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TOWARDS THE DETECTION OF MULTIPLE REFLECTIONS IN TIME-DOMAIN EM INVERSE SCATTERING OF MULTI-LAYERED MEDIA

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Abstract—In this paper, a new theoretical approach for the classification of multiple reflections in time-domain e.m. inverse scattering of multi-layered media is presented. The existence of multiples limits the capabilities of inversion algorithms, thus suitable identification and suppression techniques should be applied to reduce this undesired effect. Assuming a scenario composed of loss-less and non-dispersive media, and providing an accurate time delay estimation (TDE) of backscattered signals, the proposed method allows not only to evaluate the presence of multiples and discriminate them from primary reflections, but also to determine their propagation paths. Preliminary tests performed on FDTD simulated data have shown its potentialities to effectively handle multiple reflections and therefore to enhance the e.m. signals backscattered by primary reflectors.

1. INTRODUCTION

The problem of multiple reflections is a well-known issue in time domain inverse scattering: the probing equipment generates a wave that propagates until it encounters a discontinuity, being partly transmitted, partly reflected; each of the two generated waves becomes a new source that may strike on other interfaces and split as well, inducing a theoretically infinite train of waves that alters the informational content of the received signal.

Multiples' suppression has become an essential operation in seismic investigations, especially within the marine environment, where

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the water layer often behaves as a wave trap, so that waves are multiply reflected between the sea surface and the sea bottom, contaminating the seismograms and thus disguising important information about subsurface reflectors [1]. Most commonly employed techniques are based on predictive deconvolution [2], a process that improves the temporal resolution of seismic data by compressing the basic seismic wavelet, and that can therefore be used to remove a significant part of the multiples. More recently, new methodologies have been devised, that employ wave-equation extrapolation [3] and artificial neural networks [4], which make no assumptions on periodicity or moveout patterns of multiples, and can cope with complex unknown environments.

Despite its close affinity with seismic observations, the problem of multiple reflections in e.m. inverse scattering has still to be addressed. In literature, in fact, the prevailing approach is to neglect the presence of multiples in order to simplify the analysis [5, 6] or apply generic clutter removal techniques [7], and only a few recent works can be cited that try to explicitly face the problem. Standard filtering techniques, such as predictive deconvolution [8] and two-dimensional $f-k$ filtering [9], are generally employed, but some application-oriented algorithms can be anyway found, as in [10], where the 'wave field prediction and removal method' have been devised to enhance GPR images for the quality assessment of back-filled grouting in a shield tunnel.

In this work, we present a new theoretical approach for multiple reflections' identification and classification in time-domain e.m. inverse scattering of multi-layered scenarios. Under the hypotheses of lossless and non-dispersive media, and accurate time delay estimation (TDE) of backscattered signals, the method allows without any *a-priori* knowledge of the scenario not only to evaluate the presence of multiples and discriminate them from primary reflections, but also to determine their propagation path, information that, as far as we know, has never been provided. It is straightforward that under different operational conditions, the overall performances are expected to worsen, and therefore further steps should be introduced within the processing chain.

The paper is organized as follows: in the next section the algorithm will be theoretically discussed, introducing the notions of diophantine equations and binary decision trees; Section 3 discusses a demonstrative example providing some preliminary results; Section 4 concludes the paper with comments and final remarks.

2. ALGORITHM DESCRIPTION

As hinted in the previous section, if a pulsed e.m. source illuminates a multi-layered object, an infinite chain of signals would be produced. In particular, having assumed L loss-less and non-dispersive layers over an infinite background (see Fig. 1), these signals can (1) remain trapped within internal strata; (2) propagate indefinitely through the background; (3) be backscattered.

At the receiver, we would then collect a signal $u(t)$ made of two separate contributions: the echoes directly reflected by the interfaces of the medium (DR), and a subset of the multiple reflections (MR) generated at each discontinuity:

$$u(t) = \sum_d A_d u(t - t_d) + \sum_m A_m u(t - t_m) \tag{1}$$

According to the above consideration, the rationale behind this paper is that a DR has a generic (let us say independent) arrival time t_d which is due to the dielectric and geometric characteristics of the illuminated object, while MRs feature delays t_m which can be deterministically derived as linear combinations of the time shifts between subsequent DRs.

In case of simple scenarios and/or for the processing of the very first echoes of the radargram, the relation between a MR and the DRs might be determined by means of a direct searching. However, in general, if we assume that at a certain instant t^* the receiver has already sensed n different direct reflections (DR_1, DR_2, ..., DR_n

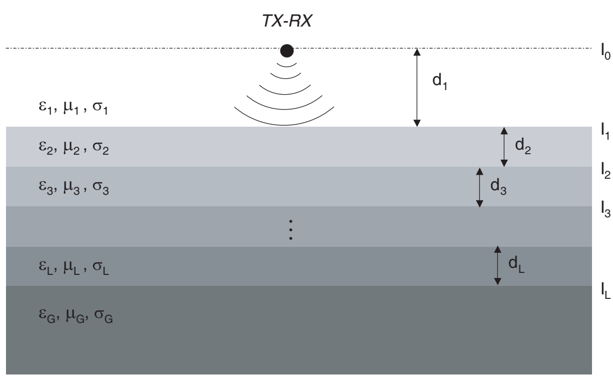

Figure 1. Test scenario: a pulsed e.m. source (TX) illuminates a multi-layered object and a receiver (RX) measures the backscattered radiation.

backscattered by interface $I_1, I_2, \ldots, I_n$ at times $t_1, t_2, \ldots, t_n$), and a new incoming echo is arriving, we can easily obtain that in case of a MR this arrival time can be expressed as:

$$t^* = t_0 + a_1(t_1 - t_0) + a_2(t_2 - t_1) + \ldots + a_n(t_n - t_{n-1}) \\ a_1, a_2, \ldots, a_n \in \mathbb{N} \tag{2}$$

with t_0 the initial acquisition time.

It is worth mentioning that the condition for an independent arrival time holds true only for $a_1, a_2, \ldots, a_n = 1$. In this case, in fact, Eq. (2) reduces to:

$$t^* = t_0 + (t_1 - t_0) + (t_2 - t_1) + \ldots + (t_n - t_{n-1}) = t_n$$

More in general, Eq. (2) suggest that it would be possible to process a generic received signal and establish — only provided that the delay for each component of the signal can be detected (this is not the scope of this paper) — (A) whether a received pulse represents a direct interface reflection or a multiple echo and (B) its propagation path.

The two phases of the algorithm will be discussed hereafter in detail, focusing on their processing scheme.

2.1. MR Echo Classification

Due to its form, the relation between the arrival time of a MR and the time shifts of the already received DR belongs to a particular family of linear equations called *Diophantine equations* [11], which have the form:

$$\mathbf{a}^T\mathbf{x} = b, \qquad \mathbf{a}, \mathbf{x} \in \mathbb{Z}^n,\ b \in \mathbb{Z} \tag{3}$$

Over the Integers, infinitely many vectors $\mathbf{a}$ satisfy Eq. (3), but when lower and/or upper bounds are imposed, the equation can be either inconsistent or have a finite number S of solutions [12]. In particular, when searching for non-negative solutions, we can borrow from combinatorics some useful results. The main counting theorem, in fact, states [13] that this number can be interpret as the coefficient of the term z^b of the complex valued function

$$\phi(z) = \prod_{k=1}^{n} (1 - z^{a_k})^{-1} \tag{4}$$

with a_k each element of vector $\mathbf{a}$. ϕ is a meromorphic function which has poles located on the unit circle $|z| = 1$ and is analytic at the origin [14], so it can be expanded to a power series of the form

$$\phi(z) = \sum_{q=0}^{\infty} f_q z^q, \quad \text{for } |z| < 1 \tag{5}$$

Eq. (5) is a Laurent series with centerpoint at the origin and whose coefficient f_q has the form

$$f_q = \frac{1}{2\pi i} \int_\gamma \frac{\phi(\zeta)d\zeta}{\zeta^{q+1}} \tag{6}$$

with γ any loop that winds once counterclockwise about the centerpoint.

The number of solutions S can be therefore seen as the residue of the function $\phi(z)/z^{b+1}$ at the origin, with b the constant term of Eq. (3):

$$S = f_b = \operatorname{Res}\left(\frac{\phi(z)}{z^{b+1}}\right) \tag{7}$$

For a detailed discussion on the computation of Eq. (7), the reader can refer to [15].

As regards the evaluation of the full set of solutions satisfying Eq. (2), we here recall a methodology which employs the basis reduction algorithm and guarantees a polynomial processing time [16].

The keypoint of the method is that solving a system of linear Diophantine equations,

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \qquad \mathbf{A} \in \mathbb{Z}^{m\times n}, \quad \mathbf{x} \in \mathbb{Z}^n, \quad \mathbf{b} \in \mathbb{Z}^m \tag{8}$$

which Eq. (3) is a special case of, is equivalent to study whether or not the term $\mathbf{b}$ belongs to the lattice L of the matrix $\mathbf{A}$, defined as the set of all integer linear combinations of its columns $\mathbf{a}_j$:

$$L = \left\{ \sum_{j=1}^{n} \alpha_j \mathbf{a}_j : \ \alpha_j \in \mathbb{Z}, \ 1 \leq j \leq n \right\} \tag{9}$$

To solve the problem above, the approach requires first the construction of the matrix $\mathbf{R}$, whose columns form a basis for lattice L, formulated as:

$$\mathbf{R} = \begin{pmatrix} \mathbf{I}^n & \mathbf{0}^{n\times 1} \\ \mathbf{0}^{1\times n} & N_1 \\ N_2\,\mathbf{A} & -N_2\,\mathbf{b} \end{pmatrix}$$

It can be proven that, if a solution to Eq. (8) exists, and the numbers N_1 and N_2 are chosen large enough [16], the

$(n+1) \times (n-m+1)$ submatrix $\hat{\mathbf{R}}$ of reduced basis obtained after applying the basis reduction algorithm in [17] would have the form:

$$\hat{\mathbf{R}} = \begin{pmatrix} \mathbf{X}_H^{n\times(n-m)} & \mathbf{x}_{NH} \\ \mathbf{0}^{1\times(n-m)} & N_1 \end{pmatrix}$$

where $\mathbf{x_{NH}} \in \mathbb{Z}^n$ is an integer vector satisfying $\mathbf{A}\mathbf{x}_{NH} = \mathbf{b}$ and $\mathbf{X}_H$ is a $n{\times}(n-m)$ matrix whose linearly independent columns $\mathbf{x}_{H,j}$ satisfy $\mathbf{A}\mathbf{x}_{H,j} = \mathbf{0}$.

Observing that $\mathbf{A}(\mathbf{x}_{NH} + \mathbf{x}_H) = \mathbf{b}$, we can obtain the complete solution $\mathbf{x}$ of Eq. (8) — or a proof of its infeasibility — by adding the particular solution $\mathbf{x}_{NH}$ to any linear integer combination of the columns of $\mathbf{X}_H$:

$$\mathbf{x} = \mathbf{x}_{NH} + \sum_{j=1}^{n-m} \lambda_j \mathbf{x}_{H,j}, \qquad \lambda_j \in \mathbb{Z} \tag{10}$$

Nevertheless, such results cannot directly apply to Eq. (2), since the lower bound $\mathbf{x} \geq 0$ must be first enforced.

To this end, the method is finalized with a branching algorithm that branches on linear combinations of vectors of $\mathbf{X}_H$ and discards the negative solutions.

2.2. MR Path Reconstruction

Once the coefficients that satisfy Eq. (2) for a certain t^* have been found according to the procedure described in the previous section, it is possible to reconstruct the propagation path of the multiple reflections which have featured that specific arrival time.

The particular nature of the problem (at each discontinuity, the propagating signal is split into two opposite waves), hints that a suitable approach can be based on the construction of a Binary Decision Tree (BDT) [18].

Therefore, starting from the root vertex (corresponding to the transmitting antenna), a BDT is built with 2^B terminations, where

$$B = 2\sum_{i=1}^{n} a_i \tag{11}$$

The links between the root vertex and the end nodes represent all the possible combinations of paths that a travelling signal can trace in B 'segments' (term which is here used to denote wave shifts per layer in the upward or downward direction). Among all the combinations provided by the tree, only a few of them can be physical solutions to our problem.

To discard the meaningless paths, it is therefore necessary to translate into physical conditions the information provided by the coefficients a_n.

To this end, let us assign to each branch b of the tree the scalar value:

$$c_b = \begin{cases} +1 & \text{in case of downward branch} \\ -1 & \text{in case of upward branch} \end{cases} \tag{12}$$

It is straightforward that each path connecting the root with the end nodes can be represented by a vector $\mathbf{p} = (0, p_1, p_2, \dots, p_B)$ whose components are given by the sum of the values of each branch:

$$p_b = \sum_{i=1}^{b} c_i \qquad \text{with} \quad b = 1, 2, \dots, B \tag{13}$$

This kind of formulation allows to directly visualize the interface level (expressed as an integer number, with the source at level 0) reached by the corresponding signal at each step of its path.

For the sake of clarity, let us assume that, according to the classification provided by phase A, we need the propagation path of the echo featuring two coefficients $a_1 = 1$ and $a_2 = 2$. Its arrival time would therefore be a certain $t^* = 1 \cdot t_1 + 2 \cdot (t_2 - t_1)$, with $t_0 = 0$. In other words, the wave has travelled for 2 segments in air and 4 segments within the first layer, i.e., it has covered twice the I_0–I_1 distance (d_1) and four times the thickness of the first layer (d_2).

The related binary tree, partly depicted in Fig. 2, would therefore have 64 end nodes, since $B = 2(1 + 2) = 6$. As already noticed, only a subset of these 64 paths that link the root with the terminations has an effective physical meaning, so we need a suitable procedure to discard unwanted branches.

- As first step, we are allowed to directly discard the whole upper part of the tree, which describes the signals departing toward the upward direction from the antenna, which would not provide any backscattering.
- Then we have to force the waves, originated at the root level ($I_0 = 0$), to return to the same level (i.e., only signals backscattered to the antenna are selected), without crossing interfaces $I_0 = 0$ and $I_2 = 2$. According to our formulation, all the paths $\{\mathbf{p} : p_7 \neq 0 \cup p_b < 0 \cup p_b > 2\}$ can be disregarded.
- Finally, the constraints given by the 'echo classification' step must be enforced, which set the correct sequence of segments traced by the echoes. In fact, if we had as unique boundary the number of segments ($B = 6$), without considering the proper order of layer crossings, we would include wrong solutions.

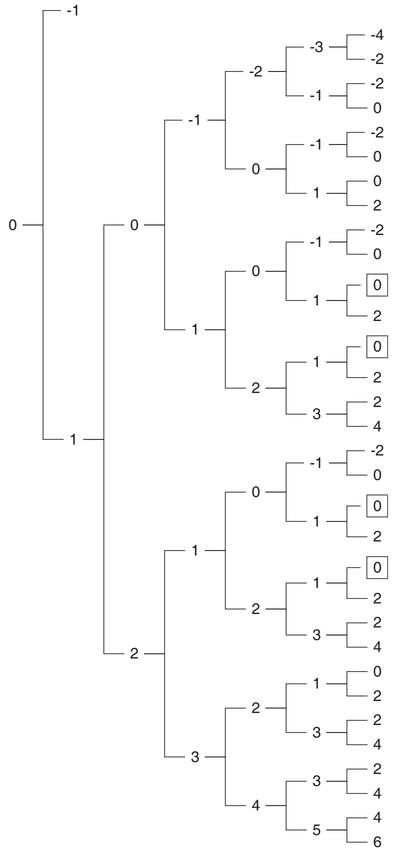

Figure 2. Binary Decision Tree associated to coefficient $a_1 = 1$, $a_2 = 2$. Paths linking TX (level 0) and RX (level 0) are marked with an external box.

For instance, in our example, the paths of six branches that successfully passed the first two steps are those marked with an external box in Fig. 2.

According to Eq. (13), they can be expressed as vectors whose components represent the level (interface) reached by the wave during its propagation:

$$\mathbf{p}_1 = (0,1,0,1,0,1,0) \quad \mathbf{p}_3 = (0,1,2,1,0,1,0)$$
$$\mathbf{p}_2 = (0,1,2,1,2,1,0) \quad \mathbf{p}_4 = (0,1,0,1,2,1,0)$$

Nevertheless, three of them — $\mathbf{p}_1$, $\mathbf{p}_3$, $\mathbf{p}_4$ — do not correspond to effective solutions, since they respectively represent a signal reflected three times by the antenna and the first layer (six

segments in air), and two symmetrical echoes that travel four segments in air and two segments within the first layer. According to our mathematical scheme, they would therefore be the solutions of Eq. (2) featuring the integer coefficients $\{a_1 = 3, a_2 = 0\}$ and $\{a_1 = 2, a_2 = 1\}$.
To avoid this commission error, it can be introduced an additional processing step based on the consideration that each vector $\mathbf{p}$ holds B ordered pairs of the kind $\{p_{b-1}, p_b\}$, for $b = 1, \ldots, B$. In fact, if we define $O_{j,k}$ as the occurrence of the string $\{j, k\}$ within a vector, the final propagation path candidates are only those vectors enforcing the following condition:

$$O_{i-1,i} + O_{i,i-1} = 2a_i \qquad 1 \leq i \leq n \tag{14}$$

In the shown example, the only vector satisfying the conditions $O_{0,1} + O_{1,0} = 2 \cdot 1$ and $O_{1,2} + O_{2,1} = 2 \cdot 2$ is $\mathbf{p}_2$, which is therefore the solution of the problem.

3. RESULTS

The operational involvements of the described technique will be shown presenting the solution of a demonstrative test case. Let us consider a scenario consisting of a stack of two slabs and a bulk illuminated by a pulsed e.m. source and with the following properties:

$$\begin{array}{lllll} L_1: & \varepsilon_1 = \ 1, & \mu_1 = 1, & \sigma_1 = 0 \text{ S/m}, & \\ L_2: & \varepsilon_2 = \ 6, & \mu_2 = 1, & \sigma_2 = 0 \text{ S/m}, & d_2 = 0.08\,\text{m} \\ L_3: & \varepsilon_3 = 16, & \mu_3 = 1, & \sigma_3 = 0 \text{ S/m}, & d_3 = 0.049\,\text{m} \\ L_G: & \varepsilon_G = 2, & \mu_G = 1, & \sigma_G = 0 \text{ S/m} & \end{array} \tag{15}$$

The above specifications have been arbitrarily chosen only to easily discuss the whole processing chain; in fact, such scenario will produce a backscattered signal composed of four echoes, to be classified as:

1) At time 2.33 ns†, 1 direct reflection:
   - DR_1 reflected by I_1
2) At time 3.64 ns, 1 direct reflection:
   - DR_2 reflected by I_2
3) At time 4.94 ns, 1 direct and 1 multiple reflection:
   - DR_3 reflected by I_3

† the theoretical value of t_n, in case of normal incidence, can be expressed as:

$$t_n = \frac{2}{c} \sum_{i=1}^{n} d_i \sqrt{\varepsilon_i} \qquad c = \text{speed of light in freespace}$$

  - MR reflected by I_2, I_1, I_2
4) At time 6.25 ns, 4 multiple reflections:
  - MR reflected by $I_3, I_2 I_3$
  - MR reflected by I_2, I_1, I_2, I_1, I_2
  - MR reflected by I_2, I_1, I_3
  - MR reflected by I_3, I_1, I_2

The e.m. characterization of the scenario has been performed with GPRMax [19], a FDTD-based simulation software. More in detail, along with the design of a set of media according to specifications in (15), the source has been modeled within a 2D environment as a current wire excited by a differentiated gaussian pulse of central frequency of 2 GHz, at distance $d_1 = 0.35$ m from I_1; the backscattered signal is then sensed by an ideal probe that measures the e.m. field at the desired lattice point. The computational volume has been discretized in 1×1 mm cells, which means that, for the Courant-Friedrichs-Lewy condition [20], the time increment dt is bound to the value of $2.357 \cdot 10^{-12}$ s. The number of iterations has been set to 3200, for an overall simulated time of 7.5 ns.

The signal to be processed is represented in Fig. 3:

1) First of all, it is scanned to detect the arrival time of the first echo (many techniques could be used to this end, e.g., see [21], which proposes a development of super-resolution methods particularly suitable also for the detection of overlapping signals). Due to the specific geometry of the problem, this first signal is assured to be the first layer's bounce and therefore it can be directly classified as the first direct reflection, DR_1. Its arrival time, expressed in terms

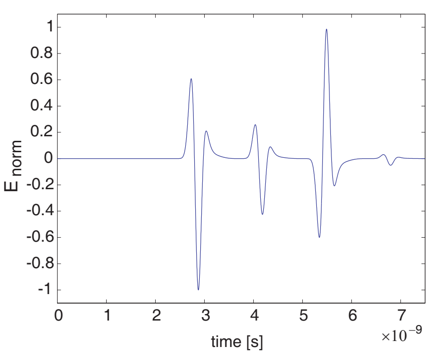

Figure 3. FDTD simulated signal.

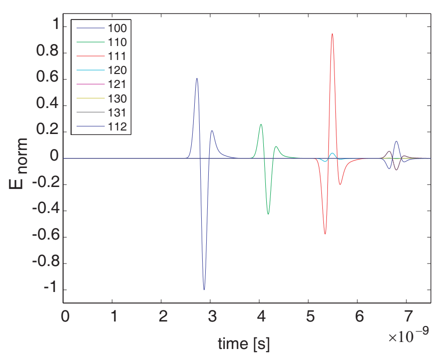

Figure 4. Contributions to the overall signal outlined in different colors, depending on their propagation paths.

of simulation time steps, corresponds to sample $n_1 = 990$. This number, multiplied by dt, actually matches the expected value of 2.33 ns. It is straightforward that the unique path associated to DR_1 is $\mathbf{p}_{1,1} = (0, 1, 0)$ featuring the integer coefficient $\{a_1 = 1\}$.

2) The second echo has a delay $n_2 = 1544$. By simply observing that n_2 is not a multiple of n_1, we can positively classify the signal as DR_2, the reflection from interface I_2. This signal is represented by the coefficients $\{a_1 = 1, a_2 = 1\}$, and can be associated to the path $\mathbf{p}_{2,1} = (0, 1, 2, 1, 0)$.

3) The third echo is received at $n_3 = 2098$. To check whether this number is a linear combination of n_1 and n_2, i.e., to solve the equation

$$990a_1 + 554a_2 = 2098 \qquad a_1, a_2 \geq 0 \tag{16}$$

it must be applied the 'echo classification' procedure shown in Section 2.1.

The set of solutions of Eq. (16) can be found by constructing the initial matrix

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & N_1 \\ 990N_2 & 554N_2 & -2098N_2 \end{pmatrix}$$

and then applying the basis reduction algorithm, obtaining

$$\hat{\mathbf{R}} = \begin{pmatrix} -277 & 1 \\ 495 & 2 \\ 0 & N_1 \end{pmatrix}$$

As can be noticed, Eq. (16) is consistent and has a particular solution $\mathbf{x}_{NH}^T = (1, 2)$. This is actually the unique solution of the problem ($S = 1$), since any multiple of the vector $\mathbf{x}_H^T = (-277, 495)$ would violate the constraint of positive coefficients. As regards path reconstruction, the case $\{a_1 = 1, a_2 = 2\}$ has been already studied in Section 2.1, providing a single vector path $\mathbf{p}_{3,1} = (0, 1, 2, 1, 2, 1, 0)$.

It is worth mentioning that the identification of a MR cannot anyway exclude the presence of a new incoming DR. In the considered example, for instance, time discretization causes sample n_3 (corresponding to $t_3 = 4.95$ ns) to coincide with the arrival time of DR_3, the radiation backscattered by interface I_3. The related set of coefficients and path vector are, respectively, $\{a_1 = 1, a_2 = 1, a_3 = 1\}$ and $\mathbf{p}_{3,2} = (0, 1, 2, 3, 2, 1, 0)$.

To complete the 'echo classification' step it is therefore necessary to resolve such ambiguity, otherwise the procedure would miss a new DR to be used within the subsequent calculations.

To this end, under our hypothesis of loss-less non-dispersive media, conditions which ensure the invariance of waveforms, some techniques can be exploited that recursively reconstruct, without any *a priori* knowledge, the vertical profile of the multi-layered medium [22, 23].
At this point, the information provided by the 'path reconstruction' phase becomes essential, since it allows to derive the theoretical amplitudes of the MRs (which depend on layers' thickness and permittivity) and remove them from the overall received signal.
In this way, any residual of significant amplitude (e.g., greater than a proper threshold that takes into account computational/approximation errors) would therefore be a primary reflection and classified as subsequent DR. It is straightforward that such procedure applies also to the case of a DR and MR that partially overlap (if they were separated, a simple signal windowing would be sufficient).

4) Finally, the last detected echo is received at sample $n_4 = 2652$. Again, the classification step requires the solution of the equation

$$990a_1 + 554a_2 + 554a_3 = 2652 \qquad a_1, a_2, a_3 \geq 0 \tag{17}$$

that can be solved with the construction of the basis matrix

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & N_1 \\ 990N_2 & 554N_2 & 554N_2 & -2652N_2 \end{pmatrix}$$

then reduced to

$$\hat{\mathbf{R}} = \begin{pmatrix} 0 & -277 & 1 \\ -1 & 248 & 2 \\ 1 & 247 & 1 \\ 0 & 0 & N_1 \end{pmatrix}$$

The full set of solutions (for Eq. (17), $S = 4$) is given by suitably combining the first two columns of $\hat{\mathbf{R}}$ with the particular solution $\mathbf{x}_{NH}^T = (1, 2, 1)$. By a simple reckoning, it can be easily evinced that the vector $\mathbf{x}_{H,2}^T = (-277, 248, 247)$ must be directly discarded, since it will never provide positive coefficients.
Therefore, after applying the branching algorithm, it results that the remaining three solutions of Eq. (17) are $\mathbf{x}_{NH}^T + \mathbf{x}_{H,1}^T = (1, 1, 2)$, $\mathbf{x}_{NH}^T - \mathbf{x}_{H,1}^T = (1, 3, 0)$ and $\mathbf{x}_{NH}^T + 2\,\mathbf{x}_{H,1}^T = (1, 0, 3)$.
We here omit the computation of the BDT for these cases: the reader would easily find that the four sets of coefficients provide

the following paths for the corresponding travelling waves:

$$\begin{array}{ll} a_1 = 1, a_2 = 2, a_3 = 1 & \mathbf{p}_{4,1} = (0, 1, 2, 3, 2, 1, 2, 1, 0) \\ & \mathbf{p}_{4,2} = (0, 1, 2, 1, 2, 3, 2, 1, 0) \\ a_1 = 1, a_2 = 1, a_3 = 2 & \mathbf{p}_{4,3} = (0, 1, 2, 3, 2, 3, 2, 1, 0) \\ a_1 = 1, a_2 = 3, a_3 = 0 & \mathbf{p}_{4,4} = (0, 1, 2, 1, 2, 1, 2, 1, 0) \\ a_1 = 1, a_2 = 0, a_3 = 3 & \mathbf{p}_{4,5} = \text{NULL} \end{array}$$

Accordingly, the last echo is composed of 4 MRs, two symmetrical waves that reach layer L_3 being once reflected by interface I_3 (paths $\mathbf{p}_{4,1}$ and $\mathbf{p}_{4,2}$), a wave that penetrates to layer L_3 and is twice reflected by interface I_3 (path $\mathbf{p}_{4,3}$), and a wave which does not go beyond layer L_2 and is thrice reflected by interface I_2 (path $\mathbf{p}_{4,4}$). The last path ($\mathbf{p}_{4,5}$) could not be computed, since a zero coefficient in a middle layer has no physical meaning.

A graphical visualization of the above results is illustrated in Fig. 4, where the contributions to the overall signal are outlined in different colors, depending on their travelling paths. Such information could be directly used to enhance the received signal, pointing out the actual reflections from interfaces.

4. CONCLUSION

The aim of this paper is the development of a new theoretical approach for the improvement of e.m. inverse scattering by means of multiple reflection classification. The effect of multiples, in fact, results within the received signal as additive noise which generally interferes with the response of primary reflectors and therefore limits the capabilities of inversion algorithms. To avoid this inconvenience, a classification technique which exploits the basis reduction algorithm and binary decision trees has been presented.

A preliminary test performed on FDTD simulated data has shown its potentialities in discriminating and classifying multiple reflections, information that could be directly exploited to point out the primary reflections within the overall signal and boost the inversion process.

The algorithm's key features can be summarized as follows:

- general-purpose
- no *a priori* knowledge of the observed scenario required
- reconstruction of wave propagation paths
- only time delay estimation (TDE) required

Although no major issues have emerged within a simulated environment (as already said, the only limitation regards loss-less and non-dispersive media), we expect that, in case of experimental samples, an inaccurate evaluation of signals' time delays will reduce the overall performances.

To this end, we are currently working on a robust TDE algorithm to be embedded within the processing scheme, as well as on a multi-view approach that could be used to reduce undesired noisy effects. Future works will be therefore devoted to assess the effectiveness of the method within the processing of on-field data.

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