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APPLICATION OF ELECTROMAGNETIC RECIPROCITY PRINCIPLE TO THE COMPUTATION OF SIGNAL COUPLING TO MISSILE-LIKE STRUCTURES

K. Yegin*

Department of Electrical and Electronics Engineering, Yeditepe University, Room A-610, Kayisdagi, Istanbul 34755, Turkey

Abstract—Lorentz Reciprocity principle is often used to describe electrical networks and reception/radiation properties of antennas residing in a linear, time-invariant, and symmetric medium. In its reaction integral form, it is usually conceived as a mathematical tool to prove electromagnetic relations. However, reciprocity, more than a mathematical tool, can be used as a powerful alternative to convert a penetration problem into a radiation one for numerical computations and measurements. We review the reciprocity formulation and show simple steps on how to apply reciprocity to penetration problems. Numerical calculations for a wire probe (antenna) inside missile-like structure are carried out for both radiation and its reciprocity formulated penetration problems, and it is shown numerically that results from both methods are identical. One of the advantages of this indirect formulation is that the radiation properties of the structure can be easily measured contrary to the direct measurement of the penetrated signal inside the structure.

1. INTRODUCTION

In classical electromagnetism, Lorentz Reciprocity principle is conceived as the interchange of time-harmonic sources and their resultant electromagnetic fields in a medium where its permittivity and permeability tensors are time-invariant, linear and symmetric. Reciprocity principle serve a good basis for the proof of various aspects electromagnetic systems such as symmetry in Green's functions [1], scattering matrix formulation of microwave networks [2], propagating modes in waveguide structures [3], and identical reception and radiation property of antennas [4].

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* Corresponding author: Korkut Yegin (kyegin@yeditepe.edu.tr).

Application of reciprocity principle to recast an original problem to an alternate one can also be very useful for certain type of problems [5–10]. Electromagnetic signal coupling to a perfect electric conductor (PEC) through a small opening due to an external source is one of them [11–12]. In addition, measurement of received signal inside the PEC object can be very tedious and limited. This is a common problem in the assessment electromagnetic compatibility and interference tests of devices. On the other hand, assessment of the level of interference caused by the interaction of the penetrated electromagnetic field with the inside circuitry is extremely important to circuit designers who must assure proper operation of internal electronic systems. A detailed analysis for the interaction of a general body with an electromagnetic field and the determination of penetrated fields can be very complex and highly demanding of resources. Fortunately, many structures of practical interest can be modeled as axi-symmetric (or body of revolution), e.g., satellites, missiles, etc., at least to a first order approximation of their actual shape. The theoretical analysis and the numerical solution procedure for electromagnetic field coupling to a general axi-symmetric structure have received much attention in the past and very efficient computational tools have been developed [13–15]. In this investigation, reciprocity is employed to compute the signal at an interior load which terminates a coaxial guide (transmission line) connected to a wire probe antenna mounted axi-symmetrically on a missile-like structure. This is carried out for both dipole and plane wave illuminations.

A typical example for an axi-symmetric structure, a mock missile, excited by a known electric field is illustrated in Fig. 1(a). Mock missiles serve a good basis for the analysis because they are designed to be immune to jamming signals and only a small amount of exterior signal can leak in. The received signal strength for specified illumination and load is of primary interest. The direct approach to solving this problem would be to determine the current induced on the wire probe and the body due to plane wave illumination or elementary dipole excitation and, subsequently, use this current to compute the signal at Y_L. Typically, Y_L represents the admittance of the inside circuitry seen from the coaxial guide. This current could be found as the solution of an integral equation which must account for the load at the terminal end of the coax and its effect at the annular aperture formed where the coax joins the top-end of the mock missile. The indirect approach adopted here makes use of the reciprocity theorem and allows one to compute the signal at the coax terminal load from knowledge of the field radiated by the mock missile, under the condition that the excitation results from a current generator impressed at the terminal end of the coax. This solution procedure utilizes a simpler integral equation. It is simpler because the integral equation is not a vector equation as it would be if the direct approach were followed, and the surface current to be computed possesses rotational symmetry.

Also, in the direct approach one typically needs to solve the scattering problem for each angle of illumination in order to determine the signal strength at the receiving end of the probe. This entails additional work if the computations are carried out for many degrees of illumination angle. The same results can be obtained in a much more efficient way by using the indirect approach, in which case the integral equation is to be solved only once and the computed electric current on the structure is repeatedly used to evaluate the signal strength that would exist in the reception problem for every desired angle of illumination. Hence, the computational complexity of the original problem is considerably reduced. Of course, one can improve the efficiency of the direct approach if one recognizes that the axially located thin wire couples only to the zero order axi-symmetric currents.

2. RECIPROCITY PRINCIPLE

In its simplest form the reciprocity theorem states that a response of a system to a source is unchanged when source and measurer are interchanged [16]. The general form for the time harmonic fields ($e^{j\omega t}$ time variation) can be derived from Maxwell's equations. Let $\mathbf{E}_a$, $\mathbf{H}_a$ and $\mathbf{E}_b$, $\mathbf{H}_b$ be the electric and magnetic fields generated by the sources $\mathbf{J}_a$, $\mathbf{M}_a$ and $\mathbf{J}_b$, $\mathbf{M}_b$, of the same frequency and existing in the same linear medium. Then, the Lorentz reciprocity theorem is

$$\begin{aligned} &-\oint\!\!\!\oint_S (\mathbf{E}_a \times \mathbf{H}_b - \mathbf{E}_b \times \mathbf{H}_a) \cdot \hat{\mathbf{n}} dS \\ &= \iiint_V (\mathbf{E}_a \cdot \mathbf{J}_b + \mathbf{H}_b \cdot \mathbf{M}_a - \mathbf{E}_b \cdot \mathbf{J}_a - \mathbf{H}_a \cdot \mathbf{M}_b) dV \end{aligned} \tag{1}$$

where V is the volume containing the sources and S is the closed surface bounding this volume.

The axi-symmetric structure of Fig. 1(a) can be either excited by a plane wave with an arbitrary angle of incidence or by an elementary dipole moment $I\ell$ represented by

$$\mathbf{J}_d = I\ell\delta\left(\mathbf{r} - \mathbf{r}_d\right)\hat{\mathbf{l}} \tag{2}$$

and located at an arbitrary point $\mathbf{r}_d$ in space. An admittance Y_L terminates the end of the coax remote from the antenna and inside the mock-missile. The desired result, as mentioned above, is the signal induced in the terminating admittance due to either plane wave or dipole illumination. In order to apply reciprocity theorem, we now consider a second source and resulting radiated field. This source is an ideal current generator of I_g impressed at the terminal end of the coax. This current generator, located very close to the admittance Y_L, produces a signal at the coax which, in turn, excites the antenna and mock-missile and gives rise to a radiated field which we call $\mathbf{E}_g$, as suggested in Fig. 1(b). The transmission line model of the coax and the admittance Y_L is shown in Fig. 2. The current generator is taken to be a volume current of density

$$\mathbf{J}_g = -\delta\left(z - z_g\right)\left[\frac{I_g}{2\pi\rho}\right]\hat{\boldsymbol{\rho}} \tag{3}$$

impressed near the end of the wire probe at position $z = z_g$ as illustrated in Fig. 1(b).

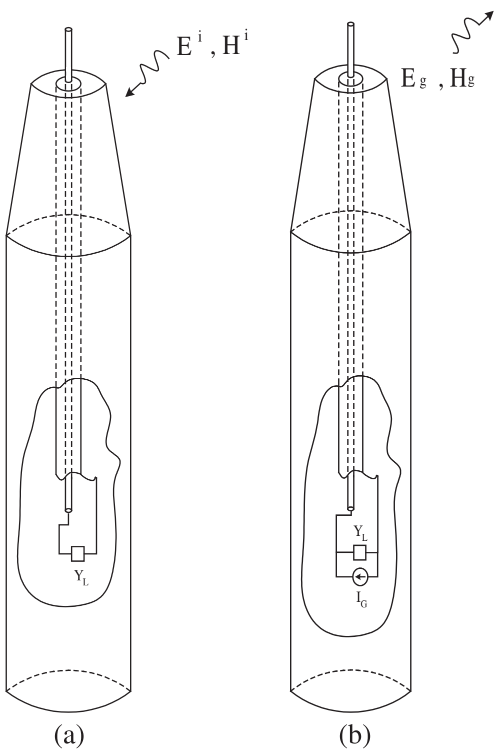

Figure 1. The mock-missile. (a) Excited by a plane wave, reception problem. (b) Excited through a coaxial probe, radiation problem.

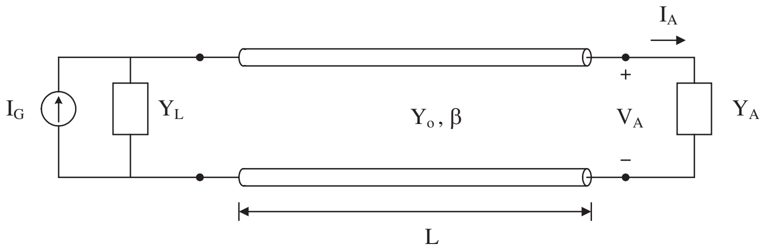

Figure 2. Transmission line model of the coax, the current generator, load Y and antenna Y_A.

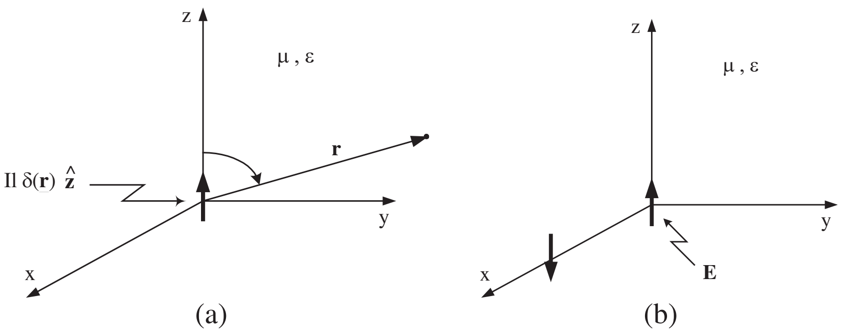

Figure 3. (a) A z directed dipole at the origin. (b) A $-z$ directed dipole on x axis in the far zone creates a plane wave at the origin.

To apply reciprocity, the form of the sources must be known explicitly. In the case of an elementary dipole, the source is expressed as in (2). In the case of a plane wave with an arbitrary angle of incidence, however, the source representation is not as obvious as that of an elementary dipole. It must be that source which produces the plane wave of interest in the local vicinity of the body. For a dipole current moment of $I\ell\hat{\mathbf{z}}$ at the origin as illustrated in Fig. 3(a), it is easy to show that in the far zone the electric field is

$$\mathbf{E}_\theta = j\omega\,\mu I\ell\frac{e^{-jkr}}{4\pi r}\sin\theta \tag{4}$$

where ω is the angular frequency, μ is magnetic permeability of the medium, and k represents the wave number. Except on the z axis when E_θ above is zero, the r component of $\mathbf{E}$ is a factor $1/r$ smaller than E_θ, so, if the z axis case is excepted, the E_θ above represents the total far-zone electric field. On the x axis, $E_\theta(= -E_z)$ becomes

$$E_\theta = j\omega\mu I\ell\frac{e^{-jkx}}{4\pi x}\sin\theta \tag{5}$$

This has all the properties of a plane wave in the vicinity of the x axis in the far zone. It is easy to show that a $-x$ directed dipole on the x axis in the far zone, as depicted in Fig. 3(b), creates a plane wave at the origin with its $\mathbf{E}$ vector in the z direction. The strength and the phase of this plane wave is

$$E = jk\eta I\ell \frac{e^{jkx}}{4\pi x} \tag{6}$$

where η represents intrinsic impedance of the medium. We generalize this notion to an arbitrary incidence angle. A θ^i directed dipole at a point $\left(r^i, \theta^i, \phi^i\right)$ in the far-zone creates a plane wave in the vicinity of the origin with strength

$$\mathbf{E} = jk\eta I\ell \frac{e^{jkr^i}}{4\pi r^i}\,\hat{\boldsymbol{\theta}}^i \tag{7}$$

where the plane wave and the unit vector $\hat{\boldsymbol{\theta}}^{\,i}$ are as illustrated in Fig. 4(a) and $r^{\,i}$ is the distance from the origin to the location of the dipole. With the dipole located in the xz plane and in the $\hat{\boldsymbol{\theta}}^{\,i}$ direction, the incident wave travels inward along the ray in the xz plane as depicted in Fig. 4(b). If the dipole field is designated $\mathbf{E}^d$, then the electric field due to the dipole $\mathbf{J} = I\ell\delta\left(r - r^i\right)\hat{\boldsymbol{\theta}}^i$ is,

$$\mathbf{E}^d = jk\eta I\ell \frac{e^{jkr^i}}{4\pi r^i}\,\hat{\boldsymbol{\theta}}^i \tag{8}$$

where $\mathbf{r}^i = r^i\,\hat{\mathbf{r}}^i$, in which $\hat{\mathbf{r}}^i$ is the outward unit vector along the ray defined by $\left(\theta^i, \phi^i = 0\right)$. If we adjust $I\ell$ to be

$$I\ell = \frac{4\pi}{jk\eta} r^i \varepsilon_\theta^i, \tag{9}$$

then the dipole creates an electric field

$$\mathbf{E}^d = \varepsilon_\theta^i\, e^{jkr^i}\,\hat{\boldsymbol{\theta}}^{\,i} \tag{10}$$

in the vicinity of origin. The electric field $\mathbf{E}^d$ has a strength ε_θ^i, propagates along a ray defined by $r^{\,i}$, and is directed in the $\hat{\boldsymbol{\theta}}^{\,i}$ direction. In summary to create a plane wave of strength ε_θ^i in the $\hat{\boldsymbol{\theta}}^{\,i}$ direction in the local vicinity of the origin, one can place a dipole of current moment given in (9) at $\left(r^{\,i}, \theta^{\,i}, \phi^{\,i} = 0\right)$ in the far-zone. The direction of the dipole is $\hat{\boldsymbol{\theta}}^{\,i}$ and the plane wave propagates inwardly along the ray from the origin to $\left(r^{\,i}, \theta^{\,i}, 0\right)$. The volume current density of this dipole is

$$\mathbf{J}_p = I\ell\delta\left(\mathbf{r} - \mathbf{r}^i\right)\hat{\boldsymbol{\theta}}^i. \tag{11}$$

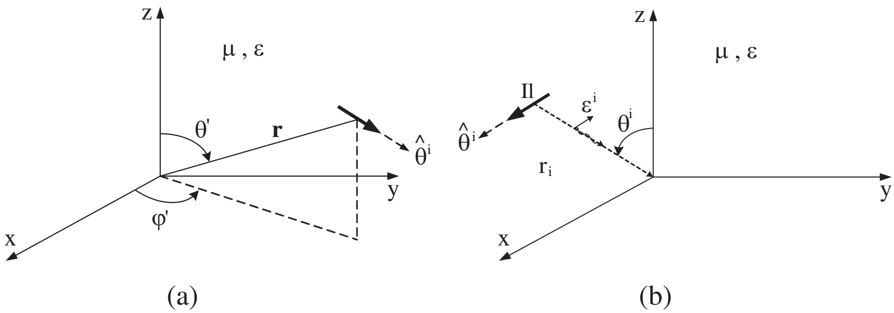

Figure 4. (a) $\hat{\boldsymbol{\theta}}^i$ directed dipole at the far zone. (b) $\hat{\boldsymbol{\theta}}^i$ directed dipole on xz plane at the far zone.

Having obtained appropriate source representations for both types of excitation, we can now apply the reciprocity theorem by considering two different experiments. In the first, the remote source radiates a field causing a voltage V_L to appear across the load admittance Y_L depicted in Fig. 1(a). Second, the source is removed and current I_g is impressed at the coax terminals by a current generator as suggested in Fig. 1(b). This generator excites the coax-fed wire antenna and the mock-missile and creates an exterior electric field designated $\mathbf{E}_g$. Upon application of (1) to these sources and fields, one obtains

$$-\oiint_S (\mathbf{E}_g \times \mathbf{H} - \mathbf{E} \times \mathbf{H}_g) \cdot \hat{\mathbf{n}} dS = \iiint_V (\mathbf{E}_g \cdot \mathbf{J} - \mathbf{E} \cdot \mathbf{J}_g) dV \qquad (12)$$

where V is the region in which all the sources are enclosed and S is the closed surface bounding this region. $\mathbf{J}_g$ and $\mathbf{J}$ are the sources that produce ($\mathbf{E}_g$, $\mathbf{H}_g$) and ($\mathbf{E}$, $\mathbf{H}$), respectively. V is the volume bounds the load inside the structure, outside the wire probe antenna and the body of the axi-symmetric structure, and inside an imaginary sphere at infinity. Let $S = S_\infty + S_{PEC}$, where S_∞ is the sphere at infinity and S_{PEC} is the remainder of S. Since the wire probe antenna structure and the axi-symmetric structure are perfectly electric conductors, the surface integral over S_{PEC} is zero, and, due to the radiation condition, the integral over the sphere at infinity S_∞ is zero too. Therefore the surface integral in (12) is zero and one is left with

$$\iiint_V (\mathbf{E}_g \cdot \mathbf{J} - \mathbf{E} \cdot \mathbf{J}_g) dV = 0 \qquad (13)$$

where $\mathbf{E}$ is the electric field caused by a plane wave or an elementary dipole illuminating the structure and $\mathbf{J}_g$ is given by (3). The evaluation of (13) may be carried out for both types of excitation. Let's first consider the dipole case. Since the elementary dipole $\mathbf{J}_d = I\ell\delta\left(\mathbf{r}-\mathbf{r}_d\right)\hat{\mathbf{l}}$ radiating in the presence of the structure is a delta function, the first term of (13) is evaluated as

$$\iiint\limits_V \mathbf{E}_g \cdot \mathbf{J}_d dV = \iiint\limits_V \mathbf{E}_g \cdot \hat{\mathbf{l}}\left[I\ell\delta\left(\mathbf{r}-\mathbf{r}_d\right)\right]dV = I\ell\mathbf{E}_g\left(\mathbf{r}_d\right)\cdot\hat{\mathbf{l}}. \quad (14)$$

For the case of an incident plane wave with electric field strength ε_θ^i in the local vicinity of the origin in $\hat{\boldsymbol{\theta}}^i$ direction, we consider an elementary dipole given by (11) in the $\hat{\boldsymbol{\theta}}^i$ direction and the first term of (17) is

$$\iiint\limits_V \mathbf{E}_g \cdot \mathbf{J}_p\, dV = \iiint\limits_V \mathbf{E}_g \cdot \hat{\mathbf{q}}^i \left[I\ell\delta\left(\mathbf{r}-\mathbf{r}^i\right)\right]dV = I\ell\mathbf{E}_g\left(\mathbf{r}^i\right)\cdot\hat{\mathbf{q}}^i. \quad (15)$$

The coax is operated in a typical way in which all higher order modes are below cut-off. Thus, the fields and the currents in the coax are circularly symmetric which implies that $\mathbf{E}$ is independent of ϕ. Since $\mathbf{J}_g$ given by (3) is also a delta function, the second term in (13) simplifies immediately to

$$\begin{aligned}\iiint\limits_V \mathbf{E}\cdot\mathbf{J}_g dV &= -I_g\int\limits_{-\pi}^{\pi}\int\limits_a^b \frac{1}{2\pi\rho}\hat{\rho}\cdot\mathbf{E}\left(\rho,z_g\right)\rho d\rho d\phi \\ &= -I_g\int\limits_{-\pi}^{\pi}\int\limits_a^b \frac{1}{2\pi}\mathbf{E}_\rho\left(\rho,z_g\right)d\rho d\phi = -I_g V_g \quad (16)\end{aligned}$$

where a is the radius of the inner conductor and b is radius the outer conductor and V_g is the potential of the coax center conductor relative to that of the outer conductor and it is expressed as

$$V_g = \int\limits_a^b E_\rho\left(\rho, z_g\right)d\rho. \quad (17)$$

From (14) and (16) it is clear that one can obtain V_g, the voltage created across the coax by the dipole, from the knowledge of elementary dipole excitation and the electric field $\mathbf{E}_g$ which results from the current generator applied at the coax terminals. This V_g due to dipole excitation is denoted as V_g^d. Finally, (14) and (16) are inserted to the integrals involved in reciprocity theorem given by (13), one arrives at

$$V_g^d = -I\ell\,\hat{\mathbf{l}}\cdot\mathbf{E}_g(\mathbf{r}_d)/I_g. \quad (18)$$

Similarly, using (15) and (16) in (13), one is able to obtain V_g, from the knowledge of plane wave illumination and the electric field $\mathbf{E}_g$ and this V_g is denoted as V_g^p and it is given by

$$V_g^p = -I\ell\,\hat{\boldsymbol{\theta}}^i \cdot \mathbf{E}_g(\mathbf{r}^i)/I_g. \tag{19}$$

3. APPLICATION PROCEDURE OF RECIPROCITY PRINCIPLE

The procedure for using reciprocity to compute the signal at Y_L due to a dipole or plane wave illumination is outlined below.

Step 1: First, a voltage of one volt ($V_A = 1\,\mathrm{V}$) across the coaxial aperture is assumed. With this excitation, the current on the wire antenna and the structure are obtained numerically by solving an integral equation. From the knowledge of this current, driven by a one volt generator, one can compute the input impedance of the antenna Y_A at the base of the antenna. Since the antenna is driven by the coax, Y_A can be viewed as the terminating admittance of the end of the coax where its center conductor becomes the antenna. With the currents known on the wire and on the structure, one can easily compute the radiated electric field due to this source, i.e., $\mathbf{E}_g$ in (18) and (19). However, this is not the correct field because V_A is assumed $1\,(\mathrm{V})$. If I_g can be related to actual V_A, then the fields produced by V_A can be used in (18) and (19).

Step 2: From transmission line theory, it can be easily shown that the current generator causes the voltage V_A in the coaxial aperture at the base of the antenna to be

$$V_A = \frac{(1+\Gamma_A)\,e^{-j\beta L}}{(Y_L+Y_o)+(Y_L-Y_o)\,\Gamma_A e^{-j2\beta L}} I_g \tag{20}$$

where β is the propagation factor of the lossless transmission line and Γ_A is the reflection coefficient at the antenna base (seen by the coax). It is expressed as

$$\Gamma_A = \frac{Y_o - Y_A}{Y_o + Y_A} \tag{21}$$

where Y_o is the characteristic admittance of the line and is taken as $Y_o = 1/50\ \left(\Omega^{-1}\right)$ throughout the computations. Now, the value of I_g is known due to any V_A.

Step 3: The actual voltage driving the antenna is V_A, which now can be used to compute the actual currents on the structure, and the radiated field caused by the current generator. Recall that the currents and the fields are computed with the assumption of $V_A = 1\,(\mathrm{V})$ so one can simply scale all the currents and field values by the ratio of $V_A : 1$.

The last step is to compute the voltage V_g (V_g^d or V_g^p) across the load Y_L using (18) or (19). The final expression for V_g is

$$V_g = \frac{-I\ell\,\hat{\mathbf{u}}\cdot\mathbf{E}_g\left(\mathbf{r}_u\right)\left(1+\Gamma_A\right)e^{-j\beta L}}{V_A\left[\left(Y_L+Y_o\right)+\left(Y_L-Y_o\right)\Gamma_A e^{-j2\beta L}\right]} \tag{22}$$

where $\hat{\mathbf{u}}$ is either $\hat{\mathbf{l}}$ of (2) or $\hat{\boldsymbol{\theta}}^i$ of (11), and $\mathbf{r}_u$ is either $\mathbf{r}_d$ or $\mathbf{r}^i$. As (22) represents most general case, for numerical computations, V_A is taken 1, which enables one to calculate Y_A (and Γ_A) and V_g easily for any value of dipole location or any angle of plane wave incidence. It is important to restate that V_A can take any arbitrary value other than 1, and V_g will remain unchanged because it is scaled with V_A.

4. NUMERICAL RESULTS AND DISCUSSION

Consider the mock missile as illustrated in Fig. 5. The body of missile-like structure comprises a straight, hollow, cylindrical section with a tapered upper end and a closed bottom end. The structure possesses a bulk-head at the place where the cylindrical section joins to the nose of the mock-missile. The wire probe is mounted at the center of the bulk-head and receives signal through a circular opening on top part of the mock-missile. The antenna is actually the extension of a coaxial cable center conductor whose opposite end in the missile interior terminates in a load admittance Y_L, which in frequency domain represents the input admittance of an instrument inside the missile. The receiver is inside the missile, and thereby is shielded from the field in the exterior region. All numerical simulations are carried out using a boundary element integral equation solver (Method of Moments) for axi-symmetric structures [13–15]. The numerical results using two different formulations of the same problem are compared to each other. The direct solution technique, which solves the scattering problem, is designated as *lumped-load method* whereas the indirect solution technique which uses the reciprocity approach, is designated as *reciprocity method.* The magnitude of the received signal normalized to the incident field at 100 MHz is computed for various angles of illumination using both methods. The results are depicted in Fig. 6(a). Excellent agreement is observed between the two methods. The same computation is performed at a higher frequency, 300 MHz. The results are depicted in Fig. 6(b). Again, both methods produced almost identical results. When the values of load voltage at two different frequencies are compared to each other, it is observed that the signal coupling to load becomes a little stronger at 300 MHz that at 100 MHz.

Next, an elementary dipole illumination is considered. The dipole is located on xz plane and it is displaced along a line parallel to z

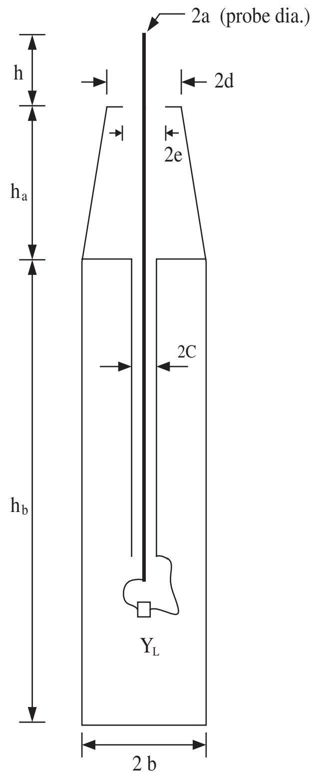

Figure 5. The cross sectional view of the structure (dimensions are: $h = 5.35\,\text{cm}$, $h_b = 118.7\,\text{cm}$, $h_a = 14.6\,\text{cm}$, $a = 0.0787\,\text{cm}$, $b = 7.875\,\text{cm}$, $c = 0.2286\,\text{cm}$, $d = 4.25\,\text{cm}$, $e = 2.25\,\text{cm}$.)

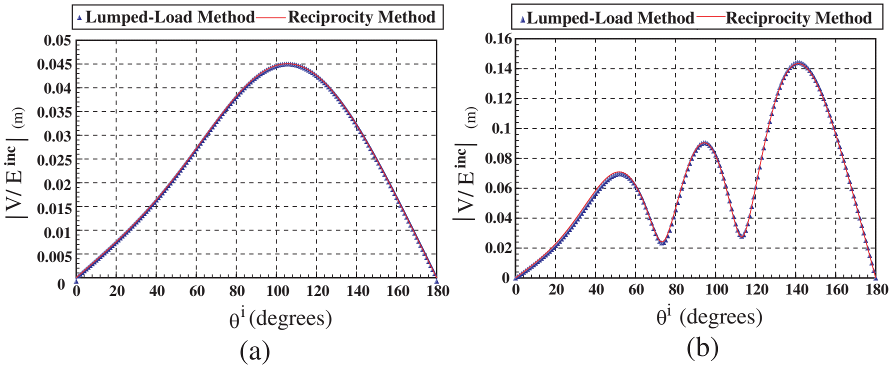

Figure 6. Plane wave illuminations. (a) 100 MHz. (b) 300 MHz.

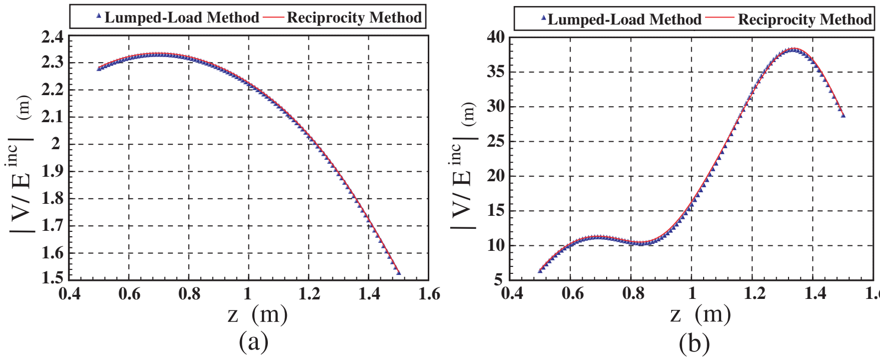

Figure 7. Dipole illumination with dipole being located on xz plane at $x = 32.875$ cm, but is displaced along a line parallel to z axis. (a) 100 MHz. (b) 300 MHz.

axis. The variation of load voltage with the dipole displacement is studied. The results are obtained at operation frequencies of 100 MHz and 300 MHz and illustrated in Figs. 7(a) and 7(b). Again an excellent agreement is observed between the two methods.

5. CONCLUSIONS

We have shown that a penetration problem can be cast into a radiation one using Lorentz reciprocity principle. An axi-symmetric mock-missile is chosen for the application and comparisons are made using direct (lumped element load) and indirect (reciprocity) methods. The indirect approach produced almost identical results with that of direct approach using scattering formulation of the problem. Consequently, we claim that reciprocity method is exact and rigorous, and it is more than a mathematical tool to prove electromagnetic relations, but rather an applied tool, particularly for the computation of signal coupling to axi-symmetric structures. Indirect formulation using reciprocity is also favorable in measurements of weak signal coupling to general structures (dielectric or perfect electric conductors) penetrated fields because radiation properties of a structure is much easier to measure than those of penetrated fields into the structure.

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