Progress In Electromagnetics Research M, Vol. 23, 167–180, 2012

CHIRAL AND/OR CHIRAL NIHILITY INTERFACES: PARAMETRIC DEPENDENCE, POWER TUNNELING AND REJECTION

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Abstract—Characteristics of reflected power from a planar interface of chiral and/or chiral nihility media have been investigated theoretically. Focus of the study is tunneling and rejection of power associated with these interfaces. Effect of polarization of incidence field and parametric dependence on reflected power have been noted. It is found from numerical results that power tunneling and rejection have strong dependency on the polarization of incidence field, angle of incidence, and chirality parameter.

1. INTRODUCTION

In optics, chiral media have been known for a long time due to the phenomena of optical activity and circular dichroism associated with it. Chiral medium is composed of numerous randomly oriented chiral objects which can never be brought into congruence with their mirror images by any translation or rotation. The effect of chirality on electromagnetic wave propagation is a rotation of the plane of a linearly-polarized wave. This phenomenon, termed as optical activity since the early nineteenth century, from the studies of Biot, Arago, and Fresnel [1–4]. Chiral media are characterized by left-handed and right-handed circularly polarized eigenwaves, each having different refractive index and phase velocity. Circular dichroism refers to the differential absorption of left and right circularly polarized light [5–7].

Constitutive relations for chiral medium [8] are given as

$$\mathbf{D} = \epsilon\mathbf{E} + i\kappa\mathbf{H}$$
$$\mathbf{B} = \mu\mathbf{H} - i\kappa\mathbf{E}$$

Received 1 December 2011, Accepted 1 February 2012, Scheduled 8 February 2012

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where ϵ, μ and κ represent permittivity, permeability, and chiralty respectively. Due to interesting properties of chiral material, propagation and radiation of electromagnetic waves in chiral medium have been studied by many authors [8–10]. Improved performance of circularly polarized antenna using semi-planar chiral metamaterial covers has been reported by Zarifi et al. [11]. The two refractive indices in the chiral media are

$$n_2^{\pm} = \sqrt{\epsilon\mu} \pm \kappa$$

Given the chirality is strong enough, negative refraction may occur for one circularly polarized wave, while for the other circular polarization the refractive index remains positive [12–17]. This gives rise to interesting phenomena that conventional negative refractive index material do not exhibit, such as negative reflection for electromagnetic waves incident onto a mirror embedded in such a medium [18]. The concept of negative refraction in a left-handed material (LHM) was first introduced by Veselago in 1968 [19]. Left-handed material are such materials in which the electric field, the magnetic field and the wave vector obey the left-hand rule.

The concept of nihility was first introduced by Lakhtakia. He introduced term "nihility" for such medium, whose $\epsilon = 0$, $\mu = 0$ [20]. Later, Tretyakov et al. [21] extended the concept of nihility for the isotropic chiral medium. He showed that isotropic chiral nihility interface has a very interesting property of double refraction: the wave is split into two circularly polarized components, such that one of them is refracted positively, but the other one is refracted negatively, like in Veselago media. The constitutive relations for chiral nihility are

$$\begin{aligned} \mathbf{D} &= i\kappa\mathbf{H} \\ \mathbf{B} &= -i\kappa\mathbf{E} \end{aligned}$$

Chiral nihility is an emerging area and a lot interesting application are discussed, such as surface wave modes in grounded chiral nihility waveguides [22, 23], focusing [24], chiral fibers [25] and use of gyrotropic chiral media [26, 27]. Fractional dual solution for chiral nihility metamaterials have been studied by Naqvi [28, 29]. Taj et al. [30], discussed behavior of the plane wave in chiral nihility-chiral nihility interface and show that there is a reflected backward as well as a refracted backward wave. Qiu et al. [31] discussed the chiral nihility effects on energy flow in case of dielectric-chiral interface and shows some critical characteristics of the effects of nihility on energy transmission and reflection, e.g., possibility of achieving a negative index of refraction. Semi-infinite periodical chiral structure consisting of alternative chiral nihility media studied by Tuz and Qiu [32] and showed that an ideal photonic bandgap though adjusting the chirality in semi-infinite chiral nihility photonics. In the present paper we have discussed the characteristics of two chiral media and nihility effects on energy flow in case of chiral-chiral interface. The reflection and transmission of power from a chiral-chiral interface, chiral-chiral nihility interface, chiral nihility-chiral and chiral nihility-chiral nihility interface with impedance and without impedance matching are discussed. Chiral nihility is considered as limiting case of chiral medium, i.e., taking very small value of relative permittivity $\epsilon_r = 10^{-5}$ and relative permeability $\mu_r = 10^{-5}$.

2. PROBLEM FORMULATION

Consider a planar interface, of two media, having infinite extent located at $z = 0$ as shown in Figure 1. The half space $z < 0$, is occupied by chiral medium having constitutive parameters $(\epsilon_1, \mu_1, \kappa_1)$, while the space $z > 0$ is filled with chiral medium having parameters $(\epsilon_2, \mu_2, \kappa_2)$. The refractive indices, wave numbers, and impedance for chiral medium $z < 0$ are

$$\begin{aligned} n_1^{\pm} &= \sqrt{\epsilon_{r1}\mu_{r1}} \pm \kappa_1, \\ k_1^{\pm} &= \omega\left(\sqrt{\epsilon_1\mu_1} \pm \kappa_1\right), \\ \eta_1 &= \sqrt{\frac{\mu_1}{\epsilon_1}}, \end{aligned}$$

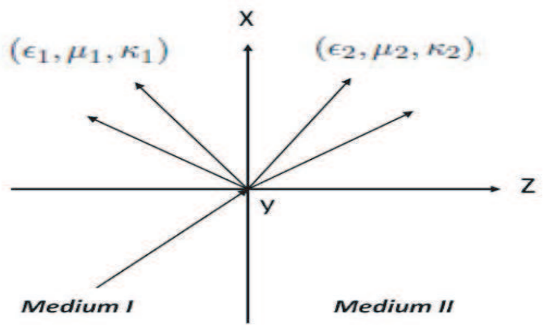


Figure 1. Reflection and transmission in chiral media: Incident RCP/LCP, Reflected LCP and RCP, Refracted LCP and RCP.

and for $z > 0$ are

$$\begin{aligned} n_2^{\pm} &= \sqrt{\epsilon_{r2}\mu_{r2}} \pm \kappa_2, \\ k_2^{\pm} &= \omega\left(\sqrt{\epsilon_2\mu_2} \pm \kappa_2\right), \\ \eta_2 &= \sqrt{\frac{\mu_2}{\epsilon_2}}. \end{aligned}$$

Hereafter half space $z < 0$ is termed as medium I and half space $z > 0$ is termed as medium II. For incidence LCP plane wave, the angles of the reflected and transmitted waves are given below

$$\begin{aligned} \theta_{\text{refLCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^- \sin\theta_{\text{inc}}}{k_1^+}\right)^2}, \\ \theta_{\text{refRCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^- \sin\theta_{\text{inc}}}{k_1^-}\right)^2}, \\ \theta_{\text{tranLCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^- \sin\theta_{\text{inc}}}{k_2^+}\right)^2}, \\ \theta_{\text{tranRCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^- \sin\theta_{\text{inc}}}{k_2^-}\right)^2}, \end{aligned}$$

and for incidence RCP plane wave the angles are modified as

$$\begin{aligned} \theta_{\text{refLCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^+ \sin\theta_{\text{inc}}}{k_1^+}\right)^2}, \\ \theta_{\text{refRCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^+ \sin\theta_{\text{inc}}}{k_1^-}\right)^2}, \\ \theta_{\text{tranLCP}} &= \cos^{-1},\sqrt{1-\left(\frac{k_1^+ \sin\theta_{\text{inc}}}{k_2^+}\right)^2}, \\ \theta_{\text{tranRCP}} &= \cos^{-1}\sqrt{1-\left(\frac{k_1^+ \sin\theta_{\text{inc}}}{k_2^-}\right)^2}. \end{aligned}$$

RCP/LCP plane wave is considered as incidence wave. The method adopted by Taj [30] are used for fields but not repeated here. Following boundary conditions must be satisfied, by the fields, at $z = 0$

$$\begin{aligned} [\mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{ref}}]_{\tan} &= [\mathbf{E}_{\text{tran}}]_{\tan}, \\ [\mathbf{H}_{\text{inc}} + \mathbf{H}_{\text{ref}}]_{\tan} &= [\mathbf{H}_{\text{tran}}]_{\tan}, \end{aligned}$$

where subscript 'inc', 'ref' and 'tran' stand for incident, reflected, and transmitted respectively while 'tan' stand for tangential components of fields. Fresnel coefficients and reflected power can be computed using above boundary conditions.

3. CHIRAL AND/OR CHIRAL NIHILITY INTERFACES

The behavior of reflected powers from a planar interface, for both types of polarization of the incident field, is investigated as a functions of angle of incidence and chirality parameter. Four different interfaces are considered in this regard. Cases of impedance matching ($\eta_1 = \eta_2$) and mismatching ($\eta_1 \neq \eta_2$) have been discussed for both polarizations. Throughout the discussion, for the case of impedance mismatching

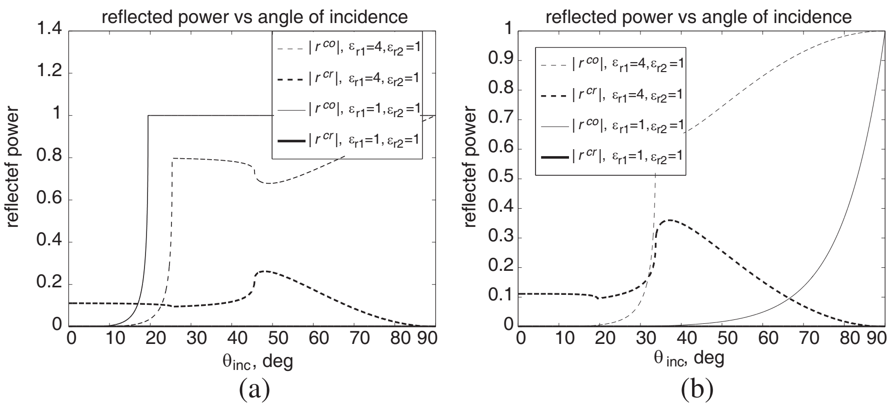


Figure 2. Reflected power verses angle of incidence (a) RCP incidence and (b) LCP incidence, when $\mu_{r1} = \mu_{r2} = 1$, for impedance mismatch $\kappa_1 = \kappa_2 = 0.25$ and for impedance matching $\kappa_1 = 0.25$, $\kappa_2 = 0.75$.

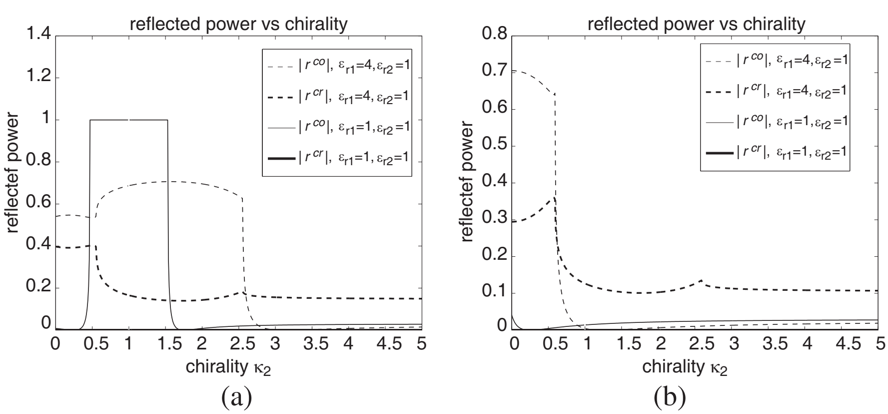


Figure 3. Reflected power verses chirality of the medium II (a) RCP incidence and (b) LCP incidence, when $\kappa_1 = 0.25$, $\mu_{r1} = \mu_{r2} = 1$, and $\theta_{\text{inc}} = 45°$.

values of constitutive parameters are arbitrarily taken as $\epsilon_{r1} = 4, \epsilon_{r2} = 1, \kappa_1 = \kappa_2 = 0.25$ while for impedance matching these parameters are $\epsilon_{r1} = 1, \epsilon_{r2} = 1, \kappa_1 = 0.25, \kappa_2 = 0.75$. Focus of analysis is the power tunneling and power rejection characteristics of the interface. It may be noted that, each figure contains two plots labeled as (a) and (b): caption labeled as (a) deals with RCP incident plane wave whereas

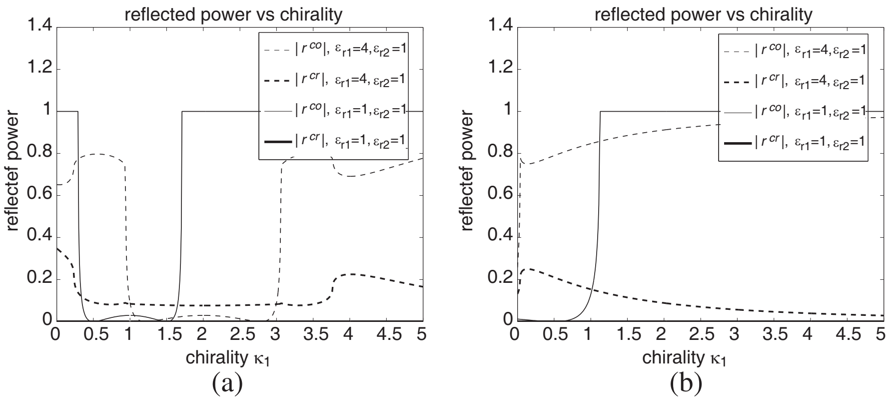


Figure 4. Reflected power verses chirality of the medium I (a) RCP incidence and (b) LCP incidence, when $\kappa_2 = 0.25$, $\mu_{r1} = \mu_{r2} = 1$, and $\theta_{\text{inc}} = 45°$.

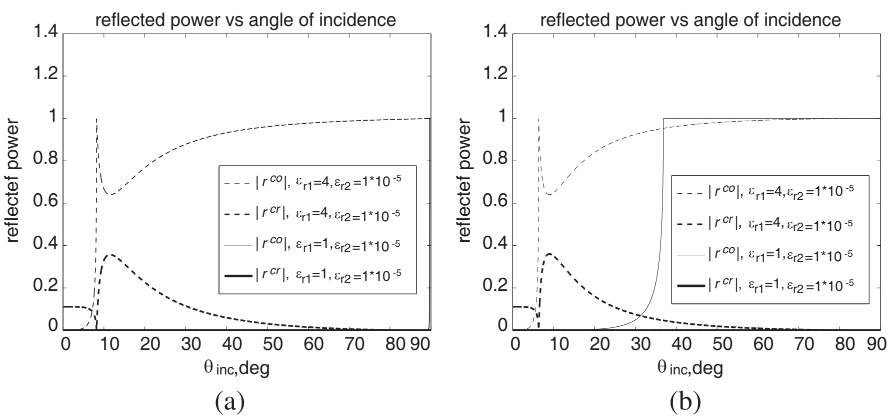
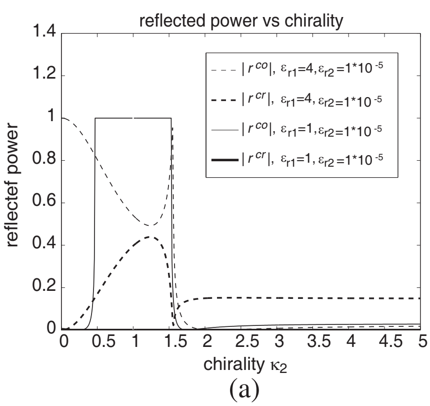


Figure 5. Reflected power verses angle of incidence (a) RCP incidence and (b) LCP incidence, when $\mu_{r1} = 1$, $\mu_{r2} = 1 * 10^{-5}$, for impedance mismatch $\kappa_1 = \kappa_2 = 0.25$, for impedance matching $\kappa_1 = 0.25$, and $\kappa_2 = 0.75$.

(b) deals with LCP incident plane wave. In each plot, solid lines correspond to impedance matching whereas dotted lines correspond to impedance mismatch.

For both types of polarization of incidence field, reflected power has only co polarized component ($|r^{co}| \neq 0, |r^{cr}| = 0$) for impedance matching case while in case of impedance mismatch there exist both co and cross components of reflected power ($|r^{co}| \neq 0, |r^{cr}| \neq 0$). The results are according to the discussion presented in [32]. For both cases of impedance (matching and mismatching) and types of polarization (LCP and RCP), complete power reflection (termed as **power rejection** hereafter) occurs at $\theta_{inc} = 90°$. These observations are obvious to understand.

First, consider a planar interface of two chiral media. Corresponding plots are shown in Figure 2 to Figure 4. The angular dependence of reflected power is shown in Figure 2. For the case of impedance matching, power rejection occurs after $\theta_{inc} = 19°$ for RCP while for LCP incidence field power rejection occurs only at $\theta_{inc} = 90°$. When $\eta_1 \neq \eta_2$, power rejection appears in form of co and cross components ($|r^{co}| + |r^{cr}| = 1$), for both types of polarization of incident field. Behavior of reflected power verses chirality of the medium II has been shown in Figure 3, for angle $\theta_{inc} = 45°$. Rejection and tunneling of power have been observed for wide range of chirality parameter. In case of impedance mismatch, power rejection ($|r^{co}| + |r^{cr}| = 1$) occurs from $\kappa_2 = 0$ to $\kappa_2 = 2.2$ for RCP incident field and $\kappa_2 = 0$ to $\kappa_2 = 0.5$ for LCP incident field. In case of impedance matching, power rejection



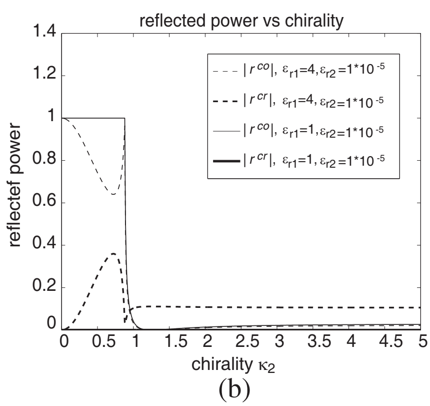
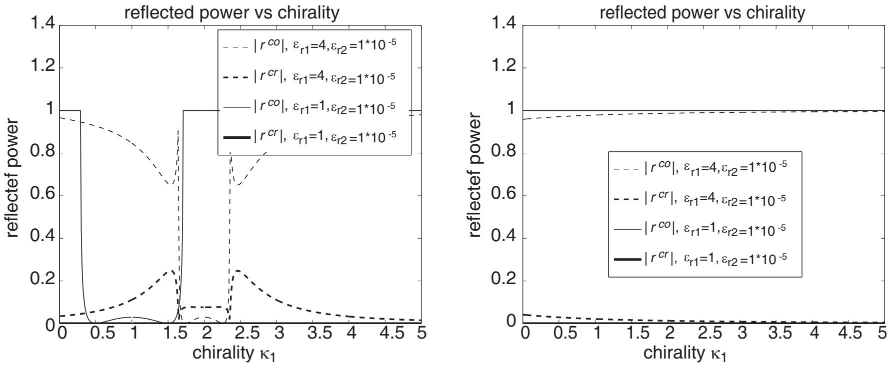
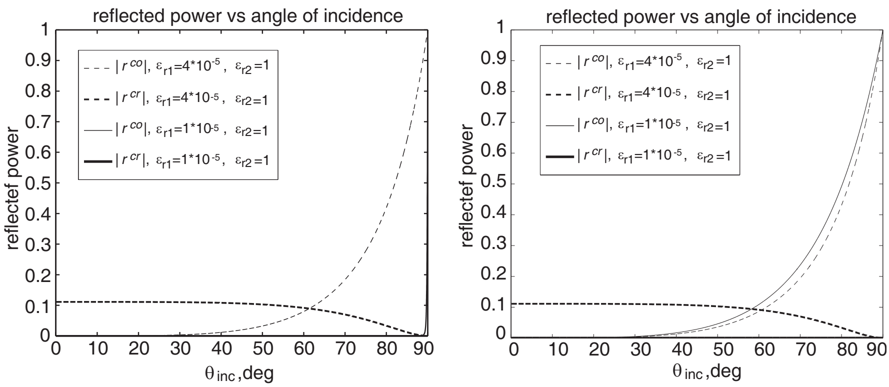


Figure 6. Reflected power verses chirality of the medium II (a) RCP incidence and (b) LCP incidence, when $\kappa_1 = 0.25$, $\theta_{\text{inc}} = 45°$, $\mu_{r1} = 1$, and $\mu_{r2} = 1 * 10^{-5}$.



(a) (b)

Figure 7. Reflected power verses chirality of medium I (a) RCP incidence and (b) LCP incidence, when $\kappa_2 = 0.25$, $\theta_{\text{inc}} = 45°$, $\mu_{r1} = 1$, and $\mu_{r2} = 1 * 10^{-5}$.



(a) (b)

Figure 8. Reflected power verses angle of incidence (a) RCP incidence and (b) LCP incidence, when $\mu_{r2} = 1$, $\mu_{r1} = 1 * 10^{-5}$, for impedance mismatch $\kappa_1 = \kappa_2 = 0.25$, for impedance matching $\kappa_1 = 0.25$, and $\kappa_2 = 0.75$.

($|r^{co}| = 1$) is observed from $\kappa_2 = 0.5$ to $\kappa_2 = 1.5$ for RCP incident field. Minor reflection of power is noted for LCP incident field in the entire range of chirality considered for the analysis. It is obvious from numerical results that higher chirality of the medium II is favorable for power tunneling for specific combination of constitutive parameters. In Figure 4 reflected power as a function of chirality of medium I, is

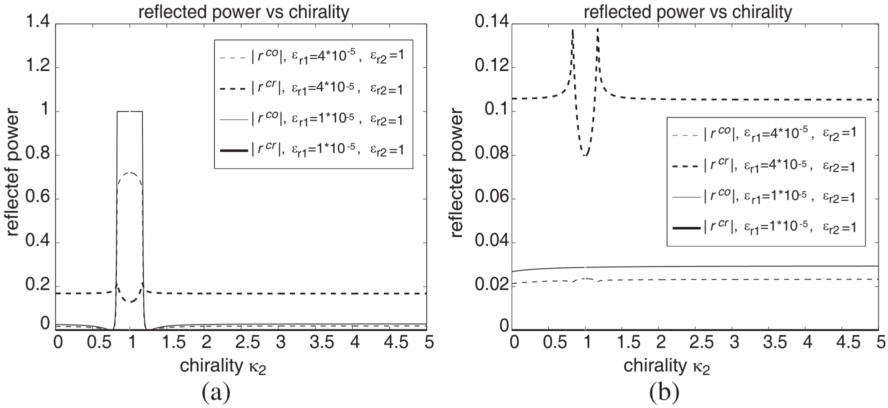


Figure 9. Reflected power verses chirality of the medium II (a) RCP incidence and (b) LCP incidence, when $\kappa_1 = 0.25$, $\theta_{\text{inc}} = 45°$, $\mu_{r2} = 1$, and $\mu_{r1} = 1 * 10^{-5}$.

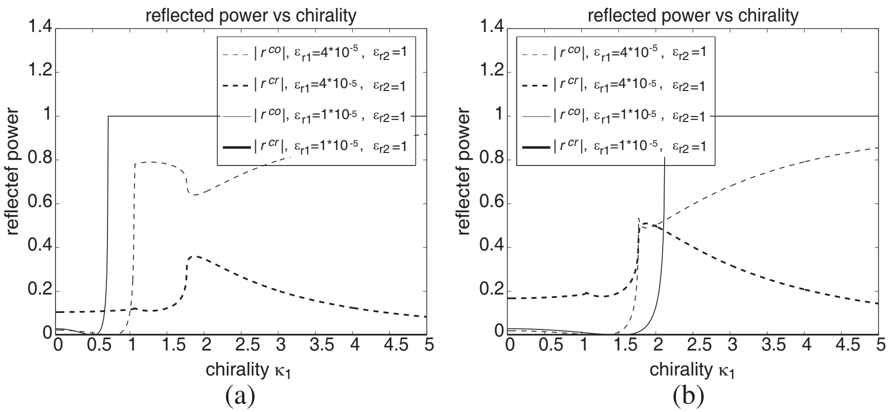
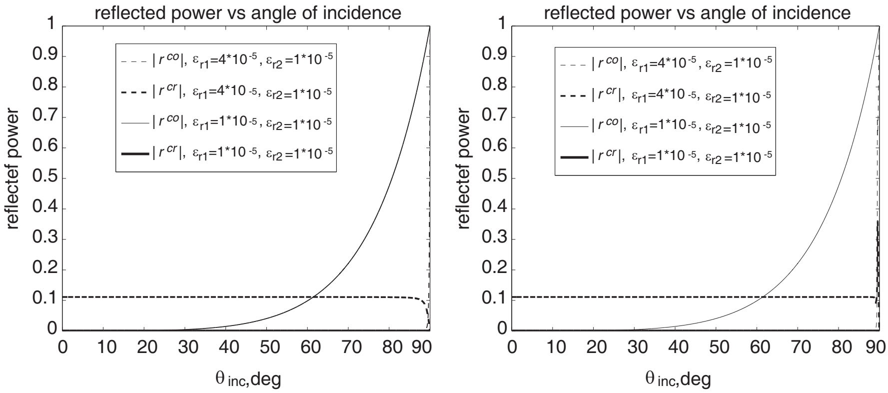


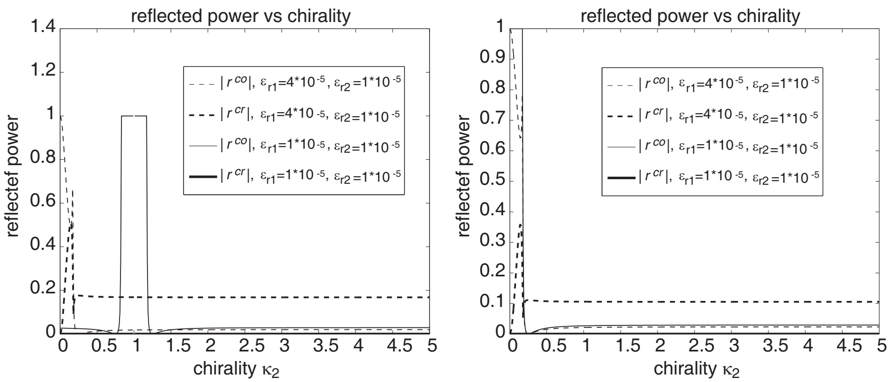
Figure 10. Reflected power verses chirality of medium I (a) RCP incidence and (b) LCP incidence, when $\kappa_2 = 0.25$, $\theta_{\text{inc}} = 45°$, $\mu_{r2} = 1$, and $\mu_{r1} = 1 * 10^{-5}$.

shown for $\theta_{inc} = 45°$. In case of impedance mismatch, minor reflection of power from $\kappa_1 = 0.75$ to $\kappa_1 = 2.75$ is observed for RCP incident field. Power rejection for whole range (0 to 5) of chirality parameter is noted for LCP incident field. For impedance matching and RCP incidence, almost complete power tunneling in terms of co polarized component, from $\kappa_1 = 0.25$ to $\kappa_1 = 1.75$, is observed. While for LCP incident field, complete power rejection after $\kappa_1 = 1$ is noted. Again we



(a) (b)

Figure 11. Reflected power verses angle of incidence (a) RCP incidence and (b) LCP incidence, when $\mu_{r1} = 1 * 10^{-5}$, $\mu_{r2} = 1 * 10^{-5}$, for impedance mismatch $\kappa_1 = 0.25$, $\kappa_2 = 0.25$, for impedance matching $\kappa_1 = 0.25$, and $\kappa_2 = 0.75$.

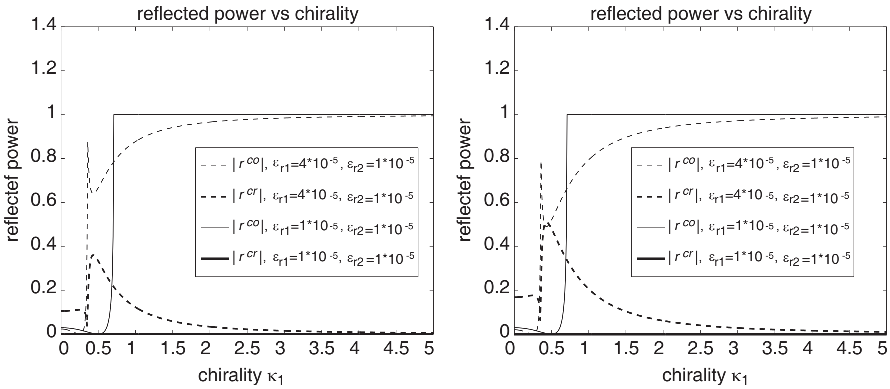


(a) (b)

Figure 12. Reflected power verses chirality of the medium II (a) RCP incidence and (b) LCP incidence, when $\kappa_1 = 0.25$, $\mu_{r1} = 1 * 10^{-5}$, $\mu_{r2} = 1 * 10^{-5}$, and $\theta_{\text{inc}} = 45°$.

can see a wide band of chirality of the medium I, for power tunneling and rejection for both polarizations of incident field.

Nihility is introduced in medium II and behavior of reflected field for both polarizations of incident fields has been presented in Figure 5 to Figure 7. Power rejection and tunneling also occurs in this situation. Absence of reflected power for impedance matching case considered



(a) (b)

Figure 13. Reflected power verses chirality of the medium I (a) RCP incidence and (b) LCP incidence, when $\kappa_2 = 0.25$, $\mu_{r1} = 1 * 10^{-5}$, $\mu_{r2} = 1 * 10^{-5}$, and $\theta_{\text{inc}} = 45°$.

in Figure 5(a) and complete reflection of power for both cases of impedance in Figure 6(b) may have interesting applications.

Figure 8 to Figure 10 deal with situation when medium I is chiral nihility and medium II is chiral. In Figure 11 to Figure 13, it is assumed that both half spaces are of chiral nihility material. Only cross component of reflected power in Figure 11 may be useful for applications concerning selection of polarization.

4. CONCLUSION

Response of a planar interface due to chiral and/or chiral nihility interface are discussed for different sets of constitutive parameters. We have considered four situations: chiral-chiral interface, chiral-chiral nihility interfaces, and chiral nihility-chiral nihility interface. Major focus of the analysis is power tunneling and rejection. In case of impedance mismatch, power reflection of cross component is obtained which is in contrast with the results obtained for dielectric chiral interface. Selection of co and/or cross polarization components of reflected power can also be achieved. It has been revealed that it is easier to realize an ideal band of chirality and angle of incident for power tunneling and rejection. These phenomena may be influenced through polarization of incident field and variation of constitutive parameters.

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