PROVISOS FOR CLASSIC LINEAR OSCILLATOR DESIGN METHODS. NEW LINEAR OSCILLATOR DESIGN BASED ON THE NDF/RR$_\mathbf{T}$

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Abstract—In this paper, the classic oscillator design methods are reviewed, and their strengths and weaknesses are shown. Provisos for avoiding the misuse of classic methods are also proposed. If the required provisos are satisfied, the solutions provided by the classic methods (oscillator start-up linear approximation) will be correct. The provisos verification needs to use the NDF (Network Determinant Function). The use of the NDF or the most suitable RR_T (Return Relation Transponse), which is directly related to the NDF, as a tool to analyze oscillators leads to a new oscillator design method. The RR_T is the "true" loop-gain of oscillators. The use of the new method is demonstrated with examples. Finally, a comparison of NDF/RR_T results with the HB (Harmonic Balance) simulation and practical implementation measurements prove the universal use of the new methods.

1. INTRODUCTION

The oscillators are fundamental elements for all RF and microwave systems, as Radar and Electronic Warfare Systems. They are one of the circuits types that have more problems with their design process. Nowadays, the linear simulation, as a first approximation, is widely used for RF and microwave oscillator design [1–3]. Non linear simulation needs more computational resources than linear simulation,

and the non linear models for active devices must be available, but they are not always available. In some cases they are not accurate enough. However, it is necessary, before starting a non linear simulation, to have a good approximation of the frequency and start-up conditions. To conclude, it is desirable to have a good background in non linear approximation and simulation [2, 4], and in some cases, knowledge in non linear solution stability [5] is necessary, mainly if the harmonic balance is used.

One of the most important reasons for using the linear simulation in oscillator design is that it is quicker and simpler than the non linear and more suitable for tuning the circuit. It is only necessary for these linear simulations to have the S parameters or the linear model of the active device. These linear models are much easier to get than the non linear ones. A simple linear oscillator model and quick simulation give us the chance of looking for new topologies [2, 6]. On the other hand, the linear simulation can only estimate the oscillation frequency, gain margin and oscillator Q_L (loaded quality factor), but neither the output power nor phase noise or harmonic levels can be estimated. Therefore, nowadays oscillator design methodology consists on a first linear simulation step, following with harmonic balance and transient simulations and optimizations.

In this paper, as a first step the classic oscillator linear analysis methods are described. Then, their strengths and weaknesses are analyzed and the provisos for their proper use defined. In a second part, it is demonstrated that the proposed linear method is the unique method for universal use. Then, the use of the proposed method is demonstrated with two practical examples. And as a last step the conclusions are exposed.

2. PROVISOS FOR CLASSIC LINEAR OSCILLATOR DESIGN METHODS

Linear oscillator analysis design techniques can be divided into two groups: Loop gain [1, 2, 7, 8] and reference plane [9–12]. Negative resistance, negative conductance and reflection coefficient (for microwave circuits) are members of the second group. Each group have numerous advantages and disadvantages described in the following sections.

These methods have been used by different authors, but the required conditions for their proper use have been omitted. These conditions are defined in this paper.

2.1. Reference Plane Methods

The reference plane methods define an imaginary plane which divides the circuit into two sub-circuits. The resonator sub-circuit is located on one side of the plane and the active sub-circuit on the other side. The use of these methods is preferred when the resonator can be identified as a dipole and the feedback is not easily identified. If the feedback is identified, the loop-gain method is usually the chosen one [2]. The reference plane methods have the main advantage of being suitable for RF topologies, mainly for microwave and distributed elements circuits. The circuits with distributed elements, even with the use of Alechno's virtual ground concept [13], are difficult or impossible to analyze by the loop-gain method.

All circuits can be analyzed by $Z/Y/\Gamma$ network functions. The network functions are preferred as they include all the system poles, but the general transfer functions (defined by $Z/Y/\Gamma$ parameters) do not include all the system poles.

The condition for a stable oscillation and proper start-up is that the network function has a unique pair of complex poles in the s domain Right Half Plane (RHP). The factorized network function defines a time response (Equation (1)), for the p-pole and with k-multiplicity.

$$L^{-1}\left[\frac{a_k}{(s-s_p)^k}\right] = \frac{a_k \cdot t^{k-1} \cdot e^{s_p \cdot t}}{(k-1)!} \tag{1}$$

Following the same procedure, it is possible to obtain the conclusion that if there is more than one pair of poles in the RHP the solution will be quasi-periodic [5]. Neither the quasi-periodic solutions nor the positive real poles are desirable for oscillators.

The reference plane methods are: negative resistance, negative conductance and reflection coefficient. All these methods divide the oscillator into resonator and active parts. Both sub-circuits must be analyzed by impedance, admittance or reflection coefficient ($Z/Y/\Gamma$), as defined in Figure 1.

The classic interpretation of the oscillators, when they are analyzed by reference plane methods, is that the resonator defines the oscillation frequency and that the active sub-circuit compensates the resonator losses. This interpretation is based on the Kurokawa's first harmonic approximation or descriptive function [14]. This classic interpretation is easy to understand and usually shown on reference books with a parallel resonator in parallel with a negative conductance generator and a serial resonator in serial with a negative resistance generator [2, 3, 9].

The network function is a more formal and powerful method for oscillator analysis. The procedure is similar for the three types
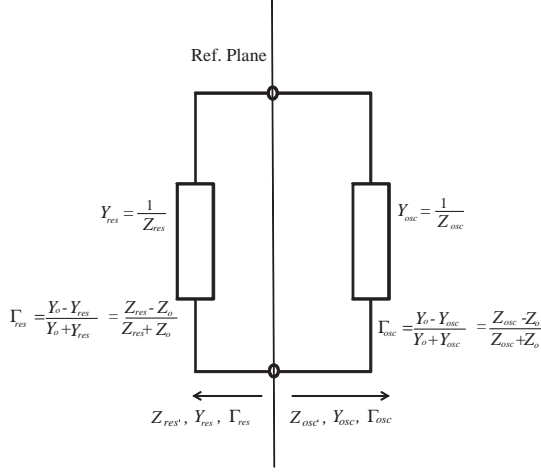
Figure 1. Oscillator reference plane division.

of network functions, so only the impedance network function is described. The network function is obtained on a negative conductance circuit by injecting current at one node on the reference plane with an ideal current generator. The network function includes all the poles from the sum of the numerators of the admittance of both sub-circuits (right and left) [15]. From Figure 1, the impedance network function is defined (Equation (2)) where I_g is the injected current, V the circuit response, and Z the inverse of the sum of the two sub-circuits admittances.

$$V = Z \cdot I_g$$
$$Z = \frac{1}{Y_{res} + Y_{osc}} \tag{2}$$

If the circuit is a proper oscillator, Z must have only a pair of complex poles in the RHP. The poles of Z are defined by the zeros of Y_T (Equation (3)), where Y_T is the characteristic function. The classic oscillation condition is defined as $\Im(Y_T) = \Im(Y_{res} + Y_{osc}) = 0$ and $\Re(Y_T) = \Re(Y_{res} + Y_{osc}) < 0$. This oscillation condition can be extended, by means of the descriptive function, to the Kurokawa's first harmonic approximation [14]. With the first harmonic approximation, the oscillation start-up condition, oscillation stability and minimum noise can be defined (see Table 1). In equations of Table 1, V is the voltage amplitude at reference plane (for the first harmonic), ω the frequency, V_0 the voltage amplitude at oscillation condition, and ω_0

the oscillation frequency.

$$Y_T = Y_{res} + Y_{osc} \tag{3}$$

Following the same procedure, it is possible to get the conditions for the negative resistance and reflection coefficient cases (see Table 2 and Table 3) [15].

Even if the first harmonic approximation is an attractive method, it is important to remember that the first harmonic approximation is an approximation and more accurate when the signal of the oscillator is a purer tone. The harmonic level is lower when the poles Q_L is higher and the amplifier device working in a quasi-A class. These two conditions are usually related. One of the most important issues to have in mind to decide the reference plane position is to choose a point where a serial or parallel resonant circuit is clearly defined. With this plane position, the equations in Tables 1, 2 and 3 are valid because the resonator

Table 1. Negative conductance oscillator design.

Parameter	Expression
Characteristic Equation	$Y_T(V, \omega) = Y_{osc}(V) + Y_{res}(\omega) = 0$
Oscillation Condition	$Y_T(V_0, \omega_0) = Y_{osc}(V_0) + Y_{res}(\omega_0) = 0$
Stability	0 to π clockwise cross of $-Y_{osc}(V)$ with $Y_{res}(\omega)$
Minimum Noise	$\pi/2$ clockwise cross of $-Y_{osc}(V)$ with $Y_{res}(\omega)$

Table 2. Negative resistance oscillator design.

Parameter	Expression
Characteristic Equation	$Z_T(I, \omega) = Z_{osc}(I) + Z_{res}(\omega) = 0$
Oscillation Condition	$Z_T(I_0, \omega_0) = Z_{osc}(I_0) + Z_{res}(\omega_0) = 0$
Stability	0 to π clockwise cross of $-Y_{osc}(V)$ with $Y_{res}(\omega)$
Minimum Noise	$\pi/2$ clockwise cross of $-Z_{osc}(I)$ with $Z_{res}(\omega)$

Table 3. Reflection coefficient oscillator design.

Parameter	Expression
Characteristic Equation	$\Gamma_T(A, \omega) = -\dfrac{1}{\Gamma_{osc}(A)} + \Gamma_{res}(\omega) = 0$
Oscillation Condition	$\Gamma_T(A_0, \omega_0) = -\dfrac{1}{\Gamma_{osc}(A_0)} + \Gamma_{res}(\omega_0) = 0$
Stability	0 to π clockwise cross of $\dfrac{1}{\Gamma_{osc}(A)}$ with $\Gamma_{res}(\omega)$
Minimum Noise	$\pi/2$ clockwise cross of $\dfrac{1}{\Gamma_{osc}(A)}$ with $\Gamma_{res}(\omega)$

Table 4. Reference plane oscillator design.

Parameter	Expression	
Impedance	$R_{osc}(\omega_0) + R_{res}(\omega_0) < 0$	$X_{osc}(\omega_0) + X_{res}(\omega_0) = 0$
Admittance	$G_{osc}(\omega_0) + G_{res}(\omega_0) < 0$	$B_{osc}(\omega_0) + B_{res}(\omega_0) = 0$
Reflection Coefficient	$\mid \Gamma_{osc}(\omega_0) \mid \cdot \mid \Gamma_{res}(\omega_0) \mid > 1$	$\Phi_{osc}(\omega_0) + \Phi_{res}(\omega_0) = 0$

sub-circuit response varies mainly with the frequency and because the active sub-circuit response varies with the signal amplitude. This plane position also guarantees a low harmonic level. This representation and plane position are usually shown on oscillator design bibliography, but they are difficult to achieve in real oscillator circuits. These reference plane properties are valid for negative resistance, negative admittance and reflection coefficient methods.

As indicated previously, many references [2, 3] consider the reference plane methods as completely valid, but the additional conditions to assure the proper use of these methods have not been defined in most of the cases. So, it is necessary to add auxiliary conditions to the ones in Table 4 to assure the proper use of these methods.

The conditions in Table 4 are neither necessary nor sufficient to guarantee the proper oscillator start-up as they have just been defined at the oscillation frequency. These conditions are a simplification of the Nyquist analysis. The necessary and sufficient condition for a proper oscillator start-up is the existence of a pair of complex poles in the RHP. To have access to the s domain network function of the circuit is difficult or even impossible for RF and MW circuits. A frequency analysis of the network function is the most common approach, which is the Nyquist Analysis.

The described conditions are only valid if some additional conditions are also satisfied. These Z and Y additional conditions are that the cross over the real axis must be only at a unique frequency and that the imaginary part must change from negative to positive on the crossing. The additional conditions for the Γ case are that the cross over the real axis must be only at a unique frequency and that the imaginary part of the sum of the phases must change from positive to negative on the crossing. Simple circuits, as a serial resonator with a negative resistance generator and a parallel resonator in parallel with a negative conductance generator, comply with these additional conditions. The common unconditional trust in Table 1 and Table 2 is strengthened with the use of these simple circuit models on bibliography. Jackson's work [16] is very interesting because it

demonstrates that the conditions in Table 4 for the reflection coefficient case are neither necessary nor sufficient for the oscillator start-up.

The Nyquist plots only provide information about the difference of zeros and poles of the analyzed function, so it is necessary to have additional conditions to assure the correct analysis of the oscillator. From the works of some authors [15, 17–19] it is possible to define the additional conditions for the correct use of these methods for stability analysis. The additional conditions for the right and left sub-circuits for the Z, Y and Γ methods are defined in the following sections.

2.1.1. Negative Conductance Oscillator

The negative conductance oscillator is analyzed by the Z network function. For the negative conductance criteria the simulator is suitable for getting the Y_T frequency response. The frequency response of Y_T is the application of the Nyquist criteria to it. The Nyquist criteria provides information of $(NZ - NP)$, where NZ is the number of zeros of Y_T in the RHP and NP the number of poles of Y_T in the RHP. For a positive $(NZ - NP)$, this value is the number of clockwise encirclements of the origin when the frequency goes from $-\infty$ to $+\infty$. The Nyquist analysis of Y_T will show the number of zeros of Y_T if Y_T does not have any poles in the RHP, so it is necessary to guarantee that Y_T does not have any poles in the RHP before testing it using the Nyquist analysis.

Y_T must not have any visible or hidden poles in the RHP. As Y_{res} is passive, the required condition to assure that Y_T does not have any poles in the RHP is that Y_{osc} does not have visible or hidden poles in the RHP. It is important to remember that as Y_{osc} is a reduced function from the active sub-circuit, it can have some hidden poles. If the network has any hidden poles, the hidden poles can invalidate the Y_T Nyquist analysis and make the oscillator operate in a wrong way.

To apply the Normalized Determinant Function (NDF) to a network formed by the negative conductance generator sub-circuit with a short-circuit on its port is the only way to assure that Y_{osc} does not have any (visible or hidden) poles in the RHP.

The $(Y_{osc}+Y_{res})$ Nyquist analysis (if the NDF analysis of the short-circuited Y_{osc} is complied) provides information about the Z poles, but it may not provide accurate information about oscillation frequency because the position of the RHP poles of Y_T depends on the reference plane position. The higher the Q_L of the poles $(Q_L(p) = \Im(p)/\Re(p))$ is, the nearer the $\Im(Y_{osc} + Y_{res})$ cross over the real axis will be to the oscillation frequency.

When the transistor gain is compressed, by decreasing g_m as Kurokawa defines the first harmonic approximation, Y_T crosses over

zero. At the zero crossing it will be $\Im(Y_{res} + Y_{osc}) = 0$ and $\Re(Y_{res} + Y_{osc}) = 0$, and this is the oscillation frequency at first harmonic approximation. The g_m compression for first harmonic approximation is more valid for FET than for BJT.

2.1.2. Negative Resistance Oscillator

The negative resistance oscillator is analyzed by the Y network function. The same analysis as in the previous case can be performed with $Z_T = (Z_{osc} + Z_{res})$. In this case, it is necessary that Z_T does not have any poles in the RHP for a proper Nyquist analysis. As Z_{res} is a passive network, the poles can only be from Z_{osc}. The NDF analysis of the negative resistance generator sub-circuit with its port terminated with an open circuit is necessary to assure that Z_{osc} does not have any poles in the RHP.

The Nyquist analysis of Z_T (if Z_{osc} complies with the NDF requirement) provides information about the existence of Y visible or hidden poles in the RHP, but, as in the previous case, the cross over the real axis of $\Im(Z_{res} + Z_{osc})$ is not exactly the oscillation frequency. The reference plane position also modifies the $\Im(Z_{res} + Z_{osc})$ cross frequency over the positive real axis. The g_m compression displaces the $\Im(Z_{res} + Z_{osc})$ cross over zero and when $\Im(Z_{res} + Z_{osc}) = 0$ and $\Re(Z_{res} + Z_{osc}) = 0$ the cross frequency is the first harmonic approximation oscillation frequency.

2.1.3. Reflection Coefficient Oscillator

For this analysis, the Γ_{osc} and Γ_{res} frequency response is analyzed with a circuit analysis software. Then the $1 - (\Gamma_{osc} \cdot \Gamma_{res})$ or $\Gamma_{osc} \cdot \Gamma_{res}$ (looking for +1 encirclement instead of 0) is analyzed by Nyquist. Nyquist analysis provides information of $NZ - NP$, where NZ is the number of zeros in the RHP and NP the number of poles in the RHP.

The necessary and sufficient condition for proper oscillator start-up is the existence of a unique pair of complex poles in the RHP of Γ (Equation (4)).

$$\Gamma(s) = \frac{\Gamma_{osc}(s)}{1 - \Gamma_{osc}(s) \cdot \Gamma_{res}(s)} \tag{4}$$

As Γ_{res} is a passive network, it does not have any poles in the RHP. It is necessary to assure that Γ_{osc} does not have any visible or hidden poles in the RHP before performing the Nyquist analysis of $1 - (\Gamma_{res} \cdot \Gamma_{osc})$. The only possible way to assure that Γ_{osc} does not have any poles in the RHP is to calculate the NDF of the active sub-circuit with its port terminated with Z_0.

As with the negative resistance and admittance cases, the cross over the real axis of $\Im(\Gamma_{osc} \cdot \Gamma_{res})$ is not the oscillation frequency and it has the same dependence with the Q_L of the poles and with the reference plane position. In this case the g_m compression also makes the cross frequency coincide with the first harmonic aproximation oscillation frequency.

Examples of circuits that comply with the conditions in Table 4 but not with the additional conditions defined on this paper are shown by the authors in [15].

To summarize, it is important to point out that the proviso must be assured before analyzing an oscillator with a linear negative conductance, negative resistance or reflection coefficient method. The provisos for negative conductance, negative resistance and reflection coefficient methods are the stability of the active sub-circuit loaded with an open-circuit, a short-circuit and Z_0, respectively. The stability of the loaded active sub-circuits is determined by the NDF plots. Some examples of erroneous classic linear analysis are shown by the authors on [15].

2.2. Loop-Gain Method

The Loop-Gain method [1] is equivalent to the reference plane methods. It also analyzes the Nyquist plot of a network function, but the loop-gain method is suitable to calculate the gain margin and Q_L. The Q_L is related to the phase noise, and the Q_L and gain margin are related to the start-up time [1, 2, 20]. If the oscillator is analyzed by a plane reference method, these two parameters, phase-noise and start-up time, are not easily estimated, but they can be easily estimated with the loop-gain method. Another advantage of the loop-gain method is that it is useful for studying new oscillator topologies [20, 21].

Equation (5) is the equation of a general feed-back system. The main problem to use this equation is that the analytical expression in Laplace domain of RF and MW systems is difficult or even impossible to get. Most designers use the Nyquist criteria for linear oscillator start-up conditions analysis [22]. Simplifications of Nyquist analysis, which are not valid for all cases, as the Barkhausen criteria, are sometimes used.

$$X_0(s) = \frac{G(s)}{1 + G(s) \cdot H(s)} \cdot X_i(s) \tag{5}$$

The Nyquist criteria are used to determine the stability of Equation (5) by determining the zeros of the characteristic equation (Equation (6)). These zeros are the poles of the feed-back Equation (5)). The Nyquist criteria determine the difference of zeros

and poles in the RHP of the characteristic equation. The system will have a periodic solution if it has a pair of complex poles in the RHP. This condition must be verified by using the Nyquist plot analysis.

$$1 + G(s) \cdot H(s) \tag{6}$$

The loop-gain method [1, 2] tries to identify a feed-back structure as the one shown in Figure 2. Randall and Hock [1] obtain the network function as a function of Z parameters, and later it is transformed as a function of S parameters. Randall and Hock's feed-back equation has gained a great importance thanks to its S parameters formulation.

$$\frac{I_o}{I} = \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}}$$
$$\frac{I_o}{I} = \frac{S_{22} - S_{11} + 2S_{21} - S_{12}S_{21} + S_{11}S_{22} - 1}{1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22})} \tag{7}$$

The system in Figure 2 will be unstable if I_o/I has a pair of complex poles in the RHP. To determine the number of poles of the network function the zeros of the Characteristic Function (CF), Equation (8), are analyzed by Nyquist criteria. To assure a correct Nyquist analysis of the CF, it is necessary to guarantee that none of the S parameters have any poles in the RHP. It is also necessary that the I_o/I numerator does not have any visible or hidden poles in the RHP. The best way to verify this condition is to analyze the circuit in Figure 2 with both ports Z_0 loaded with the NDF [17, 18]. This condition assures the correct solution of the CF Nyquist analysis but is not remarked by Randall and Hock [1].

$$CF = 1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22}) \tag{8}$$

The open-loop stability condition of the oscillator is similar to the Rollet proviso [19] for amplifier design methods that use the K [23] or
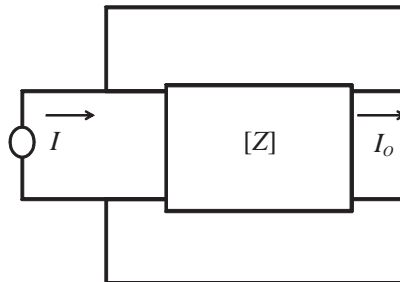

Figure 2. Randall feed-back diagram.

μ [24, 25] parameters. It is necessary to be able to perform a Nyquist analysis.

The CF predicts the oscillator start-up properly when the required conditions are satisfied, but it has the disadvantage of being dependent of the open loop point. This CF problem was solved by Randall and Hock with the definition of a new function [1]. This function is extracted from the eigenvalues of an infinite chain of quadrupoles. This new function (Equation (9)) solves the problem and the partial solutions proposed by Alechno [7, 8].

$$G_L = \frac{Z_{21} - Z_{12}}{Z_{11} + Z_{22} - 2Z_{12}}$$
$$G_L = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}} \tag{9}$$

Randall and Hock's loop-gain function (Equation (9)) is an attractive tool, but it needs to verify a set of additional conditions as the CF needs. The additional conditions are required to guarantee the correct determination of the RHP poles when Nyquist criteria is applied to G_L. This loop-gain function has, as the CF, dependence with the virtual ground position, defined by Alechno [13]. The virtual ground concept makes possible the loop-gain analysis of any circuit topology. The G_L dependence with the virtual ground position makes the G_L provides different solutions (oscillation frequency, Q_L, gain margin). The necessity of testing the additional conditions and the multiple solutions occurrence with virtual ground position are illustrated with examples by the authors.

Equation (7) is rewritten as Equation (10) for defining the G_L additional conditions.

$$\frac{I_o}{I} = \frac{\left(\dfrac{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12} - S_{22} + S_{11}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}\right)}{\left(1 - \dfrac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}\right)} \tag{10}$$

The G_L, Equation (9), can be found in the denominator of Equation (10). Randall and Hock's gain is obtained from the eigenvalues of an open loop loaded with Z_0. In our case it is possible to obtain a gain loop, Equation (11), thanks to its similarity to the expression of a feed-back system.

$$G_L = 1 - \left(1 - \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}\right)$$
$$= \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}} \tag{11}$$

The G_L has the advantage, in addition to the loop open point invariance, of providing the CF for $G_L = 1$. When $G_L = 1$, the poles are located in the imaginary axes and this is first harmonic stable oscillation condition.

Equation (10) is rewritten in function of G_L as Equation (12). And from this equation it is possible to define the additional conditions for Randall and Hock's G_L equation.

$$\frac{I_o}{I} = \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}} = \frac{-\dfrac{G_L\,(S_{22} - S_{11})}{S_{21} - S_{12}} + 1}{1 - G_L} \quad (12)$$

It is necessary that the RHP poles of the system are only from the $1 - G_L$ zeros. So, it is necessary to guarantee that neither G_L nor Equation (12) numerator have any poles in the RHP. When these conditions are fulfilled, the G_L Nyquist analysis is suitable to predict the oscillation condition. The Nyquist plot of G_L must clockwise encircle the +1 to predict a proper oscillation condition.

With the analysis of Equations (10) and (12), it can be guaranteed that the poles of the system are only the zeros of $1 - G_L$ if the following conditions are satisfied:

- None of the S parameters have any poles in the RHP. This condition is guaranteed if the NDF analysis of the circuit with both ports loaded with Z_0 does not clockwise encircle the origin.
- The Nyquist analysis of TF (Equation (13)) assures that it does not have any zeros in the RHP.

$$TF = 1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12} \quad (13)$$

The cross over the real axis of the G_L Nyquist plot predicts the presence of poles but it does not predict accurately the oscillation frequency. The cross frequency is nearer to the oscillation frequency when the Q_L of the poles is higher. But, even the cross frequency is the same for different open loop points, and it is important to remember that the cross frequency and gain margin are different for each virtual ground position.

Now that the additional conditions for G_L proper analysis have been defined, it can be extended to the first harmonic approximation. The characteristic equation, the oscillation condition and the minimum noise conditions are defined in Table 5. The G_L is divided into an active gain G_{osc} and a resonator circuit gain G_{res}. The A variable is the control variable, normally the incident wave when G_L is defined as a function of S parameters, and ω is the frequency. A_o and ω_o are the control variable and the frequency at the stable oscillation condition. It is important to remember that the conditions in Table 5 are defined

Table 5. Loop gain oscillator design.

Parameter	Expression
Characteristic Equation	$1 - G_L(A,\omega) = -\dfrac{1}{G_{osc}(A)} + G_{res}(\omega) = 0$
Oscillation Condition	$1 - G_L(A_o,\omega_o) = -\dfrac{1}{G_{osc}(A_o)} + G_{res}(\omega_o) = 0$
Stability	0 to π clockwise cross of $\dfrac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$
Minimum Noise	$\pi/2$ clockwise cross of $\dfrac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$

for the first harmonic approximation, so they will be more precise when the A signal is a purer tone.

To summarize, it is important to point out that the provisos for the use of Randall and Hock's equation or any of the simplifications are the verification of the stability of the open loop circuit with both ports loaded with Z_0 and that TF does not have any zeros in the RHP. The stability of the loaded open circuit is determined with the use of the NDF Nyquist plot.

3. PROPOSED METHOD

The most significant conclusion that can be obtained from the study of the additional conditions of the plane reference methods and gain loop method is that it is necessary to use the NDF to guarantee the applicability of any method. The question that must be done is: "Why is not the NDF directly used for oscillator linear design?". This question is the start point for the new proposed method. It will show that this new method solves many of the problems of the plane reference and gain loop methods.

The NDF is the quotient of the network determinant and the normalized network determinant, Equation (14). The normalized network determinant is obtained by disabling all the active devices of the network.

$$NDF = \frac{\triangle(s)}{\triangle_o(s)} \tag{14}$$

The NDF properties were described by Platzer [18]. The Nyquist plot analysis of the NDF is suitable for determining in one step the number of poles in the RHP of a network. Each clockwise encirclement of the zero for positive frequencies indicates the existence of a pair of conjugated complex poles in the RHP. On this way, the condition for

a start-up and proper operation of an oscillator is the existence of just one clockwise encirclement of the zero for positive frequencies. The NDF has an asymptotic behaviour towards +1, which is useful for determining the analysis upper frequency limit.

The NDF can be calculated by using the Return Relation (RR) suggested by Bode [26], as it has been described by Plazter [18]. The NDF calculation from the RR is shown in Equation (15). The RR_i is the Return Relation of the i dependent generator while the $i-1$ previous dependent generators have been disabled. It is necessary to have a linear model of the transistor to have access to the ports of the amplifier element (controlled current source). If the available model is a Spice or S parameters data file, then the linear model must be extracted from it. A linear model of a generic transistor without parasitic elements is shown in Figure 3. This model is suitable for NDF calculation using the Bode's RR, as the internal generator of the transistor is accessible. The parasitic elements of the transistor can be considered as part of the passive subcircuit of the oscillator.

$$NDF = \prod_{i=0}^{n} (RR_i + 1) \tag{15}$$

If the oscillator circuit is redrawn as in Figure 4, then it is possible to calculate the NDF from the RR function. The terminals of the dependent generator are accessible and all the other elements, even the chips and case parasitics, are included on the feedback network, this access to the transistor of an oscillator circuit is shown in Figure 5. A problem of this NDF calculation method is that a linear model of the transistor is required, but this linear model is necessary for all linear stability analysis methods as it was suggested by Jackson [19].

The NDF predicts the oscillation frequency without transistor compression for Kurokawa's first harmonic approximation. It is also suitable for the calculation of the Q_L of the circuit as the NDF is
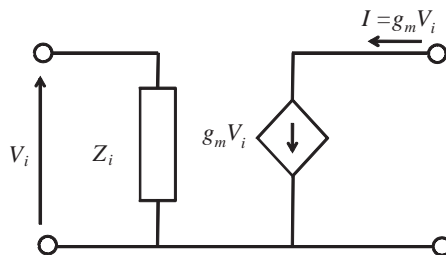

Figure 3. Generic intrinsic transistor linear model.

directly related with the RR. In order to simplify the calculus the RR_T is defined in Equation (16).

$$RR_T = -RR = RR_{osc} \cdot RR_{res} = g_m \cdot H(\omega) \qquad (16)$$

The RR_T is the "true" way to consider an oscillator as a feed-back system. The NDF separates the feed-back network from the amplifier function, as shown in Figure 4, so the RR_T can be considered as the "true" loop-gain. Considering the RR_T as an open loop gain
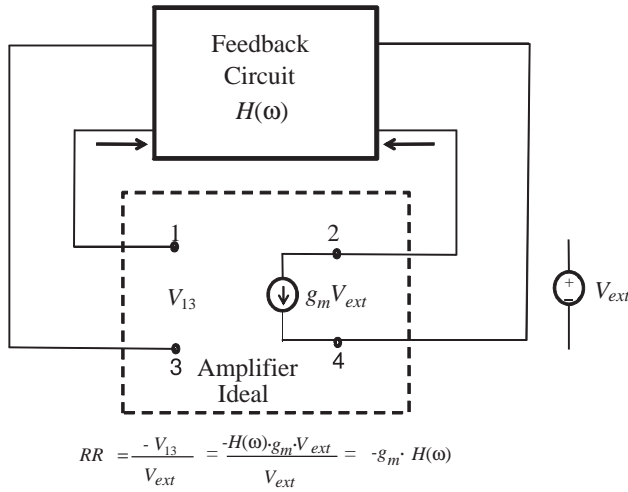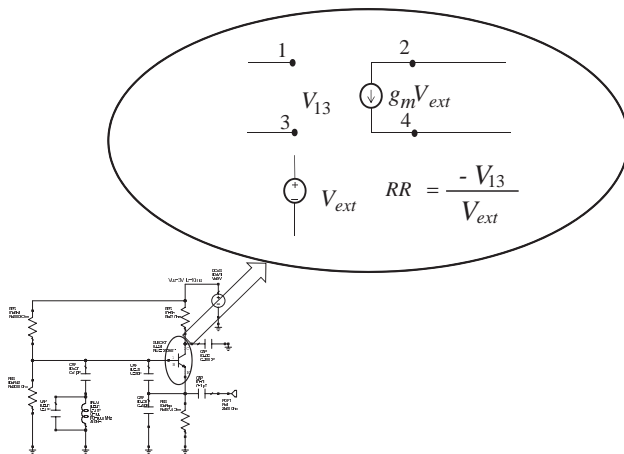

Figure 4. Oscillator model for RR calculation.

Figure 5. Circuit transistor model for RR calculation.

Table 6. RR_T oscillator design.

Parameter	Expression
Characteristic Equation	$1-RR_T\left(V,\omega\right)=-\dfrac{1}{RR_{osc}(V)}+RR_{res}\left(\omega\right)=-\dfrac{1}{g_m\left(V\right)}+H(\omega)=0$
Oscillation Condition	$1-RR_T\left(V_o,\omega_o\right)=-\dfrac{1}{RR_{osc}\left(V_o\right)}+RR_{res}(\omega_o)=-\dfrac{1}{g_m(V_o)}+H(\omega_o)=0$
Stability	0 to π clockwise cross of $\dfrac{1}{RR_{osc}\left(V\right)}$ with $RR_{res}\left(\omega\right)$
Minimum Noise	$\pi/2$ clockwise cross of $\dfrac{1}{RR_{osc}\left(V\right)}$ with $RR_{res}\left(\omega\right)$

and in a similar way as in Table 5, the characteristic equation, the oscillation condition and the minimum noise conditions are defined in Table 6. RR_T is decomposed into the active contribution of the loop, RR_{osc}, and the passive contribution of the loop, RR_{res}. The active contribution only considers the dependent generator ($RR_{osc} = g_m$) and the passive contribution includes the resonator and all the other passive elements ($RR_{res} = H(\omega)$). The V variable is the control variable of the dependent generator, and ω is the frequency. V_o and ω_o are the control variable and the frequency at the stable oscillation condition. It is important to remember that these conditions are defined for first harmonic approximation, so they will be more precise when the V signal is a purer tone, the required compression level is lower and the resonator bandwidth is smaller.

The proposed method for oscillator quasi-linear analysis is to use the Nyquist plot of the NDF/RR_T. If the RR_T Nyquist plot has a unique clockwise encirclement of the $+1$ (or of the zero if the NDF is used) the plot cross over the real axis is the oscillation frequency. The gain margin is determined by the distance between the real axis cross and the $+1$ and the Q_L is determined by the phase variation speed at the cross.

The main advantages of the NDF/RR_T method are:

- It provides the oscillation frequency at first harmonic approximation without requiring transistor compression. The cross over the real axis is always at the same frequency without g_m dependence. This independence with g_m is because g_m is only a multiplicative constant of $H\left(\omega\right)$.

- It can be used for any oscillator topologies, whereby, it is not necessary to identify the resonator as a dipole or quadrupole, as required by the loop-gain and reference plane methods.

- It is not necessary to redraw the circuit for a more suitable

distribution. The Alechno's virtual ground has been applied as a tool to make possible the use of the loop-gain method for multiple topologies. The virtual ground requires a thorough redraw of the circuit. An additional limitation of the loop-gain method is that some circuits, as the ones that use transmission lines, cannot be redrawn using virtual ground because they do not have explicit ground.

- The solution is unique and it has not dependence with the virtual ground. The solution is always the correct, but with classic methods, even if the provisos are fulfilled, it cannot be guaranteed because the classic methods solutions depend on the virtual ground position.

- The upper frequency of analysis can be assured thanks to the NDF asymptotic behaviour towards +1. By this way, it can be guaranteed that only a pair of conjugated poles in the RHP exist (condition for proper operation of an oscillator).

- The oscillator gain margin and Q_L can be estimated. As the RR_T is the "true" open loop-gain, the Q_L is determined from the gain variation speed. The reference plane methods cannot determine the Q_L as the gain is not calculated. The loop-gain method provides different gain solutions for each virtual ground position.

- The phase noise for a first harmonic approximation can be estimated and optimized as the Q_L is estimated without ambiguity.

4. PRACTICAL EXAMPLES

Two oscillators usually analyzed by reference plane methods have been designed: a capacitive emitter feed-back common collector oscillator and an inductive base feed-back common base oscillator. These circuits are shown in Figure 6 and Figure 20. The simulations have been performed with complete circuits that include all parasitic elements and microstrip lines, but the schematic shown on this paper are simplifications without parasitic elements to improve readability. A low cost and medium noise Siemens BFR380F transistor is used for these circuits.

AWR Microwave Office has been used as simulation software. This tool has an internal NDF function, but the authors have developed a script that has been used in order to generalize the method. This script automatizes the procedure shown in Figure 5.
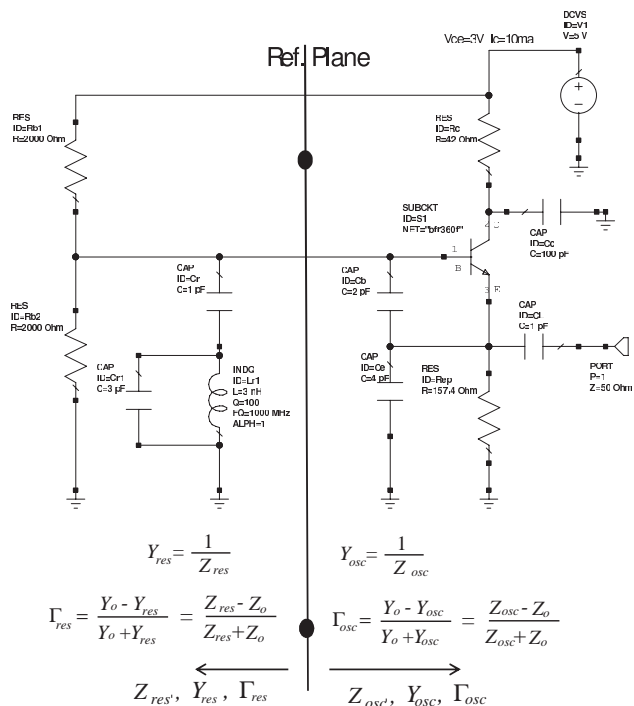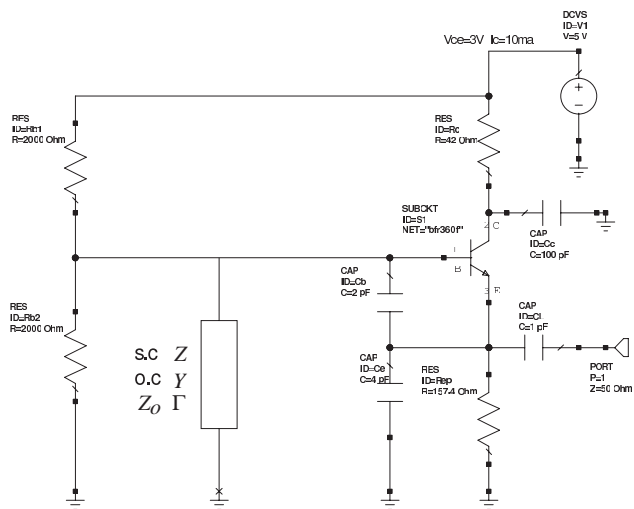
$$Y_{res} = \frac{1}{Z_{res}} \qquad\qquad Y_{osc} = \frac{1}{Z_{osc}}$$

$$\Gamma_{res} = \frac{Y_o - Y_{res}}{Y_o + Y_{res}} = \frac{Z_{res} - Z_o}{Z_{res} + Z_o} \qquad \Gamma_{osc} = \frac{Y_o - Y_{osc}}{Y_o + Y_{osc}} = \frac{Z_{osc} - Z_o}{Z_{osc} + Z_o}$$

$$\overleftarrow{Z_{res},\ Y_{res},\ \Gamma_{res}} \qquad \overrightarrow{Z_{osc},\ Y_{osc},\ \Gamma_{osc}}$$

Figure 6. Common collector oscillator.

Figure 7. Common collector oscillator proviso verification.

4.1. Example 1. Common Collector Oscillator (Negative Resistance Oscillator)

Before starting an analysis using any reference plane method, it is required to perform the verification of the necessary conditions. This analysis consists on the analysis of the plot of the NDF of the right sub-circuit of Figure 6. The NDF must be calculated for the negative resistance generator circuit, Figure 7. This circuit must be loaded with an open-circuit for admittance network function analysis, with a

(a)

(b)

(c)
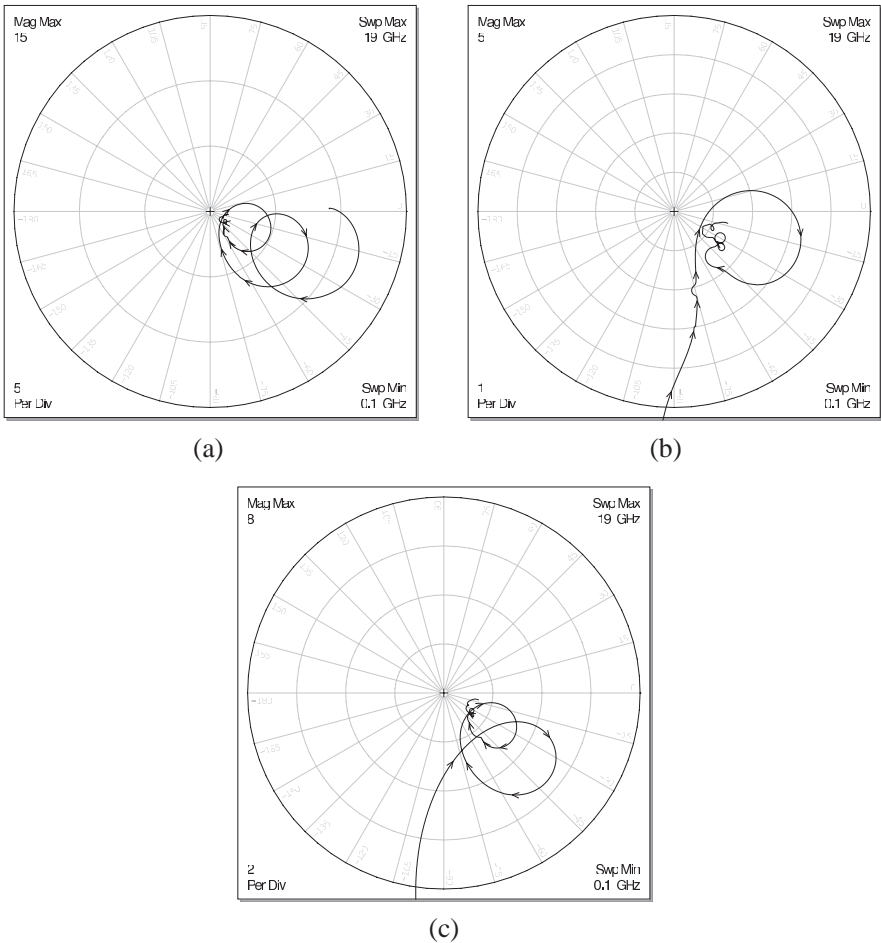
Figure 8. Common collector oscillator (a) open-circuit, (b) short-circuit, and (c) Z_0 loaded Proviso tests.

short-circuit for impedance network function and with Z_0 for reflection coefficient network function.

The NDF plots for open-circuit, short-circuit and Z_0 load conditions do not encircle the origin, Figure 8. As the origin is not encircled in any case, it is possible to analyze this circuit using admittance, impedance and reflection coefficient network functions.

This circuit is a negative resistance generator and usually analyzed by the admittance network function. In other words, the circuit is analyzed by the calculation of its impedances. The Nyquist plots for the three network functions are shown in Figures 9, 10 and 11.
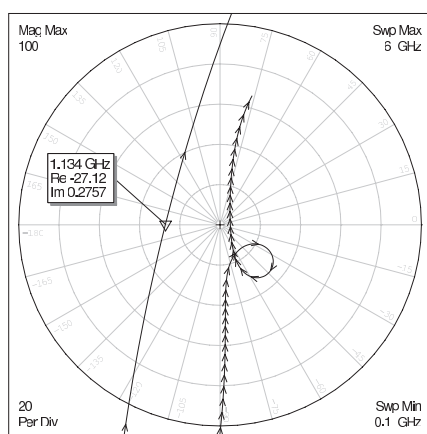
Figure 9. Common collector oscillator impedance Nyquist plot.
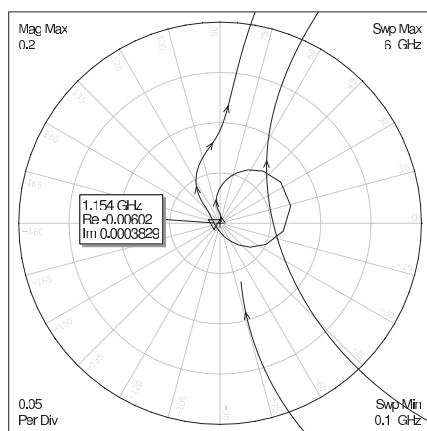
Figure 10. Common collector oscillator admittance Nyquist plot.

The three plane reference network functions predict an oscillation near to 1.14 GHz, but the predicted frequencies are different for each case. The plane reference methods are not suitable for the estimation of the oscillator Q_L, gain margin or start-up time.

This circuit can be prepared for open-loop analysis [2]. The chosen open point is between the emitter and the center point of the feed-back capacitors (see Figure 12).
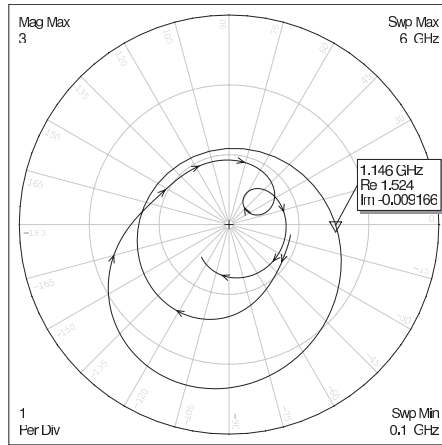

Figure 11. Common collector oscillator reflection coefficient Nyquist plot.
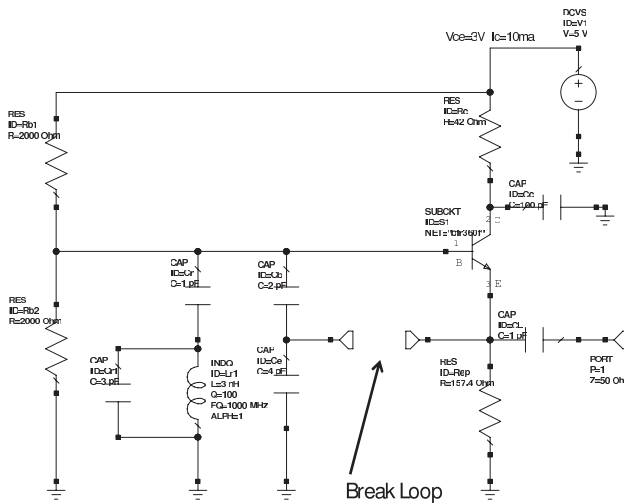
Figure 12. Common collector oscillator for G_L analysis.

Before analyzing the circuit in Figure 12, it is necessary to test the required additional conditions. The proviso is tested with the NDF plot of the open-loop circuit loaded with Z_0 on both ports. This plot is shown in Figure 13. This plot shows that there are a pair of poles in the RHP, so the G_L analysis will not provide correct information about the circuit oscillation condition. As the NDF proviso indicates that the G_L analysis will be incorrect, it is not necessary to study the TF.
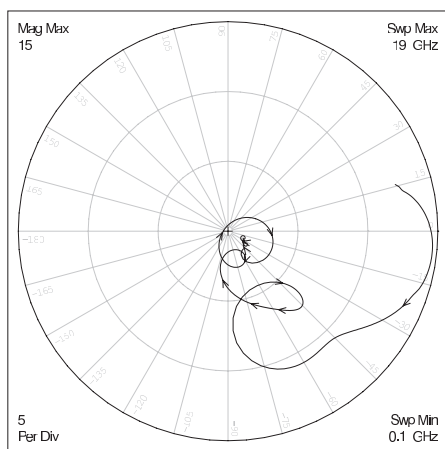
Figure 13. Common collector oscillator proviso plot for G_L analysis.
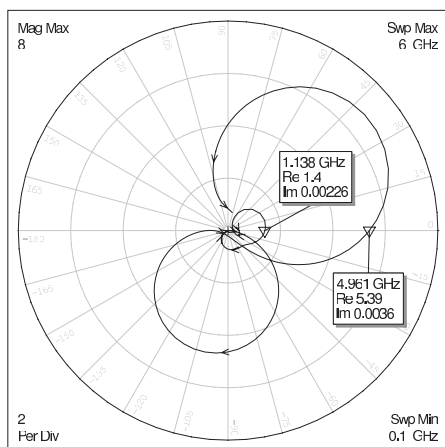
Figure 14. Common collector oscillator G_L Nyquist plot.

The G_L analysis is performed only for illustrative purposes, even it is known that the GL analysis will provide wrong information. The Nyquist G_L plot is shown in Figure 14. This plot crosses, with real part higher than 1, two times the real axis, one time clockwise and the other counterclockwise. The $+1$ is not encircled, so the G_L analysis predicts a non-oscillating condition.

The circuit has been analyzed with reference plane and loop-gain methods. The first method has estimated the oscillation frequency, but the gain margin and Q_L cannot be estimated by using it. The second method, using Randall and Hock's gain, has concluded that the circuit does not have an oscillation condition. Now, the NDF/RR_T method will be used and the different linear methods solutions will be compared with the Harmonic Balance (HB) and experimental results.

The proposed method is applied to the circuit in Figure 6. The RR_T plot is shown in Figure 15. This analysis predicts a unique pair of poles in the RHP at $1.12\,\text{GHz}$. This frequency is similar to the one predicted by the reference plane methods. It is important to remember that the NDF/RR_T method estimates the first harmonic approximation oscillation frequency, and it is not necessary to compress the g_m of the transistor as with the reference plane and loop-gain methods. With the NDF/RR_T method is possible to calculate the Q_L of the circuit, Equation (17). As it is possible to estimate Q_L, the phase noise of the oscillator can be approximated with the Lesson expression [27], or the Everard improved one [28, 29]. The Q_L is shown in Figure 16, and the Q_L estimated value for the oscillation frequency
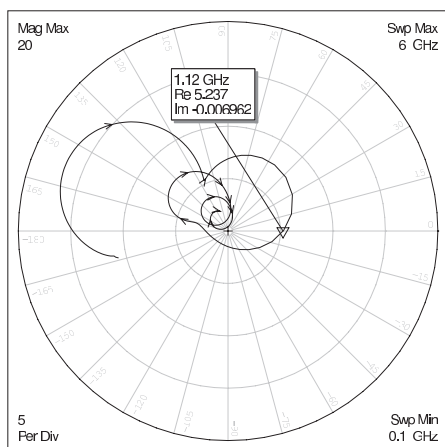

Figure 15. Common collector oscillator NDF/RR_T Nyquist plot.

is 28.

$$Q_L = -\frac{\omega}{2} \cdot \frac{d}{d\omega} \arg\left(RR_T\left(\omega\right)\right) = -\frac{f}{2} \cdot \frac{d}{df} \arg\left(RR_T\left(f\right)\right) \qquad (17)$$

The measured and HB simulated spectra are shown in Figure 17. The HB oscillation frequency is $1.129\,\mathrm{GHz}$, similar to the one obtained with the NDF/RR_T method. The measured and HB simulated phase noises are shown in Figure 18. The two phase noises have a great concordance. The measured floor noise in Figure 18 is the measurement equipment floor noise (HP E4446A Spectrum Analyzer) and not the oscillator floor noise. A summary of the most significant parameters of the oscillator are shown in Table 7.

Table 7. Common collector oscillator parameters.

Parameter	Value
Output Frequency	$1.143\,\mathrm{GHz}$
Phase Noise	$1\,\mathrm{kHz}\ -86.44\,\mathrm{dBc/Hz}$
	$10\,\mathrm{kHz}\ -112.1\,\mathrm{dBc/Hz}$
	$100\,\mathrm{kHz}\ -133.7\,\mathrm{dBc/Hz}$
Output Power	$> -5.63\,\mathrm{dBm}$

The Q_L can be calculated with Equation (18) from the HB simulation of the phase noise, Figure 18. The Q_L is similar to the one estimated by the NDF/RR_T method with Equation (17), which is shown in Figure 16.
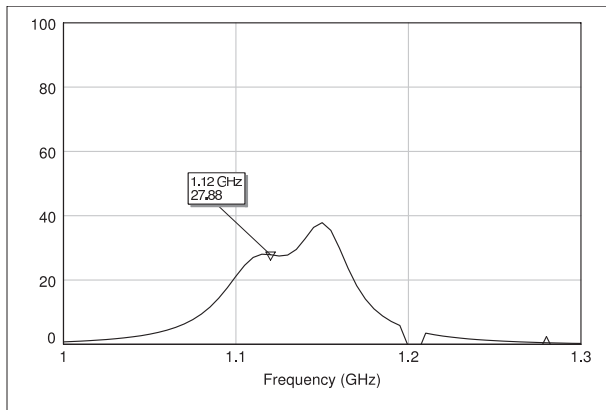

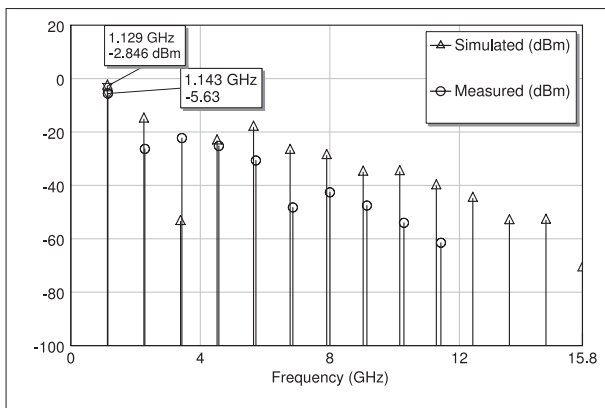**Figure 16.** Common collector oscillator Q_L.

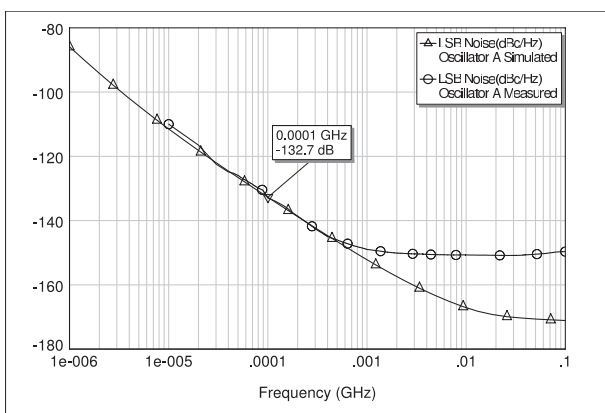**Figure 17.** Common collector oscillator HB and measured spectrum.

Figure 18. Common collector oscillator HB and measured phase noise.

$$Q_L \approx \frac{f_0}{f_m} = \frac{1.129\,\text{GHz}}{0.035\,\text{GHz}} \approx 32 \tag{18}$$

The manufactured oscillator is shown in the photo in Figure 19.

To summarize, the RR_T method has predicted the oscillation condition and it gets an accurate estimation of the oscillation frequency and the Q_L. The loop-gain method (G_L) has not predicted the oscillation condition. The reference plane methods have predicted the oscillation condition, but they are not suitable for gain margin and Q_L estimation.
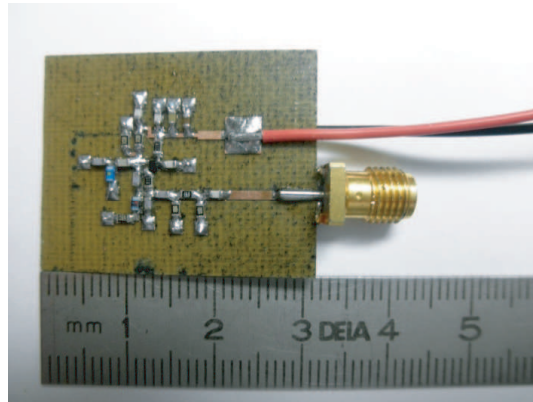
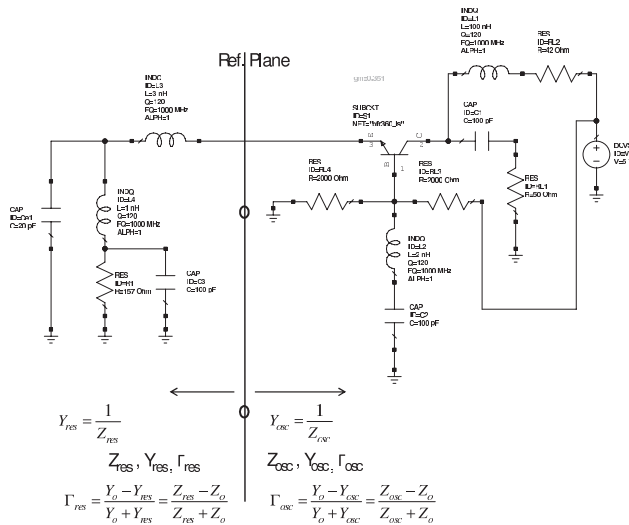**Figure 19.** Common collector oscillator photo.

$$Y_{res} = \frac{1}{Z_{res}}$$

$$Z_{res}, Y_{res}, \Gamma_{res}$$

$$\Gamma_{res} = \frac{Y_o - Y_{res}}{Y_o + Y_{res}} = \frac{Z_{res} - Z_o}{Z_{res} + Z_o}$$

$$Y_{osc} = \frac{1}{Z_{osc}}$$

$$Z_{osc}, Y_{osc}, \Gamma_{osc}$$

$$\Gamma_{osc} = \frac{Y_o - Y_{osc}}{Y_o + Y_{osc}} = \frac{Z_{osc} - Z_o}{Z_{osc} + Z_o}$$

Figure 20. Common base oscillator.

4.2. Example 2. Common Base Oscillator (Negative Conductance Oscillator)

The second example is a negative conductance oscillator, Figure 20. In this case, it is not possible to define a suitable feed-back path for open-loop G_L analysis. This impossibility appears because the feed-back includes the internal base-emitter capacitance. For this reason, this circuit model is usually analyzed by a reference plane method. In

this case the sub-circuits admittances are analyzed and the impedance network function is used. It is difficult to define the oscillator Q_L when the plane reference methods are used. Without information of the Q_L, the phase noise of the oscillator cannot be estimated neither optimized. The NDF method and the RR_T are suitable for analyzing the circuit on a similar way as the loop-gain. With the NDF/RR_T method the oscillator Q_L is available and it can be optimized for a phase noise improvement.

The RR_T analysis of the circuit in Figure 20 is shown in Figure 21. The trace encircles clockwise the $+1$, then the circuit has a pair of complex poles in the RHP. The poles frequency is 1.27 GHz.
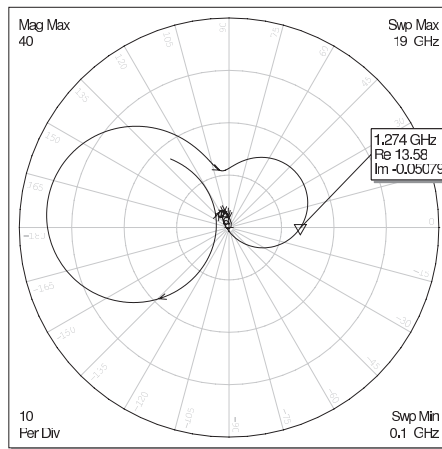

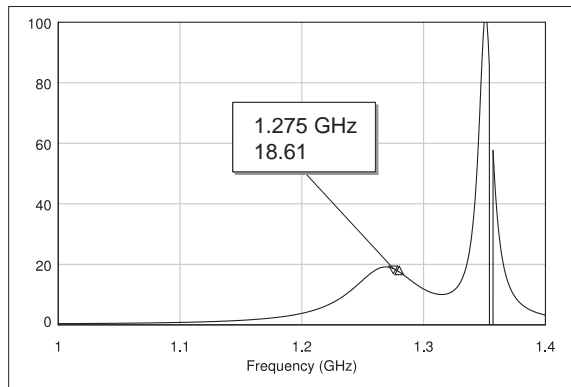**Figure 21.** Common base oscillator NDF/RR_T Nyquist plot.

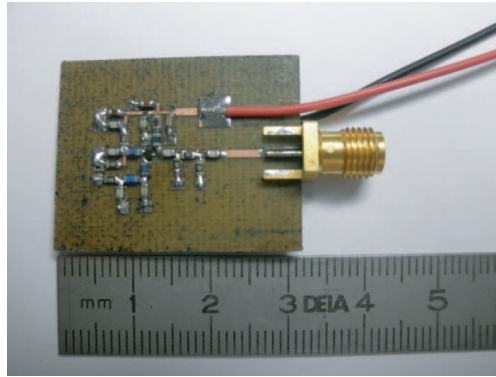**Figure 22.** Common base oscillator Q.

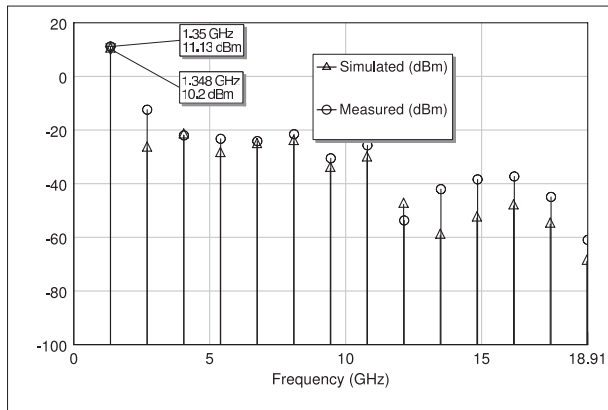**Figure 23.** Common base oscillator photo.

Figure 24. Common base oscillator HB and measured spectrum.

The Q_L is calculated from the RR_T plot using Equation (17) and shown in Figure 22. The Q_L at the oscillation frequency has a value of 18.

This oscillator has been manufactured and shown in the photo in Figure 23.

The oscillator measured and HB simulated spectra are shown in Figure 24. The measured and HB simulated phase noises are shown in Figure 25. As on the previous example, the simulated and measured phase noises have a great concordance.

A summary of the most significant parameters of the oscillator are shown in Table 8.
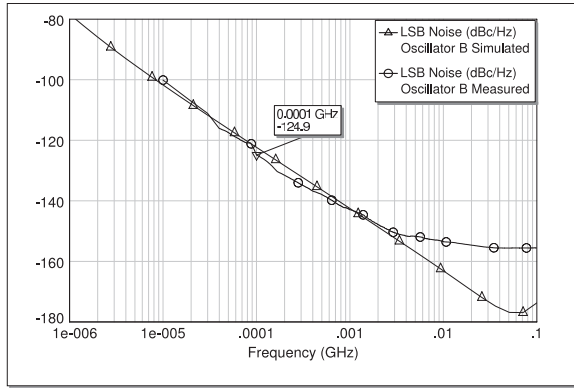
Figure 25. Common base oscillator HB and measured phase noise.

Table 8. Common base oscillator parameters.

Parameter	Value
Output Frequency	1.318 GHz
Phase Noise	1 kHz − 76.5 dBc/Hz
	10 kHz −101 dBc/Hz
	100 kHz −123.8 dBc/Hz
Output Power	> 11.13 dBm

5. CONCLUSIONS

This paper focuses on a new method for quasi-linear (descriptive function) oscillator design. This new method is based on the use of the NDF/RR_T. It has been demonstrated that the classic reference plane and loop-gain methods can provide wrong solutions. They cannot be considered universal design methods. These problems and wrong solutions are solved with the proposed method.

The classic method chosen for the analysis of a circuit is determined by experience with each topology. But the proper choice of the most suitable method is not guaranteed. The NDF must be used as it is necessary to test some additional conditions before any classic method is used. These additional provisos have been defined for each method as some authors defined amplifiers provisos.

The necessity of using the NDF for the verification of provisos of the classic methods makes the NDF/RR_T a natural method. The proposed NDF/RR_T method does not need any additional proviso, so it is always the correct method for any topology.

The use of the loop-gain method depends on the topology; however the provided solution is not always correct. Some topologies cannot be analyzed with the loop-gain but can be analyzed with plane reference methods. These methods do not provide information about the oscillator Q_L and gain margin. Without these two parameters, the phase noise and start-up time cannot be estimated, neither optimized.

The proposed NDF/RR_T method is suitable as a general method for all topologies and provides information about the oscillator Q_L and gain margin. The NDF/RR_T method is a general linear method for oscillator design which is suitable for phase noise estimation and optimization. The good results of this method have been demonstrated and compared with the results of the classic methods and the measurements of two practical examples.

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