Progress In Electromagnetics Research M, Vol. 24, 15–27, 2012

SCATTERING OF AN ARBITRARILY ORIENTED ELECTRIC DIPOLE FIELD FROM AN INFINITELY LONG DB CIRCULAR CYLINDER

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Abstract—Analytic expressions for the scattered magnetic vector potential from an infinitely long DB circular cylinder are presented. An arbitrarily oriented electric dipole is considered as a source of excitation that induces surface currents on the DB circular cylinder. Approximate far field expressions for magnetic vector potential are also derived in this setting. Numerical results of the scattering from the DB cylinder are also presented and compared with those of the PEC cylinder.

1. INTRODUCTION

In electromagnetics, variety of boundary conditions are used to characterize the scattering from objects of varying materials and geometry. For a perfect electric conductor (PEC) boundary, condition forces the tangential components of total electric field to be zero. Its dual case is the perfect magnetic conductor (PMC) with vanishing tangential components of the total magnetic field. PEC and PMC are special cases of more general impedance surface which can be described through the dyadic relation between the tangential components of electric and magnetic fields. In connection with metamaterials studies, more complex metasurfaces have also been suggested. Among those are the the perfect electromagnetic conductor (PEMC) surface and the so-called DB boundary with its generalization [1]. It is useful to mention here that all boundaries except DB, described above, deal with the tangential components of electric and/or magnetic fields. In contrast,

Received 1 December 2011, Accepted 1 March 2012, Scheduled 12 March 2012

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the DB boundary incorporates conditions on the normal components of flux densities **D** and **B**. The idea of DB boundary conditions was first proposed by Rumsey [2]. The boundary conditions for the DB interface are given below [3].

$$\hat{\mathbf{n}} \cdot \mathbf{D} = 0, \quad \hat{\mathbf{n}} \cdot \mathbf{B} = 0$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the surface.

The DB boundary has some interesting properties including the one which states that surface becomes transparent for the normal incident plane wave. Hence, DB boundary can act as a spatial filter [3]. Such a filter would allow penetration of incoming electromagnetic waves with normal incidence (regardless of the polarization) only. All the waves with other angles of incidence will experience total reflection from a DB boundary. Lindell and Sihvola showed a planar DB boundary, when excited by transverse electric (TE) and transverse magnetic (TM) modes in free space, behaves like a PEC interface for TE mode of excitation and PMC interface for TM mode of excitation, respectively [4]. In [5], Lindell and Sihvola discussed a spherical resonator with DB boundary conditions. In [6], circular waveguide with DB boundary conditions has been studied by Lindell and Sihvola. Naqvi, et al. in [7] observed that DB interface is a reflector which can yield non-zero power propagation in chiral nihility metamaterial ($\epsilon = 0$, $\mu = 0$, $\kappa \neq 0$) where κ represents chirality parameter and ϵ and μ are relative permittivity and permeability of the medium, respectively. Despite simplicity in analysis, DB boundary has remarkable in various applications including the subject of invisibility cloak [8, 9].

Scattering of electromagnetic waves from a circular cylinder has been studied by various researchers during past few decades [10–18]. The cylinder is considered a model representative scatterer and it has been frequently used as a reference to characterize the peculiar and distinctive scattering properties of various targets like aircrafts, submarines and missiles etc.. In this background, the scattering of dipole field from cylinders is a topic of great practical interest [14–18]. It has been shown by Carter [14] that the directional properties of linear antennas are affected greatly in the presence of circular cylinder of proper radius. Carter in [14] argued that when an array of such antennas is fed with proper phase more directive patterns are obtained. Luke derived similar results using Green function method [15]. In [16], Wait considered a radial dipole in cylindrical wedge region and compared theoretical and experimental radiations patterns in the principle plane. Basso et al. determined the dipole scattering amplitude in momentum space [17]. Illahi, et al. discussed the electromagnetic scattering from an infinitely long perfect electric conductor (PEC) cylinder/perfect electromagnetic conductor (PEMC) cylinder due to an arbitrarily oriented dipole [18, 19]. Illahi, et al. also discussed that the scattering of dipole field from PEMC circular cylinder has many applications as a non reciprocal scatterer and polarization control of antenna.

The aforementioned importance of DB boundary, circular cylinder and dipole motivated the authors to analyze the problem of electromagnetic scattering from a DB circular cylinder. For this purpose an electric dipole is taken as a source of excitation. To simplify our problem, we have considered an infinitely long DB circular cylinder. Electric dipole produces surface currents on DB circular cylinder. The scattered magnetic vector potential is calculated at the far-zone region after applying the method of steepest descent.

2. PROBLEM FORMULATION

A DB circular cylinder of radius a and infinite extent in z-direction is placed in free space, as depicted in Figure 1. The constitutive relations for fields in free space are given as

$$\mathbf{D} = \varepsilon_o \mathbf{E} \qquad \mathbf{B} = \mu_o \mathbf{H}$$

where μ_o and ϵ_o represent permittivity and permeability, of free space, respectively. An electric dipole located at $\mathbf{r_o} = (\rho_o, \phi_o, z_o)$ is considered as a source of excitation.

We employ the concept of vector potential to study the above scattering problem. We refer to the magnetic vector potential, in

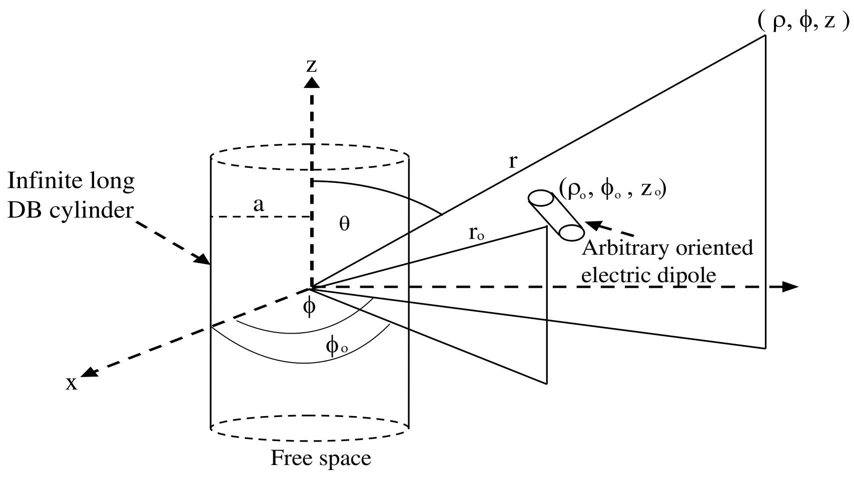

Figure 1. Scattering of an arbitrarily oriented electric dipole field from infinitely long DB circular cylinder.

the absence of DB cylinder, as the primary magnetic vector potential, while the contribution due to the presence of cylinder is termed as the secondary magnetic vector potential. The magnetic vector potential is related to the electric and magnetic fields by the following relations

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -j\omega\left\{\mathbf{A} + \frac{1}{k^2}\nabla\nabla \cdot \mathbf{A}\right\} \tag{1}$$

where $k = \omega\sqrt{\mu_o \epsilon_o}$ and ω denotes angular frequency. The above fields related with the magnetic vector potential must satisfy the Maxwell equations

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \mathbf{H} \qquad \nabla \times \mathbf{H} = j\omega\epsilon_0 \mathbf{E} + \mathbf{J}\delta(\mathbf{r} - \mathbf{r_0}) \tag{2}$$

In the above equations, $\mathbf{J}$ is the current density of the source and $\delta(\mathbf{r} - \mathbf{r_0})$ represents the Dirac's delta function describing the location of the dipole source. The radiated (primary) magnetic vector potential for such a source is derived in [18, 19]. We find, in this paper, the magnetic vector potential in the presence of DB cylinder. This can be accomplished by determining the so-called secondary magnetic vector potential due to the presence of DB circular cylinder. For this purpose, each component of magnetic vector potential is expressed in terms of Fourier transform with respect to z-parameter as

$$A_l(\rho, \phi, z) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi - \phi_o)\Big)\int_{-\infty}^{\infty}\tilde{A}_{ln}(\rho, h)\exp\Big(-jh(z - z_o)\Big)dh, \quad l = \rho, \phi, z \tag{3}$$

where $\tilde{A}_{ln}(\rho, h)$ is the Fourier transformed coefficients of magnetic vector potential. The case of ρ-directed electric dipole is presented first.

2.1. ρ-Directed Dipole Magnetic Vector Potential Analysis

Consider a ρ-directed electric dipole located at point $\mathbf{r_o} = (\rho_o, \phi_o, z_o)$ as a source of excitation. The Fourier transformed magnetic vector potential is divided into two parts as follows [18, 19]

$$\tilde{A}_{\rho n}(\rho, h) = \tilde{A}^0_{\rho n}(\rho, h) + \tilde{A}^1_{\rho n}(\rho, h) \tag{4}$$

$$\tilde{A}_{\phi n}(\rho, h) = \tilde{A}^0_{\phi n}(\rho, h) + \tilde{A}^1_{\phi n}(\rho, h) \tag{5}$$

where $\tilde{A}^0_{\rho n}(\rho, h)$ and $\tilde{A}^0_{\phi n}(\rho, h)$ represent the transformed functions in the absence of DB cylinder and have been derived in [18, 19], whereas $\tilde{A}^1_{\rho n}(\rho, h)$ and $\tilde{A}^1_{\phi n}(\rho, h)$ are those pertaining to the presence of the DB cylinder. These later functions are considered as secondary magnetic vector potential contribution. The primary magnetic vector potentials as given in [18, 19] are

$$\tilde{A}^0_{\rho n}(\rho,h) = \frac{\mu J_\rho}{4j}\left\{J'_n(\chi\rho)H'^{(2)}_n(\chi\rho_0)+\frac{n}{\chi\rho}J^{(2)}_n(\chi\rho)\frac{n}{\chi\rho_0}H^{(2)}_n(\chi\rho_0)\right\} \quad (6)$$

$$\tilde{A}^0_{\phi n}(\rho,h) = \frac{\mu J_\rho}{4}\left\{\frac{n}{\chi\rho_0}H^{(2)}_n(\chi\rho_0)J'_n(\chi\rho)+H'^{(2)}_n(\chi\rho_0)\frac{n}{\chi\rho}J_n(\chi\rho)\right\}. \quad (7)$$

The expression for the secondary magnetic vector potential contribution can be written as

$$\tilde{A}^1_{\rho n}(\rho,h) = \frac{\mu J_\rho}{4j}\left\{a_n H'^{(2)}_n(\chi\rho)+b_n\frac{n}{\chi\rho}H^{(2)}_n(\chi\rho)\right\} \quad (8)$$

$$\tilde{A}^1_{\phi n}(\rho,h) = \frac{\mu J_\rho}{4}\left\{c_n H'^{(2)}_n(\chi\rho)+d_n\frac{n}{\chi\rho}H^{(2)}_n(\chi\rho)\right\} \quad (9)$$

with $\chi = \sqrt{k^2-h^2}$, and a_n, b_n, c_n, d_n the unknown coefficients to be determined by applying the boundary conditions at $\rho = a$, i.e.,

$$\hat{\mathbf{n}}\cdot\left(\mathbf{D}_{inc}+\mathbf{D}_{sca}\right) = 0 \quad (10)$$

$$\hat{\mathbf{n}}\cdot\left(\mathbf{B}_{inc}+\mathbf{B}_{sca}\right) = 0 \quad (11)$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the surface. The electric and magnetic flux densities $\mathbf{D}$ and $\mathbf{B}$ can be obtained using (2). By applying the boundary conditions (10)–(11) at $\rho = a$ the unknown coefficients are found to be

$$a_n = \frac{-\left\{a^2\chi^2 J'''_n(\chi a)+a\chi J''_n(\chi a)+\left(k^2a^2-1\right)J'_n(\chi a)\right\}H'^{(2)}_n(\chi\rho_o)}{a^2\chi^2 H'''^{(2)}_n(\chi a)+\chi a H''^{(2)}_n(\chi a)+\left(k^2a^2-1\right)H'^{(2)}_n(\chi a)} \quad (12)$$

$$b_n = -\frac{\frac{n^2}{\rho_0}H^{(2)}_n(\chi\rho_0)\left\{aJ''_n(\chi a)-\frac{1}{\chi}J'_n(\chi a)+\left(\frac{k^2a}{\chi^2}\right)J_n(\chi a)\right\}}{n\chi a H''^{(2)}_n(\chi a)-nH'^{(2)}_n(\chi a)+\left(\frac{nk^2a}{\chi}\right)H^{(2)}_n(\chi a)} \quad (13)$$

$$c_n = -\frac{\frac{n}{\rho_o}H^{(2)}_n(\chi\rho_o)\left\{aJ''_n(\chi a)-\frac{1}{\chi}J'_n(\chi a)\right\}}{\chi a H''^{(2)}_n(\chi a)-H'^{(2)}_n(\chi a)} \quad (14)$$

$$d_n = -\frac{nH'^{(2)}_n(\chi\rho_o)\left\{J'_n(\chi a)-\left(\frac{2}{\chi a}\right)J_n(\chi a)\right\}}{nH'^{(2)}_n(\chi a)-\left(\frac{2n}{\chi a}\right)H^{(2)}_n(\chi a)} \quad (15)$$

After incorporating the values of a_n, b_n, c_n and d_n in Equations (8) and (9) and then using these equations together with the Equations (4)–(7) in Equation (3), the magnetic vector potentials for two regions in case of ρ-directed dipole can be written as

Case 1 ($\rho < \rho_o$)

$$A\rho = \frac{\mu J_\rho}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n'(\chi\rho)H_n'^{(2)}(\chi\rho_o)+\frac{n}{\chi\rho}J_n(\chi\rho)\frac{n}{\chi\rho_o}$$
$$H_n^{(2)}(\chi\rho_o) + a_n H_n'^{(2)}(\chi\rho)+b_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

$$A_\phi = \frac{\mu J_\rho}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{\frac{n}{\chi\rho_o}H_n^{(2)}(\chi\rho_o)J_n'(\chi\rho)+H_n'^{(2)}(\chi\rho_o)\frac{n}{\chi\rho}$$
$$J_n(\chi\rho) + c_n H'^{(2)}(\chi\rho) + d_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

Case 2 ($\rho > \rho_o$)

$$A\rho = \frac{\mu J_\rho}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n'(\chi\rho_o)H_n'^{(2)}(\chi\rho)+\frac{n}{\chi\rho_o}J_n(\chi\rho_o)\frac{n}{\chi\rho}$$
$$H_n^{(2)}(\chi\rho)+a_n H_n'^{(2)}(\chi\rho)+b_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

$$A_\phi = \frac{\mu J_\rho}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{\frac{n}{\chi\rho_o}H_n'^{(2)}(\chi\rho)J_n(\chi\rho_o)+H_n^{(2)}(\chi\rho)\frac{n}{\chi\rho}$$
$$J_n'(\chi\rho_o)+c_n H'^{(2)}(\chi\rho) + d_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

2.2. ϕ-directed Dipole Magnetic Vector Potential Analysis

The primary vector potentials in case of ϕ-directed dipole can be given as [18, 19]

$$\tilde{A}^0_{\rho n}(\rho, h) = -\frac{\mu J_\phi}{4}\left\{\frac{n}{\chi\rho_0}J_n'(\chi\rho)H_n^{(2)}(\chi\rho_0)+\frac{n}{\chi\rho}J_n(\chi\rho)H_n'^{(2)}(\chi\rho_0)\right\} \quad (16)$$

$$\tilde{A}^0_{\phi n}(\rho, h) = \frac{\mu J_\phi}{4j}\left\{H_n'^{(2)}(\chi\rho_0)J_n'(\chi\rho)+\frac{n}{\chi\rho}J_n(\chi\rho)\frac{n}{\chi\rho_0}H_n'^{(2)}(\chi\rho_0)\right\} \quad (17)$$

Therefore, the corresponding secondary magnetic vector potentials are

$$\tilde{A}^1_{\rho n}(\rho, h) = -\frac{\mu J_\phi}{4}\left\{e_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho) + f_n H_n'^{(2)}(\chi\rho)\right\} \quad (18)$$

$$\tilde{A}^1_{\phi n}(\rho, h) = \frac{\mu J_\phi}{4j}\left\{g_n H_n'^{(2)}(\chi\rho) + h_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\right\} \quad (19)$$

A similar procedure has been adopted as mentioned above in ρ-directed dipole case for determining the unknown coefficients e_n, f_n, g_n, h_n, in this case as well . The application of boundary conditions (10) and (11) yield the following result

$$e_n = -\frac{\frac{n}{\rho_o}H_n^{(2)}(\chi\rho_o)\left\{\chi a^2 J_n'''(\chi a) + aJ_n''(\chi a) + \left(\frac{k^2a^2-1}{\chi}\right)J_n'(\chi a)\right\}}{\chi^2 a^2 H_n'''^{(2)}(\chi a) + \chi a H_n''^{(2)}(\chi a) + \left(k^2a^2-1\right)H_n'^{(2)}(\chi a)} \quad (20)$$

$$f_n = -\frac{nH_n'^{(2)}(\chi\rho_o)\left\{a\chi J_n''(\chi a) - J_n'(\chi a) + \left(\frac{k^2a}{\chi}\right)J_n(\chi a)\right\}}{na\chi H_n''^{(2)}(\chi a) - nH_n'^{(2)}(\chi a) + \left(\frac{nk^2a}{\chi}\right)H_n^{(2)}(\chi a)} \quad (21)$$

$$g_n = -\frac{H_n'^{(2)}(\chi\rho_o)\left\{\chi a J_n''(\chi a) - J_n'(\chi a)\right\}}{\chi a H_n^{(2)\prime\prime}(\chi a) - H_n'^{(2)}(\chi a)} \quad (22)$$

$$h_n = -\frac{\frac{n^2}{\rho_o}H_n'^{(2)}(\chi\rho_o)\left\{\frac{1}{\chi}J_n'(\chi a) - \left(\frac{2}{\chi^2 a}\right)J_n(\chi a)\right\}}{nH_n'^{(2)}(\chi a) - \left(\frac{2n}{\chi a}\right)H_n^{(2)}(\chi a)} \quad (23)$$

The magnetic vector potentials for two regions in case of ϕ-directed dipole can be obtained analogous to that of ρ-directed dipole. The results are as follows

Case 1 ($\rho < \rho_o$)

$$A\rho = \frac{-\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{\frac{n}{\rho\chi}J_n(\chi\rho)H_n'^{(2)}(\chi\rho_o) + \frac{n}{\chi\rho_o}$$
$$\times H_n^{(2)}(\chi\rho_o)J_n'(\chi\rho) + f_n\frac{n}{\chi\rho}H^{(2)}(\chi\rho) + e_n H_n'^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

$$A_\phi = \frac{-j\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n'(\chi\rho)H_n'^{(2)}(\chi\rho_o) + \frac{n}{\chi\rho_o}H_n'^{(2)}(\chi\rho_o)$$
$$\times\frac{n}{\chi\rho}J_n(\chi\rho) + g_n H'^{(2)}(\chi\rho) + h_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

Case 2 ($\rho > \rho_o$)

$$A\rho = \frac{-\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{\frac{n}{\rho_o\chi}J_n(\chi\rho_o)H_n'^{(2)}(\chi\rho) + \frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)$$
$$\times J_n'(\chi\rho_o) + f_n\frac{n}{\chi\rho}H^{(2)}(\chi\rho) + e_n H_n'^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

$$A_\phi = \frac{-j\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n'(\chi\rho_o)H_n'^{(2)}(\chi\rho) + \frac{n}{\chi\rho_o}H_n'^{(2)}(\chi\rho)$$

$$\times\frac{n}{\chi\rho}J_n(\chi\rho_o)+g_nH_n'^{(2)}(\chi\rho)+h_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}\exp\Big(-jh(z-z_o)\Big)dh$$

2.3. z-directed Dipole Magnetic Vector Potential Analysis

In the case of z-directed electric dipole, the primary vector potential [18, 19] and the corresponding secondary magnetic vector potential can be given as

$$\tilde{A}_{zn}^{(0)}(\rho,h) = \frac{\mu_0 J_z}{4j}J_n(\chi\rho)H_n^{(2)}(\chi\rho_0) \tag{24}$$

$$\tilde{A}_{zn}^{(1)}(\rho,h) = \frac{\mu_0 J_z}{4j}q_nH_n^{(2)}(\chi\rho) \tag{25}$$

where q_n is the unknown scattering coefficient and is obtained using procedure as mentioned above for ρ and ϕ-directed dipole cases. The application of the boundary conditions (10)–(11) leads to the following unknown scattering coefficient

$$q_n = -\frac{J_n'(\chi a)H_n^{(2)}(\chi\rho_o)}{H_n'^{(2)}(\chi a)} \tag{26}$$

Therefore using (3), the total magnetic vector potential (primary + secondary) can be written as

Case 1 $(\rho < \rho_o)$

$$A_z = \frac{\mu_o J_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n(\chi\rho)H_n^{(2)}(\chi\rho_o)+q_nH_n^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\Big(-jh(z-z_o)\Big)dh$$

Case 2 $(\rho > \rho_o)$

$$A_z = \frac{\mu_o J_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{J_n(\chi\rho_o)H_n^{(2)}(\chi\rho)+q_nH_n^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\Big(-jh(z-z_o)\Big)dh$$

2.4. Far-zone Magnetic Vector Potential Analysis

For far-zone magnetic vector potential the integral of type $\int_{-\infty}^{\infty}(.)\exp(-jh(z-z_o))dh$ has been solved using steepest descent method following the approach in [18, 19]. In the analysis asymptotic expression of Hankel function is used with transformation

$$\rho = r\sin\theta, \quad z-z_o = r\cos\theta, \quad \chi = k\sin\theta,$$
$$h = k\cos\theta \quad \text{and} \quad H'^{(2)}{}_n(\chi\rho) \simeq -jH_n^{(2)}(\chi\rho)$$

2.4.1. ρ-directed Dipole

$$A\rho = \frac{\mu J_\rho}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{a_n H_n'^{(2)}(\chi\rho)+b_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\Big(-jh(z-z_o)\Big)dh$$
$$\simeq \frac{-j\mu_o J_\rho\exp(-jkr)}{4\pi r}\sum_{n=-\infty}^{\infty}a_n(ka\sin\theta)\exp\Big(jn(\phi-\phi_o)+\frac{jn\pi}{2}\Big)$$

$$A_\phi = \frac{\mu J_\rho}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{c_n H'^{(2)}(\chi\rho)+d_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\left(-jh(z-z_o)\right)dh$$
$$\simeq\frac{\mu_o J_\rho\exp(-jkr)}{4\pi r}\sum_{n=-\infty}^{\infty}c_n(ka\sin\theta)\exp\Big(jn(\phi-\phi_o)+\frac{jn\pi}{2}\Big)$$

2.4.2. ϕ-directed Dipole

$$A\rho = \frac{-\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{f_n\frac{n}{\chi\rho}H^{(2)}(\chi\rho)+e_n H_n'^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\Big(-jh(z-z_o)\Big)dh$$
$$\simeq \frac{j\mu_o J_\phi\exp(-jkr)}{4\pi r}\sum_{n=-\infty}^{\infty}e_n(ka\sin\theta)\exp\Big(jn(\phi-\phi_o)+\frac{jn\pi}{2}\Big)$$

$$A_\phi = \frac{-j\mu_o J_\phi}{8\pi}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty}\Big\{g_n H'^{(2)}(\chi\rho)+h_n\frac{n}{\chi\rho}H_n^{(2)}(\chi\rho)\Big\}$$
$$\times\exp\Big(-jh(z-z_o)\Big)dh$$
$$\simeq \frac{j\mu_o J_\phi\exp(-jkr)}{4\pi r}\sum_{n=-\infty}^{\infty}g_n(ka\sin\theta)\exp\Big(jn(\phi-\phi_o)+\frac{jn\pi}{2}\Big)$$

2.4.3. z-directed Dipole

$$A_z = \frac{\mu_o J_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big(jn(\phi-\phi_o)\Big)\int_{-\infty}^{\infty} q_n H_n^{(2)}(\chi\rho_o)\exp\Big(-jh(z-z_o)\Big)dh$$
$$\simeq \frac{\mu_o J_z \exp(-jkr)}{4\pi r}\sum_{n=-\infty}^{\infty} q_n(ka\sin\theta)\exp\Big(jn(\phi-\phi_o)+\frac{jn\pi}{2}\Big)$$

3. NUMERICAL RESULTS AND CONCLUDING REMARKS

In this section numerical results are depicted through Figures 2 to 6.

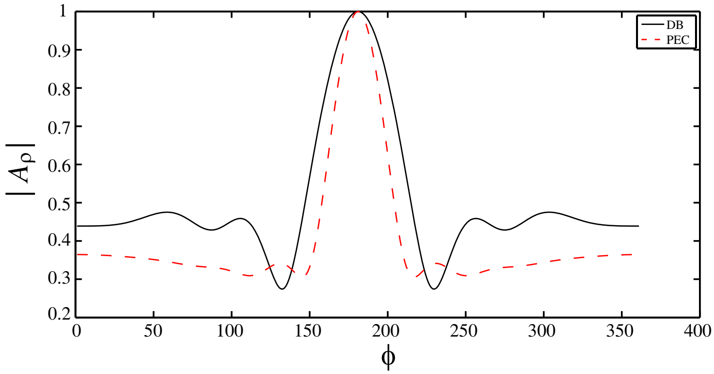

Figure 2. ρ-directed electric dipole with $ka = 5, \theta = 60$ and $\phi_o = 0$.

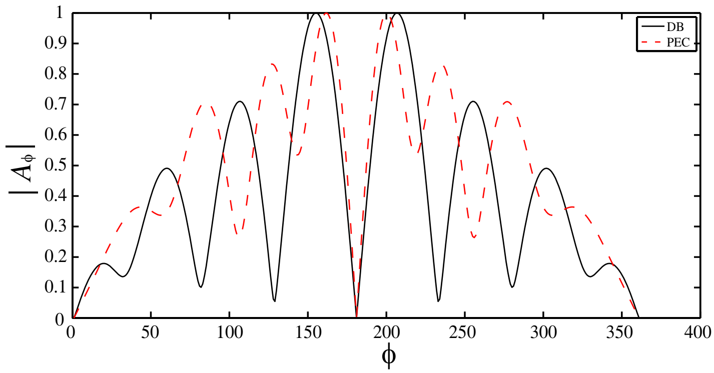

Figure 3. ρ-directed electric dipole with $ka = 5, \theta = 60$ and $\phi_o = 0$.

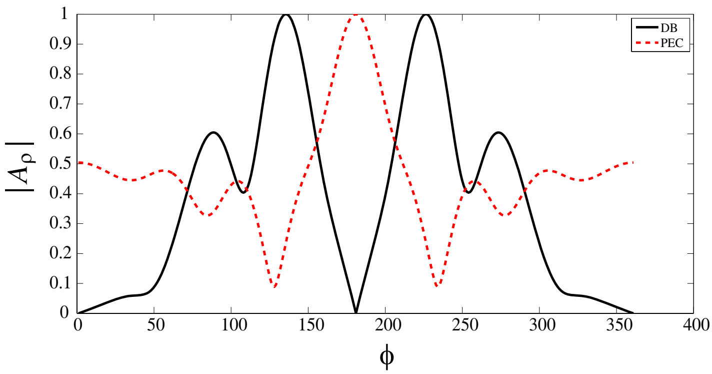

Figure 4. ϕ-directed electric dipole with $ka = 5, \theta = 60$ and $\phi_o = 0$.

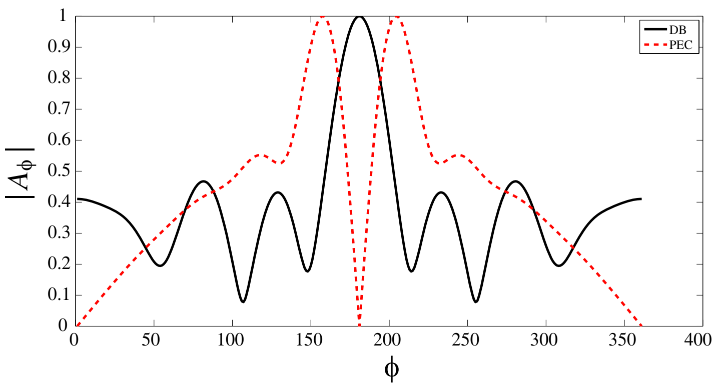

Figure 5. ϕ-directed electric dipole with $ka = 5, \theta = 60$ and $\phi_o = 0$.

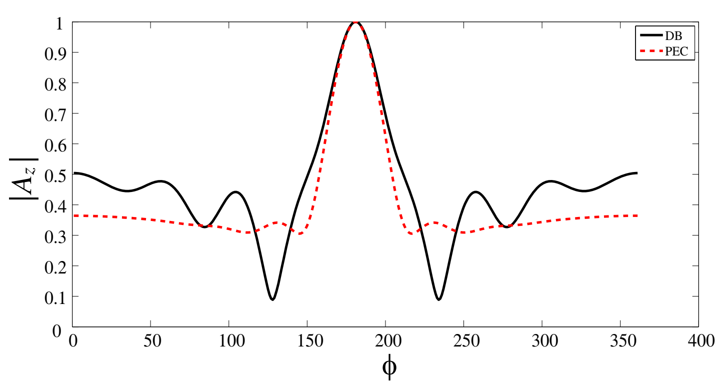

Figure 6. z-directed electric dipole with $ka = 5, \theta = 60$ and $\phi_o = 0$.

The expressions for magnetic vector potential are simplified in the far-zone region using steepest descent method and applying asymptotic approximations on cylindrical wave functions. Potentials are plotted by taking $ka = 5$, $\phi_0 = 0$ and $\theta = 60$. It has been observed from numerical results that when dipole is ρ-directed or z-directed, behavior of DB and PEC cylindrical boundaries is similar at $\phi = 180$, i.e., their null and maxima correspond to same locations. On the other hand, when the dipole is ϕ-directed the behaviors are exactly opposite to each other, i.e., at $\phi = 180$ null of PEC cylindrical interface correspond to the maxima of DB cylindrical interface and vice versa. Moreover, when dipole is z-directed, magnetic vector potential for both DB and PEC cylindrical boundaries has some finite values at $\phi = 0$ and 360 whilst for ϕ-directed electric dipole shows opposite behavior. Furthermore, it is observed from numerical results, when electric dipole is ρ-directed, DB and PEC cylindrical boundaries have some finite and null values of magnetic potential at $\phi = 0$ and 360, respectively. These new results may have potential applications in antenna design.

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