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# SIMPLE SKEWON MEDIUM REALIZATION OF DB BOUNDARY CONDITIONS

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**Abstract**—Considering the class of bi-isotropic media, a special case called the class of simple skewon (SS) media is defined. The SS medium depends on a single parameter. A plane wave incident on a planar interface of an SS medium is shown to reflect as from a DB boundary with vanishing normal components of D and B field vectors. This offers another possibility to realize the DB boundary conditions in terms of a medium interface. The same property is shown to apply for curved boundaries as well.

## 1. INTRODUCTION

The most general linear electromagnetic medium can be defined in terms of 36 parameters, either in terms of four medium dyadics as [1, 2]

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \overline{\overline{\epsilon}} & \overline{\overline{\xi}} \\ \overline{\overline{\zeta}} & \overline{\overline{\mu}} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \tag{1}$$

in the three-dimensional representation for Gibbsian vector fields, or in terms of a single medium dyadic as

$$\mathbf{\Psi} = \overline{\overline{\mathsf{M}}}|\mathbf{\Phi}, \tag{2}$$

in four-dimensional differential-form representation [3, 4]. In the latter case the field two-forms $\mathbf{\Psi}$ and $\mathbf{\Phi}$ can be expressed as

$$\mathbf{\Psi} = \mathbf{D} - \mathbf{H} \wedge \mathbf{d}\tau, \quad \mathbf{\Phi} = \mathbf{B} + \mathbf{E} \wedge \mathbf{d}\tau, \tag{3}$$

in terms of three-dimensional (spatial) two-forms $\mathbf{D}$, $\mathbf{B}$ and one-forms $\mathbf{H}$, $\mathbf{E}$. $\tau = ct$ is the normalized time. The medium dyadic $\overline{\overline{\mathsf{M}}}$

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corresponds to a $6 \times 6$ matrix in any basis expansion of field two-forms. According to Hehl and Obukhov [5], the medium dyadic $\overline{\overline{\mathsf{M}}}$ can be naturally (independently of any basis system) decomposed in three parts as

$$\overline{\overline{\mathsf{M}}} = \overline{\overline{\mathsf{M}}}_1 + \overline{\overline{\mathsf{M}}}_2 + \overline{\overline{\mathsf{M}}}_3, \tag{4}$$

where the dyadics are respectively called principal, skewon and axion components of $\overline{\overline{\mathsf{M}}}$. The axion part is a multiple of the unit dyadic which in the notation of [4] has the form

$$\overline{\overline{\mathsf{M}}}_3 = M_3 \overline{\overline{\mathsf{I}}}^{(2)T}, \tag{5}$$

while both $\overline{\overline{\mathsf{M}}}_1$ and $\overline{\overline{\mathsf{M}}}_2$ are trace-free dyadics. The components $\overline{\overline{\mathsf{M}}}_1$ and $\overline{\overline{\mathsf{M}}}_2$ can be defined so that the dyadics contracted by a quadrivector $\mathbf{e}_N$ as $\mathbf{e}_N \lfloor \overline{\overline{\mathsf{M}}}_1$ and $\mathbf{e}_N \lfloor \overline{\overline{\mathsf{M}}}_2$ are respectively symmetric and antisymmetric. Properties of these components are discussed in [5].

The total number of 36 parameters is distributed by the three components so that the principal part $\overline{\overline{\mathsf{M}}}_1$, corresponding to a trace-free symmetric $6 \times 6$ matrix has 20 parameters, the skewon part $\overline{\overline{\mathsf{M}}}_2$, corresponding to an antisymmetric $6 \times 6$ matrix, has 15 parameters, and the axion part $\overline{\overline{\mathsf{M}}}_3$, has 1 parameter.

A medium consisting only of its axion parameter, $\overline{\overline{\mathsf{M}}} = \overline{\overline{\mathsf{M}}}_3$ in (5), has been called PEMC, perfect electromagnetic conductor, because it is a generalization of both PMC ($M_3 = 0$) and PEC ($1/M_3 = 0$), [6]. A material consisting only of its skewon parameter, $\overline{\overline{\mathsf{M}}} = \overline{\overline{\mathsf{M}}}_2$ was introduced in [7] and its properties were considered in [8, 9].

## 2. SIMPLE SKEWON (SS) MEDIUM

Let us consider the medium known as bi-isotropic medium and defined by four scalar parameters in the Gibbsian vector form (1) as

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}. \tag{6}$$

Another presentation for this medium is

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} \alpha & \epsilon' \\ \mu^{-1} & \beta \end{pmatrix} \begin{pmatrix} \mathbf{B} \\ \mathbf{E} \end{pmatrix}. \tag{7}$$

The parameters of (6) and (7) have the relations

$$\alpha = \xi\mu^{-1}, \tag{8}$$

$$\epsilon' = \epsilon - \xi\mu^{-1}\zeta, \tag{9}$$

$$\mu^{-1} = \mu^{-1}, \tag{10}$$

$$\beta = -\mu^{-1}\zeta. \tag{11}$$

One should notice that while $\mu$ is the same quantity in both representations, $\epsilon$ is not, whence it is denoted by $\epsilon'$ in (7).

Working through the four-dimensional analysis, omitted here, the parameters $\epsilon'$, $\mu^{-1}$ can be shown to form the principal part of the medium while $\alpha$ and $\beta$ together form the axion and skewon parts. Expressing

$$\alpha = N + M, \qquad \beta = N - M, \tag{12}$$

one can actually show that $M$ corresponds to the axion part, and $N$ to the skewon part, of the bi-isotropic medium.

A bi-isotropic medium consisting of its principal part, $\alpha = \beta = 0$, is just the ordinary isotropic medium defined by the parameters $\mu$, $\epsilon' = \epsilon$. Similarly, a bi-isotropic medium consisting of its axion part only, satisfying $\mu^{-1} = 0$, $\epsilon' = 0$ and $\alpha = -\beta = M$, equals the PEMC medium. It can be defined by the medium conditions

$$\mathbf{D} = M\mathbf{B}, \qquad \mathbf{H} = -M\mathbf{E}. \tag{13}$$

It is noteworthy that (13) cannot be directly expressed in the form (6) because $\mu$ has no finite value.

The last simple special case, a bi-isotropic medium consisting of its skewon part only, $\mu^{-1} = \epsilon' = 0$, $\alpha = \beta = N$, has not been widely studied. The conditions

$$\mathbf{D} = N\mathbf{B}, \qquad \mathbf{H} = N\mathbf{E} \tag{14}$$

are quite similar to those of the PEMC, (13). Since, instead of 15 parameters, there is only a single scalar parameter in the definition (14), let us call it by the name simple skewon medium or SS medium for short. Conditions of the type (14) were previously formulated in [10], [5] Equation (D.1.117) and, together with the axion term, in [11] Equation (75).

Since it is difficult to express the SS medium conditions in the form (6), let us consider obtaining (14) as a limit of (8)–(11) as

$$\alpha = \xi\mu^{-1} = N, \tag{15}$$

$$\epsilon' = \epsilon - \xi\mu^{-1}\zeta = \epsilon_o/q, \tag{16}$$

$$\mu^{-1} = \mu^{-1} = 1/(\mu_o q), \tag{17}$$

$$\beta = -\mu^{-1}\zeta = N, \tag{18}$$

which obviously yields (14) for $q \to \infty$. From these we can solve the bi-isotropic parameters (6) as

$$\begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} = q \begin{pmatrix} -N^2\mu_o + \epsilon_o/q^2 & N\mu_o \\ -N\mu_o & \mu_o \end{pmatrix}, \tag{19}$$

which can be recognized as an isotropic chiral medium [12], with $N$ representing the chiral parameter of the medium. One may notice that for $q \to \infty$ all four parameters become infinite in (19). The term $\epsilon_o/q^2$ is necessary for the matrix to have an inverse for finite $q$, as was recognized for the similar representation of the PEMC in [13].

## 3. INTERFACE OF SS MEDIUM

It is known from previous analyses that the PEMC has strange properties as a medium, but its interface serves as a boundary with certain interesting properties [14, 15]. Since the SS medium appears equally strange, let us study its properties at the interface. Let us assume an isotropic half space $z < 0$ with parameters $\mu_o$, $\epsilon_o$ bounded by a planar interface $z = 0$ of an SS medium half space $z > 0$ and a plane wave incident to the interface

$$\begin{pmatrix} \mathbf{E}^i(\mathbf{r}) \\ \mathbf{H}^i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}^i \\ \mathbf{H}^i \end{pmatrix} e^{-jk_z z} e^{-jk_x x}. \tag{20}$$

The reflected wave has the form

$$\begin{pmatrix} \mathbf{E}^r(\mathbf{r}) \\ \mathbf{H}^r(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}^r \\ \mathbf{H}^r \end{pmatrix} e^{jk_z z} e^{-jk_x x}, \tag{21}$$

with

$$k_z^2 + k_x^2 = k_o^2 = \omega^2 \mu_o \epsilon_o. \tag{22}$$

Because the tangential components of $\mathbf{E}$ and $\mathbf{H}$ vectors are continuous over the boundary, at $z = 0$ they satisfy

$$\mathbf{u}_z \times \left(\mathbf{H}^i + \mathbf{H}^r\right) = N\mathbf{u}_z \times \left(\mathbf{E}^i + \mathbf{E}^r\right). \tag{23}$$

Similarly, from the continuity of the the normal components of the $\mathbf{D}$ and $\mathbf{B}$ vectors we obtain

$$\mathbf{u}_z \cdot \left(\mathbf{D}^i + \mathbf{D}^r\right) = N\mathbf{u}_z \cdot (\mathbf{B}^i + \mathbf{B}^r). \tag{24}$$

Now we can write from the Maxwell equations

$$\mathbf{u}_z \cdot \left(\mathbf{u}_x k_x \times \mathbf{E}^i\right) = \omega \mathbf{u}_z \cdot \mathbf{B}^i, \tag{25}$$

$$\mathbf{u}_z \cdot (\mathbf{u}_x k_x \times \mathbf{E}^r) = \omega \mathbf{u}_z \cdot \mathbf{B}^r, \tag{26}$$

$$\mathbf{u}_z \cdot \left(\mathbf{u}_x k_x \times \mathbf{H}^i\right) = -\omega \mathbf{u}_z \cdot \mathbf{D}^i, \tag{27}$$

$$\mathbf{u}_z \cdot (\mathbf{u}_x k_x \times \mathbf{H}^r) = -\omega \mathbf{u}_z \cdot \mathbf{D}^r. \tag{28}$$

Combining (25) and (26) we have

$$k_x \mathbf{u}_x \cdot \mathbf{u}_z \times \left(\mathbf{E}^i + \mathbf{E}^r\right) = -\omega \mathbf{u}_z \cdot \left(\mathbf{B}^i + \mathbf{B}^r\right), \tag{29}$$

while combining (27), (28) and taking (23) and (24) into account yields

$$k_x\mathbf{u}_x \cdot \mathbf{u}_z \times \left(\mathbf{E}^i + \mathbf{E}^r\right) = \omega\mathbf{u}_z \cdot \left(\mathbf{B}^i + \mathbf{B}^r\right). \tag{30}$$

From (29), (30), (23) and (24) we conclude that the following conditions must be satisfied by the fields of any plane wave at the interface of the SS half space:

$$\mathbf{u}_y \cdot \left(\mathbf{E}^i + \mathbf{E}^r\right) = 0, \quad \mathbf{u}_y \cdot \left(\mathbf{H}^i + \mathbf{H}^r\right) = 0, \tag{31}$$

$$\mathbf{u}_z \cdot \left(\mathbf{B}^i + \mathbf{B}^r\right) = 0, \quad \mathbf{u}_z \cdot \left(\mathbf{D}^i + \mathbf{D}^r\right) = 0. \tag{32}$$

To summarize, the total fields $\mathbf{E}$ and $\mathbf{H}$ have no component orthogonal to the $\mathbf{k}$ vector plane while the $\mathbf{D}$ and $\mathbf{B}$ vectors have no components orthogonal to the planar interface of the SS medium. Since the latter property is independent of the $\mathbf{k}$ vector of the plane wave and linear in the $\mathbf{D}$ and $\mathbf{B}$ fields, it is valid for any combination of plane waves, in short, for any fields in the form

$$\mathbf{n} \cdot \mathbf{D} = 0, \quad \mathbf{n} \cdot \mathbf{B} = 0, \tag{33}$$

where the unit normal vector $\mathbf{n}$ equals $\mathbf{u}_z$ in the present case. It should be noted that this property is also independent of the parameter $N$ of the SS medium. Finally, one should also note that, since the axion part of the medium dyadic is known to be inactive for the plane wave, the same DB conditions are also obtained for the simple skewon-axion medium defined by (12).

Boundary conditions of the form (33) were probably first introduced in [16] and proved to yield unique solutions to boundary-value problems in [17, 18]. The conditions (33) have more recently been called DB conditions [19]. They have proved important in constructing electromagnetic cloaking structures [20–22]. It has also been shown that objects with certain symmetry properties and defined by DB boundary conditions have zero backscattering, i.e., they cannot be seen by the monostatic radar [23].

In [16] it was shown that DB conditions can be realized by an anisotropic medium with zero components $\epsilon_{zz}$ and $\mu_{zz}$ in the permittivity and permeability dyadics. Also, it was shown in [24] that six-parameter uniaxial skewon-axion media have the same property, making DB boundary conditions at the planar interface. Thus, the present analysis has added the possible realizations with yet another medium. Although the SS medium is a special case of the six-parameter medium of [24] the analysis was actually not valid for certain special cases including the present SS medium. Also, it was limited to planar boundaries. In Appendix A it is shown that the conditions (33) are valid for more general curved interfaces of SS media.

## 4. FIELDS IN THE SS MEDIUM

Let us assume that a plane wave reflected from the interface $z = 0$ creates a transmitted plane wave in the SS medium half space $z > 0$,

$$\begin{pmatrix} \mathbf{E}^t(\mathbf{r}) \\ \mathbf{H}^t(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \mathbf{E}^t \\ \mathbf{H}^t \end{pmatrix} e^{-jk_z^t z} e^{-jk_x x}. \tag{34}$$

From continuity, $k_x$ equals that of the incident wave. The Maxwell equations for the plane wave fields, with the conditions (14) inserted, are

$$(\mathbf{u}_z k_z^t + \mathbf{u}_x k_x) \times \mathbf{E}^t - \omega \mathbf{B}^t = 0 \tag{35}$$

$$(\mathbf{u}_z k_z^t + \mathbf{u}_x k_x) \times N\mathbf{E}^t + \omega N \mathbf{B}^t = 0. \tag{36}$$

Multiplying the first equation by $N$ and subtracting we obtain

$$\mathbf{B}^t = 0, \qquad \mathbf{D}^t = 0 \tag{37}$$

in the SS medium. Also, we have

$$\mathbf{u}_y \cdot \mathbf{E}^t = 0, \qquad \mathbf{u}_y \cdot \mathbf{H}^t = 0 \tag{38}$$

and

$$k_z^t \mathbf{u}_x \cdot \mathbf{E}^t - k_x \mathbf{u}_z \cdot \mathbf{E}^t = 0, \tag{39}$$

$$k_z^t \mathbf{u}_x \cdot \mathbf{H}^t - k_x \mathbf{u}_z \cdot \mathbf{H}^t = 0. \tag{40}$$

Although $\mathbf{u}_x \cdot \mathbf{E}^t$ and $\mathbf{u}_x \cdot \mathbf{H}^t$ as well as $k_x$ are determined by the incident field, $k_z^t$ and $\mathbf{u}_z \cdot \mathbf{E}^t$ cannot be determined because there is no dispersion equation from which $k_z^t$ could be solved. This is similar to the PEMC medium in which the fields are not unique. However, the fields outside the SS medium are unique.

Unique fields can be obtained by considering The SS medium as a limiting case of the bi-isotropic medium (19) with $q \to \infty$. The plane-wave equation for the transmitted field $\mathbf{E}^t$ can be easily derived as

$$\mathbf{k}^t \times \left(\mathbf{k}^t \times \mathbf{E}^t\right) + 2\omega\mu_o N q \mathbf{k}^t \times \mathbf{E}^t + k_o^2 \mathbf{E}^t = 0. \tag{41}$$

For finite $q$ this implies

$$\mathbf{k}^t \cdot \mathbf{E}^t = k_z^t \mathbf{u}_z \cdot \mathbf{E}^t + k_x \mathbf{u}_x \cdot \mathbf{E}^t = 0. \tag{42}$$

Inserting this in (41) we obtain the dispersion equation

$$\left(\mathbf{k}^t \cdot \mathbf{k}^t - k_o^2\right)^2 + k_o^2 q^2 (2N\eta_o)^2 \mathbf{k}^t \cdot \mathbf{k}^t = 0. \tag{43}$$

This has two solutions $\mathbf{k}_\pm^t \cdot \mathbf{k}_\pm^t$ which, after some algebra, for $q \to \infty$ can be written as

$$\mathbf{k}_+^t \cdot \mathbf{k}_+^t \to -k_o^2 q^2 (2N\eta_o)^2, \qquad \mathbf{k}_-^t \cdot \mathbf{k}_-^t \to -k_o^2/q^2 (2N\eta_o)^2. \tag{44}$$

Because $k_{\pm x} = k_x$ is finite, we have

$$k_{+z} \rightarrow j2qk_oN\eta_o, \qquad k_{-z} \rightarrow -jk_x, \tag{45}$$

In both cases (41) yields

$$\mathbf{E}^t_\pm \cdot \mathbf{E}^t_\pm = (\mathbf{u}_x \cdot \mathbf{E}^t_\pm)^2 + (\mathbf{u}_z \cdot \mathbf{E}^t_\pm)^2 = 0, \tag{46}$$

whence the eigenfields are circularly polarized. From (42) and (45) we have

$$\mathbf{u}_z \cdot \mathbf{E}^t_+ \rightarrow 0, \quad \Rightarrow \qquad \mathbf{E}^t_+ \rightarrow 0, \quad \mathbf{H}^t_+ \rightarrow 0. \tag{47}$$

Since also $\mathbf{k}_+ \cdot \mathbf{D}_+ = 0$, $\mathbf{k}_+ \cdot \mathbf{B}_+ = 0$, there is no coupling to this eigenfield through the interface.

Because of (45) the eigenfield $\mathbf{E}^t_-$ decays exponentially for $z \rightarrow \infty$ and

$$\omega\mathbf{B}^k_- = \mathbf{k}^t_- \times \mathbf{E}^t_- \rightarrow 0, \qquad \omega\mathbf{D}_- \rightarrow 0, \tag{48}$$

whence DB conditions are valid at the interface.

## 5. CONCLUSION

A class of media has been defined under the name simple skewon (SS) media. Any SS medium is a special case of bi-isotropic media and depends on a single parameter $N$. It was shown that a plane wave incident at a planar interface of an SS medium is reflected as from a DB boundary for any parameter $N$ value. Thus, the planar DB-boundary conditions can be realized by an interface of an SS medium. Fields inside the SS medium are unique if the medium is defined as a limiting case of a bi-isotropic medium. The same property was also shown to be valid for curved boundaries. The material realization of an SS medium is a topic for future study. A promising direction to look for such a realization would be the use of strongly resonating helices embedded into a dielectric matrix making an isotropic chiral medium with large parameters $\epsilon$, $\mu$, $\xi$, $\zeta$. The materialization scheme of chiral nihility through an ensemble of interacting helical scatterers described in [27] could be used as a starting point.

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## APPENDIX A. CURVED INTERFACE OF SS MEDIUM

Let us consider the interface $x_3 = 0$ of an isotropic medium $(\mu, \epsilon)$ in $x_3 > 0$ and SS medium $x_3 < 0$, where $x_3(\mathbf{r})$ is a coordinate function in a curvilinear coordinate system $x_1,\ x_2,\ x_3$. Maxwell equations in the isotropic medium side of the interface can be written as

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}, \tag{A1}$$
$$\nabla \times \mathbf{H} = j\omega\mathbf{D}. \tag{A2}$$

The components of the Maxwell equations parallel to $\mathbf{u}_3$ can be expressed in coordinate expansions as [25, 26]

$$\frac{1}{h_1 h_2}(\partial_{x_1}(h_2 E_2) - \partial_{x_2}(h_1 E_1)) = -j\omega B_3, \tag{A3}$$
$$\frac{1}{h_1 h_2}(\partial_{x_1}(h_2 H_2) - \partial_{x_2}(h_1 H_1)) = j\omega D_3. \tag{A4}$$

Because of continuity of the tangential components $E_1,\ E_2,\ H_1,\ H_2$ and the normal components $B_3,\ D_3$ across the interface, the fields satisfy the conditions of the SS medium as

$$\eta_o D_3 = NB_3, \quad \eta_o H_1 = NE_1, \quad \eta_o H_2 = NE_2. \tag{A5}$$

Thus, we can rewrite the Equation (A4) at the interface as

$$\frac{N}{h_1 h_2}(\partial_{x_1}(h_2 E_2) - \partial_{x_2}(h_1 E_1)) = j\omega NB_3. \tag{A6}$$

Multiplying (A3) by $B$ and subtracting (A6) and (A3) we obtain $2NB_3 = 0$, whence we arrive at the conditions

$$B_3 = \mathbf{n}\cdot\mathbf{B} = 0, \qquad D_3 = \mathbf{n}\cdot\mathbf{D} = 0, \tag{A7}$$

which coincide with (33). Thus, also the curved interface of an SS medium acts as a DB boundary.

## APPENDIX B. WAVES IN THE BI-ISOTROPIC MEDIUM

Let us briefly consider plane waves in the special bi-isotropic medium defined by the medium parameter matrix (19) following the notation of [12]. The chirality parameter denoted by $\kappa$ can be identified as

$$\kappa = jqN\eta_o, \tag{B1}$$

whence for $q \to \infty$ we have $|\kappa| \to \infty$. Substituting from (19) we obtain

$$n = \sqrt{\mu\epsilon/\mu_o\epsilon_o} = \sqrt{1+\kappa^2} \to \kappa + \frac{1}{2\kappa}. \tag{B2}$$

Thus, the two circularly polarized eigenwaves $\mathbf{E}_\pm$ propagate as in two isotropic media with respective effective refraction factors defined by

$$n_\pm = n \pm \kappa, \quad n_+ \to 2\kappa, \quad n_- \to 1/2\kappa. \tag{B3}$$

Because of $|n_+| \to \infty$ and $|n_-| \to 0$ for $q \to \infty$, the two eigenwaves see two very different media.

The medium appears lossless when $n_+$ and $n_-$ have real values, which requires that $N$ must be imaginary. For real $N$, the $n_+$ and $n_-$ are imaginary with opposite signs which means that the medium is passive for one of the waves and active for the other one. It may be difficult to realize such a medium. On the other hand, imaginary $N$ in Gibbsian representation corresponds to an operator in the four-dimensional definition of the SS medium in which case $N$ is a multiple of differentiation in time.

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