d) + \varepsilon_x K_z \cos(K_{z1}d)} \quad (11)$$

$$G^h = \frac{1}{i\omega\varepsilon_0} \frac{-K_0^2 \sin(K_{z1}d)}{iK_z \sin(K_{z1}d) + K_z^h \cos(K_{z1}d)} \quad (12)$$

And  $K_{z1} = K_0 \cos(K_z d)$  and  $i = \sqrt{-1}$ .

One of the main problems with the computational procedure is to overcome the complicated time-consuming task of calculating the Green's functions in the procedure of resolution by the moment method. The choice of the basis function is very important for a fast convergence to the true values.

The basis functions selected in this study were proposed by Chen et al. [5]:

$$\begin{aligned} J_x(m, n) = \sqrt{3} \left[ l \sin\left(\frac{2\pi lx}{\sqrt{3}a}\right) \cos\left(\frac{2\pi(m-n)y}{3a}\right) \right. \\ \left. + m \sin\left(\frac{2\pi mx}{\sqrt{3}a}\right) \cos\left(\frac{2\pi(n-l)y}{3a}\right) \right. \\ \left. + n \sin\left(\frac{2\pi nx}{\sqrt{3}a}\right) \cos\left(\frac{2\pi(l-m)y}{3a}\right) \right] \quad (13) \end{aligned}$$

$$\begin{aligned} J_y(m, n) = (m-n) \cos\left(\frac{2\pi lx}{\sqrt{3}a}\right) \sin\left(\frac{2\pi(m-n)y}{3a}\right) \\ + (n-l) \cos\left(\frac{2\pi mx}{\sqrt{3}a}\right) \sin\left(\frac{2\pi(n-l)y}{3a}\right) \\ + (l-m) \cos\left(\frac{2\pi nx}{\sqrt{3}a}\right) \sin\left(\frac{2\pi(l-m)y}{3a}\right) \quad (14) \end{aligned}$$

$l, m$  and  $n$  are numbers which design propagation modes.

However, we have developed analytically the calculus of the two dimensional Fourier-transform current components on the triangular patch and we have obtained the following results:

$$\begin{cases} \tilde{J}_x = I_{1x} + I_{2x} + I_{3x} \\ \tilde{J}_y = I_{1y} + I_{2y} + I_{3y} \end{cases} \quad (15)$$

where:

$$\begin{cases} I_{1x} = \frac{\sqrt{3}l}{4i} (I_{11x} + I_{12x} - I_{13x} - I_{14x}) \\ I_{2x} = \frac{\sqrt{3}m}{4i} (I_{21x} + I_{22x} - I_{23x} - I_{24x}) \\ I_{3x} = \frac{\sqrt{3}n}{4i} (I_{31x} + I_{32x} - I_{33x} - I_{34x}) \end{cases} \quad (16)$$

$$\begin{cases} I_{1y} = \frac{\sqrt{3}l}{4i}(I_{11y} + I_{12y} - I_{13y} - I_{14y}) \\ I_{2y} = \frac{\sqrt{3}m}{4i}(I_{21y} + I_{22y} - I_{23y} - I_{24y}) \\ I_{3y} = \frac{\sqrt{3}n}{4i}(I_{31y} + I_{32y} - I_{33y} - I_{34y}) \end{cases} \quad (17)$$

The terms  $I_{vwx}$  and  $I_{vwy}$  are given by:

$$\begin{aligned} I_{vwx} = I_{vwy} = U_{vw} & \left[ \sin c(aK_y + Y_{vw}) - \frac{i \cos(aK_y + Y_{vw})}{aK_y + Y_{vw}} + \frac{i}{aK_y + Y_{vw}} \right] \\ & \left[ \sin c\left(\frac{\sqrt{3}}{2}aK_x + \frac{a}{2}K_y + X_{vw}\right) - \frac{i \cos\left(\frac{\sqrt{3}}{2}aK_x + \frac{a}{2}K_y + X_{vw}\right)}{\frac{\sqrt{3}}{2}aK_x + \frac{a}{2}K_y + X_{vw}} \right. \\ & \left. + \frac{i}{\frac{\sqrt{3}}{2}aK_x + \frac{a}{2}K_y + X_{vw}} \right] \end{aligned} \quad (18)$$

where:  $v = 1, 2, 3$  and  $w = 1, 2, 3, 4$ .

Explicit expressions of the coefficients  $U_{vw}$  are given in Appendix A.

Virtually all this work has been done with entire domain basis functions for the current on the patch. For the resonant patch, entire domain expansion currents lead to fast convergence and can be related to a cavity model type of interpretation [8, 9]. The currents can be defined using a sinusoid basis functions defined on the whole domain, without the edge condition, these currents associated with the complete orthogonal modes of the magnetic cavity. Both  $x$  and  $y$  directed currents were used, with the following forms. Since the chosen basis functions approximate the current on the patch very well for conventional microstrips, only one or two basis functions are generally used for each current component.

The electric field integral equation is deduced by applying the boundary conditions requiring that the current distribution vanish on the triangular patch; Using Eqs. (10) and (15), cascading the matrices by simple multiplication; the following matrix form is obtained:

$$\bar{A} = \begin{bmatrix} (\bar{A}_1)_{N*N} & (\bar{A}_2)_{N*M} \\ (\bar{A}_3)_{M*N} & (\bar{A}_4)_{M*M} \end{bmatrix} \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} \quad (19)$$

$N$  and  $M$  are entire numbers.

It should be noted that the roots of the characteristic equation given by (19) are complex, Muller's algorithm has been employed to compute the roots and hence to determine the resonant frequency.

Finally the complex resonant frequency  $f = f_{re} + if_{im}$  result from the resolution of the equation:

$$\det(A_{ij}) = 0 \quad (20)$$

Stationary phase evaluation yields convenient and useful results for the calculation of antenna patterns [1]. The scattered far zone electric field from the patch can then be found in spherical coordinates with components  $E_\theta$  and  $E_\phi$ , the results are of the form:

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = iK_0 \frac{e^{iK_0 r}}{2\pi r} \begin{bmatrix} -G^e & 0 \\ 0 & G^h \cos \theta \end{bmatrix} \begin{bmatrix} \tilde{J}^e \\ \tilde{J}^h \end{bmatrix} \quad (21)$$

$K_x, K_y$  are evaluated at the stationary phase point as:

$$K_x = K_0 \sin \theta \cos \phi \quad (22a)$$

$$K_y = K_0 \sin \theta \sin \phi \quad (22b)$$

### 3. NUMERICAL RESULTS AND DISCUSSIONS

Although the proposed approach can give results for several resonant modes according to different parameters of the antenna, only analysis for the real part of the resonant frequency are presented in this paper for the fundamental mode  $TM_{10}$ , focusing on the effect of an uniaxially anisotropic substrate on the frequency and the far field considering the variation of permittivity and its thickness. In this study several patch dimensions are also investigated and tested. However, the influence of uniaxial anisotropy in the substrate on the resonant frequency and radiation pattern of an equilateral triangular patch antenna with sidelength  $a = 1.86$  cm and substrate thickness  $d = 0.159$  cm, for different pairs of relative permittivity  $(\epsilon_x, \epsilon_z)$  is shown in Table 1.

**Table 1.** Resonant frequency values for the isotropic, positive and negative uniaxial anisotropic substrates for rectangular and equitriangular patches.

Uniaxial anisotropy type	Relative Permittivity $\epsilon_x$	Relative Permittivity $\epsilon_z$	Rectangular patch [10] Resonant Frequency GHz	Triangular patch Resonant Frequency GHz
Isotropic	2.35	2.35	8.6360	8.6680
Isotropic	7.0	7.0	5.2253	5.6486
Positive	1.88	2.35	8.7241	8.7252
Négative	2.82	2.35	8.5537	8.6036
Négative	8.4	7.0	5.1688	5.5502
Positive	5.6	7.0	5.2869	5.8825

From the Table 1 it can be observed that the positive uniaxial anisotropy slightly increases the resonant frequency, while the negative uniaxial anisotropy slightly decreases both resonant frequency. These results are logical comparing with those of reference [10] for a rectangular patch antenna having the same parameters (patch surface and substrate thickness) using the moment method. Noting that, both rectangular and triangular microstrip antennas have generally the same behaviour and radiation characteristics [11], with some differences on the values of the resonant frequency, this must be improved in the following section.

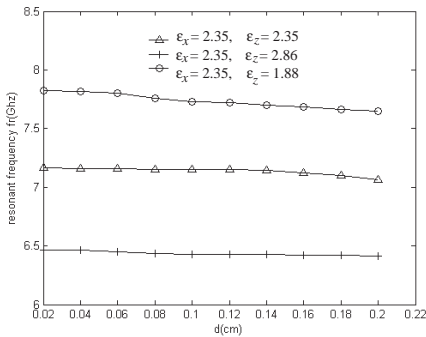
However, we can see that our calculated resonant frequency values are slightly more important than the rectangular patch. The results shown in Table 2 for the numerical data presented by [10] and [12] for a rectangular patch antenna with dimensions  $(1.9 \times 2.29) \text{ cm}^2$  and substrate thickness of  $d = 0.159 \text{ cm}$  are compared with our computed results for an equitriangular patch having the same surface of the rectangular patch, so with side length  $a = 3.17 \text{ cm}$ . The obtained results show that when the permittivity along the optical axis  $\varepsilon_z$  is changed and  $\varepsilon_x$  remains constant the resonant frequency changes drastically, on the other hand, a slight shift is observed in the resonant frequency when the permittivity  $\varepsilon_x$  is changed and  $\varepsilon_z$  remains constant, these behaviors agree very well with those obtained by [12], with a light increase in the values compared to the rectangular patch antenna. Notice that this increase is enough important as it is about the case of the negative anisotropy.

Figures 2 and 3 show the influence of uniaxial anisotropy in the substrate on the resonant frequency, when we change the substrate

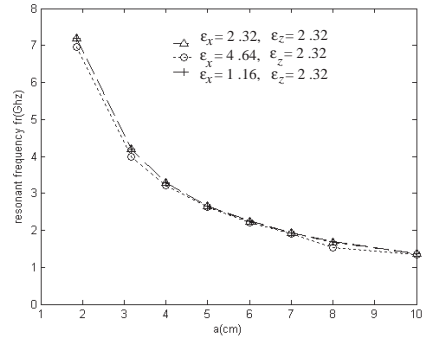
**Table 2.** Dependence of resonant frequency on relative permittivity  $(\varepsilon_x, \varepsilon_z)$ .

$\varepsilon_x$	$\varepsilon_z$	Anisotropic ratio $AR = \varepsilon_x/\varepsilon_z$	Resonant frequencies (Ghz)		
			Results of [12] (rectangular patch)	Results of [10] (rectangular patch)	Our results (triangular patch)
2.32	2.32	1	4.123	4.121	4.1418
4.64	2.32	2	4.042	4.041	4.1344
2.32	1.16	2	5.476	6.451	5.8660
1.16	2.32	0.5	4.174	4.171	4.1963
2.32	4.64	0.5	3.032	3.028	3.0885





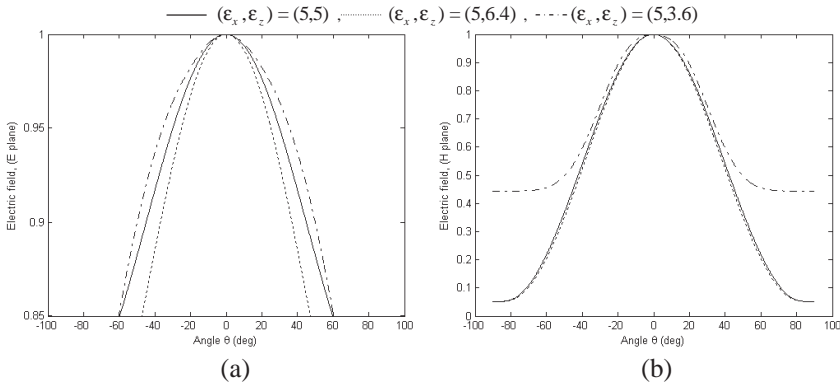
**Figure 2.** Variation of the resonant frequency of an equi-triangular patch antenna versus substrate thicknesses for different permittivity pairs: isotropic  $(\epsilon_x, \epsilon_z) = (2.35, 2.35)$  positive uniaxial anisotropy  $(\epsilon_x, \epsilon_z) = (2.35, 2.86)$  and negative uniaxial anisotropy  $(\epsilon_x, \epsilon_z) = (2.35, 1.88)$ .



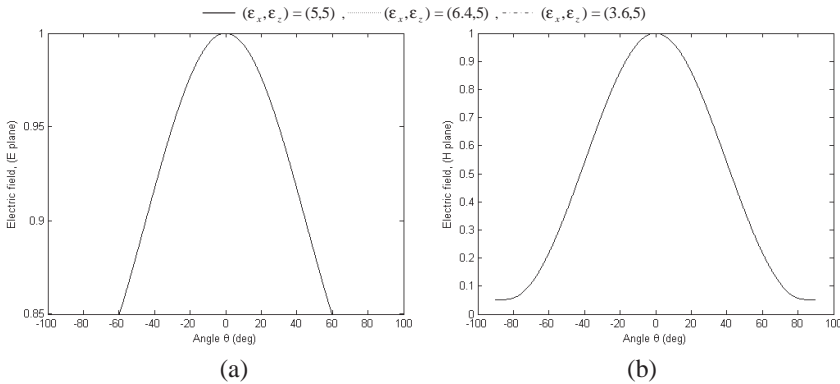
**Figure 3.** Variation of the resonant frequency of an equi-triangular patch antenna versus the dimensions of the patch for different permittivities: isotropic  $(\epsilon_x, \epsilon_z) = (2.32, 2.32)$  positive uniaxial anisotropy  $(\epsilon_x, \epsilon_z) = (1.16, 2.32)$  and negative uniaxial anisotropy  $(\epsilon_x, \epsilon_z) = (4.64, 2.32)$ .

thickness or the dimensions of the patch for different cases; the isotropic ( $\epsilon_x = \epsilon_z$ ), positive uniaxial anisotropic ( $\epsilon_x < \epsilon_z$ ) and negative uniaxial anisotropic substrates ( $\epsilon_z < \epsilon_x$ ). For the first case (Figure 2), a triangular patch antenna with side length  $a = 1.86$  cm printed on a single substrate with different thicknesses is considered. The obtained results can be seen to be the same as discussed previously in the case of the resonant frequency. moreover we can notice from Figure 3 which represents the variation of the frequency of resonance according to dimensions of the triangular patch, that in the case of the permittivity along the optical axis is maintained constant, the values are very close to the values of the isotropic case especially in the case of the positive uniaxial anisotropy. We can conclude that the permittivity  $\epsilon_z$  along the optical axis is the significant and most important factor in determining of the resonant frequency.

Figures 4(a), (b) and 5(a), (b) represent the influence of uniaxial anisotropy in the substrate on the radiation field displayed as a function of the angle  $\theta$  at  $\phi = 0$  ( $E$  plane) and  $\phi = \frac{\pi}{2}$  ( $H$  plane) respectively, at the frequency 5.074 GHz and where the isotropic, positive uniaxial anisotropic and negative uniaxial anisotropic substrates are considered for the two proposed cases (when  $\epsilon_z$  is changed or if  $\epsilon_x$ ). An equilateral triangular patch antenna with sidelength  $a = 1.86$  cm and



**Figure 4.** Radiation pattern versus the angle  $\theta$  for the isotropic, positive uniaxial anisotropic and negative uniaxial anisotropic substrates when changing  $\epsilon_z$ . (a) In the  $E$  plane. (b) In the  $H$  plane.



**Figure 5.** Radiation pattern versus the angle  $\theta$  for the isotropic, positive uniaxial anisotropic and negative uniaxial anisotropic substrates when changing  $\epsilon_x$ . (a) In the  $E$  plane. (b) In the  $H$  plane.

substrate thickness  $d = 0.2$  cm is considered. Generally we can deduce the remarks that previously concerning the resonant frequency, the permittivity along the optical axis acts considerably on the radiation of the triangular patch antenna according to Figure 4. On the other hand uniaxial anisotropy does not have a considerable effect on the radiation characteristics of the antenna according to Figure 5 (the curves are almost identical).

#### 4. CONCLUSION

The obtained results confirm that the use of the uniaxial anisotropy substrates significantly affects the characterization of the microstrip patch antennas. Numerical results indicate that the resonant frequency and the radiation field of an equilateral triangular patch antenna are sensitive to the variations of the permittivity of an uniaxial anisotropic substrate, in the case of the positive or negative anisotropy.  $\epsilon_z$  permittivity has a stronger effect than  $\epsilon_x$ . Moreover, the resonant frequency's values of the rectangular patch are relatively lower than that of the triangular patch antenna.

The accuracy of the method was checked by performing a set of results in terms of resonant frequencies and radiation patterns. The analysis presented here can also be extended to study other parameters characterizing the triangular patch antennas with various structures.

#### APPENDIX A. EXPLICIT EXPRESSIONS FOR THE COEFFICIENTS INTRODUCED IN SECTION 2

$$U_{vw} = \frac{\sqrt{3}}{2} a^2 e^{-i\left(\frac{\sqrt{3}}{2} a K_x + \frac{a}{2} K_y\right)} e^{-iX_{vw}} \quad (A1)$$

where

$$X_{vw} = \begin{bmatrix} -\frac{\sqrt{3}}{4} a\alpha - \frac{a}{2}\beta & -\frac{\sqrt{3}}{4} a\alpha + \frac{a}{2}\beta & \frac{\sqrt{3}}{4} a\alpha - \frac{a}{2}\beta & \frac{\sqrt{3}}{4} a\alpha + \frac{a}{2}\beta \\ -\frac{\sqrt{3}}{4} a\alpha' - \frac{a}{2}\beta' & -\frac{\sqrt{3}}{4} a\alpha' + \frac{a}{2}\beta' & \frac{\sqrt{3}}{4} a\alpha' - \frac{a}{2}\beta' & \frac{\sqrt{3}}{4} a\alpha' + \frac{a}{2}\beta' \\ -\frac{\sqrt{3}}{4} a\alpha'' - \frac{a}{2}\beta'' & -\frac{\sqrt{3}}{4} a\alpha'' + \frac{a}{2}\beta'' & \frac{\sqrt{3}}{4} a\alpha'' - \frac{a}{2}\beta'' & \frac{\sqrt{3}}{4} a\alpha'' + \frac{a}{2}\beta'' \end{bmatrix} \quad (A2)$$

$$Y_{vw} = \begin{bmatrix} -a\beta & a\beta & -a\beta & a\beta \\ -a\beta' & a\beta' & -a\beta' & a\beta' \\ -a\beta'' & a\beta'' & -a\beta'' & a\beta'' \end{bmatrix} \quad (A3)$$

The parameters  $\alpha$ ,  $\beta$ ,  $\alpha'$ ,  $\beta'$ ,  $\alpha''$ ,  $\beta''$  are given as following:

$$\begin{aligned} \alpha &= \frac{2\pi l}{\sqrt{3}a}, & \beta &= \frac{2\pi(m-n)}{3a} \\ \alpha' &= \frac{2\pi m}{\sqrt{3}a}, & \beta' &= \frac{2\pi(n-l)}{3a} \\ \alpha'' &= \frac{2\pi n}{\sqrt{3}a}, & \beta'' &= \frac{2\pi(l-m)}{3a} \end{aligned}$$

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