INHOMOGENEOUS PLANAR LAYERED CHIRAL MEDIA: ANALYSIS OF WAVE PROPAGATION AND SCATTERING USING TAYLOR'S SERIES EXPANSION

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Abstract—In this paper, an analytic frequency domain method based on Taylor's series expansion approach is introduced to analyze inhomogeneous planar layered chiral media for an arbitrary linear combination of TM and TE polarizations. In the presented method, electromagnetic parameters of inhomogeneous chiral media and also the electric and magnetic fields are expressed using Taylor's series expansion. Finally, the validity of the method is verified considering some special types of homogeneous and inhomogeneous chiral media and comparison of the obtained results from the presented method with the exact solutions.

1. INTRODUCTION

The interaction of electromagnetic fields with chiral media has attracted many scientists and engineers over the years. The term chiral media was first used by Jaggard et al. in 1979 [1], who defined chiral media as consisting of macroscopic chiral objects randomly embedded in a dielectric. The word chiral describes something that is handed, i.e., an object whose mirror image cannot be produced solely by rotating and translating the original object. In addition to pioneering studies, recently, there is rapid development on the study of electromagnetic wave propagation in chiral media, such as chiral plate [2], chiral slab [3], electromagnetic scattering with chiral objects [4–15], and coating with chiral material for reducing radar cross-section of targets [16–20]. More recently, chiral nihility as a special case of chiral media with many applications has attracted increasing attention [21–32].

The time-harmonic constitutive relations of an isotropic and homogeneous chiral medium assuming $e^{j\omega t}$ as time dependence are given by [33]:

$$\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E} - j\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{H}, \quad \mathbf{B} = j\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{E} + \mu_r\mu_0\mathbf{H}, \qquad (1)$$

where ε_0 and μ_0 are the permittivity and permeability of vacuum, ε_r and μ_r are the relative permittivity and permeability of the chiral medium, respectively, and κ is the chirality parameter. Chiral media have two important properties: the first one is optical (or electromagnetic) activity, which can rotate the polarization plane of a linearly polarized wave propagating through it; and the second property is circular dichroism, which is attributed to the different absorptivity of the right and left circularly polarized waves (RCP and LCP) inside chiral medium. The wave equation in a homogeneous chiral medium is:

$$\nabla^2 \mathbf{E} + 2\frac{\kappa\omega}{c_0}\nabla \times \mathbf{E} + \frac{\omega^2}{c_0^2}\left(\mu_r\varepsilon_r - \kappa^2\right)\mathbf{E} = 0 \qquad (2)$$

where c_0 and ω are the speed of light in vacuum and the angular frequency, respectively. It can be easily seen that the RCP and LCP waves are the eigenpolarization of the wave equation in a homogeneous chiral medium [33].

The study of wave propagation in inhomogeneous chiral media, which have some applications in the polarization correction of the lens and aperture antennas [34, 35], is much more complicated than homogeneous chiral media. In order to study of wave propagation and electromagnetic scattering from inhomogeneous non-chiral media, several approaches have been presented such as Richmond method [36, 37], solving Riccati equation [38], full-wave analysis [39–41], rigorous full vectorial analysis [42], finite difference method [43], using of Taylor's series expansion [44], using of Fourier series expansion [45], method of moments [46], equivalent source method [47], and cascading thin linear layers method [48]. In this study, a general method to frequency domain analysis of inhomogeneous planar layered chiral media is introduced. The presented analytic approach is based on the use of Taylor's series expansion. In order to verify the validity of the presented method, some special types of homogeneous and inhomogeneous chiral media are considered and the obtained results from the presented method are compared with the exact solutions.

2. PROPAGATION OF PLANE WAVES IN INHOMOGENEOUS CHIRAL MEDIA

In order to obtain the solution for the reflection and transmission co-efficients and even internal electromagnetic field of an inhomogeneous

planar layered chiral media, the problem illustrated in Figure 1 have to be solved. As previously mentioned, a linearly polarized wave propagating in chiral media undergoes a rotation of its polarization, and so TE and TM waves scattered by or transmitted through chiral media are coupled. Since, it is assumed that a plane wave with an arbitrary polarization (an arbitrary linear combination of TM ($E_i^{\parallel}$) and TE ($E_i^{\perp}$) polarizations):

$$\mathbf{E}_i = \left[E_i^{\parallel} \left(\cos(\theta_0)\hat{x} - \sin(\theta_0)\hat{z} \right) + E_i^{\perp}\hat{y} \right] e^{-j(k_x x + k_z z)} \tag{3}$$

where $k_x = (\omega/c_0)\sin(\theta_0)$, and $k_z = (\omega/c_0)\cos(\theta_0)$, is obliquely incident with incident angle of θ_0 from free space onto an inhomogeneous chiral slab which occupies the region $0 \le z \le t$. The chiral layer has inhomogeneous material parameters $\varepsilon(z, \omega) = \varepsilon_0 \varepsilon_r(z, \omega)$, $\mu(z, \omega) = \mu_0 \mu_r(z, \omega)$, and $\kappa(z, \omega)$ which are assumed to be z and frequency dependent. Henceforth, for simplicity, the frequency dependence of these parameters is not shown.

Substituting the constitutive equations, Equation (1), into Faraday's and Ampere's laws, respectively, the differential equations describing inhomogeneous chiral layer are given by:

$$\frac{\partial E_x}{\partial z} = \frac{\omega \kappa(z)}{c_0} E_y - j k_x E_z - j\omega \mu_0 \mu_r(z) H_y \tag{4}$$

$$\frac{\partial E_y}{\partial z} = -\frac{\omega \kappa(z)}{c_0} E_x + j\omega \mu_0 \mu_r(z) H_x \tag{5}$$
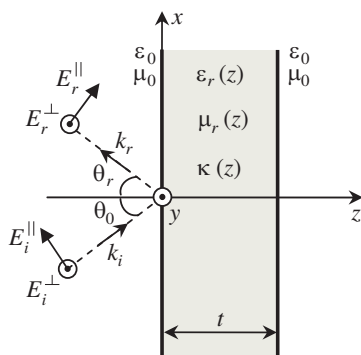

Figure 1. A typical inhomogeneous planar layered chiral media exposed to an incident plane wave with an arbitrary linear combination of TM ($E_i^{\parallel}$) and TE ($E_i^{\perp}$) polarizations.

$$-jk_x E_y = \frac{\omega\kappa(z)}{c_0} E_z - j\omega\mu_0\mu_r(z)H_z \qquad (6)$$

$$\frac{\partial H_x}{\partial z} = j\omega\varepsilon_0\varepsilon_r(z)E_y + \frac{\omega\kappa(z)}{c_0} H_y - jk_x H_z \qquad (7)$$

$$\frac{\partial H_y}{\partial z} = -j\omega\varepsilon_0\varepsilon_r(z)E_x - \frac{\omega\kappa(z)}{c_0} H_x \qquad (8)$$

$$-jk_x H_y = j\omega\varepsilon_0\varepsilon_r(z)E_z + \frac{\omega\kappa(z)}{c_0} H_z \qquad (9)$$

in which the following conditions

$$\frac{\partial}{\partial y} = 0, \quad \frac{\partial}{\partial x} = -jk_x \qquad (10)$$

have been used; because the planar structure is of infinite extent along the y-direction, and so the derivative of the fields with respect to the y variable vanishes. In addition, in the inhomogeneous chiral layer, k_x must take on the same value as in free space in order to satisfy the boundary conditions on tangential fields at the boundaries. By eliminating E_z and H_z from Equations (4)–(9), one can write:

$$\frac{\partial E_x}{\partial z} = \frac{\omega}{c_0}\kappa(z)\left(1 - \frac{\sin^2(\theta_0)}{\kappa^2(z) - \varepsilon_r(z)\mu_r(z)}\right) E_y$$
$$-j\omega\mu_0\mu_r(z)\left(1 + \frac{\sin^2(\theta_0)}{\kappa^2(z) - \varepsilon_r(z)\mu_r(z)}\right) H_y \qquad (11)$$

$$\frac{\partial E_y}{\partial z} = -\frac{\omega}{c_0}\kappa(z)E_x + j\omega\mu_0\mu_r(z)H_x \qquad (12)$$

$$\frac{\partial H_x}{\partial z} = j\omega\varepsilon_0\varepsilon_r(z)\left(1 + \frac{\sin^2(\theta_0)}{\kappa^2(z) - \varepsilon_r(z)\mu_r(z)}\right) E_y$$
$$+\frac{\omega}{c_0}\kappa(z)\left(1 - \frac{\sin^2(\theta_0)}{\kappa^2(z) - \varepsilon_r(z)\mu_r(z)}\right) H_y \qquad (13)$$

$$\frac{\partial H_y}{\partial z} = -j\omega\varepsilon_0\varepsilon_r(z)E_x - \frac{\omega}{c_0}\kappa(z)H_x \qquad (14)$$

Furthermore, there are four boundary conditions enforcing continuity of tangential electric and magnetic fields at the two boundaries. According to Figure 1, at $z = 0$ one can write:

$$E_i^{\parallel}\cos(\theta_0) + E_r^{\parallel}\cos(\theta_0) = E_x(0), \quad E_i^{\perp} + E_r^{\perp} = E_y(0) \qquad (15)$$

$$-\frac{1}{\eta_0}E_i^{\perp}\cos(\theta_0) + \frac{1}{\eta_0}E_r^{\perp}\cos(\theta_0) = H_x(0), \quad \frac{1}{\eta_0}E_i^{\parallel} - \frac{1}{\eta_0}E_r^{\parallel} = H_y(0) \qquad (16)$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance in the free space. By eliminating $E_r^{\parallel}$ and $E_r^{\perp}$ from these equations, we can write:

$$E_x(0) + \eta_0 \cos(\theta_0) H_y(0) = 2E_i^{\parallel} \cos(\theta_0) \tag{17}$$

$$E_y(0) - \frac{\eta_0}{\cos(\theta_0)} H_x(0) = 2E_i^{\perp}. \tag{18}$$

Similarly, the boundary conditions at $z = t$, are given by:

$$E_x(t) - \eta_0 \cos(\theta_0) H_y(t) = 0 \tag{19}$$

$$E_y(t) + \frac{\eta_0}{\cos(\theta_0)} H_x(t) = 0 \tag{20}$$

By solving the system of differential equations including the Equations (11)–(14), the reflection and transmission coefficients can be obtained.

3. SOLUTION TO THE SYSTEMS OF EQUATIONS USING TAYLOR'S SERIES APPROACH

It can be clearly seen that solving analytically the system of differential equations describing inhomogeneous chiral layer is very challenging. Thus, in this section, we discuss the use of Taylor's series expansion to solve the system of differential equations. Assuming material parameters of inhomogeneous chiral layer could be expanded using Taylor's series approach, one can write:

$$\varepsilon_r(z) = \sum_{n=0}^{\infty} E_n \left(\frac{z}{t}\right)^n \tag{21}$$

$$\mu_r(z) = \sum_{n=0}^{\infty} M_n \left(\frac{z}{t}\right)^n \tag{22}$$

$$\kappa(z) = \sum_{n=0}^{\infty} K_n \left(\frac{z}{t}\right)^n \tag{23}$$

where E_n, M_n, and K_n are known coefficients. For convenience, it is better to expand the impedance, the admittance, and the cross-coupling terms as follows:

$$Z(z) = j\omega\mu_0\mu_r(z) = \sum_{n=0}^{\infty} Z_n \left(\frac{z}{t}\right)^n \tag{24}$$

$$Y(z) = j\omega\varepsilon_0\varepsilon_r(z) = \sum_{n=0}^{\infty} Y_n \left(\frac{z}{t}\right)^n \tag{25}$$

$$C(z) = \frac{\omega}{c_0}\kappa(z) = \sum_{n=0}^{\infty} C_n \left(\frac{z}{t}\right)^n \tag{26}$$

$$\frac{1}{\kappa^2(z) - \varepsilon_r(z)\mu_r(z)} = \sum_{n=0}^{\infty} A_n \left(\frac{z}{t}\right)^n \tag{27}$$

where Z_n, Y_n, C_n, A_n are also known coefficients. It should be noticed that Equation (27) is valid only if the use of Taylor's series expansion is possible. Afterwards, using the Taylor's series expansion of the electric and magnetic fields, one can write:

$$E_x(z) = \sum_{n=0}^{\infty} Ex_n \left(\frac{z}{t}\right)^n, \quad E_y(z) = \sum_{n=0}^{\infty} Ey_n \left(\frac{z}{t}\right)^n \tag{28}$$

$$H_x(z) = \sum_{n=0}^{\infty} Hx_n \left(\frac{z}{t}\right)^n, \quad H_y(z) = \sum_{n=0}^{\infty} Hy_n \left(\frac{z}{t}\right)^n \tag{29}$$

where Ex_n, Ey_n, Hx_n, and Hy_n are unknown coefficients, which have to be determined. In order to determine the unknown coefficients of Taylor's series expansions of electric and magnetic fields, Equations (24)–(29) have to be substituted in Equations (11)–(14), which gives:

$$\frac{1}{t}\sum_{n=0}^{\infty}(n+1)Ex_{n+1}\left(\frac{z}{t}\right)^n$$
$$= \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} C_p Ey_q \left(\frac{z}{t}\right)^{p+q}\left(1 - \sin^2(\theta_0)\sum_{r=0}^{\infty} A_r \left(\frac{z}{t}\right)^r\right)$$
$$- \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} Z_p Hy_q \left(\frac{z}{t}\right)^{p+q}\left(1 + \sin^2(\theta_0)\sum_{r=0}^{\infty} A_r \left(\frac{z}{t}\right)^r\right) \tag{30}$$

$$\frac{1}{t}\sum_{n=0}^{\infty}(n+1)Ey_{n+1}\left(\frac{z}{t}\right)^n$$
$$= -\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} C_p Ex_q \left(\frac{z}{t}\right)^{p+q} + \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} Z_p Hx_q \left(\frac{z}{t}\right)^{p+q} \tag{31}$$

$$\frac{1}{t}\sum_{n=0}^{\infty}(n+1)Hx_{n+1}\left(\frac{z}{t}\right)^n$$
$$= \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} Y_p Ey_q \left(\frac{z}{t}\right)^{p+q}\left(1 + \sin^2(\theta_0)\sum_{r=0}^{\infty} A_r \left(\frac{z}{t}\right)^r\right)$$

$$+ \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} C_p H y_q \left(\frac{z}{t}\right)^{p+q} \left(1 - \sin^2(\theta_0) \sum_{r=0}^{\infty} A_r \left(\frac{z}{t}\right)^r \right) \quad (32)$$

$$\frac{1}{t} \sum_{n=0}^{\infty} (n+1) H y_{n+1} \left(\frac{z}{t}\right)^n$$

$$= - \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} Y_p E x_q \left(\frac{z}{t}\right)^{p+q} - \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} C_p H x_q \left(\frac{z}{t}\right)^{p+q}. \quad (33)$$

Since the coefficients of a Taylor expansion of a function are unique, it follows that if two functions are equal, their Taylor's series coefficients must agree. This argument justifies equating the coefficients terms with the same power in two sides of Equations (30)–(33) and leads to the following recursive relations:

$$Ex_{n+1} = \frac{t}{n+1} \left[\sum_{p=0}^{n} C_{n-p} Ey_p - \sin^2(\theta_0) \sum_{p=0}^{n} \sum_{q=0}^{n-p} C_{n-p-q} A_q Ey_p \right.$$

$$\left. - \sum_{p=0}^{n} Z_{n-p} Hy_p - \sin^2(\theta_0) \sum_{p=0}^{n} \sum_{q=0}^{n-p} Z_{n-p-q} A_q Hy_p \right] \quad (34)$$

$$Ey_{n+1} = \frac{t}{n+1} \left[- \sum_{p=0}^{n} C_{n-p} Ex_p + \sum_{p=0}^{n} Z_{n-p} Hx_p \right] \quad (35)$$

$$Hx_{n+1} = \frac{t}{n+1} \left[\sum_{p=0}^{n} Y_{n-p} Ey_p + \sin^2(\theta_0) \sum_{p=0}^{n} \sum_{q=0}^{n-p} Y_{n-p-q} A_q Ey_p \right.$$

$$\left. + \sum_{p=0}^{n} C_{n-p} Hy_p - \sin^2(\theta_0) \sum_{p=0}^{n} \sum_{q=0}^{n-p} C_{n-p-q} A_q Hy_p \right] \quad (36)$$

$$Hy_{n+1} = \frac{t}{n+1} \left[- \sum_{p=0}^{n} Y_{n-p} Ex_p - \sum_{p=0}^{n} C_{n-p} Hx_p \right] \quad (37)$$

Equations (34)–(37) for $n = 0, 1, 2, \ldots$ give a system of infinitive coupled equations. Moreover, substituting Equations (28) and (29) into the boundary conditions, Equations (17)–(20) gives:

$$Ex_0 + \eta_0 \cos(\theta_0) Hy_0 = 2E_i^{\parallel} \cos(\theta_0) \quad (38)$$

$$Ey_0 - \frac{\eta_0}{\cos(\theta_0)} Hx_0 = 2E_i^{\perp} \quad (39)$$

$$\sum_{n=0}^{\infty} [Ex_n - \eta_0 \cos(\theta_0) Hy_n] = 0 \qquad (40)$$

$$\sum_{n=0}^{\infty} \left[Ey_n + \frac{\eta_0}{\cos(\theta_0)} Hx_n \right] = 0. \qquad (41)$$

Truncating Taylor's series expansions at the positive integer N, Equations (34)–(37) for $n = 0, 1, 2, \ldots, N-1$, and Equations (38)–(41) give a $(4N + 4) \times (4N + 4)$ system of coupled equations. Thus, to find the unknown coefficients, a system of coupled equations should be solved, either by an iterative procedure or by the inverse matrix method. It should be noticed that the necessary condition for the convergence of the solutions is the capability of expressing each of the electric and magnetic material parameters, i.e., Equations (24)–(27), by a converged Taylor's series expansion at all points on the region $0 \le z \le t$.

Once, unknown coefficients of Taylor's series expansions were determined, reflection and transmission coefficients could be identified. It should be noticed that due to the chiral nature of medium, the co- and cross-reflections and also co- and cross-transmissions should be considered. The co- and cross-reflection coefficients of the planar layered inhomogeneous chiral media at $z = 0$ can be expressed based on the Taylor's series coefficients of the electric field using Equation (28) as the following:

$$R_{TE\text{-}TE} = \left[\frac{E_r^{\perp}}{E_i^{\perp}} \right]_{E_i^{\parallel}=0} = \frac{E_y(0) - E_i^{\perp}}{E_i^{\perp}} = \frac{Ey_0}{E_i^{\perp}} - 1 \qquad (42)$$

$$R_{TM\text{-}TM} = \left[\frac{E_r^{\parallel}}{E_i^{\parallel}} \right]_{E_i^{\perp}=0} = \frac{\left[E_x(0) - E_i^{\parallel} \cos(\theta_0) \right] / \cos(\theta_0)}{E_i^{\parallel}}$$

$$= \frac{Ex_0}{E_i^{\parallel} \cos(\theta_0)} - 1 \qquad (43)$$

$$R_{TE\text{-}TM} = \left[\frac{E_r^{\perp}}{E_i^{\parallel}} \right]_{E_i^{\perp}=0} = \frac{E_y(0)}{E_i^{\parallel}} = \frac{Ey_0}{E_i^{\parallel}} \qquad (44)$$

$$R_{TM\text{-}TE} = \left[\frac{E_r^{\parallel}}{E_i^{\perp}} \right]_{E_i^{\parallel}=0} = \frac{E_x(0) / \cos(\theta_0)}{E_i^{\perp}} = \frac{Ex_0}{E_i^{\perp} \cos(\theta_0)} \qquad (45)$$

Similarly, the co- and cross-transmission coefficients of the planar layered inhomogeneous chiral media at $z = t$ can easily be expressed

based on the Taylor's series coefficients of the electric field. For instance, the co-transmission coefficients are given by:

$$T_{TE\text{-}TE} = \left[\frac{E_t^\perp}{E_i^\perp}\right]_{E_i^\parallel = 0} = \frac{E_y(t)}{E_i^\perp} = \frac{1}{E_i^\perp}\sum_{n=0}^{N} Ey_n \tag{46}$$

$$T_{TM\text{-}TM} = \left[\frac{E_t^\parallel}{E_i^\parallel}\right]_{E_i^\perp = 0} = \frac{E_x(t)/\cos(\theta_0)}{E_i^\parallel} = \frac{1}{E_i^\parallel\cos(\theta_0)}\sum_{n=0}^{N} Ex_n. \tag{47}$$

4. EXAMPLES, RESULTS AND DISCUSSIONS

In this section, three types of homogeneous and inhomogeneous chiral layer are considered for analysis of the wave propagation and reflection and transmission coefficients using the proposed approach. The first and second examples have exact solutions and can be used to verify the accuracy of the proposed method based on the Taylor's series expansion approach. The results presented in this section are obtained by solving the system of coupled Equations (34)–(41) using the inverse matrix method.

4.1. Example 1 (Homogeneous Chiral Slab)

Consider a homogeneous chiral slab with thickness of $t = 0.2\,\mathrm{m}$, and the electromagnetic parameters as the following:

$$\varepsilon_r(z) = 4 \tag{48}$$

$$\mu_r(z) = 1 \tag{49}$$

$$\kappa(z) = 1.5 \tag{50}$$

illuminated by oblique incident of a linearly polarized plane wave (TE^z or TM^z) with unity amplitude, and the excitation frequency of $1\,\mathrm{GHz}$. The problem of the plane wave propagation through an infinite homogeneous chiral slab was analytically discussed in Ref. [3]. The amplitudes of co- and cross-reflection and co-transmission coefficients obtained from the exact solution and the presented method with $N = 40$ versus the angle of incidence are compared in Figure 2. It can be seen that for $N = 40$ or greater, the obtained solutions from the presented method are in the excellent agreement with the exact solutions. Further studies show that as the thickness of the inhomogeneous chiral slab with respect to the wavelength increases, the necessary number of unknown coefficients increases.
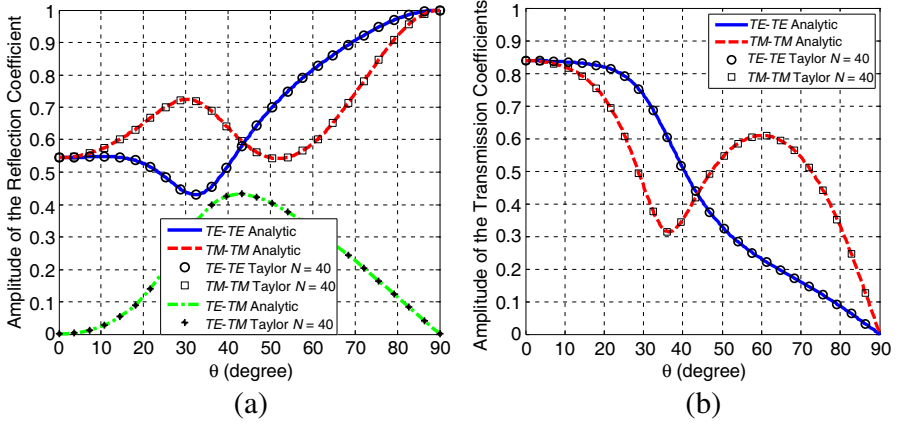
Figure 2. Reflection and transmission coefficients as a function of incident angle θ for homogeneous chiral slab. (a) Co- and cross-reflection coefficients. (b) Co-transmission coefficients.

4.2. Example 2 (Inhomogeneous Chiral Nihility Slab)

In the second example, the propagation of a plane wave through an infinite inhomogeneous chiral nihility slab of thickness $t = 0.2$ m in which both the permittivity and permeability are equal to zero [26], is considered. Assume a plane wave with TE^z polarization, unity amplitude, and the frequency of 1 GHz illuminates normally the assumed inhomogeneous chiral nihility slab whose the electromagnetic parameters are as the following:

$$\varepsilon_r(z) \quad \to 0 \tag{51}$$
$$\mu_r(z) \quad \to 0 \tag{52}$$
$$\kappa(z) \quad = \exp(z) \tag{53}$$

The exact solution of this problem is presented in the Appendix A. The amplitudes of the transverse components of the electric field in the chiral nihility slab obtained from the exact solution and the proposed method based on Taylor's series expansion with $N = 40$ are shown in Figures 3(a) and 3(b). Apparently, there is an excellent agreement between the results from two different methods. It should be also mentioned that the existence of x-component of electric field in the chiral nihility slab is due to the polarization rotation of incident wave.

4.3. Example 3 (Inhomogeneous Chiral Slab)

In prior examples, the validity of the proposed method was verified. In the third example, the problem of scattering from an inhomogeneous
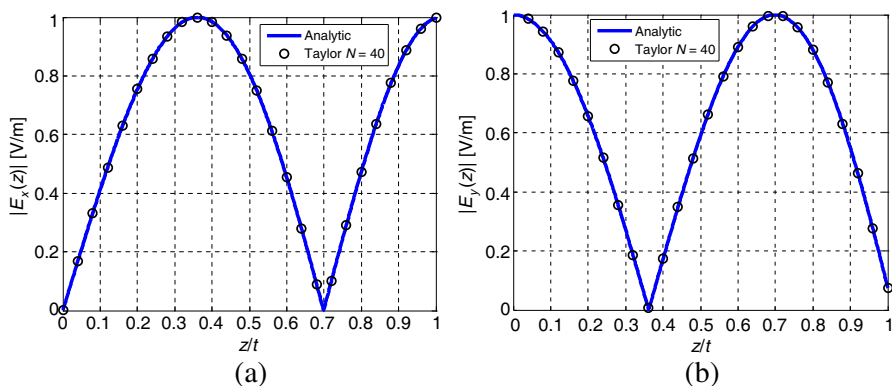
Figure 3. Amplitudes of the transverse components of the electric field in inhomogeneous chiral nihility slab for TE^z polarization, obtained from exact solution and from the presented method with $N = 40$. (a) $E_x(z)$, and (b) $E_y(z)$.
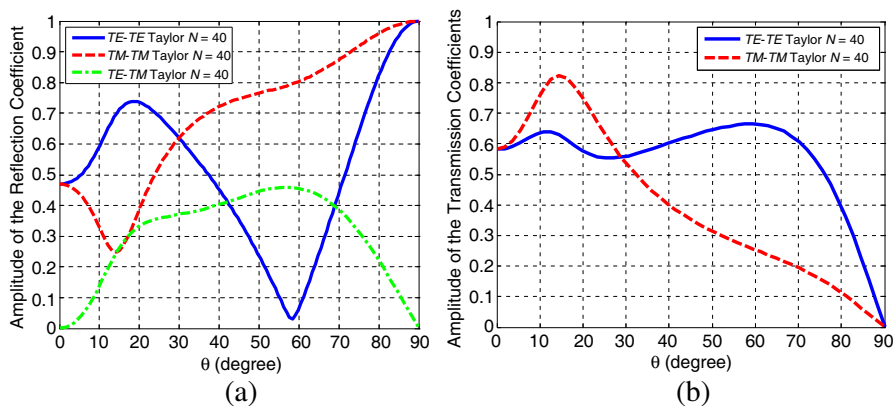

Figure 4. Reflection and transmission coefficients as a function of incident angle θ for inhomogeneous chiral slab. (a) Co- and cross-reflection coefficients. (b) Co-transmission coefficients.

chiral slab with thickness of $t = 0.2\,\mathrm{m}$, which does not have an exact solution and its relative permittivity, relative permeability, and chirality parameter have the following typical profiles:

$$\varepsilon_r(z) = 4z \tag{54}$$

$$\mu_r(z) = 1 \tag{55}$$

$$\kappa(z) = \frac{1}{1 + Qz} \tag{56}$$

which Q is an arbitrary constant, illuminated by oblique incident of a linearly polarized plane wave (TE^z or TM^z) with unity amplitude, and the excitation frequency of 1 GHz. Assuming $Q = 0.5$, the amplitudes of co- and cross-reflection and co-transmission coefficients obtained from the presented method with $N = 40$ versus the angle of incidence are shown in Figures 4(a) and 4(b).

Finally, according the discussed examples, it can be concluded that the proposed method for analyzing the inhomogeneous chiral planar layered is efficiently applicable to such structures, whose electromagnetic parameters and the cross-coupling term, i.e., Equation (27), can be expanded using Taylor's series expansion.

5. CONCLUSIONS

A new analytic frequency domain approach was presented to solve the problem of the propagation of electromagnetic waves in inhomogeneous chiral media with arbitrary variations of dielectric permittivity, permeability, and cross-coupling terms which could be expanded using Taylor's series approach. Using this method, the problem is then equivalent to the solution of a system of coupled equations. In order to verify the validity of the presented method, it was then used to solve the scattering problem from the three special types of homogeneous and inhomogeneous chiral slabs. The examples showed that the obtained results from the presented method are in the excellent agreement with the exact solutions. This method is very simple and fast, and can be also generalized to solve the problem of the propagation of electromagnetic waves in complex inhomogeneous bi-isotropic media. In the future, it is expected that chiral inhomogeneous planar layers can be optimally designed as microwave absorbers in a desired frequency range and incidence angle range.

APPENDIX A.

In this section, the exact electric and magnetic fields in an inhomogeneous chiral nihility slab are determined. Using $\varepsilon_r(z) \to 0$ and $\mu_r(z) \to 0$ in Equations (11)–(14), one can obtain the following differential equation for E_x:

$$\frac{d^2 E_x(z)}{dz^2} - \frac{\frac{d}{dz}\left(\kappa(z) - \frac{\sin^2\theta_0}{\kappa(z)}\right)}{\kappa(z) - \frac{\sin^2\theta_0}{\kappa(z)}}\frac{dE_x(z)}{dz}$$

$$+ \left(\frac{\omega\kappa(z)}{c_0}\right)^2\left(1 - \frac{\sin^2\theta_0}{\kappa^2(z)}\right)E_x(z) = 0 \qquad \text{(A1)}$$

which E_x can be also replaced with H_x. It is seen that solving this differential equation describing inhomogeneous chiral layer analytically is difficult. For convenience, we consider Equation (A1) in the case of normal incidence, i.e., $\theta_0 = 0$, which gives:

$$\frac{d^2 E_x(z)}{dz^2} - \frac{1}{\kappa(z)} \frac{d\kappa(z)}{dz} \frac{dE_x(z)}{dz} + \left(\frac{\omega\kappa(z)}{c_0}\right)^2 E_x(z) = 0 \qquad \text{(A2)}$$

The general solution of this differential equation can be expressed as the following:

$$E_x(z) = B_1 \sin\left(\frac{\omega}{c_0} \int \kappa(z)dz\right) + B_2 \cos\left(\frac{\omega}{c_0} \int \kappa(z)dz\right) \qquad \text{(A3)}$$

and similarly:

$$H_x(z) = B_3 \sin\left(\frac{\omega}{c_0} \int \kappa(z)dz\right) + B_4 \cos\left(\frac{\omega}{c_0} \int \kappa(z)dz\right) \qquad \text{(A4)}$$

which B_1, B_2, B_3, B_4 are unknown coefficients which have to be determined using boundary conditions. Now, considering Example 2 and boundary conditions, Equations (17)–(20), we have:

$$B_1 = -0.5, \quad B_2 = -0.866, \quad B_3 = -0.0023, \quad B_4 = 0.0013 \qquad \text{(A5)}$$

Once, unknown coefficients are determined, the transverse components of electric field in the chiral nihility slab could be identified.

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