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COMPRESSIVE SENSING BASED PARAMETER ESTIMATION FOR MONOSTATIC MIMO NOISE RADAR

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Abstract—The novelty of this letter is that it capitalizes on noise waveform to construct measurement operator at the transmitter and presents a new method of how the analogue to digital converter (ADC) sampling rate in the monostatic multiple-input multiple-output (MIMO) noise radar can be reduced — without reduction in waveform bandwidth — through the use of compressive sensing (CS). The proposed method equivalently converts the measurement operator problems into radar waveform design problems. The architecture is particularly apropos for signals that are sparse in the target scene. In this letter, Estimates of both target directions and target amplitudes using CS for monostatic MIMO noise radar are presented. Sparse bases are constructed using array steering vectors. Orthogonal least squares (OLS) algorithm for reconstruction of both target directions and target amplitudes is implemented. Finally, the conclusions are all demonstrated by simulation experiments.

1. INTRODUCTION

Noise radars transmit random or pseudorandom signals and apply coherent reception to achieve low probability of interception (LPI) and low probability of detection (LPD). Noise radars have the unique property that allows them to achieve high resolution in both range and Doppler which can be independently controlled by varying the bandwidth and integration time, respectively. They also have excellent resistance to jamming and interference. Another advantage of noise radars is their ability to efficiently share the frequency spectrum, thereby allowing a number of noise radars to operate over the same

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frequency band with minimal cross-interference. This spectrally parsimonious feature can be used to integrate several noise radars to a network centric platform. Therefore, many research studies have been carried out on this topic [1–4].

Unlike a conventional transmit beamforming radar system that uses highly correlated waveforms, a MIMO radar system transmits multiple independent waveforms via its antennas [5, 6]. The MIMO radar system is advantageous in both widely separated antennas scenario and collocated antennas scenario. In the first scenario, the transmit antennas are located far apart from each other relative to their distance to the target, which make the radar system offer considerable advantages for estimation of target parameters, such as location and velocity. In the second scenario, the transmit antennas and receive antennas are located close to each other relative to the target that all antennas view the same aspect of the target, which enables the MIMO radar to achieve superior resolution in terms of direction finding. The latter scenario, which is adopted in this letter, performs estimation of both target directions and target amplitudes for monostatic MIMO radar using CS.

In this letter, the monostatic noise radar concept is extended to an array of M_t transmit antennas and N_r receive antennas. When independent noise sources are transmitted from each antenna the approach may be viewed as a special case of monostatic MIMO radar and direction finding may be derived. In this case, the monostatic MIMO noise radar is equipped with M_t transmit and N_r receive antennas that are close to each other relative to the targets, so that the RCS does not vary between the different paths. The phase differences induced by transmit and receive antennas can be exploited to form a long virtual array with M_tN_r elements. This enables the monostatic MIMO noise radar system to achieve superior spatial resolution as compared to a traditional noise radar system.

CS is a new paradigm in signal processing that trades sampling frequency for computing power and allows accurate reconstruction of signals sampled at rates many times less than the conventional Nyquist frequency, received considerable attention recently and has been applied successfully in diverse fields. The theory of CS states that a K-sparse signal η of length N can be recovered exactly from few measurements with high probability via linear programming. Let Ψ denote the basis matrix that spans this sparse space, and Φ a measurement matrix. The convex optimization problem arising from CS is formulated as follows

$$\min \|\eta\|_1, \quad \text{subject to} \quad X = \Phi\Psi\eta + E \tag{1}$$

where η is a sparse vector with K principal elements, and the remaining elements can be ignored; Φ is an $M \times N$ matrix incoherent with Ψ and $M \ll N$. E denotes the interference term which is a complex random vector with correlation matrix G, where G is the $N \times N$ matrix.

CS techniques offer a framework for the detection and allocation of sparse signals for radar with a reduced number of samples [7–9]. The application of compressive sensing to MIMO radar system has been investigated quite intensively in recent years [10–12]. The problem discussed in [10] is of the targets angular separation and reduction of the physical array elements required for the system [10] uses CS to reduce the number of real receiving elements so as to obtain a sparse MIMO array. The sensing matrix is obtained from the conventional digital beam forming matrix by selecting only a subset of rows corresponding the sparse MIMO receive channels. In [11], the DOA estimation for MIMO radar in a distributed scenario is proposed. The transmitted waveforms in MIMO radar are known at each receive antennas, so that each receive antenna can construct the basis matrix locally, without the knowledge of the received signal at other antennas. In [12], CS approach to accurately estimate properties (position, velocity) of multiple targets was exploited for MIMO radar. The sampled outputs of the matched filter at the receivers are used to estimate the positions and velocities of multiple targets using MIMO radar systems with widely separated antennas by employing sparse modeling and compressive sensing.

In this letter, we present one specific scenario in which our proposed system improves the performance considerably. The treatment of monostatic MIMO noise radar focuses on estimation of both target locations and target amplitudes, ignoring range and Doppler effects. Since the number of targets is typically smaller than the number of snapshots that can be obtained, estimation of both target locations and target amplitudes can be formulated as the recovery of a sparse vector using CS. Unlike the scenario considered in [7–11] and [12], in the proposed monostatic MIMO noise radar system it capitalizes on random or pseudo-random waveform to construct measurement operator at the transmitter and convert the measurement operator problems into radar waveform design problems.

In the next sections, after a statement of the monostatic MIMO radar scenario, the three prime issues involving possibility knowledge are investigated: first, the orthogonal random waveforms are proposed for the compressive sampling which is based on the principle of CS; then the sparse bases composed of the signal steering vectors is studied; lastly, CS reconstruction algorithm is selected to estimate both target directions and target amplitudes. We also provide simulation results to show that the proposed approach can accomplish the accuracy

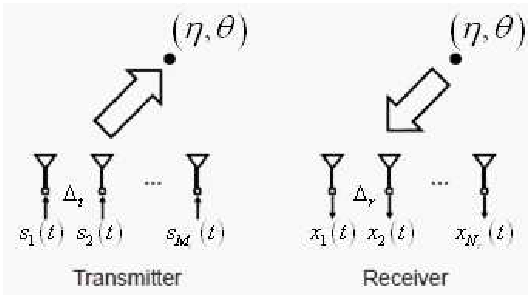


Figure 1. Monostatic MIMO noise radar scenario.

estimation in the MIMO noise radar system by using far fewer samples than existing conventional methods, such as amplitude and phase estimation (APES) and generalized likelihood ratio test (GLRT) [6].

2. SIGNAL MODEL FOR MIMO NOISE RADAR

In this section, we describe a signal model for the MIMO radar. The model focuses on the effect of the target spatial properties ignoring range and Doppler effects. For simplicity, we consider a monostatic MIMO radar system, shown in Figure 1, with an M_t-element transmit array and an N_r-element receive array, both of which are closely spaced uniform linear arrays (ULA). The targets and antennas all lie in the same plane. Assume that the inter-element spaces of the transmit and receive arrays are denoted by Δ_t and Δ_r, respectively (see Figure 1). The targets appear in the far-field of transmit and receive arrays.

At the transmit site, M_t different bandlimited and random noise signal transmitted are modeled as

$$s_{m_t}(t) = u_{m_t}(t) \exp\left(i 2\pi f_0 t\right) \tag{2}$$

where $1 \leq m_t \leq M_t$. f_0 is the center frequency of the waveform. $u_{m_t}(t) = \mu_{m_t}(t) + i\upsilon_{m_t}(t)$. $\mu_{m_t}(t)$ and $\upsilon_{m_t}(t)$ are stationary, orthogonal random process with bandwidth B. Let $s_{m_t}(t)$ denote the waveform transmitted by the m_t-th transmit antenna.

$$a_{m_t}(\theta) = \exp\left(i(2\pi/\lambda)(m_t - 1)\Delta_t \sin\theta\right)$$

is the transmit array steering vector, where λ denotes the wavelength.

$$A(\theta) = [a_1(\theta), a_2(\theta), \ldots, a_{M_t}(\theta)]$$

is the transmitted signal steering matrix.

$$b_{n_r}(\theta) = \exp\left(i(2\pi/\lambda)(n_r - 1)\Delta_r \sin\theta\right)$$

is the steering vector of the receive array.

$$B(\theta) = [b_1(\theta), b_2(\theta), \ldots, b_{N_r}(\theta)]$$

is the received signal steering matrix. The backscatter from a point target observed at the n_r-th receiver, $(1 \leq n_r \leq N_r)$, is given by

$$x_{n_r}(t) = Sb_{n_r}(\theta)A^T(\theta)\eta + e_{n_r}(t) \tag{3}$$

where $(\cdot)^T$ denotes the transpose. $S = [s_1(t), s_2(t), \ldots, s_{M_t}(t)]$. η is the complex amplitude proportional to the radar-cross-section (RCS) of the point target, θ is the azimuth angle, $e_{n_r}(t)$ denotes the interference-plus-noise term. The unknown parameters, to be estimated from $x_{n_r}(t)$, are direction parameter θ and amplitude parameter η.

3. COMPRESSIVE SENSING FOR MIMO NOISE RADAR

The proposed approach for monostatic MIMO noise radar is based on two key observations. First, there exists a small number of targets, the unknown parameters θ are sparse in the angle space. Hence, the property of the target (azimuth angles) is specified by η. If η_k is the state element of the k-th target, we define $\eta_k = \alpha_k$. Otherwise, let $\eta_k = 0$. Second, modulated version of the stationary random process transmitted as radar waveforms $s_{m_t}(t)\,(1 \leq m_t \leq M_t)$ form a measurement matrix $\Phi_{n_r} = [s_1, s_2, \ldots, s_{M_t}]$ that is incoherent with the frequency base $\Psi_{n_r}\,(1 \leq n_r \leq N_r)$ (the signal steering matrix) that sparsify or compress the above mentioned classes of point targets reflectivity functions η. By combining these observations we can both eliminate the matched filter in the radar receiver and lower the receiver A/D converter bandwidth using CS principles. Consider a new design for a radar system that consists of the following components. The transmitter is the same as in a classical MIMO noise radar; the transmit antenna emit the bandlimited and random signal. However, the receiver does not consist of a matched filter and high-rate A/D converter but rather only a low-rate A/D converter that operates not at the Nyquist rate but at a rate proportional to the target sparsity (see Figure 2).

By CS theory, we can construct a basis matrix Ψ_{n_r} for the n_r-th receive antenna as

$$\Psi_{n_r} = b_{n_r}(\theta)A^T(\theta) \tag{4}$$

The goal is to estimate η for all the K targets. Now, we discretize the target angle space into a grid of N possible values $\eta = [\eta_1, \eta_2, \ldots, \eta_N]^T$, which is a sparse target state vector. A non-zero element with index n in η indicates that there is a target at the angles θ_n. Also, considering

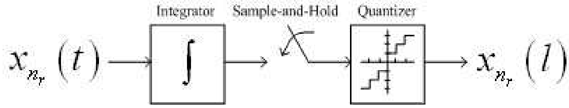


Figure 2. CS-based MIMO noise radar receivers for the transmitters in Figure 1 perform neither matched filtering nor high-rate analog-to-digital conversion.

the discrete-time waveform and discrete-angle space, we have linear projections of the received signal at the n_r-th antenna as

$$x_{n_r}(l) = \Phi_{n_r} \Psi_{n_r} \eta + e_{n_r}(l) \tag{5}$$

where $1 \leq l \leq L$ and L is the snapshot number. Placing the output of N_r receive antennas, i.e., $x_1, x_2, \ldots, x_{N_r}$, in measurement vector $X = [x_1(1), \ldots, x_1(L), x_2(1), \ldots, x_{N_r}(L)]^T$ we have

$$X = \Phi \Psi \eta + E \tag{6}$$

where X is the $N_r L \times 1$ virtual data vector associated with the monostatic MIMO noise radar. $\Phi = \text{diag}(\Phi_1, \Phi_2, \ldots, \Phi_{N_r})$ is a $N_r L \times M_t N_r$ diagonal matrix of the discrete-time waveform and $\text{diag}(\cdot)$ represents the diagonalization operation. $\Psi = [\Psi_1^T, \Psi_2^T, \ldots, \Psi_{N_r}^T]^T$ is a $M_t N_r \times N$ basis matrix. $E = [e_1^T, e_2^T, \ldots, e_{N_r}^T]^T$ is a $N_r L \times 1$ matrix of the noise term.

Our goal is to estimate both target directions and target amplitudes. In other words, we want to solve for the vector η in (6). Solving for η in (6) can be viewed as an ordinary inverse problem. Compressed sensing reconstruction algorithms such as minimum mean squared error (MMSE) estimation [13], and iterative subspace identification (ISI) [14] are well-known for solving this type of problems. However, OLS method is a powerful and simple tool for solving this type of problems. In this letter we apply OLS method in the monostatic MIMO noise radar to reconstruct the sparse target scene vector η in (6). The numerical example of the recovery results will be shown in Sec. 4. It shows that the compressed sensing based monostatic MIMO noise radar system has good estimation results both in target directions and target amplitudes and has a good robustness.

4. SIMULATION RESULTS

In this section, the simulation is carried out to illustrate the correctness and the performance of the proposed method. The monostatic MIMO noise radar system, with uniform linear array (ULA) in which the

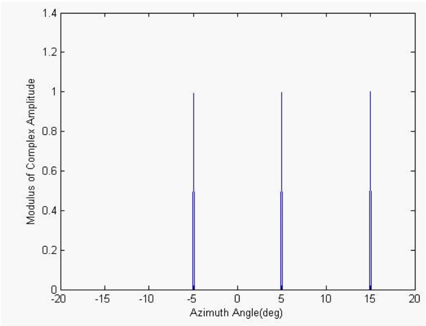


Figure 3. The estimated azimuth angles of the targets.

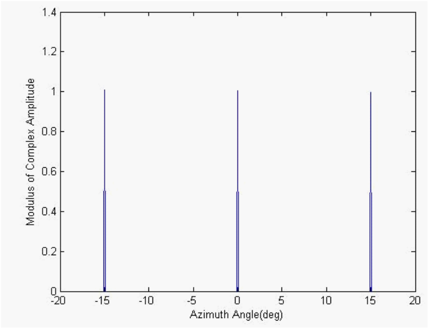


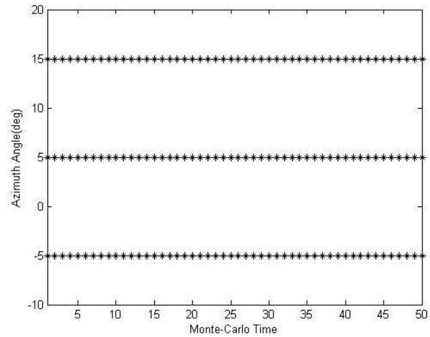
Figure 4. The estimated azimuth angles of the targets.

half-wavelength spacing between adjacent antennas is used both for transmitting and for receiving. The transmitted waveforms with a bandwidth of 100 MHz are orthogonal and random signals. The transmitted pulse width is 55 μs, and the pulse repeat period is 500 μs.

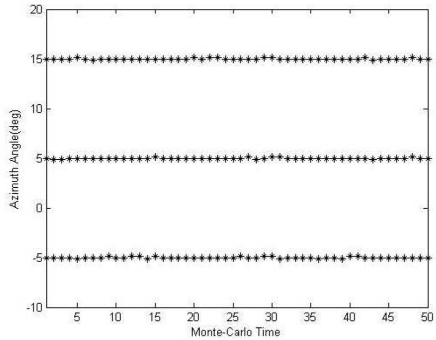
Simulation 1: $M_t = 45$ and $N_r = 8$, are considered. Assume that three targets locate at $\theta_1 = -5.00^\circ$, $\theta_2 = 5.00^\circ$ and $\theta_3 = 15.00^\circ$ with the same complex amplitudes $\alpha_k = 1\,(1 \leq k \leq K)$, SNR $= 10\,\text{dB}$, $K = 3$. The number of snap-shots is selected as $L = 50$ during the simulation. The estimated result for azimuth angles θ of the targets are shown in Figure 3. From Figure 3 we can see that both target directions and target amplitudes can accurately be estimated by the proposed method.

Simulation 2: $M_t = 45$ and $N_r = 2$, are considered. Assume that three targets locate at $\theta_1 = -15.00^\circ$, $\theta_2 = 0.00^\circ$ and $\theta_3 = 15.00^\circ$ with the same complex amplitudes $\eta = 1$, SNR $= 10\,\text{dB}$, $K = 3$. The number of snap-shots is selected as $L = 50$ during the simulation. The estimated result for azimuth angles θ of the targets are shown in Figure 4. If two receive antennas are used, the proposed approach can also yield similar performance, but by using far fewer samples. From Figure 4 both target directions and target amplitudes can accurately be estimated by the proposed method.

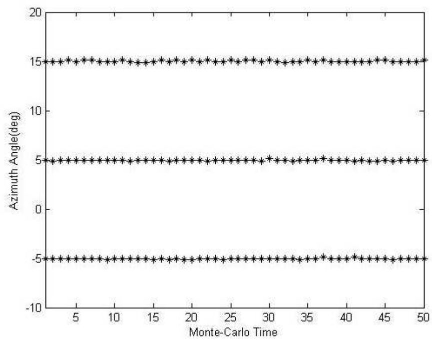
Simulation 3: $M_t = 45$ and $N_r = 8$, are considered. Assume that three targets locate at $\theta_1 = -5.00^\circ$, $\theta_2 = 5.00^\circ$ and $\theta_3 = 15.00^\circ$ with the same complex amplitudes $\eta = 1$, SNR $= 10\,\text{dB}$, $K = 3$, and the number of Monte-Carlo trials is 50. Figure 5 plots the variation of direction finding of three methods against the number of Monte-Carlo trials. The performance of the proposed method is compared with that of three methods, APES, GLRT and CS-based [11]. From Figure 5 we



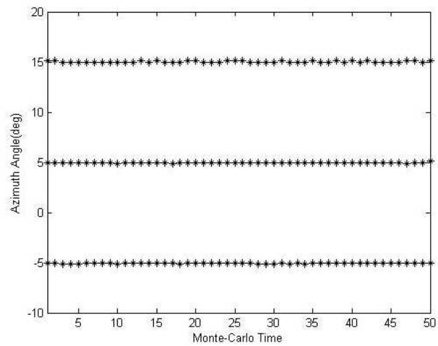
(a) Using 70 snapshots



(b) Using 256 snapshots



(c) Using 256 snapshots



(d) Using 70 snapshots

Figure 5. The estimated azimuth angles of the targets. (a) Proposed method. (b) APES method. (c) GRLT method. (d) CS-based [11].

can see that azimuth angles can accurately be estimated in these trials and the proposed method has a better robustness.

Simulation 4: $M_t = 45$ and $N_r = 8$, are considered. The number of snap-shots is selected as $L = 50$ during the simulation. Assume that two targets locate at $\theta_1 = -5.00°$ and $\theta_2 = 5.00°$ with the same complex amplitudes $\eta = 1$, $K = 2$, and the number of Monte-Carlo trials is 10000. The root-mean-square error (RMSE) of the azimuth angles of the targets versus SNR are shown in Figure 6. It can be seen from Figure 6, the proposed method has low RMSE for azimuth angles estimation.

Simulation 5: $M_t = 45$ and $N_r = 8$, are considered. According to (3) and the principles of CS, it is obvious that the more the number of snap-shots is, the better recovery signals the algorithm can find. Assume that two targets locate at $\theta_1 = -5.00°$ and $\theta_2 = 5.00°$ with

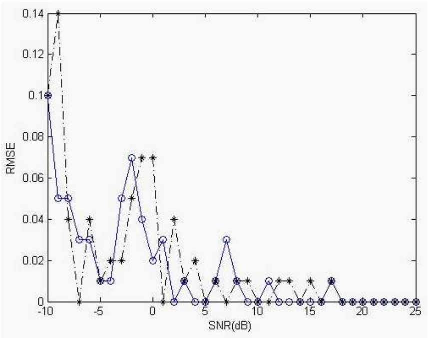


Figure 6. RMSE of azimuth angles versus SNR.

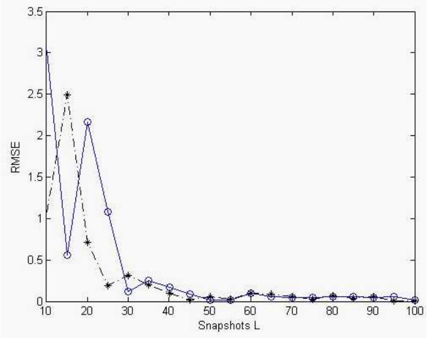


Figure 7. RMSE of azimuth angles versus snapshots.

the same complex amplitudes $\eta = 1$, $K = 2$, and the number of Monte-Carlo trials is 10000. The RMSE of the azimuth angles of the targets versus the number of snap-shots are shown in Figure 7. It can be seen from Figure 7, the proposed method has low RMSE for few samples.

5. CONCLUSION

We have proposed measurement operator at the transmitter in order to improve parameters estimation performance of CS-based monostatic MIMO noise radar for the case in which the corresponding waveform optimization method could be implemented, according to the restricted isometry property (RIP) and/or signal-to-interference ratio (SIR). The proposed method has been used to estimate both target directions and target amplitudes for the proposed monostatic MIMO noise radar systems. It is superior to these conventional methods, i.e., the APES and GLRT techniques, when fewer receive antennas are active.

We are currently working on extending this letter by developing the complete proof for the RIP and/or SIR for different random or pseudo-random waveform and by studying the sparse signals in the range-Doppler-angle space. Moreover, we will evaluate such assertions from a system engineering viewpoint.

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