Progress In Electromagnetics Research Letters, Vol. 29, 201–211, 2012

A MODIFIED CAUCHY METHOD SUITABLE FOR DUPLEXER AND TRIPLEXER RATIONAL MODELS EXTRACTION

Y. L. Zhang*, T. Su, B. Wu, J. Chen, and C. H. Liang

Science and Technology on Antennas and Microwaves Laboratory, Xidian University, Xi'an, Shaanxi 710071, China

Abstract—A modified Cauchy method which generates accurate duplexer and triplexer rational models from either measurements or electromagnetic analysis is presented in this paper. The modified Cauchy method has some advantages over the conventional Cauchy method because it takes into account the relationship between the transmission coefficients of each channel filter and reflection coefficient. It is suitable for duplexer and triplexer whose channel filters are connected through resonating junction. The total least square method is used to solve the system matrix. Synthesized numerical duplexer and triplexer examples verify the method successfully.

1. INTRODUCTION

In recent years, the generation of reduced-order polynomial from measurement or electromagnetic (EM) simulated responses have been an active topic in the filter society [1–4]. But there is few method in the literature about how to obtain the reduced order polynomial model of multiplexers, such as duplexer and triplexer. Cauchy method is a well-known and effective technique for generating reduced-order rational polynomial models from the response of a passive device [5]. The polynomial is the prerequisite of the equivalent circuit synthesis which is a fundamental requirement for computer-aided tuning.

Some effort has also been made in order to extend the Cauchy method to multidimensional functions, i.e., to functions of more than one independent variable [6–8]. In [6], an effective generic approach for computer-aided design of microwave circuits is presented. In [7],

Received 12 December 2011, Accepted 16 January 2012, Scheduled 8 February 2012
* Corresponding author: Yong-Liang Zhang (namarzhang@163.com).

the authors present a robust algorithm for extracting the Cauchy interpolation coefficients in a multidimensional rational function modeling problem. In [8], the authors proposed an optimization methodology suitable for the design of various antenna structures based on Cauchy model.

Cauchy method is a fast, convenient and accurate technique to fit rational polynomials to specified response. In [5], the conventional Cauchy method only generates the numerator polynomial coefficients in the first step, and the posterior reconstruction of the common set of poles is required. The junction in [5] is a simple shunt connection of the two filters input ports. However, the method proposed in this paper is different from the conventional method. It generates numerator and denominator coefficients simultaneously. The transmission and reflection coefficients share a common set poles. The modified Cauchy method presented in this paper guarantees the three (for duplexers) or four (for triplexers) rational polynomials to have the same poles. And the junction is a resonator which agrees with the physical model.

This paper is organized as follows. In Section 2, the modified Cauchy method for duplexer is presented. This new technique is valid either for lossless or lossy responses, and guarantees all rational polynomials have the same poles. In Section 3, the modified Cauchy method for triplexer is presented. In Section 4, both duplexer and triplexer numerical examples verify the new technique separately. A conclusion is drawn in Section 5.

2. MODIFIED CAUCHY METHOD FOR DUPLEXERS

Cauchy method is well known in the literature. A three-port network can be described by its scattering parameters S_{11}, S_{21} and S_{31}. The scattering parameters for the architecture shown in Fig. 1 can be

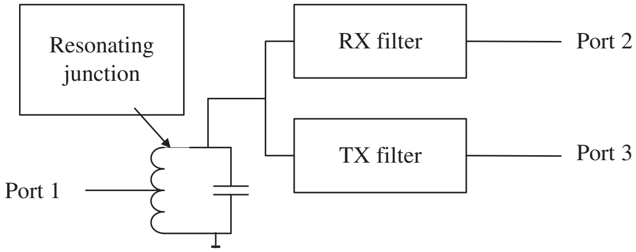

Figure 1. Architecture of duplexer connected through common resonating node.

expressed in the normalized low-pass frequency domain as follows.

$$\begin{cases} S_{11} = \dfrac{N(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nRX+nTX+1} a_{1k}s^k}{\sum\limits_{k=0}^{nRX+nTX+1} b_k s^k} \\ S_{21} = \dfrac{P_{RX}(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nzRX+nTX} a_{2k}s^k}{\sum\limits_{k=0}^{nRX+nTX+1} b_k s^k} \\ S_{31} = \dfrac{P_{TX}(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nRX+nzTX} a_{3k}s^k}{\sum\limits_{k=0}^{nRX+nTX+1} b_k s^k} \end{cases} \tag{1}$$

where, nRX is the number of poles of RX channel filter, $nzRX$ the number of finite transmission zeros of RX channel filter, nTX the number of poles of TX channel filter, and $nzTX$ the number of finite transmission zeros of TX channel filter. We could obtain following consideration about the scattering parameters.

1. Channel filters of the duplexer share a common resonator, because they are connected through the common resonator. The total degree of denominator is $(nRX + nTX + 1)$. The numerator degree of S_{11} is the same as the denominator.

2. Because of the interaction between RX and TX channel filters, the numerator degrees of S_{21} and S_{31} are $nzRX + nTX$ and $nRX + nzTX$ respectively.

Given a set of Ns sample frequency points, equations in (1) can be rewritten as

$$\begin{cases} \sum\limits_{k=0}^{nRX+nTX+1} a_{1k}s_i^k - S_{11}(s_i) \sum\limits_{k=0}^{nRX+nTX+1} b_k s_i^k = 0 \\ \sum\limits_{k=0}^{nzRX+nTX} a_{2k}s_i^k - S_{21}(s_i) \sum\limits_{k=0}^{nRX+nTX+1} b_k s_i^k = 0 \\ \sum\limits_{k=0}^{nRX+nzTX} a_{3k}s_i^k - S_{31}(s_i) \sum\limits_{k=0}^{nRX+nTX+1} b_k s_i^k = 0 \end{cases}, \tag{2}$$

where, $i = 1, 2, \ldots, Ns$.

Using the matrix notation, (2-2) can be rewritten as

$$\begin{cases} [V_{nRX+nTX+1} \quad -S_{11}V_{nRX+nTX+1}] \begin{bmatrix} a_1 \\ b \end{bmatrix} = 0 \\ [V_{nzRX+nTX} \quad -S_{21}V_{nRX+nTX+1}] \begin{bmatrix} a_2 \\ b \end{bmatrix} = 0 \\ [V_{nRX+nzTX} \quad -S_{31}V_{nRX+nTX+1}] \begin{bmatrix} a_3 \\ b \end{bmatrix} = 0 \end{cases}, \tag{3}$$

where,

$$\begin{aligned} a_1 &= [a_{1,nRX+nTX+1} \dots a_{1,0}]^T, \\ a_2 &= [a_{2,nzRX+nTX} \dots a_{2,0}]^T, \\ a_3 &= [a_{3,nRX+nzTX} \dots a_{3,0}]^T \\ b &= [b_{nRX+nTX+1} \dots b_0]^T, \\ S_{k1} &= diag\{S_{k1}(s_i)\}, \quad k = 1,\, 2,\, 3. \end{aligned}$$

and V_m is a decreasing-power mth-order Vandermonde matrix whose size is Ns-by-$(m+1)$.

The equations in (2)–(3) can be combined into one new system.

$$\begin{bmatrix} V_{nRX+nTX+1} & 0_{Ns,nzRX+nTX+1} & 0_{Ns,nRX+nzTX+1} & -S_{11}V_{nRX+nTX+1} \\ 0_{Ns,nRX+nTX+1+1} & V_{nzRX+nTX} & 0_{Ns,nRX+nzTX+1} & -S_{21}V_{nRX+nTX+1} \\ 0_{Ns,nRX+nTX+1+1} & 0_{Ns,nzRX+nTX+1} & V_{nRX+nzTX} & -S_{31}V_{nRX+nTX+1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ b \end{bmatrix} = 0; \tag{4}$$

The total number of the unknowns is

$(nRX+nTX+2+nzRX+nTX+1+nRX+nzTX+1+nRX+nTX+2)$,

we use the total least square (TLS) method to solve the system matrix. In order to guarantee the system matrix has solution, the TLS method requires that Ns must be greater than or equal to

$(nRX+nTX+2+nzRX+nTX+1+nRX+nzTX+1+nRX+nTX+2-1)$,

that is

$$3*(nRX+nTX)+nzRX+nzTX+5.$$

The system matrix is different from that in [5]. The coefficients of numerators and denominator can be solved with TLS (total least square) method simultaneously, without requiring the reconstruction of the network poles. And the unitary condition is not strictly required. That is to say $|S_{11}|^2+|S_{21}|^2+|S_{31}|^2<1$ for lossy system.

3. MODIFIED CAUCHY METHOD FOR TRIPLEXER

The architecture of triplexer connected through common resonating junction is shown in Fig. 2.

And the scattering parameters can be expressed as follows.

$$\begin{cases} S_{11} = \dfrac{N(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nRX+nMX+nTX+1} a_{1k}s^k}{\sum\limits_{k=0}^{nRX+nMX+nTX+1} b_k s^k} \\ S_{21} = \dfrac{P_{RX}(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nzRX+nMX+nTX} a_{2k}s^k}{\sum\limits_{k=0}^{nRX+nMX+nTX+1} b_k s^k} \\ S_{31} = \dfrac{P_{MX}(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nRX+nzMX+nTX} a_{3k}s^k}{\sum\limits_{k=0}^{nRX+nMX+nTX+1} b_k s^k} \\ S_{41} = \dfrac{P_{TX}(s)}{D(s)} = \dfrac{\sum\limits_{k=0}^{nRX+nMX+nzTX} a_{4k}s^k}{\sum\limits_{k=0}^{nRX+nMX+nTX+1} b_k s^k} \end{cases} \quad (5)$$

where, nRX is the number of poles of RX channel filter, $nzRX$ the number of finite transmission zeros of RX channel filter, nMX the number of poles of MX channel filter, $nzMX$ the number of finite

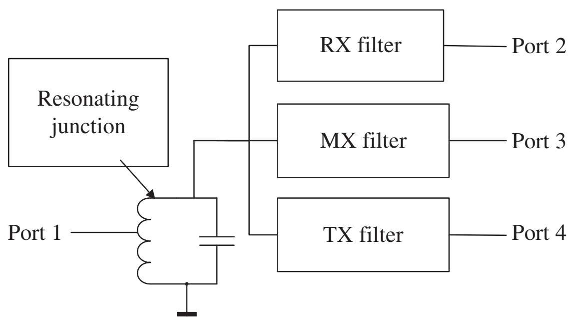

Figure 2. Architecture of triplexer connected through common resonating junction.

transmission zeros of MX channel filter, nTX the number of poles of TX channel filter, and $nzTX$ the number of finite transmission zeros of TX channel filter. Because channel filters share a common resonator, the total number degree of denominator is $nRX + nMX + nTX + 1$. The numerator degree of S_{11} is the same as the denominator. Due to the interaction between three channel filters, the numerator degrees of S_{21}, S_{31} and S_{41} are $nzRX + nMX + nTX$, $nRX + nzMX + nTX$ and $nRX + nMX + nzTX$, respectively.

Given a set of Ns sample frequency points, equations in (5) can be rewritten as

$$\begin{cases} \sum_{k=0}^{nRX+nMX+nTX+1} a_{1k}s_i^k - S_{11}(s_i) \sum_{k=0}^{nRX+nMX+nTX+1} b_k s_i^k = 0 \\ \sum_{k=0}^{nzRX+nMX+nTX} a_{2k}s_i^k - S_{21}(s_i) \sum_{k=0}^{nRX+nMX+nTX+1} b_k s_i^k = 0 \\ \sum_{k=0}^{nRX+nzMX+nTX} a_{3k}s_i^k - S_{31}(s_i) \sum_{k=0}^{nRX+nMX+nTX+1} b_k s_i^k = 0 \\ \sum_{k=0}^{nRX+nMX+nzTX} a_{4k}s_i^k - S_{41}(s_i) \sum_{k=0}^{nRX+nMX+nTX+1} b_k s_i^k = 0 \end{cases} \tag{6}$$

where $i = 1, 2, \ldots, Ns$,

Using the matrix notation, (6) can be expressed as

$$\begin{cases} [V_{nRX+nMX+nTX+1} \quad -S_{11}V_{nRX+nMX+nTX+1}] \begin{bmatrix} a_1 \\ b \end{bmatrix} = 0 \\ [V_{nzRX+nMX+nTX} \quad -S_{21}V_{nRX+nMX+nTX+1}] \begin{bmatrix} a_2 \\ b \end{bmatrix} = 0 \\ [V_{nRX+nzMX+nTX} \quad -S_{31}V_{nRX+nMX+nTX+1}] \begin{bmatrix} a_3 \\ b \end{bmatrix} = 0 \\ [V_{nRX+nMX+nzTX} \quad -S_{41}V_{nRX+nMX+nTX+1}] \begin{bmatrix} a_4 \\ b \end{bmatrix} = 0 \end{cases} \tag{7}$$

where,

$$\begin{aligned} a_1 &= [a_{1,nRX+nMX+nTX+1} \cdots \; \cdots a_{1,0}]^T, \\ a_2 &= [a_{2,nzRX+nMX+nTX} \cdots \; \cdots a_{2,0}]^T, \\ a_3 &= [a_{3,nRx+nzMX+nTX} \cdots \; \cdots a_{3,0}]^T, \\ a_4 &= [a_{4,nRX+nMX+nzTX} \cdots \; \cdots a_{4,0}]^T, \end{aligned}$$

$$\begin{aligned} b &= [b_{nRX+nMX+nTX+1} \cdots \ \ldots b_0]^T, \\ S_{k1} &= diag\{S_{k1}(s_i)\} \quad k = 1,\, 2,\, 3,\, 4. \end{aligned}$$

and V_m is a decreasing-power mth-order Vandermonde matrix whose size is Ns-by-$(m+1)$. The equations in (7) can be combined into one new system.

$$\left[\begin{matrix} V_{nRX+nMX+nTX+1} & 0_{nzRX+nMX+nTX+1} & 0_{nRX+nzMx+nTX+1} \\ 0_{nRX+nMX+nTX+2} & V_{nzRX+nMX+nTX} & 0_{nRX+nzMX+nTX+1} \\ 0_{nRX+nMX+nTX+2} & 0_{nzRX+nMX+nTX+1} & V_{nRX+nzMX+nTX} \\ 0_{nRX+nMX+nTX+2} & 0_{nzRX+nMX+nTX+1} & 0_{nRX+nzMX+nTX+1} \end{matrix}\right.$$

$$\left.\begin{matrix} 0_{nRX+nMX+nzTX+1} & -S_{11}V_{nRX+nMX+nTX+1} \\ 0_{nRX+nMX+nzTX+1} & -S_{21}V_{nRX+nMX+nTX+1} \\ 0_{nRX+nMX+nzTX+1} & -S_{31}V_{nRX+nMX+nTX+1} \\ V_{nRX+nMX+nzTX} & -S_{41}V_{nRX+nMX+nTX+1} \end{matrix}\right] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b \end{bmatrix} = 0 \qquad (8)$$

The total number of the unknowns is

$$\begin{aligned} &nRX+nMX+nTX+2+nzRX+nMX+nTX+1+nRX+nzMX \\ &+nTX+1+nRX+nMX+nzTX+1+nRX+nMX+nTX+2, \end{aligned}$$

here the total least square (TLS) method is also used to solve the system matrix, In order to guarantee the system has solution, the TLS method requires that Ns must be greater or equal to

$$\begin{aligned} &nRX+nMX+nTX+2+nzRX+nMX+nTX+1+nRX+nzMX \\ &+nTX+1+nRX+nMX+nzTX+1+nRX+nMX+nTX+2-1, \end{aligned}$$

that is

$$4*(nRX+nMX+nTX)+nzRX+nzMX+nzTX+6.$$

All coefficients can be obtained only in one step.

4. NUMERICAL EXAMPLES: DUPLEXER AND TRIPLEXER

4.1. Synthesized Test Duplexer

The synthesized scattering parameters of duplexer are used in order to test the performance of the novel modified Cauchy method proposed in this paper. We use the method in [9, 10] to determine the characteristic polynomials of the test duplexer. The modified Cauchy method can be used here to reconstruct the same polynomials. Then the rational responses are compared between the synthesized and reconstructed polynomials.

Table 1. The relative error between extracted and original coefficients of tested duplexer.

$E_{rel}\,(a_1)$	$E_{rel}\,(a_2)$	$E_{rel}\,(a_3)$	$E_{rel}\,(b)$
7.3268e-006	2.7197e-008	6.0128e-008	3.3395e-005

In the duplexer example, the synthesized duplexer is carried out in a normalized frequency domain. In this example, the RX channel filter has eight poles. The transmission zeros are $-j1.221$, $j0.1518$, $j0.2006$, $j0.3451$, and the return loss is 22 dB. The TX channel filter has seven poles. The transmission zeros are $-j0.3761$, $-j0.1692$, $-j0.1006$, and the return loss is 22 dB. The novel approach to the synthesis of microwave duplexer in [9] is used to obtain the original characteristic polynomials. The number of complex unknowns (the total number of the extracted characteristic polynomials coefficients) of the duplexer system is 58, and the number of sampled frequency points is 80. The synthesized duplexer response and the reconstructed polynomial model responses are presented in Fig. 3. The red solid lines represent the synthesized response, and other lines represent the extracted polynomials response.

In fact, the responses obtained from the polynomials computed with all method are different from the original one. In order to express the difference between the two responses, the relative error is presented. The notation $E_{rel}\,(x)$ refers to the 2-norm relative error between the original coefficients and extracted solutions for the parameter x. the details are shown in Table 1.

From the table above, it can be seen that the difference between the extracted and original coefficient is so small that it can be neglected. The example shows the validity of the modified Cauchy method for duplexer.

4.2. Synthesized Test Trilexer

In the triplexer example, the synthesized triplexer is carried out in a normalized frequency domain, as in [11]. The example of the test triplexer is specified as follows: the RX channel filter has five poles. The transmission zeros are placed at $-j1.5134$, $j0.0104$, and the return loss is 20 dB. The MX channel filter has six poles. The transmission zeros are placed at $-j0.7454$, $j0.7545$, and the return loss is 20 dB. The TX channel filter has five poles. The transmission zeros are placed at $j0.2597$, $j1.2443$, and the return loss is also 20 dB. We use the synthesis method in [11] to obtain the original characteristic polynomials of the triplexer. The number of complex unknowns (the total number of

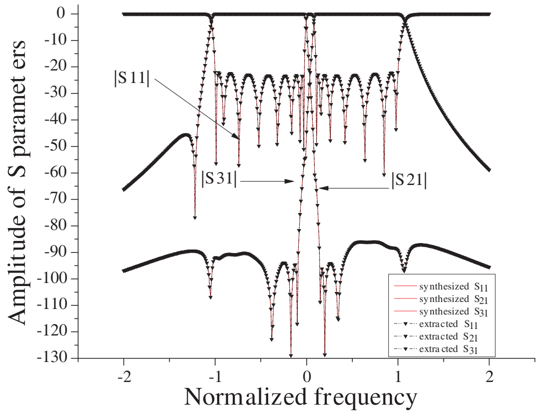

Figure 3. Attenuation and return loss of the duplexer.

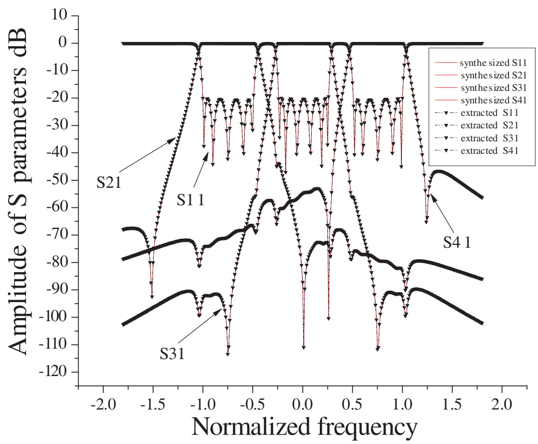

Figure 4. Attenuation and return loss of the triplexer.

the extracted characteristic polynomials coefficients) of the triplexer system is 77, and the number of sampled frequency points is 80.

The synthesized and extracted responses are showed in Fig. 4. The red lines represent the synthesized response, and the other lines represent the extracted polynomial response. We can see that the extracted polynomial response agrees with original response well. The relative error between the original and extracted coefficients is shown in Table 2.

We could see that there is almost no difference between the synthesized and extracted polynomials responses. A very satisfactory

Table 2. The relative error between extracted and original coefficients of tested triplexer.

$E_{rel}\,(a_1)$	$E_{rel}\,(a_2)$	$E_{rel}\,(a_3)$	$E_{rel}\,(a_4)$	$E_{rel}\,(b)$
2.5364e-007	3.7133e-010	3.8113e-011	5.3391e-010	5.2902e-007

agreement between the original and extracted coefficients can be observed. And the relative error is very small and can be neglected. The validity is shown in this triplexer example.

5. CONCLUSION

A modified Cauchy method suitable for microwave duplexer and triplexer whose channel filter is connected through the resonator junction is presented. Compared with the conventional technique, the modified technique has two advantages: first, it can generate the numerator and denominator coefficients in one step, without requiring the reconstruction the common poles; second, the new method can handle the lossy case. Two numerical examples are presented. The extracted rational polynomials responses show the validity of this new technique. The modified Cauchy method can also be applied to the same type of multiplexers (channel filters connected through a resonator junction).

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (NSFC) under project No. 60901031 and the Fundamental Research Funds for the Central Universities No. 72005477.

REFERENCES

1. Wang, R. and J. Xu, "Extracting coupling matrix and unloaded Q from scattering parameters of lossy filters," *Progress In Electromagnetics Research*, Vol. 115, 303–315, 2011.
2. Wang, R., J. Xu, C. L. Wei, M.-Y. Wang, and X.-C. Zhang, "Improved extraction of coupling matrix and unloaded Q from S-parameters of lossy resonators," *Progress In Electromagnetics Research*, Vol. 120, 67–81, 2011.
3. Garcia-Lamperez, A., T. K. Sarkar, and M. Salazar-Palma, "Generation of accurate rational models of lossy systems using the

Cauchy method," *IEEE Microw. Wireless Compon. Lett.*, Vol. 14, No. 10, 490–492, Oct. 2004.

4. Macchiarella, G. and D. Traina, "A formulation of the Cauchy method suitable for the synthesis of lossless circuit models of microwave filter from lossy measurements," *IEEE Microw. Wireless Compon. Lett.*, Vol. 16, No. 5, 243–245, May 2006.
5. Traina, D. and G. Macchiarella, "Robust formulation of the Cauchy method suitable for microwave duplexers modeling," *IEEE Trans. on Microw. Theory and Tech.*, Vol. 55, No. 5, 974–982, May 2007.
6. Peik, S. F., R. R. Mansour, and Y. L. Chow, "Multidimensional Cauchy method and adaptive sampling for an accurate microwave circuit modeling," *IEEE Trans. on Microw. Theory and Tech.*, Vol. 46, No. 12, 2364–2371, Dec. 1998.
7. Basl, P. A. W., R. H. Gohary, M. H. Bakr, and R. Mansour, "Modeling of electromagnetic responses using a robust multidimensional Cauchy interpolation technique," *IET Microwaves, Antennas & Propagation*, Vol. 4, No. 11, 1955–1964, 2010.
8. Shaker, G. S. A., M. H. Bakr, N. Sangary, and S. Safavi-Naeini, "Accelerated antenna design methodology exploiting parameterized Cauchy models," *Progress In Electromagnetic Research B*, Vol. 18, 279–309, 2009.
9. Macchiarella, G. and S. Tamiazzo, "Novel approach to the synthesis of microwave diplexers," *IEEE Trans. on Microw. Theory and Tech.*, Vol. 12, 4281–4290, Dec. 2006.
10. Macchiarella, G. and S. Tamiazzo, "Synthesis of star-junction multiplexers," *IEEE Trans. on Microw. Theory and Tech.*, Vol. 58, 3732–3741, Dec. 2010.
11. Macchiarella, G. and S. Tamiazzo, "Design of triplexer combiners for base stations of mobile communications," *IEEE MTT-S Int. Microw. Symp. Dig.*, 429–432, Anheim, CA, May 2010.