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MODIFIED DOA ESTIMATION METHODS WITH UNKNOWN SOURCE NUMBER BASED ON PROJECTION PRETRANSFORMATION

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Abstract—In this paper, our purpose is to develop methods that have high resolution and robustness in the presence of unknown source number, array error, snapshot deficient, and low SNR. The DOA (Direction-Of-Arrival) estimation with unknown source number methods referred as MUSIC-like and SSMUSIC-like methods have shown high resolution in the snapshot deficient and low SNR scenario. However, they need to take several times of fine search on the full space, which bring about high computational complexities. Thus, modified methods are proposed to reduce computational complexities and improve performances further. In the modified methods, we priori use conventional beamforming to get the rough distribution of signals' angle, which helps to reduce computational complexity and connect the technique of projection pretransformation. Then through projection pretransformation, original methods are further simplified and improved. As demonstrated in computer simulations, the modified DOA estimation with unknown source number methods shows not only higher resolution in the snapshot deficient and lower SNR scenario, but also more robustness against array errors. Although the proposed methods cannot replace the array calibration completely, they reduce the requirement of calibration accuracy. Combined with these advantages, it has been shown that the new methods are more suitable in engineering.

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1. INTRODUCTION

The problem of Direction-Of-Arrival (DOA) estimation has been receiving high attentions as it is a fundamental task in array signal processing with applications in Radar [1], Sonar [2], Communications [3]. The general problem formulation involves a set of signals incident on the array elements with different spatial separations. When the spatial separation between two close signals is smaller than the nominal array resolution (corresponding to the Rayleigh resolution limit of conventional beamforming), we need to find high resolution methods to enable incident signal individual identified. Among those high resolution DOA estimation methods, subspace-based methods [4–6] such as MUSIC (Multiple Signal Classification), SSMUSIC (Signal Subspace Scaled MUSIC) have received wide attention because of their relatively high resolution and computational simplicity. In the ideal environment, the estimation variance of MUSIC has been shown to converge asymptotically to Cramer-Rao lower bound as the number of snapshot increases [7]. However, the performances of subspace-based methods are highly related to many conditions.

As subspace-based methods exploit the orthogonality between noise subspace and signal subspace, the source number used to divide noise and signal subspace is very critical in its implements. In many cases, the source number provided by special source number detection methods [8–11] is underestimated or overestimated. When the source number is underestimated, some signal eigenvectors are mixed in the noise subspace. Because the presumed noise subspace and signal subspace cannot maintain orthogonality, the subspace-based methods would get wrong results or even lose targets. In [12], the effects of source number underestimation on MUSIC are illustrated in detail through theoretic analysis. Compared with the grave degradation caused by underestimation, the degradation caused by overestimation seems tender. Its main defect is that its estimating results not only contain real DOAs but also get spurious DOAs. Although the spurious results could be eliminated by other methods [13, 14], these solutions are also not very perfect.

As the subspace-based methods are handled based on the array model, their steering vectors are particularly dependent on the characteristics of the array, such as amplitudes, phases and locations. To simplify the discussion, we only consider the amplitude and phase error of the array in this paper. In practice, the array errors are inevitable even after a calibration procedure. To make the array model approach the real array characteristics as much as possible, one way is to develop special calibration methods [15–17]. Another way is to develop DOA estimation methods with inherent robustness [18, 19].

Except the mentioned conditions, the signal-to-noise ratio (SNR) and snapshot are other conditions the paper considers. In engineering, it is common to find scenarios of snapshot deficient and low SNR. In this case, the sample covariance matrix (SCM) formed from the collection of spatial snapshot is quite different from the real SCM. Then, the degradation of the obtained noise and signal subspaces makes the subspace-based methods fail to distinguish the signals from close angles.

Different from other methods which focus on improving the calibration performance [15–17], source number estimating accuracy [8–11] and resolution of close signals in the snapshot deficient scenario [5, 6], our purpose is to develop methods with high resolution and robustness in the presence of unknown source number, array error, snapshot deficient and low SNR. In [20], we have developed a simple but high resolution DOA estimating method with unknown source number, suitable for the snapshot deficient and low SNR scenario. Through the equations provided in [20], the spectrums of MUSIC and SSMUSIC could be represented approximately, which eliminate estimating the source number in advance. However, to avoid the awful impact brought by the spreading of noise eigenvalues in the snapshot deficient and low SNR scenario, a high computational complexity method is used, which needs to be simplified.

Regarded as a particular beamspace method, projection pretransformation referred in [21] could be used to enhance the robustness of subspace-based methods and reduce their computational complexities. In [22], it is used to improve the performance of adaptive beamforming. In this paper, on its basis, the methods proposed in [20] are further modified to get lower computational complexity and better performance.

The rest of this paper is organized as follows. The signal model and several relevant DOA estimation algorithms are introduced in Section 2. Modified methods based on projection pretransformation are illustrated in Section 3. To verify the validity of modified methods, computer simulations in numerous different situations are conducted in Section 4. Finally, we make conclusions in Section 5.

2. SIGNAL MODEL AND RELEVANT ALGORITHMS

Consider that P independent narrowband signals from the directions of $\{\theta_i\}$ arrive at an arbitrary array of M sensors under an additive white gaussian noise environment (the signals and noises are uncorrelated), where $\{\theta_i\}$ denotes the incident angle of the ith signal. The received noisy signals can be expressed as:

$$X(t) = \sum_{i=1}^{P} a(\theta_i)s_i(t) + n(t) = A(\theta)s(t) + n(t) \tag{1}$$

where $X(t)$, $s(t)$, $n(t)$ are the vectors of the received signals, incident signals, and the additive noise, and A is $M \times P$ matrix $A(\theta) = [a(\theta_1), a(\theta_2), \ldots, a(\theta_P)]$. Here $a(\theta_i)$ is the steering vector of the array toward the direction θ_i. Considering that the array exists amplitude and phase errors, we denote the amplitude and phase errors of the ith sensor as α_i and β_i, respectively. Thus, the data of array outputs are rewritten as

$$X(t) = G\Phi A(\theta)s(t) + n(t) \tag{2}$$

where G, Φ are diagonal matrices and their ith diagonal elements G_{ii} and Φ_{ii} are α_i and $e^{j\beta_i}$.

The RCM R is given by

$$R = E[x(t)x^H(t)] = AR_S A^H + \sigma^2 I \tag{3}$$

where R_S, σ^2, I denote the signal covariance matrix, noise power, and identity matrix, respectively. Besides, $E[\cdot]$ denotes the statistical expectation and H the conjugate transpose. The eigendecomposition of matrix R yields

$$R = \sum_{i=1}^{M} \lambda_i u_i u_i^H = \sum_{i=1}^{P} \lambda_i u_i u_i^H + \sum_{i=P+1}^{M} \lambda_i u_i u_i^H \tag{4}$$

where λ_i and u_i are the ith eigenvalue and eigenvector, respectively. When the source number P is known in advance, the eigenvalues and eigenvectors of the RCM can be split into two sets that generate independent linear spaces: the signal subspace and noise subspace.

In spite of its limited resolution, conventional beamforming (CBF) is still being widely used in engineering for its simplicity and robustness against array errors and snapshot deficient. The CBF spectrum can be expressed as

$$P_{CBF}(\theta) = A^H(\theta)RA(\theta) \tag{5}$$

Due to the property that the noise subspace is orthogonal to the steering vectors of the signals, the high resolution of subspace-based methods is realized by looking for steering vectors as orthogonal to the noise subspace as possible.

The MUSIC algorithm spectrum can be expressed as

$$P_{MUSIC}(\theta) = \frac{1}{A^H(\theta) \sum_{i=P+1}^{M} u_i u_i^H A(\theta)} \tag{6}$$

By contrast to MUSIC, the numerator in SSMUSIC is a signal subspace function. Its spectrum can be expressed as

$$P_{SSMUSIC}(\theta) = \frac{A^H(\theta) \sum_{i=1}^{P} \frac{1}{\lambda_i - \sigma^2} u_i u_i^H A(\theta)}{A^H(\theta) \sum_{i=P+1}^{M} u_i u_i^H A(\theta)} \tag{7}$$

Through the simulations in [20], we know that the weights of signal subspace projection in SSMUSIC signal subspace function could be slightly changed. A new spectrum similar to SSMUSIC is given as

$$P_{new}(\theta) = \frac{A^H(\theta) \left(\frac{1}{\lambda_1 - \sigma^2} u_1 u_1^H + \sum_{i=2}^{P} \frac{1}{\lambda_i} u_i u_i^H \right) A(\theta)}{A^H(\theta) U_N U_N^H A(\theta)} \tag{8}$$

In standard form, the dimensions of signal subspace and noise subspace are based on the results offered by the source number estimation method. In [20], we infer approximate representations of MUSIC and SSMUSIC spectrums, in which the source number is unnecessary. In the ideal environment, we have:

$$\lambda_1 \geq \ldots \geq \lambda_P > \lambda_{P+1} = \ldots = \lambda_M = \sigma^2 \tag{9}$$

Assuming that m is selected sufficiently large, we can obtain,

$$\lim_{m \to \infty} \left(\frac{\lambda_M}{\lambda_i} \right)^m u_i u_i^H \cong \begin{cases} 0, & \text{for } i = 1, \ldots, P \\ u_i u_i^H, & \text{for } i = P+1, \ldots, M. \end{cases} \tag{10}$$

According to Equation (10), the following equations are easily derived:

$$\sum_{i=P+1}^{M} u_i u_i^H \cong \lim_{m \to \infty} \sum_{i=1}^{M} \left(\frac{\lambda_M}{\lambda_i} \right)^m u_i u_i \tag{11}$$

$$\frac{1}{\lambda_1 - \sigma^2} u_1 u_1^H + \sum_{i=2}^{P} \frac{1}{\lambda_i} u_i u_i^H \cong \frac{1}{\lambda_1 - \lambda_M} u_1 u_1^H + \sum_{i=2}^{M} \frac{1}{\lambda_i} u_i u_i^H - \frac{1}{\lambda_M} \sum_{i=P+1}^{M} u_i u_i^H \tag{12}$$

Based on Equation (11), MUSIC spectrum can be approximately represented. And SSMUSIC spectrum could be approximately represented based on Equations (8), (11)–(12). For convenience, we name the two new spectrums as MUSIC-like and SSMUSIC-like methods, respectively.

It is notable that the derivations are based on assumption (9). When the array exists, some errors or the number of snapshots is small. The eigenvalues obtained from the practical SCM can be given as: $\lambda'_1 > \ldots > \lambda'_P > \lambda'_{P+1} \geq \ldots \geq \lambda'_M$, which spread significantly. To prohibit the spreading of noise eigenvalues, we need to load a proper value λ'' to modify the eigenvalues. As λ'' decreases, the resolutions of SSMUSIC-like and MUSIC-like methods improve in the risk of producing spurious peaks. According to this feature, an empirical method is used to modify the eigenvalues in [20]. In the empirical method, the proper loading value is obtained through testing the performances of the spatial spectrums as λ'' decreases. When the spurious peaks occur, the test stops, and the corresponding value is selected as the proper loading value. However, the procedure of searching and detecting spurious peaks need to take several times of fine search on the full space, which brings about high computational complexity.

In fact, the procedure of modifying eigenvalues can be simplified when the rough directions of signals are known, because the locations in the spectrums of MUSIC-like and SSMUSIC-like methods, which produce spurious peaks are far away from the real DOAs. Moreover, the rough directions of signals could be obtained by prior information and low complexity DOA estimation methods such as CBF. Thus, we could eliminate the spurious peaks based on the results of CBF. Exploiting the results of CBF, we shrink the scanning area from full space to several separate sectors, which contain rough directions of signals. In fact, such shrinkage can not only eliminate spurious peaks, but also reduce computational complexity. Besides, it is helpful for us to use projection pretransformation to modify MUSIC-like and SSMUSIC-like methods further.

3. MODIFIED METHODS BASED ON PROJECTION PRETRANSFORMATION

Regarded as a particular beamspace method, projection pretransformation that transforms the output data of the array into the transform domain could be used to enhance the robustness of subspace-based methods. Its key point is to calculate transform matrix T in a rough known region Ω. The details of calculating T are as follows.

Step 1) Construct a correlation matrix R_Ω, which is defined as:

$$R_\Omega = \int_\Omega a(\theta)a^H(\theta) \tag{13}$$

Step 2) R_Ω could be eigendecomposited as $R_\Omega = \sum_{i=1}^{M} \bar{\lambda}_i e_i e_i^H$, where $\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq \ldots \geq \bar{\lambda}_M$. Assume a small constant ε (in this paper, $\varepsilon = 10^{-2}$) and find D significant eigenvalues of R_Ω by the formula:

$$\sum_{i=D+1}^{M} \bar{\lambda}_i / \sum_{i=1}^{M} \bar{\lambda}_i \geq \varepsilon \tag{14}$$

Step 3) The transform matrix T is consisted by eigenvectors which correspond to D significant eigenvalues. Thus, $T = [e_1, e_2, \ldots, e_D]^H$.

From the above procedure, we can draw a conclusion that the calculation of the matrix T is easy to be obtained provided that the distribution of signal angles is known. Thus, for computation in real time, the transform matrix could be calculated off-line in advance. After dividing the whole scanning area into many small sectors, we calculate all the corresponding transform matrices T and store them in computer, which is convenient for practice use.

Through transformation, the M dimension array output data $X(t)$ could be reduced to D dimension ($D < M$). The transformed datum $Y(t)$ is given as

$$Y(t) = TX(t) \tag{15}$$

Similarly, the transformed steering vector $A_T(\theta)$ and transformed SCM R_T can be expressed as:

$$A_T(\theta) = TA(\theta) \tag{16}$$

$$R_T = TRT^H \tag{17}$$

There are two principles by which projection pretransformation can improve the performances of subspace-based methods.

Firstly, transform matrix T is a unitary matrix, which satisfies $TT^H = I$. In the perspective of beamspace, if we regard T as a beamforming matrix, it can be regarded as the best beamforming matrix, which makes Beamspace MUSIC [23] have the minimum SNR resolution threshold [21].

Secondly, the errors caused by array error and snapshot deficient are reduced. Assume that the eigenvectors u_i, $\hat{u}_i$ are obtained from SCM in ideal circumstance and SCM in nonideal circumstance, respectively. So $\hat{u}_i = u_i + \delta u_i$, where δu_i presents the eigenvector error. Through transformation, we have $\bar{u}_i = Tu_i$ and $T\hat{u}_i = Tu_i + T\delta u_i$. Further, $T\hat{u}_i = \bar{u}_i + T\delta u_i$. As $\|T\delta u_i\|_2 \leq \|\delta u_i\|_2$, where $\|\cdot\|_2$ represents Euclidean norm. Hence, the transformation can suppress the errors.

Considering array errors and snapshot deficient, we eigendecomposite the matrix R_T in unideal circumstance:

$$R_T = \sum_{i=1}^{D} \hat{\lambda}_i \hat{u}_i \hat{u}_i^H = \sum_{i=1}^{P} \hat{\lambda}_i \hat{u}_i \hat{u}_i^H + \sum_{i=P+1}^{D} \hat{\lambda}_i \hat{u}_i \hat{u}_i^H \tag{18}$$

As previously mentioned, the spurious peaks could be easily eliminated, if only the rough known sector Ω is selected sufficiently small. Besides, the smaller sector Ω could improve the performance of projection pretransformation provided that the DOAs of close signals are contained. As a matter of experience, the sector that extends nearly two to three times of nominal array resolution around the rough known direction is enough in practice. On this basis, the task of loading value is reduced to realize high resolution of close signals.

Because a theoretic value is too difficult to provide, the modified loading value we provide is based on qualitative analysis. As the purpose of loading value is to make the noise eigenvalues smooth and steady, it should be less than the biggest signal eigenvalue and bigger than the least noise eigenvalue, which is restricted as: $\hat{\lambda}_D \ll \hat{\lambda}'' \ll \hat{\lambda}_1$. So a loading value is recommended as:

$$\hat{\lambda}'' = \sqrt{\sum_{i=1}^{D} \hat{\lambda}_i} \tag{19}$$

Through computer simulation, this loading value is proper in most situations. In some particular cases, the recommended value could be changed slightly.

In practice, m is replaced by a finite value. As illustrated previously in [20], in ideal environment, the distinction between the SSMUSIC-like method and SSMUSIC decreases as m increases. So do MUSIC-like method and MUSIC. However, as the parameters of m and loading value λ'' interact greatly, m and λ'' are too hard to be defined simultaneously in unideal environment. Unlike MUSIC-like and SSMUSIC-like methods, m in the modified methods could be quite flexible, as they only need to distinguish signals in the feasible areas while the problem of avoiding spurious peaks is solved by CBF. Whereas, improving performances of new methods are not obvious when m increases to a certain value. Generally, m selected between 15 and 25 is sufficient, which has been demonstrated in the simulation test.

For modified methods, except parameters in the original spectrums which need to be replaced by that in transform domain, their procedures are also different. They are implemented as follows.

Step 1) Based on prior information, divide the full space into many small sectors and calculate all the corresponding matrices T based on Equations (13)–(14). Then store all the calculated matrices in computers.

Step 2) Use CBF to scan the full space in a large step to obtain the rough directions of signals. According to the result of CBF, choose all feasible sectors and the corresponding transform matrix T.

Step 3) Choose a feasible sector and transform the data of array domain into transform domain based on Equations (15)–(17).

Step 4) Reconstruct the modified spectrums based on Equations (6), (8), (10)–(12),(18)–(19) and search for DOAs.

Step 5) Choose other sectors and repeat the Steps (3)–(4).

From the above illustrations, as the first step of modified methods could be done off-line, so their computational complexities mainly focus on SCM decomposition and searching peaks of the spectrum. Compared with the original methods, the computational complexities of these two points have been greatly reduced. Firstly, the modified methods only one time of take rough search on the full space and several times of fine search on the feasible sectors. The total computational complexity of this searching method is less than that of taking one-time search on the full space. Whereas, the empirical method needs at least three times of fine search on the full space. Secondly, through transformation, the corresponding computational complexities of SCM decomposition are reduced from $O(M^3)$ to $O(D^3)$.

4. COMPUTER SIMULATIONS

As the lower computational complexities of modified methods have been analysed in Section 3, we only consider the resolution and robustness of these methods in this section. For simplicity, SSMUSIC and MUSIC used in this section are assumed that the source number is correctly estimated and that variables in SSMUSIC-like and MUSIC-like methods remain the same as that in [20], which $m = 10$ and $k = 0.25$.

To show the improvement of the modified methods better, we arrange the simulations as follows. Firstly, we give all the mentioned spectrums (including CBF) for rough comparison in Figure 1. Secondly, as m is an important variable in the modified methods, we evaluate its impact and choose a proper value for the following simulations based on Figure 2. Lastly, to verify the improvement of resolution performance in the snapshot deficient and low SNR scenario and robustness under array error, we compare the modified methods with MUSIC, SSMUSIC, MUSIC-like method, and SSMUSIC-like method in detail in Figure 3 and Figure 4.

In simulations, assume a uniform circular array with 40 sensors, its radius is 150 meters. We adopt 16 sequential sensors of this circular array as receiving array. The center frequency of signals is 10 MHz. The Rayleigh resolution limit in this case is about 5°. In Mento-Carlo simulations, every single experiment has been run 500 times. Regarding to two closely spaced signals in a single experiment, if the estimated DOAs satisfy: $|\hat{\theta}_1 - \theta_1| + |\hat{\theta}_2 - \theta_2| < |\hat{\theta}_1 - \hat{\theta}_2|$, we define the trial of angle separation successful. On this basis, the RMSE (Root-Mean-Square-Error) is calculated as:

$$RMSE(\theta) = \sqrt{\frac{1}{K}\sum_{i=1}^{K}(\hat{\theta}^i - \theta)^2} \tag{20}$$

where K is the number of successful trials in the Mento-Carlo simulation, θ the true DOA, and $\hat{\theta}^i$ the estimated DOA of the ith trial.

Figure 1 shows that the rough impression of the mentioned methods. In this case, three narrowband signals ($\theta_1 = 66°$, $\theta_2 = 67.5°$, $\theta_3 = 85°$) with 10 dB impinge on the array, and the snapshot is 40. In the modified methods, $m = 20$. Based on the results of CBF, the whole

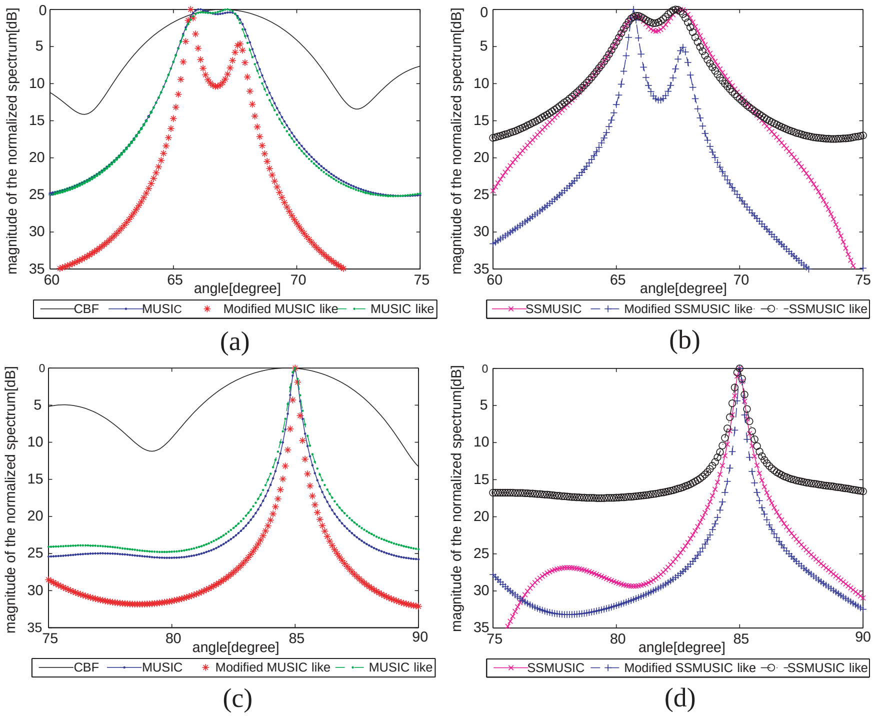

Figure 1. The normalized spectrums of the mentioned methods. (a) Performances of two close signals. (b) Performances of two close signals. (c) Performances of the third signal. (d) Performances of the third signal.

space is shrunk to two sectors: [60°, 75°], [70°, 95°]. At the same time, transform matrices T calculated respectively on this two sectors are selected. It is observed that CBF cannot resolve the two close signals completely, and MUSIC, MUSIC-like method can hardly resolve two peaks, while two peaks are resolved distinctly in SSMUSIC, SSMUSIC-like method and the modified methods. Besides, from the perspective of spectrum peak sharp degree, modified SSMUSIC-like method have the sharpest peaks, and modified MUSIC-like method have the less sharp peaks, indicating that the resolutions of modified methods have been greatly improved.

Figure 2 shows that the performances of the modified methods under different m. In this case, we mainly consider the impact of m on

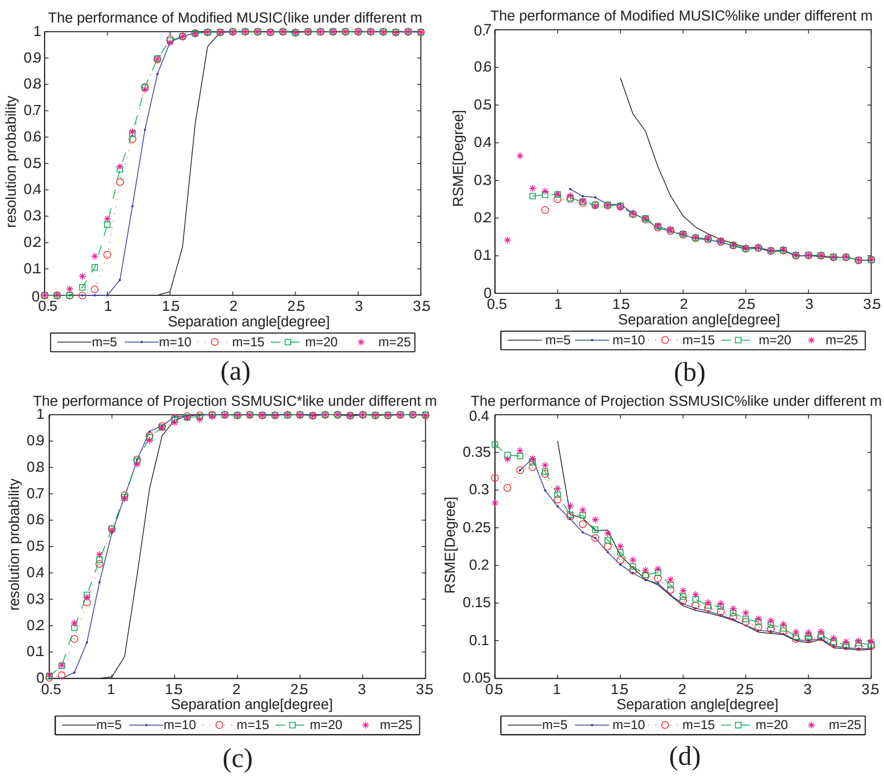

Figure 2. The performances of modified methods under different m. (a) Resolution probability of modified MUSIC-like method. (b) RSME of modified MUSIC-like method. (c) Resolution probability of modified SSMUSIC-like method. (d) RSME of modified SSMUSIC-like method.

the resolution of close signals. The snapshots and SNR are fixed at 40 and 10 dB while the angle separation of two close signals varies. From this figure, we can find that the two modified methods have similar regularity when m increases. The performances of modified methods have been improving until m increases to a certain value. From the results, m selected between 15 and 25 is sufficient to realize the best resolution. Therefore, $m = 20$ is selected in the following simulations.

Figure 3 shows that the resolution improvement of modified methods in the snapshot deficient and low SNR scenario. This figure consists of three parts, which focus on the conditions, such as the angle separation of two close signals, snapshot, SNR. Figure 3(a) and Figure 3(b) show the comparison against angle separation of two close signals while the snapshots and SNR are fixed at 40 and 10 dB. Figure 3(c) and Figure 3(d) show the comparison against the snapshot while the angle separation of two close signals and the SNR are fixed at 2° and 10 dB. Figure 3(e) and Figure 3(f) show the comparison against SNR while the number of snapshot and the angle separation of two close signals are fixed at 40 and 2°.

Assume the probability of 90% as a resolution threshold, which enables two close signals individually identified. As seen from these subfigures, we can rank modified SSMUSIC-like method, SSUMUSIC, modified MUSIC-like method, SSMUSIC-like method, MUSIC-like method, MUSIC in the order of resolution probability from high to low. Besides, the RSME of modified methods are better than other four methods. Generally speaking, the resolution of modified methods in the snapshot deficient and low SNR scenario is significant and reliable.

Based on Figure 3, we test these six methods' robustness under array errors in Figure 4, while the scenario remains the snapshot deficient and low SNR. Figure 4(a) and Figure 4(b) show the comparison against the random amplitude error of the array while the phases of the array sensors are fixed at $0rad$. Figure 4(c) and Figure 4(d) against the random phase error of the array while the amplitudes of the array sensors are fixed at 1. In this two parts, the angle separation of two close signals, snapshot and SNR are fixed at 2°, 40, and 10 dB, respectively.

In Figure 4, the performances of methods in the two simulations not only are related to the random array errors but also have the impact of small sample random noise, both of which make the resolution probabilities of these methods vary greatly. Thus, we can only get a rough impression of robustness of the six methods while the accurate information of robustness in this case needs further investigation. In [19], the authors proved that MUSIC exhibits certain inherent robustness against array model errors. From Figure 4,

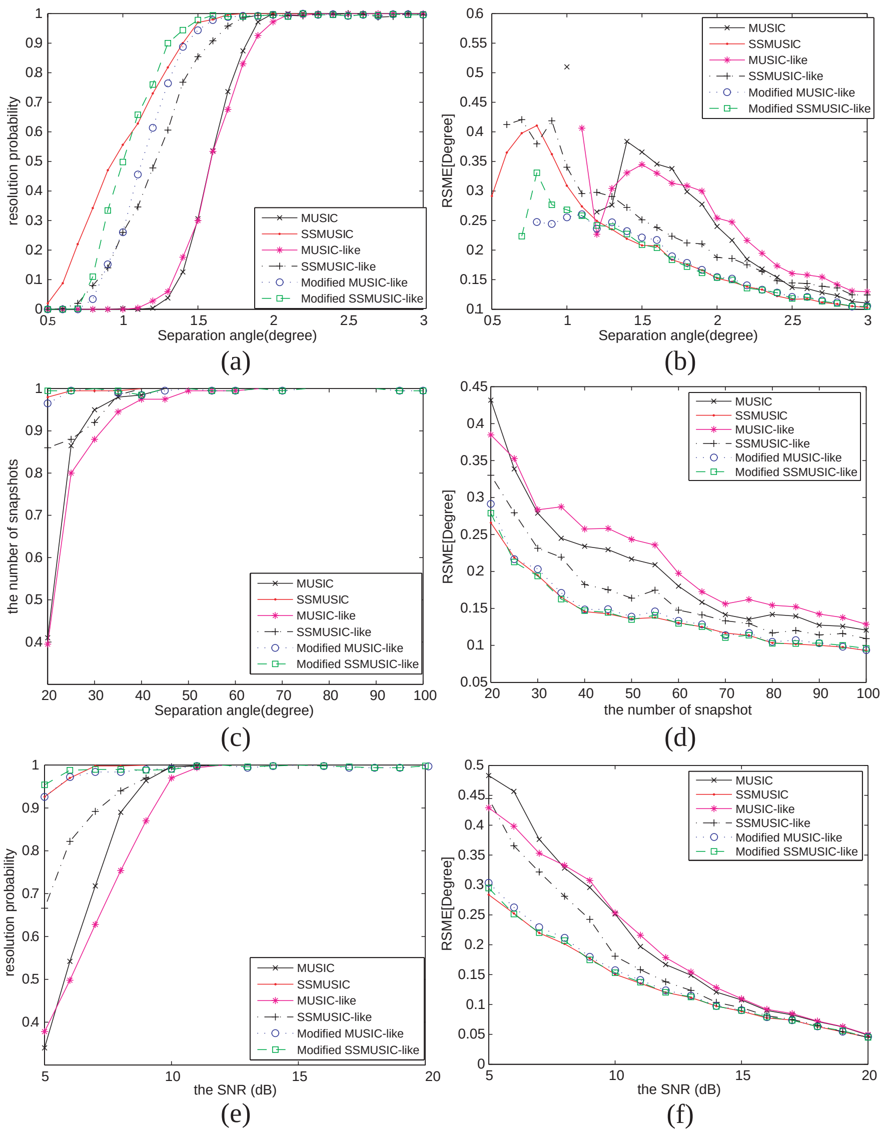

Figure 3. The resolution improvement of modified method in the snapshot defficient and low SNR scenario. (a) Resolution probability against angle separation. (b) RSME against angle separation. (c) Resolution probability against snapshot. (d) RSME against snapshot. (e) Resolution probability against SNR. (f) RSME against SNR.

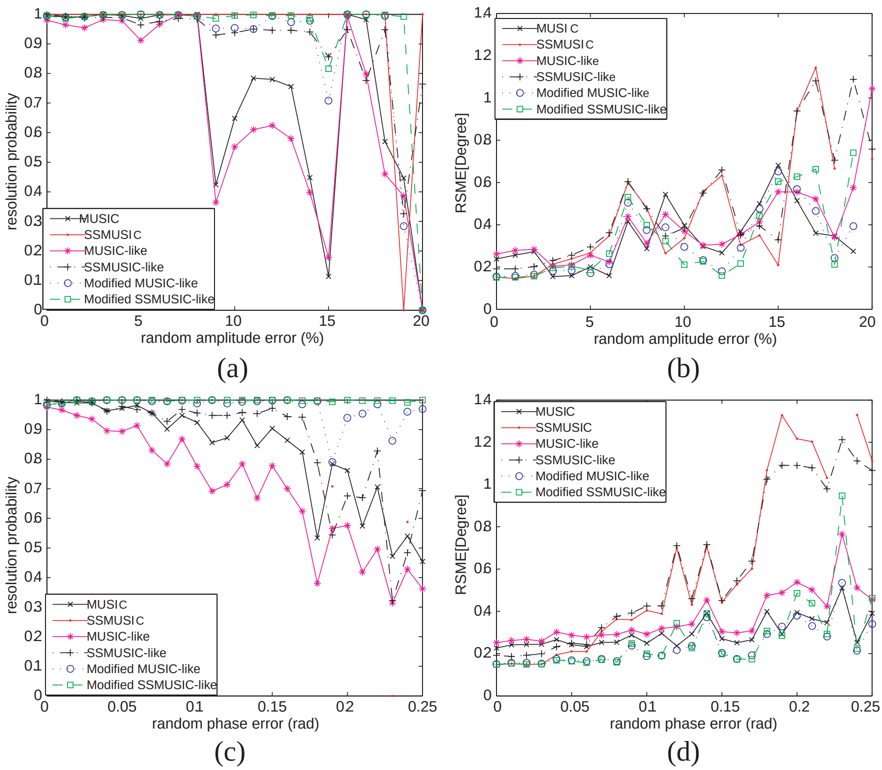

Figure 4. The comparison of robustness under array error. (a) Resolution probability against amplitude error. (b) RSME against amplitude error. (c) Resolution probability against phase error. (d) RSME against phase error.

in the perspective of resolution probability, it is observed that the modified methods and SSMUSIC exhibit much better robustness against array errors than other three methods. More specifically, SSMUSIC has better robustness against amplitude error than modified methods while modified methods are more robust against phase error. Besides, MUSIC is more robust than SSMUSIC-like and MUSIC-like methods. However, from another perspective of RSME, the higher resolution probability of SSMUSIC is at the expense of estimating accuracy. Thus, results of the modified methods are more reliable than SSMUSIC-like method, SSMUSIC, MUSIC-like method, and MUSIC in unideal circumstance. Comparatively speaking, the robustness of the modified methods is decent, although it cannot replace the array calibration completely.

5. CONCLUSION

In this paper, new modified methods are proposed to obtain less computational complexity and better performances. Through computer simulations that the conditions, such as snapshot, SNR, amplitude error and phase error of the array, are considered separately, we find that the modified DOA estimation with unknown source number methods not only show high resolution in the snapshot deficient and low SNR scenario, but also exhibit certain robustness against array errors. Although the proposed methods cannot replace the array calibration completely, they reduce the requirement of calibration accuracy. Combined with these advantages, it has been shown that the new methods are more suitable for engineering.

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