OPTIMAL SYNTHESIS OF PHASE-ONLY RECONFIGURABLE LINEAR SPARSE ARRAYS HAVING UNIFORM-AMPLITUDE EXCITATIONS

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Abstract—In a large number of applications, including communications from satellites, an optimal exploitation of the available power is of the outmost importance. As a consequence, isophoric array architectures, i.e., arrays using the same power in all the different entry points and achieving the amplifiers' maximum efficiency, are of great interest. At the same time, the easy reconfigurability of the power patterns results fundamental in order to get a full exploitation of the payload. In this paper, an innovative and deterministic approach is proposed for the optimal synthesis of linear phase-only reconfigurable isophoric sparse arrays able to commute their pattern amongst an arbitrary number of radiation modalities. The introduced perspective leads to an effective solution procedure for the fast design of antennas with high performance, and does not recur to computationally expensive global-optimization techniques. Numerical results concerning applications of actual interest and employing realistic element patterns are provided in support of the given theory.

1. INTRODUCTION

Due to the extraordinary large number of its applications, the effective design of single antennas able to radiate more than one pattern is a very long standing subject in electromagnetics [1]. In particular, amongst the different kinds of reconfigurable radiating systems, array antennas have a relevant role, as they can be controlled by means of completely-electronic techniques increasing the flexibility and the speed of the reconfiguration [1–4].

On the other hand, a new challenge has recently arisen in satellite and radar communication area. In such applications, one needs to exploit in an optimal fashion the available power, so that, when Direct Radiating Arrays (DRA) are in order, one usually requires that all the different 'entry points' of the antenna work under the same optimal conditions. Therefore, in all cases of sparse, thinned, and clustered DRA, the control points must have the same (unitary) entry power, i.e., the array must be 'isophoric' [5–8]. Moreover, in order to have DRA competitive with more usual reflector-based solutions, the number of control points must be as low as possible.

When considering in a joint fashion all the requirements above, the problem arises to synthesize an aperiodic array such that, with a common architecture or geometry, and by using just unitary entry points, one can reconfigure in an easy fashion the power pattern amongst at least two different modalities.

This contribution presents an innovative and effective approach aimed to solve such problem in the case of one-dimensional isophoric arrays. The proposed solution procedure takes advantage from the strategy proposed in [7] for the synthesis of shaped beams by means of isophoric sparse arrays, as well as from a recently introduced point of view to the synthesis of phase-only reconfigurable arrays lying on a regular lattice [9].

The paper is organized as follows. In Section 2, we briefly recall available procedures for the synthesis of uniformly-excited sparse arrays radiating single pencil or shaped beams. Then, in Section 3, we propose a new approach to the optimal design of phase-only reconfigurable isophoric arrays. Sections 4 and 5 respectively present a final optimization procedure aimed at maximizing the antennas' performance and a set of numerical results achieved in realistic conditions and concerning applications of actual interest. Conclusions close the paper.

2. FAST DESIGN OF ISOPHORIC ARRAYS RADIATING SINGLE PENCIL OR SHAPED BEAMS: STATE OF THE ART

The problem of synthesizing pencil beams by means of linear isophoric sparse arrays has been studied since the sixties of the last century, with pioneering contributions from Doyle [10], Skolnik [11], and many others, introducing interesting analytical procedures for an effective antenna design. Later, synthesis techniques based on 'global' optimization procedures have been also proposed to tackle the same problem [12–15]. Very recently, a new analytical approach with

improved performances has been defined in [16].

Roughly speaking, the methods in [10, 11, 16] can be referred as 'density taper' procedures. In fact, the amplitude tapering which is required on the aperture field in order to achieve a far field distribution with low sidelobes is emulated through a proper tapering in the spacings amongst the array elements. As a consequence, one will have a denser packing of elements in the zone where the reference aperture field is higher, and larger spacings where the reference source's amplitude is lower [16].

Later, in [17], a related approach has been introduced for the one-dimensional shaped beam case, wherein, however, only real sources are considered. At the same time, in [7], a synthesis procedure dealing with complex reference sources has been devised. Such techniques amount to achieve a fitting between the complex cumulative distributions corresponding to an 'ideal' continuous reference source and to the actual isophoric sparse array. By virtue of the Fourier Transform's properties, the enforcement of such fitting induces the minimization of the $1/u^2$ weighted distance between the reference and actual radiated fields (u being the spectral variable associated to the aperture coordinate).

By deferring the interested reader to [7, 17] for more details, two comments are worth to be done herein. First, note that, in the synthesis of one-dimensional shaped beams, different continuous sources can give rise to the same power pattern [18, 19]. Therefore, by changing the reference source, one can potentially generate different isophoric sparse arrays radiating an equivalent power pattern. Second, note that the discretization of the reference source into the isophoric array may induce some degradation of the pattern. The latter will be dependent on the source's spatial variability: the smoother the aperture field distribution, the better the fidelity to the desired pattern of the resulting array. Such circumstances play a key role in the new synthesis procedure outlined in the following Section.

3. A NEW PERSPECTIVE IN THE SYNTHESIS OF RECONFIGURABLE ISOPHORIC ARRAYS

Basically, the perspective consists in exploiting at best theoretical results and optimal solutions available in the separate synthesis of the different patterns amongst which one wants to reconfigure the array, in order to maximize the efficiency of the solution space exploration. The employed notions can be summarized as follows.

First, note that, for any fixed-geometry array, the optimal synthesis of a pencil beam subject to arbitrary sidelobe level (SLL)

constraints can be formulated as a Convex Programming (CP) problem [20]. As a consequence of convexity, the globally optimal solution of the problem is unique and can be identified without making resort to global optimization techniques. Also, by using suitable finite-dimensional expansions of the source or proper regularization techniques, these results and approaches can be extended to the synthesis of continuous aperture sources [21–23]. Finally, the density taper techniques [10, 16] can be applied to each achieved aperture field distribution in order to generate a set of array locations corresponding to the desired radiation pattern and identifiable as a point in a multidimensional space whose coordinates are the locations themselves (see Fig. 1).

Moreover, it is worth noting that the power pattern synthesis of shaped beams through linear equispaced arrays can be performed in a globally optimal fashion by recurring essentially to linear programming (LP) optimization procedures. In fact, according to [18], the square amplitude array factor of an array constituted by N antennas can be
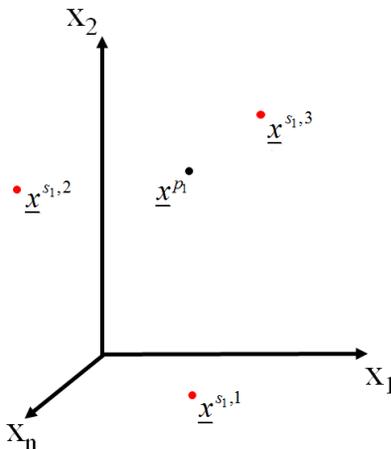

Figure 1. Representation in the space of array locations of the unique solution corresponding to a desired pencil beam (black point) and of three equivalent solutions radiating a desired shaped beam (red points).
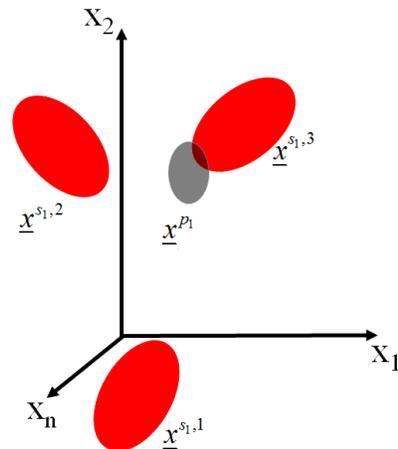
Figure 2. Pictorial representation of the effect, on the location sets depicted in Fig. 1, of a slight relaxation of the radiation requirements.

written as:

$$F\left(u'\right) = \sum_{n=-N+1}^{N-1} D_n e^{jnu'} \quad \text{with} \quad D_n = D_{-n}^* \tag{1}$$

wherein $u' = \beta d \cos\theta$ (being β the wavenumber, and d, θ the uniform spacing amongst neighboring elements and the angle with respect to the array axis, respectively). Therefore, by respectively denoting with UB and LB the upper and lower bounds for the power pattern, a *necessary and sufficient* condition for the feasibility of an electromagnetic field lying in the given mask is the existence of a set of coefficients D_n such that the properly discretized system of linear inequalities

$$\begin{cases} \sum_{n=-N+1}^{N-1} D_n e^{jnu'} \leq UB\left(u'\right) \\ \sum_{n=-N+1}^{N-1} D_n e^{jnu'} \geq LB\left(u'\right) \end{cases} \tag{2}$$

is satisfied.

In practice, by solving the system (2), one is able to ascertain *a priori*, i.e., without solving the overall synthesis problem, whether the given design problem admits a solution or not. Moreover, if the set of the solutions of system (2) is not empty, then a straightforward way exists to find the corresponding array excitations [18]. In particular, since the function in (1) is a real and positive semi-definite trigonometric polynomial of order $2N - 2$ in the auxiliary variable $w = e^{ju'}$, a factorization of the kind

$$F\left(u'\right) = f\left(u'\right) f^*\left(u'\right) \tag{3}$$

will always be available, where * denotes complex conjugation and

$$f\left(u'\right) = \sum_{n=0}^{N-1} a_n e^{jnu'} \tag{4}$$

is the needed array factor and may be generated by 2^T different sets of array excitations, T being the number of zeroes of the Schelkunoff polynomial underlying (4) and not lying on the unit circle [18]. Such excitations, denoted in (4) with a_n, can be provided by the 'zero-flipping' procedure presented in [18].

As in the case of pencil beams, the synthesis procedure can be extended to the case of continuous sources. In this case, a line-source distribution will be identified through the FFT interpolation of the excitations of a sufficiently dense equispaced array [7].

Let us suppose now that the above synthesis procedures, unified with the density taper techniques recalled in the previous Section, are separately applied to each of the desired reconfigurable patterns. At the end of the process, one will have at his/her disposal:

- a multiplicity of *equivalent* isophoric sparse arrays for each of the desired shaped patterns;
- a unique isophoric sparse array for each of the desired pencil beams.

In the multidimensional space we have previously introduced, the sets of element locations corresponding to each array will be represented by disjoint points (see Fig. 1). Since these points are generally different from the single point representing the solution for the pencil beam mode, one is not able, for the time being, to find a set of locations which is contemporarily optimal for the two (or more) patterns at hand.

In order to finally introduce the proposed procedure, it is interesting to discuss what happens when slightly relaxing the radiation requirements on the pencil beams or on the shaped beams. As small variations on constraints will induce small variations in the overall steps of the synthesis procedures, it can be argued that each of the single points of Fig. 1 will degenerate in some connected set, wherein a point of each set represents a possible solution for the corresponding pattern. As pictorially represented in Fig. 2, an interesting result comes out: the locations sets corresponding to the different kind of patterns can have an intersection if the right determination is chosen amongst the 2^T solutions of the shaped beam cases. Notably, such an intersection represents a solution (or even a set of solutions) for the reconfigurable isophoric sparse array.

Such a circumstance suggests then a simple and effective strategy to deal with the optimal synthesis of phase-only reconfigurable isophoric sparse linear arrays. In fact, once the optimal solutions to the separate synthesis problems have been found, the subsequent step is to identify, amongst all the equivalent location sets generating each shaped beam, the closest one to the points corresponding to all the other desired patterns. Saying it in other words, and temporarily focusing on the case where the designer wants to reconfigure the array radiation pattern just from a pencil to a shaped beam and vice versa, if $\underline{\mathbf{x}}^{S,k} = (\mathbf{x}_0^{S,k}, \ldots, \mathbf{x}_{N-1}^{S,k})$ and $\underline{\mathbf{x}}^P = (\mathbf{x}_0^P, \ldots, \mathbf{x}_{N-1}^P)$ respectively denote the k-th equivalent locations set for the shaped beam and the unique pencil beam's locations set, then one has to minimize over the 2^T different possible values of k the distance

$$\Phi(k) = \left\| \underline{\mathbf{x}}^{S,k} - \underline{\mathbf{x}}^P \right\| \tag{5}$$

In fact, the set of locations minimizing (5) is the one which more easily lends itself to be reconfigured between the desired shaped and pencil beams. An exemplification of the adoption of such technique is given in Fig. 3.
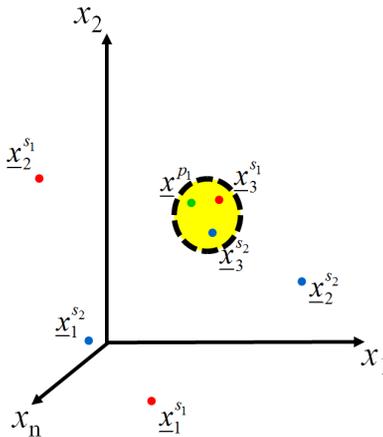

Figure 3. Representation, in the space of array locations, of: (green point) the unique solution of a pencil beam synthesis problem; (blue and red points) three equivalent solutions for two different shaped beams. The yellow set, achieved by minimizing (5), identifies the solutions most suitable for a 'common-locations' reconfiguration.

As long as T is not too large, such a step can be done in an enumerative fashion, i.e., by checking all possible solutions. Alternatively, as one has to choose a configuration amongst all the suitable zero-flipping outcomes, it can be dealt with as a binary optimization problem (on T variables) by effective solution procedures of the kind shown in [24]. Through the guidelines in [9], the procedure can be easily extended to the case of more than two shaped, pencil and difference beams.

Once the most convenient set of locations for the shaped beam has been determined, a local optimization technique can be used to perform the actual choice of the locations common to both the radiation modalities. A possible way to perform it is presented in the following Section wherein, as their absence is not guaranteed by the minimization of (5), we provide also a strategy to keep under control possible mutual coupling effects.

4. A FINAL OPTIMIZATION MAXIMIZING THE RADIATION PERFORMANCES

In order to introduce the final-optimization step of the synthesis procedure, let us suppose one wants to phase-only reconfigure the pattern of a N-elements isophoric array amongst M different radiation modalities, and let us denote with

$$E_m\left(u\right) = \sum_{n=1}^{N} e^{j\phi_{m,n}} e^{jx_{m,n}\beta u} \qquad (6)$$

the m-th desired far field distribution (wherein $u = \cos\theta$ and β is the wavenumber, θ being the angle with respect to the array axis) radiated by an isophoric sparse array determined by the selection procedure outlined in the previous Section and having locations $x_1, x_2, \ldots, x_N$ and excitation phases $\phi_1, \phi_2, \ldots, \phi_N$.

Also, let us denote with

$$\hat{E}_m\left(u\right) = \sum_{n=1}^{N} e^{j\hat{\phi}_{m,n}} e^{j\hat{x}_{m,n}\beta u} \qquad (7)$$

the m-th far field distribution radiated by the isophoric phase-only reconfigurable array. Note that, as the different patterns must correspond to arrays differing only in their excitation phases, the variables x and ϕ in (7) hold the same significance as in (6) with the additional condition

$$\hat{x}_{m,n} = \hat{x}_{m+1,n} \quad \forall n \quad \text{with} \quad m = 1, \ldots, M-1 \qquad (8)$$

Finally, if the locations and excitation phases of the reconfigurable array are expressed as

$$\hat{x}_{m,n} = x_{m,n} + \Delta x_{m,n} \quad \hat{\phi}_{m,n} = \phi_{m,n} + \Delta\phi_{m,n} \quad \forall m, n \qquad (9)$$

which is reasonable by virtue of all the reasonings of the previous Section, then the final local optimization step can be formulated in terms of the unknowns $\Delta x_{m,n}, \Delta\phi_{m,n}$ as:

minimize

$$\Omega\left(\Delta x_{m,n}, \Delta\phi_{m,n}\right) = \sum_{m=1}^{M} \alpha_m \left\| E_m\left(u\right) - \hat{E}_m\left(u\right) \right\|^2 \quad \forall u \in \left[u_m^A, u_m^B\right] \qquad (10)$$

subject to

$$\left| \hat{E}_m\left(u\right) \right|^2 \le f_m\left(u\right) \quad \forall u \notin \left[u_m^A, u_m^B\right] \qquad (11)$$

and

$$x_{m,n} + \Delta x_{m,n} = x_{m+1,n} + \Delta x_{m+1,n} \quad \forall n \quad \text{with} m = 1, \ldots, M-1 \quad (12)$$

In such a formulation, the cost functional (10) aims to enforce a similarity (in the zones defined by u_m^A and u_m^B) between the reference (6) and actual (7) patterns, while constraints (11) are devoted to fulfill upper bounds on the sidelobes (being f_m an arbitrary real and positive function). Such a formulation is more flexible and effective than simply requiring a fitting in the overall visible space. Finally, the linear constraints (12) are nothing but the consistency relationships amongst the different Δx variables.

Because of the way the unknowns enter in the cost functional (10), the latter is a multimodal one, i.e., it may have a number of different *local optima*. Therefore, if a gradient-like optimization procedure will be used to solve the problem, then the actual solution will depend on the adopted starting point. The latter, by virtue of all the above analysis and of (10)–(12), can be profitably identified in the locations' space as the center of the M-dimensional hypersphere minimizing (5) (see Fig. 3).

Interestingly, by virtue of all the above, one may assume that the location sets corresponding to the minimum value of (5) are quite near each to the other, i.e.,

$$\beta |x_{m,n} - x_{p,n}| < 1 \quad \forall n \quad \forall (m,p) \, pair \quad (13)$$

Then, if (13) is actually satisfied, which is one of the goals of the synthesis procedure (and can be checked before running the final-optimization step), it seems reasonable to assume that also the final solution will be close to the 'most convenient' separated solutions (and hence to the adopted starting point), so that

$$|\Delta \phi_{m,n}| < 1 \quad |\beta \Delta x_{m,n}| < 1 \quad \forall n, m \quad (14)$$

Provided (14) holds true, the difference between the fields in (10) can be linearized as

$$E_m(u) - \hat{E}_m(u) = -j \sum_{n=1}^{N} e^{j\phi_{m,n}} e^{jx_{m,n}\beta u} (\Delta \phi_{m,n} + \Delta x_{m,n}\beta u) \quad (15)$$

so that (10) becomes a positive semi-definite quadratic objective function. As constraints (11) become convex after linearization, and constraints (12), (14) are convex too, the overall optimization problem (10)–(12), (14) is equivalent to a CP problem, for which effective solution procedures leading to the global optimum do exist.

By so doing, as long as conditions (13) are satisfied, which is the goal of introduced selection procedure and may be a-priori ascertained,

one can be confident, without recurring to global optimization schemes, of finding the global minimum of a functional equivalent to (10). Obviously, the procedure still makes sense when conditions (13) are not satisfied, but only local optimality is ensured in such a case.

5. AN ASSESSMENT OF PERFORMANCES IN REALISTIC CONDITIONS

This Section provides an assessment of the capabilities of the proposed approach through a number of numerical examples achieved by exploiting only convex-optimization techniques and emulating (in the one-dimensional case) communications from a Geostationary Earth Orbit (GEO) satellite. For each test case, after having synthesized the elements' locations and excitation phases, the radiation performance has been evaluated by considering:

- an 'ideal' $\sin\theta$ element pattern, θ being the angle with respect to the array axis;
- an array of truncated metallic waveguides.

The real far field pattern radiated by the array of truncated waveguides, taking into account also possible mutual coupling effects, has been computed by means of a full wave numerical simulation with the *CST Microwave Studio* software, at a working frequency of 1 GHz and adopting waveguides with section $0.6\lambda \times 0.2\lambda$, being λ the wavelength in free space (see Fig. 4).

The first GEO satellite test case concerns the synthesis of a phase-only reconfigurable isophoric array able to generate a pencil beam and a flat-top beam emulating an angularly uniform coverage of the Europe. To synthesize the reference continuous sources, we have considered an uniformly-excited linear array having an extension of 14λ and
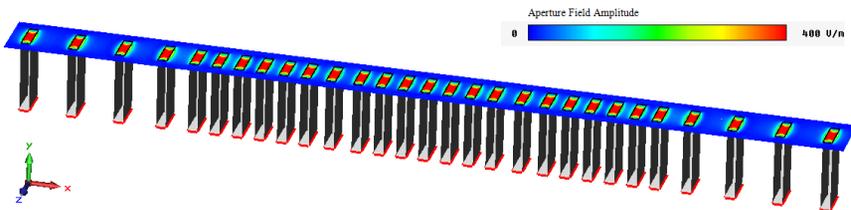

Figure 4. Array of truncated metallic waveguides adopted to evaluate the radiation performance in realistic conditions. The computed aperture field amplitude distribution is also depicted.

composed by 29 elements. The constraints on the pencil beam consist in a maximum SLL ≤ -20 dB for $\theta \leq 84.5°$ and $\theta \geq 95.5°$. Also, in order to satisfy the coverage requirements, we have enforced on the flat-top power pattern a maximum ripple not larger than ± 0.5 dB for $85.5° \leq \theta \leq 94.5°$ and a maximum SLL ≤ -20 dB for $\theta \leq 80°$ and $\theta \geq 100°$.

The reference pencil beam to exploit in the final optimization step has been synthesized by applying the method in [16] to a proper line-source distribution. The corresponding array locations and phase excitations are depicted in red colour in Figs. 5 and 6(a), respectively.
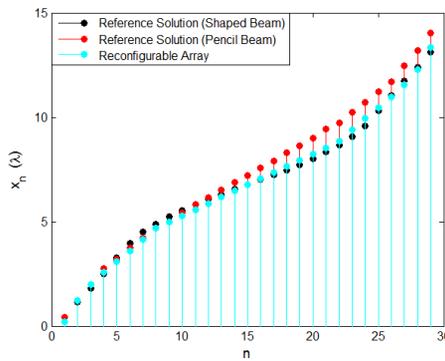

Figure 5. Isophoric array's locations set corresponding to: (red lines) reference pencil beam; (black lines) reference flat-top beam; (cyan blue lines) phase-only reconfigurable patterns.
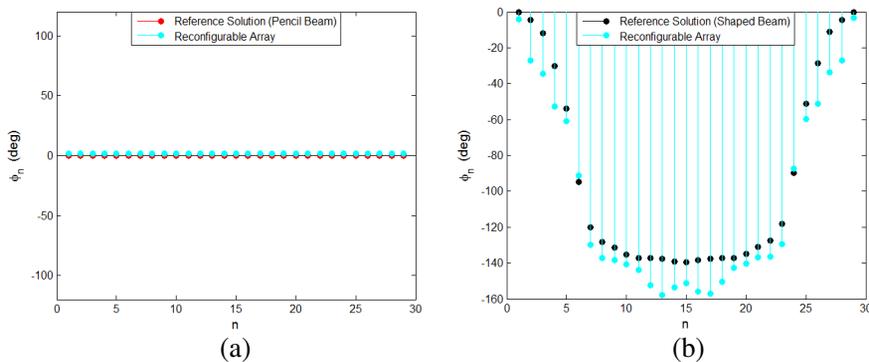

Figure 6. Isophoric array's excitation phases corresponding to: [(a), red lines] reference pencil beam; [(b), black lines] reference flat-top beam; [(a), (b), cyan blue lines] phase-only reconfigurable patterns.

The reference shaped beam has been determined by exploiting the procedures of Section 3. By so doing, different sets of locations able to fulfil the given constraints have been identified. Then, we have selected the solution which more easily lends itself to be reconfigured from the desired flat-top pattern to the pencil beam. The corresponding locations and phase excitations are depicted in black colour in Figs. 5 and 6(b), respectively. As it can be seen from Fig. 5, the locations sets corresponding to the different patterns and processed through the approach of Section 3, selected as the starting point of the final-optimization step, are very close each to the other.

Finally, the concluding optimization step has been carried out in the CP option by enforcing, in addition to (11), (12), (14), a couple of convex constraints on the spacing between neighbouring elements. The latter, in order to both keep down possible mutual coupling effects and avoid large unexploited aperture sections, has been bounded between 0.3λ and 1λ. The two achieved phase-only reconfigurable patterns, both generated by the 29-element isophoric array having the locations and excitations sets depicted in Figs. 5 and 6 (cyan blue lines), are shown in Figs. 7 and 8. In particular, such figures show the power masks of the initial synthesis problems and a comparison between the performance achieved by means of the ideal and real element patterns. As it can be seen, notwithstanding the 'losses' which are expected with respect to the reference patterns because of the reconfigurability (and unitary excitation-amplitude) requirements, by adopting a simple $\sin\theta$ element pattern all the constraints imposed on the power patterns have been fulfilled. Moreover, the field radiated by the realistic array
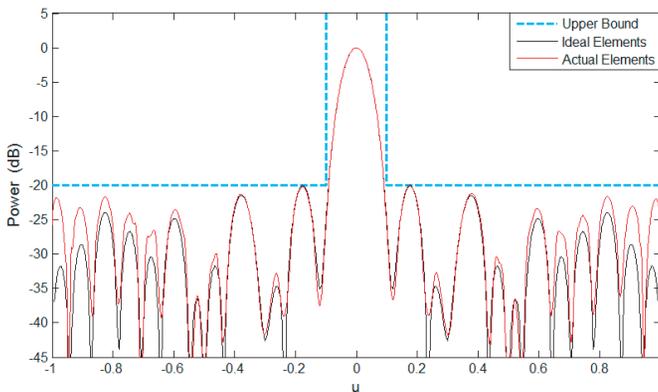

Figure 7. Pencil beam radiated by the isophoric phase-only reconfigurable arrays composed by ideal and realistic radiating elements.
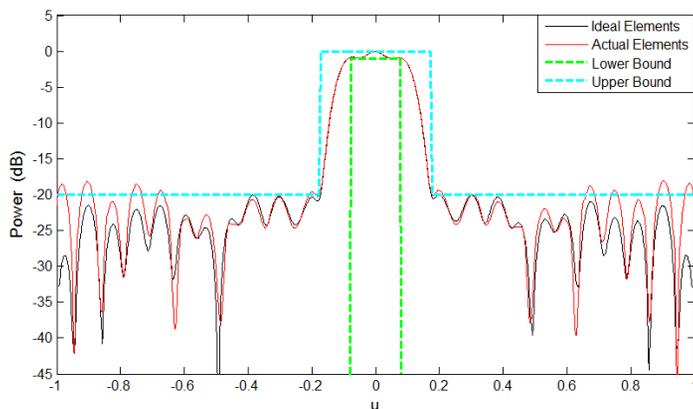
Figure 8. Flat-top pattern radiated by the isophoric phase-only reconfigurable arrays composed by ideal and real radiating elements.

of truncated waveguides results practically equal to the one of the 'ideal' architecture up to 40° from the boresight direction, while slight violations of the upper bound constraint are visible at large values of the u variable (only for the shaped beam case). Such a circumstance, rather than to residual mutual coupling, can be attributed to the fact that the element pattern is scarcely directive in the plane at hand, so that better results are expected with different elements patterns (e.g., with elements emulating a Huygens Source). Also, in telecommunications from satellites (which are the main applications for isophoric arrays), one usually does not worry about the field level outside of the Earth cone as seen from the satellite itself.

The second test case concerns the synthesis of a phase-only reconfigurable isophoric array to be mounted on a GEO satellite and such that the same amount of power density is realized on each (visible) portion of the Earth. This kind of pattern is known as 'isoflux' pattern. In such a design problem, when stating the power mask to adopt in the synthesis procedure, the Earth curvature as seen from the satellite has to be taken into account. In particular, a depression of -1.5 dB is required at the center of the coverage zone to compensate the power attenuation on the planet border. The latter is due to a 6900 km longer path, since the distance between Earth and satellite is approximately equal to 42650 km and 35790 km when considering the edge and the center of the planet, respectively.

Therefore, denoting again with θ the angle with respect to the array axis, the constraints on the power patterns have been chosen as:

- a -1.5 dB power depression for $\theta = 90°$ in the isoflux pattern;

- a maximum SLL $\leq -20\,\mathrm{dB}$ for $\theta \leq 85.5°$ and $\theta \geq 94.5°$ in the pencil beam case;
- a maximum SLL $\leq -20\,\mathrm{dB}$ for $\theta \leq 77°$ and $\theta \geq 103°$ in the shaped beam case;
- a maximum ripple not larger than $\pm 0.5\,\mathrm{dB}$ for $82° \leq \theta \leq 98°$ in the isoflux pattern.

The geometry of the adopted uniformly-excited linear array is equal to the one considered in the previous test case. The reference pencil beam exploited in the final optimization step corresponds to the array locations and phase excitations depicted in red colour in Figs. 9 and 10(a), respectively. The reference shaped power pattern has been determined by means of the procedure given in Section 3, and turned out to correspond to 64 equivalent array locations sets, very different one from each other. The locations which more easily lend themselves to be reconfigured from the desired isoflux pattern to the above pencil beam, quickly identified by means of the presented procedure, are depicted in black colour in Fig. 9. The corresponding excitation phases are shown in Fig. 10(b) (black curves). Such results have been used as the starting point of the final local-optimization procedure, which has provided the locations and excitation phases depicted in cyan blue colour in Figs. 9 and 10. Through a couple of convex constraints, the spacing between neighbouring array elements has been bounded between 0.4λ and 0.85λ. The corresponding phase-only reconfigurable patterns are shown in Figs. 11 and 12, where the mask adopted to shape the reference far-field distributions are also reported.
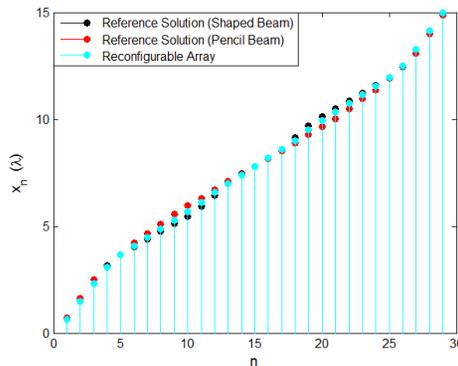

Figure 9. Isophoric array's location sets corresponding to: (red lines) reference pencil beam; (black lines) reference isoflux beam; (cyan blue lines) phase-only reconfigurable patterns.

Notably, as in the previous test case, by adopting a $\sin\theta$ element pattern all radiation constraints have been fulfilled, and the locations and phases of the phase-only reconfigurable array resulted very close to those ones corresponding to the adopted starting point. Such a circumstance points out the effectiveness of the overall synthesis procedure, and, in particular, it is an excellent a-posteriori proof of the high performance of the selection strategy presented in the previous Section. Moreover, the power pattern radiated by the array of truncated waveguides fulfils all the imposed constraints but for a
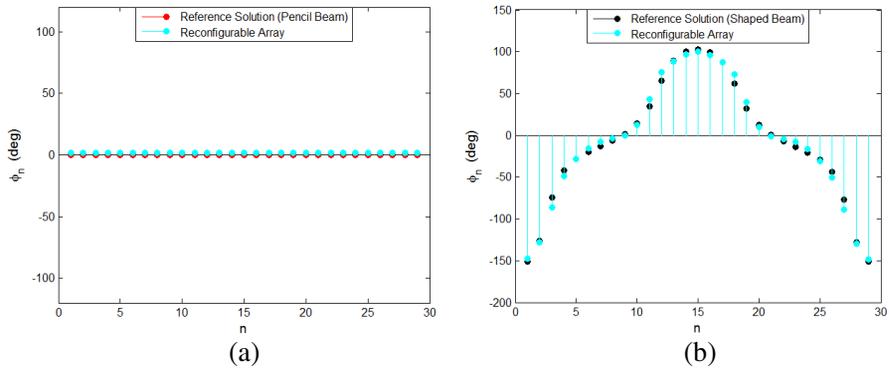

Figure 10. Isophoric array's excitation phases corresponding to: [(a), red lines] reference pencil beam; [(b), black lines] reference isoflux pattern; [(a), (b), cyan blue lines] phase-only reconfigurable patterns.
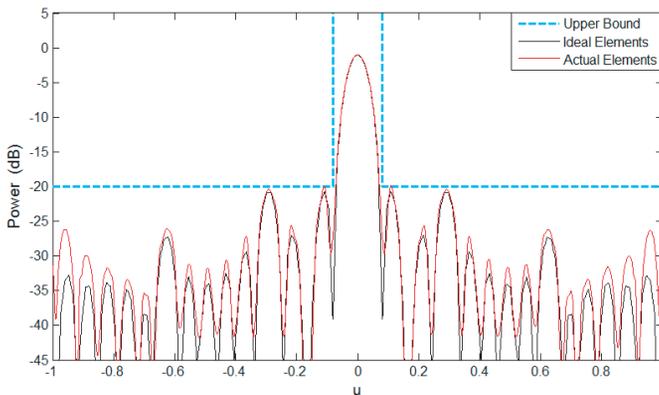

Figure 11. Pencil beam radiated by the isophoric phase-only reconfigurable array: (black curve) $\sin\theta$ element pattern embedded; (red curve) realistic waveguide element pattern embedded.
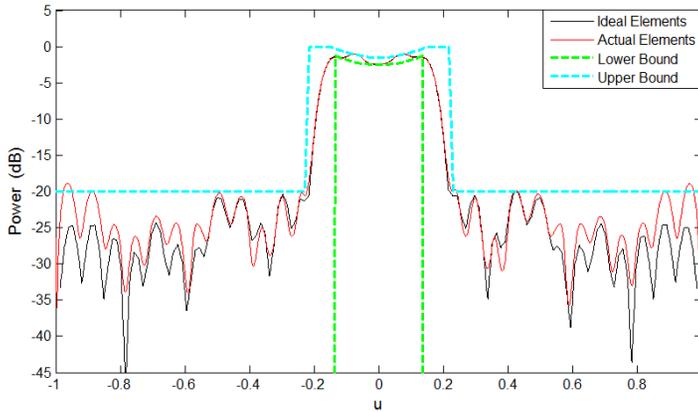
Figure 12. Isoflux pattern radiated by the isophoric phase-only reconfigurable array: (red curve) realistic performance; (black curve) $\sin\theta$ element pattern embedded.

1 dB violation at 75° from the boresight direction (visible only on the isoflux beam). Such result, considering the 'uniform amplitude' behaviour of the aperture field observed in both the test cases (see Fig. 4), as well as the low directivity of the realistic truncated metallic waveguides, proves again the capability of keeping under control the mutual coupling between array elements.

As a final comment concerning the extremely low computational burden of the overall synthesis procedure, it is worth noting that the runtime for the determination of the reconfigurable array elements' locations and excitation phases turned out to be less than 1 minute per test case (on a PC equipped with a 2.53 GHz Processor).

6. CONCLUSIONS

An innovative and effective approach to the synthesis of phase-only reconfigurable, isophoric linear sparse arrays has been presented, discussed, and corroborated by numerical examples of actual interest. The approach takes extraordinary advantage from the hidden combinatorial nature of the problem at hand, as well as from analytical results available in the separate optimal synthesis of each desired pattern. In particular, a general philosophy is introduced to efficiently and effectively explore the solution space, looking within its 'more convenient' zones, and exploiting at best the a-priori available knowledge in the synthesis scenario at hand (which is neglected in a number of published approaches).

Notably, the proposed synthesis procedure does not limit the maximum number of generated patterns, deals with both shaped and sum/difference beams, allows one to keep under control the mutual coupling effects, and achieves effective solutions through optimization techniques with a low computational burden.

Finally, it is interesting to note that, by exploiting the procedures presented in [22, 23], the approach can be extended to the case of planar arrays generating circularly symmetric patterns.

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