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# NEW CLASS OF SURFACE MAGNON POLARITONS IN ENANTIOMERIC ANTIFERROMAGNETIC STRUCTURES

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**Abstract**—A novel class of surface magnon polaritons supported in identical enantiomeric antiferromagnetic structures is presented. The surface waves arise due to bianisotropy. The existence of two distinct surface modes with unusual dispersion and polarization properties is predicted. The role of losses is investigated and the propagation length of the surface waves is determined.

## 1. INTRODUCTION

One of the most actual directions of modern solid-state spectroscopy is associated with surface polaritons. Surface waves propagate along an interface between two different media and are characterized by the fields exponentially decaying for increasing distances from the interface. Surface polaritons represent mixed excitations consisting of surface electromagnetic waves and one (or more) of infrared-active solid state excitations: surface optical phonons, plasmons, excitons, or magnons [1–6]. For historical, theoretical and experimental aspects of surface polaritons we refer to [7–15].

Among various types of surface polaritons, the surface magnon polaritons (SMP) propagating along an interface between a magnetic medium and other magnetic or nonmagnetic one, play an important role and represent a highly dynamic research area [16–22]. At present, SMP are widely used in several areas of science and technology, particularly in near-field spectroscopy of magnetic materials as well as in spintronic devices. Usually, SMP occur at a planar interface of different materials, within definite frequency intervals and only in the case when magnetic permeability of one of the contacted media is negative.

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Recent development in SMP applications at the interface of a negative-index metamaterial and a conventional medium has generated a considerable interest in surface waves in complex electromagnetic materials such as photonic crystals and bianisotropic media [23–31]. Among the complex media, bi-isotropic and bianisotropic materials which exhibit right- or left-handedness and magnetoelectric coupling are one of the most challenging substances because of the essential role of the cross polarizability effects in their electromagnetic behavior [31, 32]. In Ref. [33], the possibility of the existence of lossless surface electromagnetic waves guided by the interface between two transparent enantiomeric chiral media has been shown in the absence of frequency dispersion. In recent work [34], the properties of surface plasmon polaritons at an interface between enantiomeric chiroplasmonic media have been investigated. Surface waves predicted in [33] and [34] are caused by bianisotropy and are absent in the case of enantiomeric bi-isotropic media.

The purpose of this paper is to show the possibility of existence of a new class of SMP propagating along a planar interface between two *identical* uniaxially bianisotropic antiferromagnetic media (AFM), one of which is a mirror image of the second one. So as not to complicate the analysis taking into account magnon-plasmon coupling effects [35], we will restrict ourselves to consideration of insulating AFM. We will show that even in this relatively simple case, the predicted modes of SMP possess unusual dispersion and polarization properties and can exist at frequencies for which the real part of the magnetic permeability is positive.

The plan of the paper is as follows. In Section 2, the field structure of evanescent partial waves is investigated, dispersion relations and polarization properties of the SMP are described. Section 3 discusses the existence conditions and dispersion curves of SMP in nondissipative case. Effect of losses is examined in Section 4. Finally, conclusions are presented in Section 5.

## 2. SMP AT AN INTERFACE OF ENANTIOMERIC AFM MEDIA. PARTIAL WAVES

Let us consider a bianisotropic AFM with "easy-axis" type of magnetic anisotropy and assume that the $z = 0$ plane separates a semi-infinite AFM-medium (by definition, R-material, $z < 0$) from the identical material of opposite handedness (L-material, $z > 0$). Optic axes in both materials are supposed to be perpendicular to the interface plane. The L-material is characterized by chirality admittance tensor [31, 32]

$$\hat{\xi}_L = \xi u_z u_z, \tag{1}$$

where $u_z$ is the axial component of the unit vector. In the case of reciprocal media $\xi$ is assumed to be real, positive and nondimensional constant smaller than 1. The corresponding tensor for the R-material is given by $\hat{\xi}_R = -\hat{\xi}_L$. Both media are described by the same relative permittivity tensor

$$\hat{\varepsilon} = \varepsilon_\perp \hat{I} + \varepsilon_\| u_z u_z, \tag{2}$$

with real and positive constants $\varepsilon_\|$, $\varepsilon_\perp$ and transverse unit dyadic $\hat{I}$. We assume that the sublattice saturation magnetization $\mathbf{M}_s$, the anisotropy field $\mathbf{H}_A$ and the exchange field $\mathbf{H}_E$ are oriented in the same direction along the $z$-axis. Then dynamic permeability tensor is given by [35]

$$\hat{\mu} = \mu_\perp \hat{I} + \mu_\| u_z u_z, \tag{3}$$

where $\mu_\| = 1$,

$$\mu_\perp \equiv \mu(\omega) = 1 + \frac{\omega_L^2 - \omega_T^2}{\omega_T^2 - (\omega + i\Gamma)^2}, \tag{4}$$

$$\omega_T = \gamma \sqrt{H_A (H_A + 2H_E)} \tag{5a}$$

is the antiferromagnetic resonance frequency, $\gamma$ is the gyromagnetic ratio, $\omega$ the angular frequency of the waves, $\Gamma$ the damping parameter

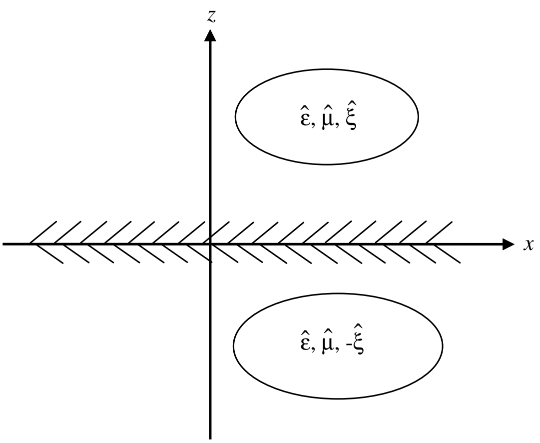


**Figure 1.** Geometry of the problem. The region $z > 0$ is occupied by L-material and the region $z < 0$ — by R-material (they are the mirror image of each other). The surface wave is propagating along the positive $x$-axis.

and $\omega_L$ the frequency of the long-wavelength longitudinal optical magnons:

$$\omega_L = \omega_T \sqrt{1+\chi}, \quad \chi \equiv M_s/H_s, \quad H_s = H_E + H_A/2. \tag{5b}$$

Usually, $\chi \ll 1$ [35]; in the following, for estimations we assume that $\chi = 0.01$. Without losing generality, we can consider surface waves which travel and are attenuated in the $x$-direction along the interface (see Figure 1) so as the wave vector $\mathbf{k} = k_0\{n, 0, iq\}$, where $k_0 = \omega/c$ is the free space wave number and $\mathrm{Re}_q > 0$. Assuming that time enters as a factor of the form $\exp(-i\omega t)$ and using constitutive relations for a bianisotropic medium

$$\mathbf{D} = \varepsilon_0 \hat{\varepsilon} \mathbf{E} - ic^{-1}\hat{\xi}\mathbf{H}, \quad \mathbf{B} = \mu_0 \hat{\mu} \mathbf{H} + ic^{-1}\hat{\xi}\mathbf{E}, \tag{6}$$

one can find evanescent solutions of the Maxwell equations

$$\mathrm{rot}\mathbf{H} = -i\omega\mathbf{D}, \quad \mathrm{rot}\mathbf{E} = i\omega\mathbf{B}, \tag{7}$$

which decrease with increasing distance from the interface plane $z = 0$. In (6), $\varepsilon_0$ and $\mu_0$ are permittivity and permeability of free space, $c = (\varepsilon_0\mu_0)^{-1/2}$.

In each medium, at a given value of the tangential wave vector component $k_x = k_0 n$, there are two evanescent partial waves with different values of the normal component $(k_z)_\pm = ik_0 q_\pm$, where

$$q_\pm^2 = -A \pm \sqrt{A^2 - C}, \tag{8a}$$

$$A = \varepsilon_\perp \mu - (\varepsilon + \mu) n^2/2(1-\zeta), \;\; C = \varepsilon_\perp \mu \left[2A - \varepsilon_\perp \mu + n^4/\varepsilon_\parallel (1-\zeta)\right], \tag{8b}$$

$$\varepsilon \equiv \varepsilon_\perp/\varepsilon_\parallel, \quad \zeta \equiv \xi^2/\varepsilon_\parallel. \tag{8c}$$

In the following we assume that $\varepsilon_\parallel > 1$, so as the parameter $\zeta < 1$. Setting, for example, $\varepsilon_\parallel = 4.9$ and $\xi = 0.7$, we obtain $\zeta = 0.1$. The electric field vector is a linear combination of the field vectors of both partial waves:

$$\mathbf{E}_L = \mathbf{E}_+^L \exp(-k_0 q_+ z) + \mathbf{E}_-^L \exp(-k_0 q_- z) \tag{9a}$$

in the space region $z > 0$ (the common term $\exp[i(k_0 nx - \omega t)]$ is emitted) and

$$\mathbf{E}_R = \mathbf{E}_+^R \exp(k_0 q_+ z) + \mathbf{E}_-^R \exp(k_0 q_- z) \tag{9b}$$

in the region $z < 0$, and analogically for the magnetic field vectors.

Using the standard boundary conditions for the tangential components of the fields in the absence of a surface current, we find two distinct modes of SMP which are described by the following dispersion relations:

$$n^2 = \varepsilon_\parallel \mu_1 \left[\frac{\mu(\omega) - \varepsilon}{\mu(\omega) - \mu_1}\right] \equiv n_\varepsilon^2 \tag{10a}$$

and

$$n^2 = \varepsilon_{\|}\mu(\omega)\left[\frac{\mu(\omega)-\varepsilon}{\mu(\omega)-\mu_2}\right] \equiv n_\mu^2, \tag{10b}$$

where

$$\mu_1 \equiv \varepsilon(1-\zeta), \quad \mu_2 \equiv \varepsilon(1-\zeta)^{-1} \tag{11}$$

and $\mu(\omega)$ is given by Equation (4).

In the SMP mode described by Equation (10a) (furthermore, for the brevity this mode is referred as $\varepsilon$-mode), the tangential components of the electric field in both R- and L-materials coincide: $\mathbf{E}_{\pm t}^L = \mathbf{E}_{\pm t}^R$, while the components of the magnetic fields are parallel but $|\mathbf{H}_{\pm t}^L| \neq |\mathbf{H}_{\pm t}^R|$. These components are polarized elliptically:

$$H_{+y}^L/H_{+x}^L = H_{-y}^L/H_{-x}^L = H_{+y}^R/H_{+x}^R = H_{-y}^R/H_{-x}^R = \rho, \tag{12a}$$

where

$$\rho = (\varepsilon_{\|}/\xi)\left[(1-\zeta)n_\varepsilon^{-2}\left(q_+ + \varepsilon_\perp \mu q_+^{-1}\right) - \mu q_+^{-1}\right] \tag{12b}$$

is a complex quantity. The polarization of the field components parallel to the $(xz)$-plane are described by the relations

$$H_{+z}^L/H_{+x}^L = -H_{+z}^R/H_{+x}^R = \rho_+, \;\; H_{-z}^L/H_{-x}^L = -H_{-z}^R/H_{-x}^R = \rho_-, \tag{13a}$$

where

$$\rho_\pm = i n_\varepsilon^{-1}\left(q_\pm + \varepsilon_\perp \mu q_\pm^{-1}\right). \tag{13b}$$

In the second mode with dispersion relation (10b) (in the following it will be referred as $\mu$-mode) the properties of the electric and magnetic fields are interchanged: the tangential magnetic field components in R- and L-materials coincide: $\mathbf{H}_{\pm t}^L = \mathbf{H}_{\pm t}^R$, while those of the electric fields are parallel, but $|\mathbf{E}_{\pm t}^L| \neq |\mathbf{E}_{\pm t}^R|$. These components are characterized by elliptical polarization too:

$$E_{+y}^L/E_{+x}^L = E_{-y}^L/E_{-x}^L = E_{+y}^R/E_{+x}^R = E_{-y}^R/E_{-x}^R = \lambda, \tag{14a}$$

where

$$\lambda = (\varepsilon_{\|}/\xi)\left[(1-\zeta)n_\mu^{-2}\left(q_+ + \varepsilon_\perp \mu q_+^{-1}\right) - \varepsilon q_+^{-1}\right]. \tag{14b}$$

Polarizations of the field components parallel to the plane $(xz)$ are characterized by

$$E_{+z}^L/E_{+x}^L = -E_{+z}^R/E_{+x}^R = \lambda_+, \quad E_{-z}^L/E_{-x}^L = -E_{-z}^R/E_{-x}^R = \lambda_-, \tag{15a}$$

where

$$\lambda_\pm = \rho_\pm n_\varepsilon/n_\mu. \tag{15b}$$

## 3. SURFACE MAGNON POLARITONS IN NONDISSIPATIVE CASE

In this section we will consider SMP in the case when the wave dissipation can be neglected, so as the damping parameter in Equation (4) is equal to zero and therefore the waves can be described by real frequencies and real tangential wave vectors. Note that in this case the polarization parameters given by Equations (12b) and (14b) are real too. It means that tangential components of the fields are polarized linearly and the field vectors in each partial wave can be written as a sum of two vectors, one of which lies in the $xy$-plane while the second one is parallel to the $z$-axis and shifted in phase by $\pi/2$ (or $-\pi/2$) with respect to the first vector. Besides, both partial waves can be true evanescent waves if normal components of the wave vectors are pure imaginary quantities: $\mathrm{Im}q_{\pm} = 0$. According to Equation (8), such a situation is only possible if the following conditions are fulfilled simultaneously:

$$A < 0, \quad 0 < C < A^2. \tag{16}$$

Furthermore in this section we restrict ourselves to the consideration of the lossless SMP for which unequalities (16) are fulfilled. We will examine dispersion properties of the $\varepsilon$- and $\mu$-modes separately.

### 3.1. $\varepsilon$-modes

Using Equation (4) with $\Gamma = 0$, the dispersion law (10a) for $\varepsilon$-mode can be rewritten as

$$n_\varepsilon^2 = n_{\varepsilon\infty}^2 \left(\omega^2 - \omega_1^2\right) / \left(\omega^2 - \omega_2^2\right), \tag{17}$$

where

$$n_{\varepsilon\infty}^2 \equiv \varepsilon_{\|}\mu_1(1-\varepsilon)/(1-\mu_1), \tag{18a}$$

$$\omega_1^2 = \left(\omega_L^2 - \varepsilon\omega_T^2\right)/(1-\varepsilon), \tag{18b}$$

$$\omega_2^2 = \left(\omega_L^2 - \mu_1\omega_T^2\right)/(1-\mu_1). \tag{18c}$$

According to Equation (17), the wave is cutoff ($k_x = 0$) at $\omega = \omega_1$ and has a resonance ($k_x \to \infty$) at $\omega = \omega_2$. It is evident that cutoff and resonance frequencies exist only if the expressions on the right-hand side of the corresponding Equation (18b) or (18c) are positive. Note that conditions (16) for $\varepsilon$-modes are only fulfilled simultaneously if

$$\mu(\omega) < \mu_1. \tag{19}$$

It means that the existence of lossless SMP is possible not only in frequency range $\omega_T < \omega < \omega_L$, where $\mu(\omega) < 0$, but also at frequencies

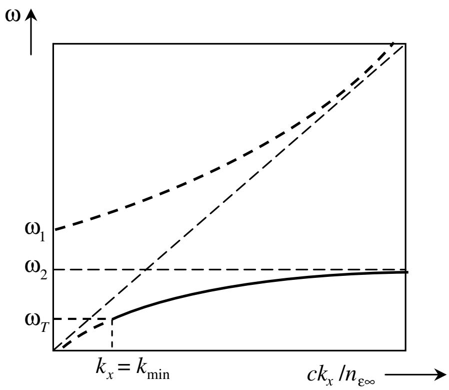


**Figure 2.** Dispersion curves of the lossless SMP (solid line) and bulk (Brewster) waves (dashed) in the case $\varepsilon < 1$.

for which $\mu(\omega) \geq 0$, namely: in the range $\omega_L \leq \omega < \omega_2$ if $\mu_1 < 1$, in the region $\omega \geq \omega_L$ if $1 \leq \mu_1 \leq 1 + \chi$, and in the regions $\omega < \omega_2 (< \omega_T)$, $\omega \geq \omega_L$ if $\mu_1 > 1 + \chi$. However, condition (19) is not sufficient for the wave existence: it can only be propagating if the right hand side of Equation (17) is positive. The precise regions of existence as well as the dispersion curves of the waves $\omega = \omega(k_x)$ are shown in Figures 2–4 in five different intervals of change of the parameter $\varepsilon = \varepsilon_\perp / \varepsilon_\parallel$.

In the case when $\varepsilon < 1$ (see Figure 2) there are two branches with $n_\varepsilon^2 > 0$. The high-frequency branch as well as the part of the low-frequency branch with $\omega < \omega_T$ correspond to the bulk (Brewster) waves with real value of the normal wave vector component $k_z$. It means that both attenuation constants $q_+$ and $q_-$ are pure imaginary ($A > C > 0$) and thus describe magnon polaritons which are not bound with the interface.

The dispersion curve of SMP (solid curve) begins at the point $(ck_{\min}/n_{\varepsilon\infty}, \omega_T)$, where

$$ck_{\min} = \omega_T [\varepsilon_\perp (1 - \zeta)]^{1/2}, \tag{20}$$

and approaches asymptotically to the straight line $\omega = \omega_2$ with increasing $k_x$. Thus, the SMP exist in the tangential wave number region

$$k_x > k_{\min} \tag{21}$$

and in the frequency range

$$\omega_T < \omega < \omega_2. \tag{22}$$

In the case of Figure 3(a), when $1 < \varepsilon < 1 + \chi$, both $\omega_1^2$ and $n_{\varepsilon\infty}^2$ are negative, so as there is a single branch with $n_\varepsilon^2 > 0$ and the

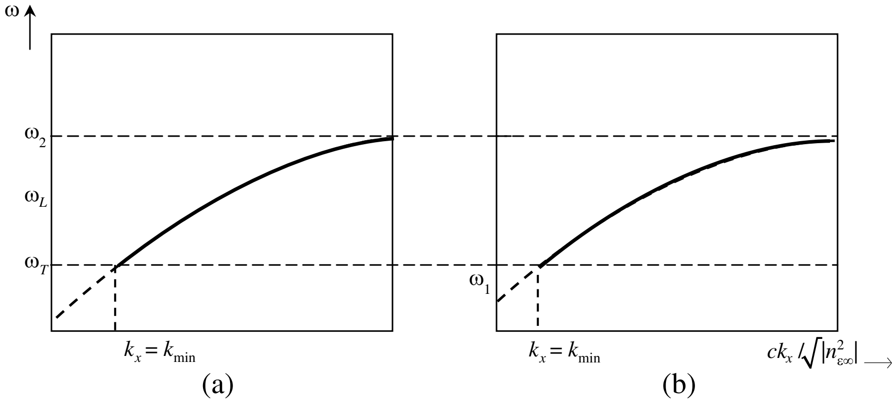


**Figure 3.** Dispersion curves in the cases (a) $1 < \varepsilon < 1 + \chi$ and (b) $1 + \chi < \varepsilon < (1 - \zeta)^{-1}$.

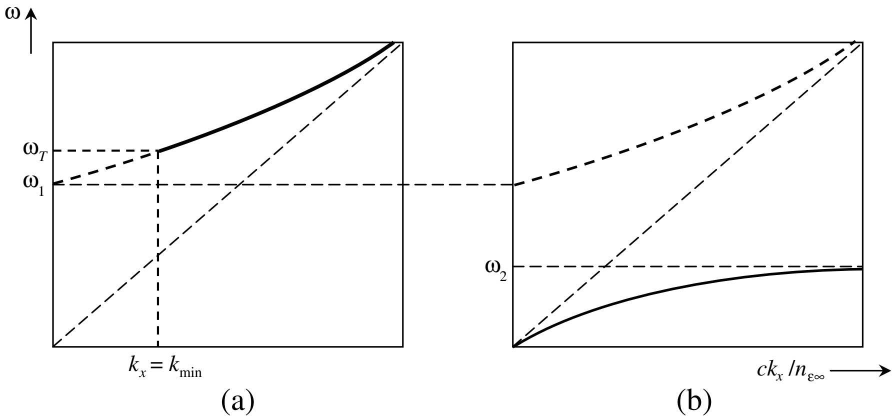


**Figure 4.** Dispersion curves (solid lines) in the intervals (a) $1 < \mu_1 < 1 + \chi$ and (b) $\mu_1 > 1 + \chi$.

dispersion curve begins at $\omega = 0$. Unlike that, in the next interval $1 + \chi < \varepsilon < (1 - \zeta)^{-1}$, where $\mu_1 < 1$, the dispersion curve begins at cut-off frequency $\omega_1$ (see Figure 3(b)). In both these cases, the section of the dispersion curve corresponding to SMP is given by the same Equations (21), (22).

It is easily to see that in the interval of $\varepsilon(1 - \zeta)^{-1} < \varepsilon < (1 + \chi)(1 - \zeta)^{-1}$, that is, when $1 < \mu_1 < 1 + \chi$ (Figure 4(a)), the wave has no resonance frequency ($\omega_2^2 < 0$). The dispersion curve begins at cut off frequency $\omega = \omega_1$ and for large values of $k_x \gg k_{\min}$ behaves asymptotically as a linear function $\omega = ck_x/n_{\varepsilon\infty}$. The frequency region of existence is given by

$$\omega > \omega_T. \tag{23}$$

Finally, in the case $\varepsilon > (1+\chi)(1-\zeta)^{-1}$ (or $\mu_1 > 1+\chi$, Figure 4(b)) there are again two branches with $n_\varepsilon^2 > 0$ but, unlike the case of Figure 2, the surface wave exists for all values of $k_x$ and for any frequency in the region

$$\omega < \omega_2. \tag{24}$$

### 3.2. $\mu$-modes

Dispersion law (9b) for lossless $\mu$-modes can be rewritten in the form

$$n_\mu^2 = \frac{n_{\mu\infty}^2 \left(\omega^2 - \omega_1^2\right) \left(\omega^2 - \omega_L^2\right)}{\left(\omega^2 - \omega_3^2\right) \left(\omega^2 - \omega_T^2\right)}, \tag{25}$$

where

$$n_{\mu\infty}^2 = \varepsilon_{\|}(1-\varepsilon)/(1-\mu_2), \tag{26a}$$

$$\omega_3^2 = \left(\omega_L^2 - \mu_2\omega_T^2\right)/(1-\mu_2), \tag{26b}$$

$\mu_2$ and $\omega_1$ are given by Equations (11) and (18b), respectively.

The wave has two cutoff frequencies at $\omega = \omega_L$ and $\omega = \omega_1$ if $\varepsilon < 1$ or $\varepsilon > 1+\chi$, and only one cutoff ($\omega = \omega_L$) if $1 < \varepsilon < 1+\chi$.

As to the resonance frequencies of the wave, there is a single resonance at $\omega = \omega_T$ if $1 < \mu_2 < 1+\chi$, and an additional one at $\omega = \omega_3$ if $\mu_2 < 1$ or $\mu_2 > 1+\chi$. It is important to note that according to Equation (10b), the wave exists ($n_\mu^2 > 0$) only at frequencies for which $\mu(\omega)$ is positive. It means that the frequency range $\omega_T < \omega < \omega_L$ is forbidden for the propagation of the wave. Note also that conditions (16) can only be fulfilled simultaneously if

$$\mu(\omega) > \mu_2. \tag{27}$$

Consequently, the limits of the frequency regions of existence as well as the number of such regions depend on the value of the parameter $\mu_2$. In fact, the wave exists in two different regions

$$\omega < \omega_T, \quad \omega > \omega_3 (> \omega_L), \tag{28a}$$

if $\mu_2 < 1$ (see Figure 5(a)), in the region $\omega < \omega_T$ if $1 \leq \mu_2 \leq 1+\chi$ (Figure 5(b)) and in the range

$$\omega_3 < \omega < \omega_T, \tag{28b}$$

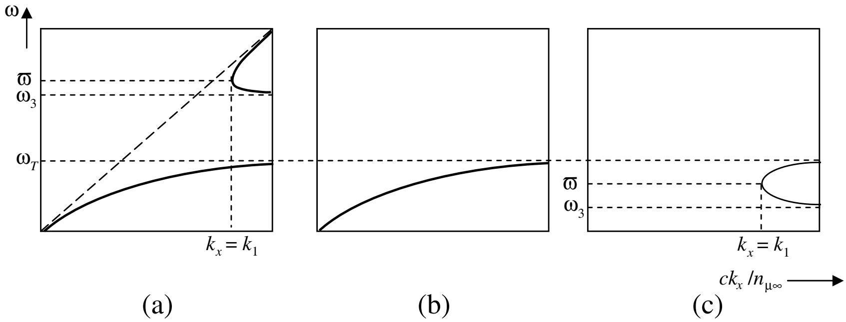


**Figure 5.** Dispersion curves of the $\mu$-modes in the cases (a) $\mu_2 < 1$, (b) $1 \leq \mu_2 \leq 1 + \chi$ and (c) $\mu_2 > 1 + \chi$.

if $\mu_2 > 1 + \chi$ and $\varepsilon > 1 + \chi$ (Figure 5(c)). In Figure 5(a), there is a single branch at $k_x < k_1$ and three branches at $k_x > k_1$, where

$$k_1 \simeq \frac{\varpi}{c}\sqrt{\frac{\varepsilon_{\|}(1-\varepsilon)g}{1-\mu_2}}, \tag{29}$$

$$\varpi \simeq \omega_3 \left[1 + \frac{\chi\left(1 + 4\mu_2\sqrt{\zeta(1-\varepsilon)^{-1}}\right)}{2(1-\mu_2+\chi)}\right]^{1/2}, \tag{30}$$

$$g = \frac{\varpi^2 - \left(\omega_L^2 + \omega_1^2\right)/2 + \left(\varpi^2 - \omega_L^2\right)\left(\varpi^2 - \omega_1^2\right)/2\varpi^2}{\varpi^2 - \left(\omega_T^2 + \omega_3^2\right)/2}. \tag{31}$$

The low-frequency branch begins at $\omega = 0$ and exists for all values of $k_x$ while both high-frequency branches begin at $\omega = \varpi$ and exist only for $k_x > k_1$. With increasing $k_x$, the dispersion curve of the highest branch increases monotonically and approaches asymptotically to the straight line $\omega = ck_x/n_{\mu\infty}$ while that of the middle branch decreases and approaches asymptotically to the straight line $\omega = \omega_3$. Thus, in the range $\omega_3 < \omega < \varpi$ SMP possess anomalous dispersion: the phase and group velocities of the wave have opposite direction.

In the case of Figure 5(b) there is only one branch for all values of $k_x$. In the case of Figure 5(c), there are two branches which begin at the point $\omega = \varpi$, $k_x = k_1$ and exist only in the region $k_x > k_1$, where former $\varpi$ given by Equation (31) should be replaced by

$$\varpi \simeq \omega_3 \left[1 + \frac{\chi\left\{1 + 8(\mu_2-1)^{-1}\left[1 - \mu_2(\varepsilon-1)(\mu_2-1)^{-1}\right]\right\}}{2(\mu_2-1-\chi)}\right]^{1/2}. \tag{32}$$

Thus, the dispersion properties of both $\varepsilon$- and $\mu$-modes of SMP are very sensitive to the ratio $\varepsilon_{\|}/\varepsilon_{\perp}$, as well as to the value of the parameters $\zeta$ and $\chi$. In particular, in the case of the $\mu$-modes the minimum permitted value $\mu_2$ of the magnetic permeability increases with increasing of the chirality parameter $\xi$, and that leads to the rough changes in the dispersion curves shown in Figures 5(a)–(c).

## 4. ROLE OF THE WAVE DISSIPATION. REFRACTION INDEX AND PROPAGATION LENGTH OF SMP

Consider now briefly the main properties of SMP in the case when damping is taken into account. Substituting Equations (4) into (10a), (10b) and separating into real and imaginary parts ($n = n' + in''$, $\mu = \mu' + i\mu''$) we obtain

$$n'^2 = \left[\beta + \sqrt{\alpha^2 + \beta^2}\right]/2, \quad n'' = \alpha/2n', \tag{33}$$

where

$$\alpha \equiv \frac{\varepsilon_{\perp}\zeta\mu_1\mu''}{(\mu' - \mu_1)^2 + \mu''^2}, \quad \beta \equiv \frac{\varepsilon_{\|}\mu_1[\mu''^2 + (\mu' - \varepsilon)(\mu' - \mu_1)]}{(\mu' - \mu_1)^2 + \mu''^2} \tag{34}$$

for the $\varepsilon$-mode and

$$\alpha \equiv \frac{\varepsilon_{\|}\mu''[|\mu|^2 + (\varepsilon - 2\mu')\mu_2]}{(\mu' - \mu_2)^2 + \mu''^2}, \quad \beta \equiv \frac{\varepsilon_{\|}\left[(\mu' + \mu_2\zeta)|\mu|^2 + (\varepsilon - 2\mu')\mu'\mu_2)\right]}{(\mu' - \mu_2)^2 + \mu''^2} \tag{35}$$

for the $\mu$-mode. In the absence of losses ($\mu'' = 0$) Equation (33) gives, naturally, $n'' = 0$ and $n^2 = \beta$. Then, using expressions for $\beta$ in Equations (34), (35) and substituting $\mu' = \mu(\omega)$, one can easily find exactly the same dispersion relations (10a) and (10b) for the lossless $\varepsilon$- and $\mu$-modes, respectively. It obviously means that the Section 3 analysis is valid if the dissipation is not strong.

The relative attenuation of the waves on the wavelength distance is given by

$$n''/n' = \alpha\left[\beta + \sqrt{\beta^2 + \alpha^2}\right]^{-1}. \tag{36}$$

Another important characteristics of the SMP is the propagation length $L(\omega)$ — the distance where the mode power decays by a factor of $1/e$. The propagation length can be calculated using the expression

$$L(\omega) = (2\mathrm{Im}k_x)^{-1} = n'/\alpha k_0. \tag{37}$$

It is evident that in the absence of losses $L(\omega) \to \infty$ while the presence of dissipation leads to a finite value of that. Besides, it leads to a finite value of the refraction index at resonance frequencies, as well as to nonzero values at cutoff frequencies. Indeed, for the frequencies far from the antiferromagnetic resonance frequency $|\omega - \omega_T| \gg \Gamma$, the propagation length can be calculating using expressions given by Equations (17) and (25) for $n'$ and setting

$$\mu' \cong \left(\omega_L^2 - \omega^2\right) / \left(\omega_T^2 - \omega^2\right), \quad \mu'' \cong 2\Gamma\omega \left(\omega_L^2 - \omega_T^2\right) / \left(\omega_T^2 - \omega^2\right)^2 \quad (38)$$

for the real and imaginary parts of $\mu(\omega)$. Let us find, for example, the propagation length of the $\varepsilon$-mode. Assuming that $\mu'' \ll \mu_1 - \mu'$, at frequencies far from the resonance frequency $\omega_2$ we obtain

$$L_\varepsilon = \frac{(1-\mu_1)^{3/2}}{2k_0\varepsilon\chi\zeta} \sqrt{\frac{(1-\varepsilon)\left(\omega^2 - \omega_1^2\right)}{\varepsilon_\perp(1-\zeta)}} \frac{\left(\omega^2 - \omega_2^2\right)^{3/2}}{\omega\Gamma\omega_T^2}. \quad (39)$$

The minimum value of the propagation length [that is the peak of the absorption coefficient $2\omega n''(\omega)/c$] appears at $\omega = \omega_2$ and is given by

$$L_\varepsilon^{\min} = \frac{c}{\omega_T \delta} \sqrt{\frac{1 + \sqrt{1+\delta^2}}{2\varepsilon_\perp(1-\zeta)}}, \quad (40a)$$

where

$$\delta = \varepsilon\zeta\chi\omega_T^2 / 2\Gamma\omega_2 (1-\mu_1)^2. \quad (40b)$$

From Equation (39) is obvious that the propagation length increases monotonically with decreasing of the chirality parameter $\zeta$. It is not difficult to show that such a conclusion is true for the $\mu$-modes too. As to finite values of the refractive index (e.g., for the $\varepsilon$-mode) at resonance and cutoff frequencies, they can be obtained using Equations (33), (34) and (38):

$$n'^2(\omega_2) = \frac{1}{2}\varepsilon_\perp(1-\zeta)\left(1 + \sqrt{1 + \frac{(\varepsilon\zeta\chi\omega_T/2\Gamma)^2}{(1-\mu_1)(1+\chi-\mu_1)}}\right), \quad (41a)$$

$$n'^2(\omega_1) \simeq \frac{\varepsilon_\parallel (1-\zeta)(1-\varepsilon)^{3/2}(1+\chi-\varepsilon)^{1/2}\Gamma}{\chi\zeta\omega_T}, \quad (41b)$$

where the cutoff ($\omega_1$) and resonance ($\omega_2$) frequencies are given by Equations (18b), (18c).

## 5. CONCLUSIONS

In conclusion, I have shown that two novel different modes of SMP can travel along the planar interface between enantiomorph bianisotropic antiferromagnetic materials of opposite handedness with optic axes perpendicular to the interface. The surface waves possess unexpected dispersion and polarization properties and exist not only in the frequency range where the transverse permeability is negative, but also when it is positive. Moreover, one of the surface modes propagates only if $\mu(\omega) > \mu_2 > 0$. The behavior of the dispersion curves, the limits of the frequency regions of existence as well as the number of such regions are very sensitive to the ratio of the dielectric constants along and across the optic axis as well as to the value of the chirality parameter $\zeta$ and magnetic parameter $\chi$ which is the ratio of the long-wavelength longitudinal optical magnon frequency $\omega_L$ and the antiferromagnetic resonance frequency $\omega_T$. A frequency region is found, where the group and phase velocities of the $\mu$-mode have opposite direction. The role of dissipation is considered, the propagation length, resonance width and relative attenuation of the surface waves on the wavelength distance are determined. It is shown that both the propagation length and the resonance width increase monotonically with decreasing of the chirality parameter.

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