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ACCURATE CALCULATION OF THE RIGHT-OF-WAY WIDTH FOR POWER LINE MAGNETIC FIELD IMPACT ASSESSMENT

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Abstract—In this work, approximate formulas are presented for computing the magnetic field intensity near electric power transmission lines. Original expressions are given for single circuit lines of any type of arrangement and double circuit lines in both super-bundle and low-reactance conductor phasing. These expressions can be used for assessing directly the Right-of-Way width of power lines related to maximum magnetic field exposure levels which may be efficiently used in environmental impact assessment. The accuracy of approximate formulas is demonstrated by comparison with exact formulas for computing the rms field distribution.

1. INTRODUCTION

The growing public concern of possible harmful effects caused by magnetic fields and difficulties in assessing the related risks have led several governments to set new regulations limiting maximum exposure levels. Because of these limitations it is necessary to quantify field intensity near electric power installations like overhead lines having the widest impact [1].

As regards the maximum allowable exposure level for the magnetic field there is not yet a worldwide recognized value and different exposure levels have been set by national regulations [2]. According to a prudent avoidance policy the Italian Law No. 36/2001 has set new restrictive standards for electromagnetic compatibility (EMC) at very low frequency in order to reduce possible long term harmful effects [3]. The following levels were introduced: (i) the *exposure*

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limit of 100 μT which is the maximum acceptable field intensity on the short term period, (ii) the *attention value* of 10 μT which is a precautional level for the long term exposure, and (iii) the *target quality value* of 3 μT which is a reference level for future installations [4]. The former limit corresponds to the level recommended by the International Commission on Non-Ionizing Radiation Protection (ICNIRP) for the short-term exposure [5], whereas the two latter levels have been introduced as a precautionary protection measure for the long-term exposure of the population and are meant as median values over the 24 hours.

The compliance to such maximum exposure limits leads to the definition of different Right-of-Way (ROW) widths, which depend on the electric and the geometric parameters of the overhead line. The determination of accurate ROW widths is very important, e.g., in environmental impact assessments or in territory planning, especially when new buildings have to be built near power installations. The guidelines CEI 106-11 issued by the Italian Electrotechnical Committee define a safety volume surrounding phase conductors in which the magnetic field intensity is higher than the maximum exposure limit set by standards (Figure 1) [6].

The main aim of this paper is to obtain approximate yet precise analytical formulas for evaluating the magnetic field intensity in proximity of both single circuit and double circuit lines in a form which can be used for a straightforward computation of the ROW width of the power line.

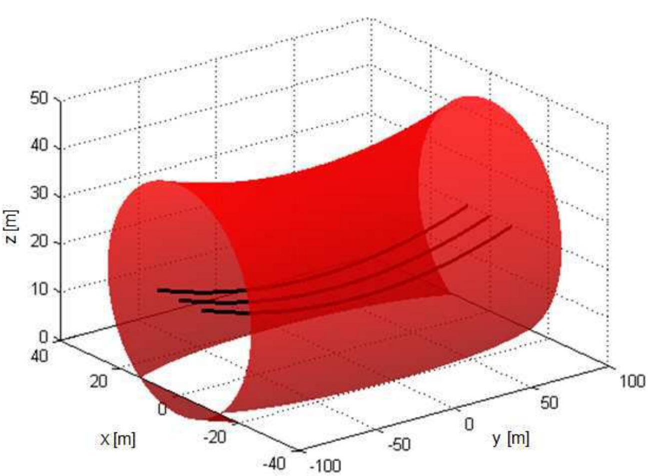

Figure 1. Example of safety volume around a power line span.

For single or double circuit lines, two dimensional numerical analysis is sufficiently accurate in the majority of practical cases and the magnetic induction is computed by using Biot-Savart's law and the superposition principle [7]. Three-dimensional models based on the numerical integration of Biot-Savart's law should be considered when the line sag is not negligible compared to its span and when one has to consider different spans of the same line or even different spans of different lines [8, 9]. Although numerical approaches represent a useful tool for computing power line environmental fields, these are not well suited for assessing the functional dependence of the field strength on electric and geometric parameters [10]. In order to obtain a simple and compact representation of the magnetic field intensity for a rapid estimate of the ROW width for transmission lines a new kind of approach has been adopted. For balanced single circuit lines exact analytical formulas can be obtained by representing elliptically polarized fields by double complex numbers [11] or complex vectors [12] whereas approximated formulas can be estimated by a multipole expansion of Biot-Savart's law [13].

It has been observed that the Complex Vector Method (CVM) — already used for analyzing plane electromagnetic waves — can be effectively used for representing time-harmonic magnetic fields. Due to a more compact representation compared to other methods proposed in the literature it has been possible to obtain a unique general expression for evaluating magnetic flux density rms value as a function of line phase-to-phase distance, distance from calculation point and current load in the case of balanced power systems [12]. Thus formulas for actual line configurations such as single circuit three-phase lines with triangle or aligned phase conductors have been derived. The CVM has then been applied to analyze magnetic field generated by multiple circuit three-phase lines, even with electrically independent circuits [14].

From these relationships approximate formulas for computing the magnetic field intensity in proximity of single and double circuit power lines are obtained. It is shown that the ROW width can be easily computed for each line configuration once the maximum exposure level is known. The level of accuracy of these relationships is given for different current loads.

2. APPROXIMATE FORMULAS FOR THE MAGNETIC FIELD INTENSITY OF THREE-PHASE LINES

It is well known from the literature that time-harmonic magnetic fields generated by electric power systems are elliptically polarized as they are generated by three phase currents varying all at the same frequency. Thus the magnetic field can be described as a rotating vector tracing an ellipse on the polarization plane. In the particular case of plane symmetry assumption, the polarization plane is orthogonal to line conductors so that Biot-Savart's law can be used to compute the magnetic field. A closed form solution for three phase lines with polygonal phase conductor arrangement has been derived in [11] by using double complex numbers with two imaginary units.

It has been shown in [12] that more general results can be obtained instead by using complex vectors to represent elliptically polarized fields. By assuming a 2-D line geometry and by using complex vector algebra, it has been possible to obtain the following expression of the magnetic flux density for multiphase power lines in any type of conductor arrangement and current load (balanced or unbalanced):

$$b = \frac{\mu}{2\pi}\sqrt{\sum_{p=1}^{n}\left[\sum_{q=1}^{n}\frac{I_p I_q \cos(\theta_{pq})}{r_p^2} - \sum_{q=p+1}^{n}\frac{I_p I_q \cos(\theta_{pq}) d_{pq}^2}{r_p^2 r_q^2}\right]}, \qquad (1)$$

where μ is the air magnetic permeability, n is the number of phases, p and q indicate phase conductors with rms currents I_p and I_q and distances from the field point r_p and r_q, d_{pq} is the phase-to-phase distance, and θ_{pq} is the phase shift. From this general expression, which provides exact field intensity values, approximate formulas suitable for estimating the ROW width of classical line arrangements are derived in the following. Equation (1) can be reduced to a more compact relationship in practical cases, when the sum of phase currents is zero, since the first term in (1) identically vanishes:

$$b = \frac{\mu}{2\pi}\sqrt{\sum_{p=1}^{n}\sum_{q=p+1}^{n} I_p I_q \cos(\theta_{pq}+\pi)\left(\frac{d_{pq}}{r_p r_q}\right)^2}. \qquad (2)$$

2.1. Three-phase Single Circuit Power Lines

The case of single circuit three-phase lines with generic phase-to-phase distance d_{12}, d_{23}, and d_{31} and balanced three-phase currents is considered. By letting $n = 3$, $\theta_{pq} = 2\pi/3$ and $I_p = I_q = I$ in the general expression (2) the rms magnetic flux density distribution can be written as:

$$b(r, I) = \frac{\mu I}{2\sqrt{2}\pi}\sqrt{\left(\frac{d_{12}}{r_1 r_2}\right)^2 + \left(\frac{d_{23}}{r_2 r_3}\right)^2 + \left(\frac{d_{31}}{r_3 r_1}\right)^2}, \qquad (3)$$

where only a few basic parameters such as the current intensity, the phase-to-phase distance, and the field-point-conductor distances are involved. This compact expression still provides an exact representation of the magnetic field intensity around a three-phase line and, in turn, makes it possible to assess the dependence of the field intensity on electrical and geometrical line parameters which is of primary importance for determining an estimate of the ROW width. It is worth noting that (3) can be used for optimizing conductor placement when designing compact line arrangements to mitigate the field intensity in proximity of overhead lines as described in [15, 16].

The most widespread used single circuit configurations are represented by lines with triangle (delta) and aligned conductors, as reported below.

2.1.1. Three-phase Lines in Delta Arrangement

Figure 2 shows a triangle phase arrangement of single circuit line (*delta line*) with constant phase-to-phase distance d equal to $\sqrt{3}s$, where s is the distance from the triangle centroid. The field calculation point is located on (r, φ) where r is the distance from the origin and φ the field point direction with respect to the horizontal axis.

With reference to Figure 2 it may be shown that the analytical expression of the magnetic flux density rms value is given by:

$$b(r, I) = \frac{3\mu I}{2\sqrt{2}\pi}\sqrt{\frac{s^4 + s^2 r^2}{r^6 - 2s^3 r^3 \cos(3\varphi) + s^6}}, \tag{4}$$

which shows that the field intensity distribution has periodical symmetry and the highest values are attained when $\varphi = 0$, $2\pi/3$, and

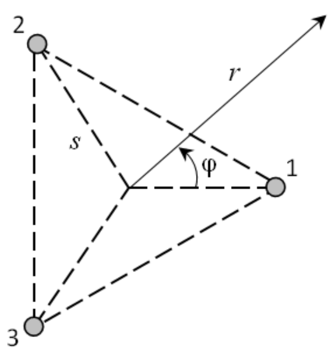

Figure 2. Triangle single-circuit line arrangement.

$4\pi/3$.

In order to obtain the ROW width for a given maximum allowed field intensity level a conservative estimate of b in proximity of the overhead line is required. It can be noted that each contour line of the rms field distribution can be approximated as a circle at sufficient distance from the line axis — i.e., *circle approximation*. By letting $d = \sqrt{3}s$ in (4) and by assuming $r \gg s$, the following expression can thus be used as a conservative approximate estimate of the field intensity as a function of the distance r:

$$b(r, I) = \frac{3\mu I}{2\sqrt{2}\pi} \frac{d}{r^2}, \tag{5}$$

which is applicable also for non-regular triangle arrangements assuming d equal to the geometric mean $d_m = \sqrt[3]{d_{12}d_{23}d_{31}}$ of the phase-to-phase distances. Note that the same expressions are derived in [11] and [13] by using a multipole expansion of Biot-Savart's law.

The accuracy of the approximate relationship can be easily assessed by comparing the circular contour lines computed with (5) and exact contour plots computed with (4). Exposure limits here considered are in the range of those reported in Italian standards, namely 3 µT and 10 µT [4].

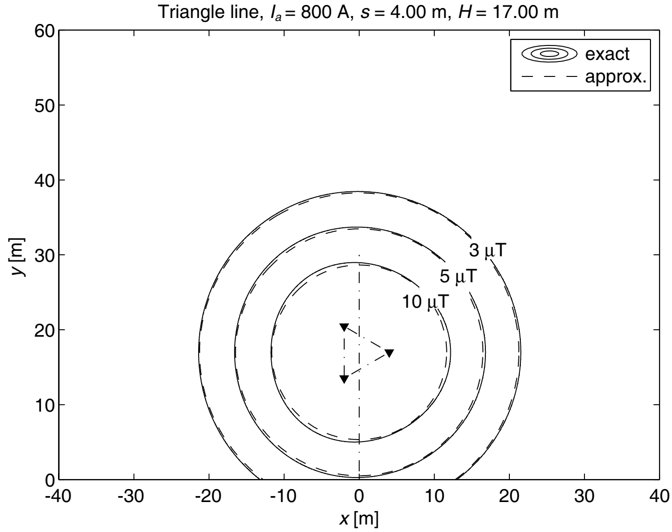

Figure 3. Contour plots of the rms magnetic flux density [µT] for a three-phase delta power line: exact profiles are in continuous line, the approximated ones in dash line.

Figure 3 shows the rms field distributions computed for a triangle power line with distance $s = 4\,\mathrm{m}$ and balanced currents $I = 800\,\mathrm{A}$ rms. It can be noted that contour plots are in good agreement also in the close vicinity of the line conductors.

2.1.2. Three-phase Lines with Aligned Conductors

Figure 4 shows a three phase single circuit line in aligned conductor arrangement (*flat line*) where s is the distance between phase conductors 1–2 and 1–3, φ is the field point direction with respect to the line axis, and r the field point distance from the central phase conductor 1.

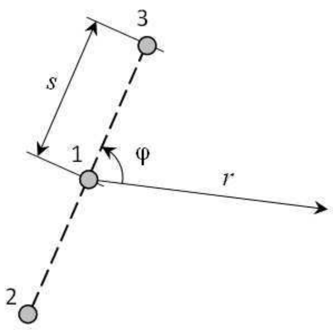

Figure 4. Aligned single-circuit line arrangement.

By letting $d_{12} = d_{31} = s$ and $d_{23} = 2s$ in (3) and expressing distances from phase conductors as a function of (r, φ) the following expression for computing the rms magnetic flux density of a flat line is obtained:

$$b(r, I) = \frac{\mu I}{2\pi r}\sqrt{\frac{s^4 + 3s^2r^2}{r^4 - 2s^2r^2\cos(2\varphi) + s^4}}, \tag{6}$$

where this time the field (with periodic symmetry) attains its maximum intensity for $\varphi = 0$, π, i.e., in the direction of the line axis.

An approximate estimate of (6) can be obtained by letting $s^2 + 3r^2 \approx 3r^2$ and by assuming $r \gg s$ in the far field region:

$$b(r, I) = \frac{\mu I}{2\pi}\frac{\sqrt{3}s}{\sqrt{r^4 - 2s^2r^2\cos(2\varphi) + s^4}}, \tag{7}$$

which still takes into account the direction of the line axis. In order to obtain a conservative estimate of the field intensity, (7) can be further

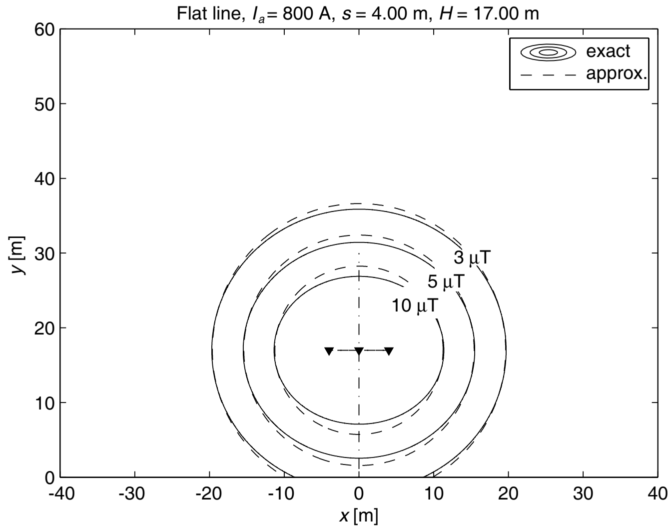

Figure 5. Contour plots of the rms magnetic flux density [μT] for a three-phase flat power line: exact profiles are in continuous line, the approximated ones in dash line.

simplified using the circle approximation as above. By letting $\varphi = 0$ (maximum field intensity) the rms value distribution becomes:

$$b(r, I) = \frac{3\mu I}{2\pi} \frac{\sqrt{3}s}{r^2 - s^2}, \tag{8}$$

which holds far enough from line phase conductors, when the field intensity distribution becomes almost independent on the direction.

The accuracy of (8) is assessed as above by comparing the contour lines computed as circles with contour plots computed with the corresponding exact relationship (6). Figure 5 shows the rms field distributions for a line in flat configuration ($s = 4$ m, $I = 800$ A rms phase currents). It can be noted that also in this case contour plots of the magnetic flux density are in good agreement also in proximity of the line conductors. The maximum discrepancy is attained along the vertical axis — passing through phase conductor 1 — but the estimate is still conservative since contour lines computed with (8) surround those computed with (6).

2.2. Double Circuit Power Lines

Magnetic field analysis increases in complexity when double circuit lines are examined, in particular if line circuits are unevenly loaded.

Analytical expressions giving the magnetic field strength of double circuit lines have already been proposed in the case of balanced three-phase systems [17]. The method of moments has been applied in [13] for devising analytical relationships which are accurate far from line conductors. By using the same approach exact expressions have been obtained in [11] for double circuit lines with polygonal conductor arrangement.

A general formula for computing the magnetic flux density strength at any distance from power line and for any conductor arrangement has been presented in [12]. By assuming balanced currents for circuits a and b, and a phase angle displacement δ between phases $1a - 1b$, the general relationship (2) can be rewritten in the following form:

$$b = \frac{\mu}{2\pi} \sqrt{b_a^2 - I_a I_b \sum_{p=1}^{3} \sum_{q=p}^{3} \cos(\theta_{pq} + \delta) \left(\frac{d_{pq}}{r_p r_q}\right)^2 + b_b^2}, \tag{9}$$

where the indexes p and q span conductors of circuits a and b, respectively, I_a and I_b are the circuit current magnitudes, b_a and b_b are the field intensities generated by each circuit independently, and θ_{pq} can assume values 0, $-2\pi/3$, $+2\pi/3$, depending on the phase ordering of line currents. It can be observed that the above relationship (9) cannot be inverted in order to obtain the ROW width as a function of the limiting exposure value, since it depends on mutual magnetic couplings between circuits.

More simple relationships can be derived by adopting the following strategy. By using the superposition principle and the triangle inequality of the Euclidean norm a conservative estimate of the rms magnetic flux density distribution of any type of double circuit power line can be obtained:

$$b = |\mathbf{B}_a + \mathbf{B}_b| \leq |\mathbf{B}_a + \mathbf{B}_b|, \tag{10}$$

where $\mathbf{B}_a$, $\mathbf{B}_b$ can be the field contributions from circuit a and b or, more in general, from different sets of three-phase (balanced) currents termed again a and b. The basic advantage in using (10) is that it does not require any information about the instantaneous phase shift between (balanced) three-phase currents of circuit a and b, which is typically not know in practice.

In the following approximate expressions for double circuit lines in both super-bundle (SB) and low-reactance (LR) phasing — which can be used to estimate the ROW width — are proposed. These are compared with the exact representation of the rms field distribution given by (9).

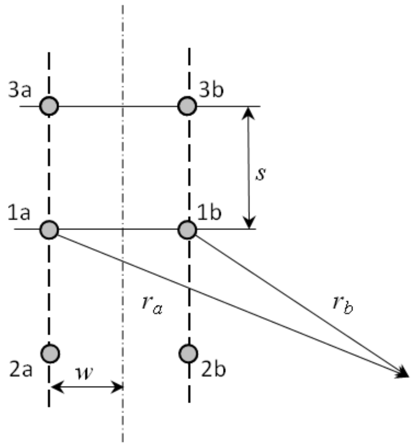

Figure 6. Super-bundle double circuit line.

2.2.1. Super-bundle Phasing

Figure 6 shows a transmission line in super-bundle phasing (*SB line*) where homologous phases are symmetrically placed with respect to the line axis (in dashed line). Subscripts a, b indicate different circuits, s the phase to phase distance, w the conductor distance from the line axis and r_a, r_b the distances between field point and circuits central conductors $1a - 1b$.

By using (10) the magnetic flux density rms value can be conservatively estimated at any point in the space as:

$$b(I_a, I_b, r_a, r_b) = \frac{\sqrt{3}s\mu}{2\pi}\left(\frac{I_a}{r_a^2 - s^2} + \frac{I_b}{r_b^2 - s^2}\right), \tag{11}$$

summing up contributions of vertical single circuit lines given by (8). At a distance greater than a few phase-spacings it may be assumed that $r_a \approx r_b \approx r$ with $r = \sqrt{r_a r_b}$ so that (11) becomes:

$$b(r, I) = \frac{\sqrt{3}s\mu}{\pi}\left[\frac{I}{(r - \Delta r)^2 - s^2}\right], \tag{12}$$

where $I = \frac{I_a + I_b}{2}$ is the average circuit current and

$$\Delta r = \left|\frac{I_a - I_b}{I_a + I_b}\right| w. \tag{13}$$

This additional term takes into account the degree of unbalance between circuits and is obtained by noting that the rms field contour

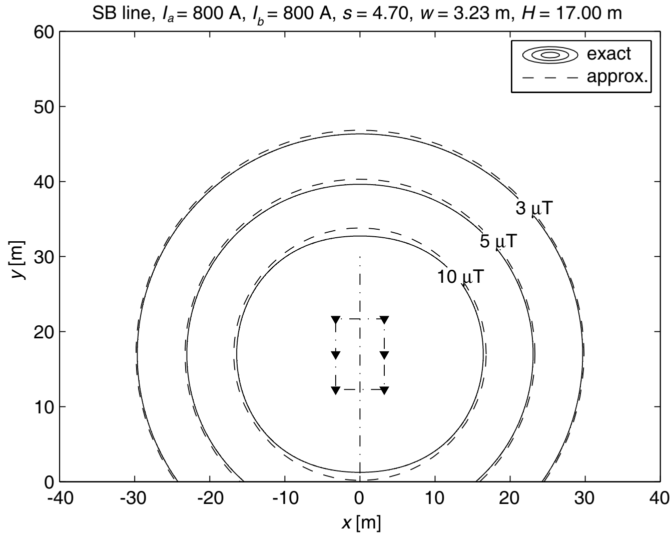

Figure 7. Contour plots of the rms magnetic flux density [μT] for evenly loaded double circuit SB line ($I_a = 800\,\mathrm{A}$, $I_b = 800\,\mathrm{A}$): exact profiles are in continuous line, approximated ones in dash line.

lines are centered around the circuit carrying the highest currents. In the limiting case $I_a = 0$ contour lines are centered around circuit b and can be exactly represented with (8); on the other hand, in the balanced case, $\Delta r = 0$.

It may be verified that the previous expressions hold accurate even with an high degree of load unbalance. As an example, the case of a double circuit SB line with $s = 4.7\,\mathrm{m}$ and $w = 3.23\,\mathrm{m}$ is considered. Figure 7 shows the rms field distribution for a balanced load configuration with $I_a = I_b = 800\,\mathrm{A}$. Figure 8 shows the same distribution for the unbalanced case with $I_a = 200\,\mathrm{A}$ and $I_b = 800\,\mathrm{A}$. It can be noticed that approximate contour lines are in good agreement with those computed with (11) and the circle assumption holds even in the near vicinity of line conductors.

2.2.2. Low-reactance Phasing

Low-reactance phasing (*LR line*) is a well-known example of low-field double circuit line design. With this type of arrangement phase conductors of circuit b are swapped with respect to the super-bundle line configuration in order to enhance the field compensation effect when circuits are evenly loaded (Figure 9). It is worth noting that such mitigating effect might cancel out when line currents are unbalanced and may lead even to higher field intensities than the SB arrangement.

In case of evenly loaded circuits ($I_a = I_b = I$) the magnetic field intensity produced by this phasing arrangement decays as $1/r^3$ and

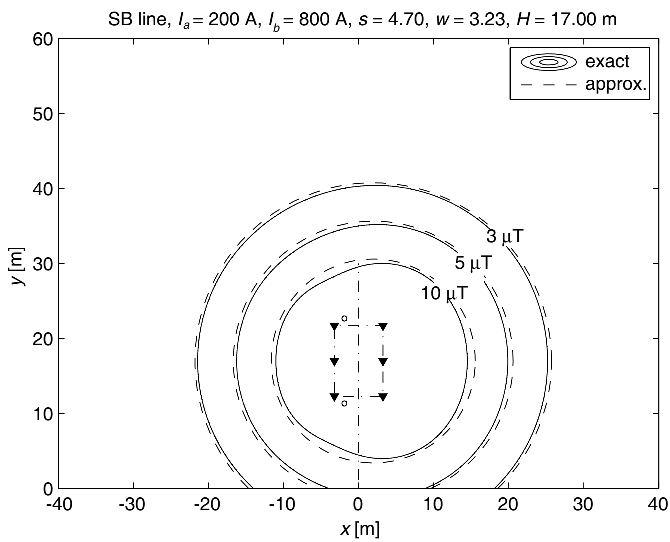

Figure 8. Contour plots of the rms magnetic flux density [μT] for unevenly loaded SB line ($I_a = 200\,\mathrm{A}$, $I_b = 800\,\mathrm{A}$): exact profiles are in continuous line, approximated ones in dash line.

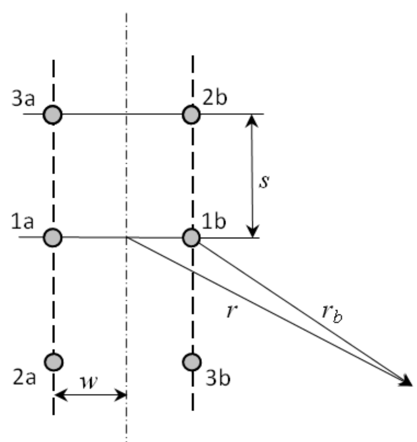

Figure 9. Low-reactance double circuit line.

may be computed with a good accuracy as [13]:

$$b(I, r) = \frac{\mu s I}{\pi} \frac{\sqrt{s^2 + 12w^2}}{r^3} \tag{14}$$

where r is the distance from the center of gravity of the conductors and s, w are the phase-to-phase distance and the distance from the line axis, as defined in Figure 9. The approximate relationship (14) does not hold any more when I_a differs from I_b since it can be proved that field intensity at far distances decays as $1/r^2$ instead of $1/r^3$.

In order to account for such uneven loading conditions the superposition principle and the triangle inequality of the Euclidean norm are used. The rms field distribution in proximity of a LR line can be conservatively estimated by summing up the contribution of an evenly loaded LR line carrying a net current $I = \frac{I_a+I_b}{2}$ and of a single circuit in vertical arrangement carrying a net current $\Delta I = \frac{|I_a-I_b|}{2}$. The following relationship as a function of r, I, and ΔI is thus obtained:

$$b(r, I, \Delta I) = \frac{\mu I}{\pi} \frac{s\sqrt{s^2 + 12w^2}}{r^3} + \frac{\mu \Delta I}{\pi} \frac{\sqrt{3}s}{r_b^2 - s^2} \tag{15}$$

where r_b is the distance from the center of circuit b (Figure 9). Equation (15) can be further simplified by assuming that $r \approx r_b$ and $s \gg r$, as

$$b(r, I, \Delta I) = \frac{\mu I}{\pi} \frac{s\sqrt{s^2 + 12w^2}}{r^3} + \frac{\mu \Delta I}{\pi} \frac{\sqrt{3}s}{r^2}. \tag{16}$$

The case of a LR double circuit line with conductor distances $s = 4.7\,\mathrm{m}$ and $w = 3.23\,\mathrm{m}$, carrying both even ($I_a = I_b = 800\,\mathrm{A}$) and uneven ($I_a = 200\,\mathrm{A}$, $I_b = 800\,\mathrm{A}$) circuit currents is considered in Figures 10 and 11, respectively. From Figure 10 it can be observed that with evenly loaded circuits — as it usually occurs in the case of LR double circuit lines — the contour plot circle approximation with radii computed by (16) is in very good agreement with the exact computation. Conversely in the less common case of unevenly loaded circuits contour plots are not symmetrical and the circle approximation holds only in the half-plane of the circuit carrying the higher current, whereas it provides a conservative estimate of the actual field distribution in the opposite half-plane (Figure 11). Note that, however, in the ROW width estimate the worst case condition should be considered — i.e., maximum distance from the line axis for a given exposure level — since loading conditions can be time-varying and the highest currents can be carried by either circuit a or b.

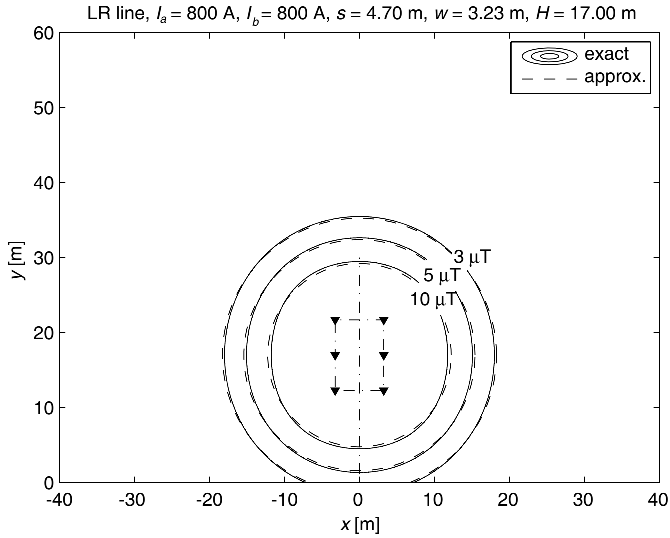

Figure 10. Contour plots of the rms magnetic flux density [μT] for evenly loaded LR line ($I_a = 800\,\mathrm{A}$, $I_b = 800\,\mathrm{A}$): exact profiles are in continuous line, approximated ones in dash line.

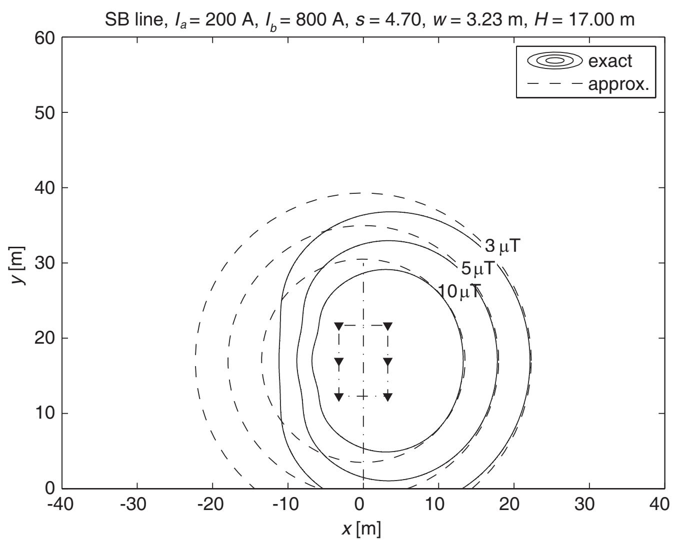

Figure 11. Contour plots of the rms magnetic flux density [μT] for unevenly loaded LR line ($I_a = 200\,\mathrm{A}$, $I_b = 800\,\mathrm{A}$): exact profiles are in continuous line, approximated ones in dash line.

3. ANALYTICAL EVALUATION OF THE RIGHT-OF-WAY WIDTH

A typical assessment procedure of the Right-of-Way width for a given limiting exposure level would be to numerically compute the rms value magnetic field distribution and trace its contour plots on a vertical plane normal to the line axis in order to verify the compatibility with neighboring residential buildings. As an example, Figure 12 reports the case of a building located 20 m apart the axis of a double circuit power line, where a maximum field intensity value of 3 μT has been considered. The ROW width is thus the vertical projection of the 3 μT — contour line.

A more simple analytical approach, still providing an accurate estimate of the ROW width, is here proposed. The formulas for the calculation of the magnetic flux density proposed in the previous section can be used to assess the distance r^* — from the center of the phase conductor arrangement — at which a given limiting value b^* is attained. The solution of this inverse problem is typically non-trivial since analytical expressions obtained for b^* have a non-linear dependency on r^*.

In the following all line arrangements discussed above are considered. For each case sufficiently simple yet accurate relationships, which can be applied on the field, are provided. Limiting exposure levels here considered are 10 μT (attention value) and 3 μT (quality

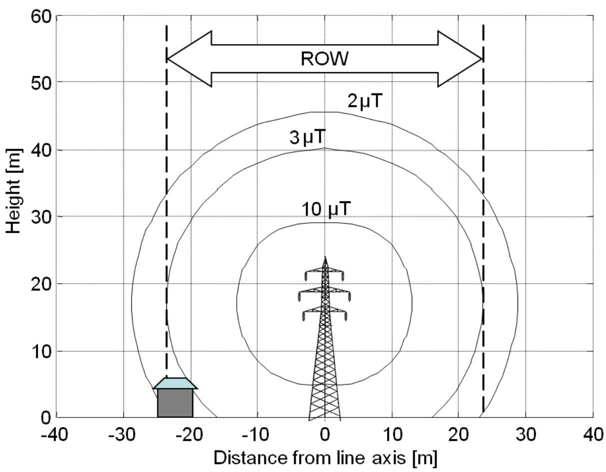

Figure 12. Example of contour plot for assessing the ROW width around a line span.

value) as it is prescribed by Italian regulations [4]. Approximate formulas to estimate the ROW width for a given field exposure limit are validated with data obtained from a numerical procedure, which resolves the inverse problem of finding the distance from the line axis at which a specific field intensity value is attained. The discrepancy between analytically r^* and numerically $\widetilde{r}^*$ computed ROW width values is calculated as:

$$e[\%] = \left|1 - \frac{r^*}{\widetilde{r}^*}\right| \cdot 100. \tag{17}$$

3.1. Three-phase Single Circuit Power Lines

It is assumed in the following that three-phase currents of a single circuit line form a balanced set so that the magnetic field intensity b is proportional to the current rms value I. The ROW width should be assessed for a fixed reference current rms value, which is typically the rated current of the power transmission line or the maximum current value given by the grid manager.

3.1.1. Three-phase Lines in Delta Arrangement

It can be noted in practice that for single circuit three-phase lines — far enough from the line center of gravity — contour lines of the magnetic flux density are almost circular so that the field intensity does not depend on direction. In that case it becomes easy to get the value of the ROW width for a given rms value $b = b^*$. By inverting (5) one obtains the following relationship, which can be used for any type of single circuit line and applies in particular to compact phase conductor arrangements exhibiting usually a non-symmetric structure:

$$r^*(b^*, I) = \sqrt{\frac{3\mu I}{2\sqrt{2}\pi}\frac{d_m}{b^*}}, \tag{18}$$

where $d_m = \sqrt[3]{d_{12}d_{23}d_{31}}$ is the average distance between phase conductors. For symmetric arrangements with conductors placed on the vertexes of an equilateral triangle the same expression can be used by letting $d = \sqrt{3}s$, where s is the distance from the line conductor center of gravity (Figure 2).

It can be observed in Figure 13 that the discrepancy between numerically and analytically computed ROW width values increases as long as current load decreases. This is because the assumption $r \gg s$ does not hold any more and it is not possible to apply multipole expansion on Biot-Savart's law yielding (5). Moreover, the level of

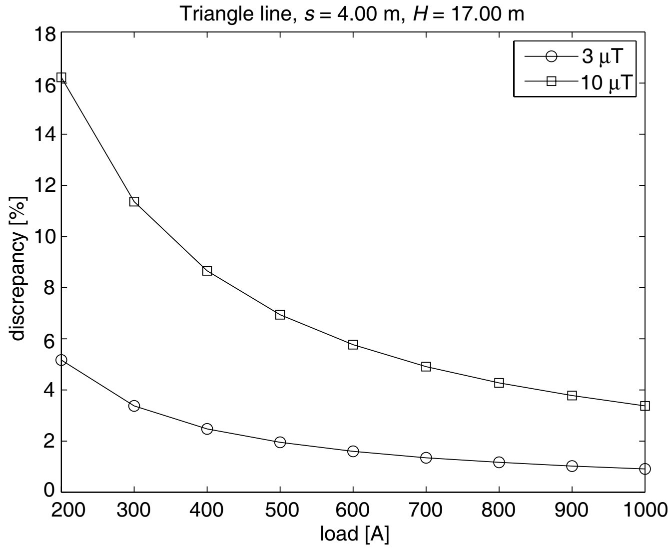

Figure 13. Discrepancy between numerical and analytical ROW width of a delta power line for different loading conditions and field exposure limits.

accuracy is very good close to the rated loading condition of 600 A rms as the contour line corresponding to the limiting exposure level (3 μT or 10 μT) is at bigger distance from the line.

3.1.2. Three-phase Lines with Aligned Conductors

With limited simplifying assumptions on the line geometry one can obtain a general expression for estimating the ROW width of single circuit lines in flat arrangement, with conductors aligned on the same axis (non necessarily vertical). The approximate relationship (7) for estimating the rms field value can be cast in the following equation to be solved in terms of r:

$$r^4 - (2s^2 \cos 2\varphi)r^2 + (s^4 - s^2 I^2/k^2) = 0, \tag{19}$$

where $k = (2\pi/\sqrt{3}\mu)b^*$ is constant for a given exposure limit. This can be easily reduced to a 2-nd order equation, yielding a unique positive solution:

$$r^* = \sqrt{s^2 \cos 2\varphi + \sqrt{(s^2 \cos 2\varphi)^2 - s^4 + s^2 I^2/k^2}}. \tag{20}$$

From this expression ROW widths for more common line geometries with horizontal and vertical aligned conductors can be deduced by

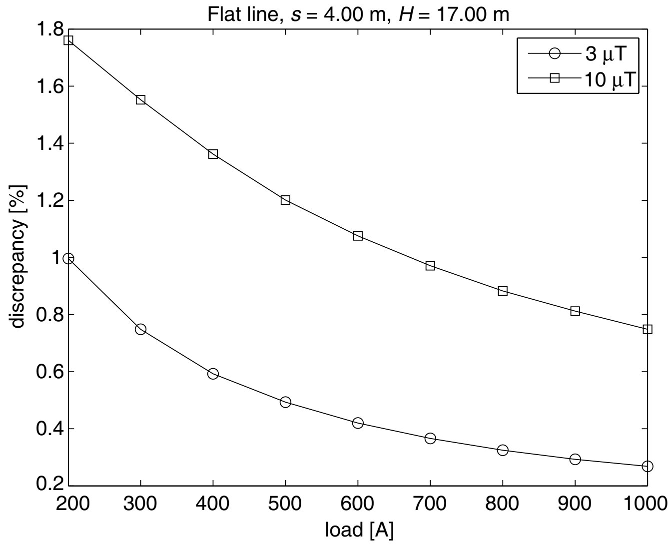

Figure 14. Discrepancy between numerical and analytical ROW width of a flat power line for different loading conditions and field exposure limits.

letting $\varphi = 0$ and $\varphi = \pi/2$, respectively. Even in that case the circle assumption for contour lines is well verified for both horizontal and vertical arrangement.

As an example, the case of a three-phase power line with horizontal aligned conductors (phase-to-phase distance $s = 4\,\text{m}$) is considered. Figure 14 shows that the discrepancy between ROW width values computed with (20) and numerically computed data is always below 2% in any load condition.

3.2. Double Circuit Power Lines

3.2.1. Super-bundle Phasing

It has been shown above that the circle approximation holds also for unevenly loaded double circuit lines in SB phasing. A conservative estimate of the ROW width can be obtained from (12) by expressing r^* as a function of the maximum field intensity b^* and the average current $I = \frac{I_a+I_b}{2}$, yielding:

$$r^*(b^*, I) = \sqrt{\frac{\mu I}{\pi}\frac{\sqrt{3}s}{b^*} + s^2} + \Delta r, \tag{21}$$

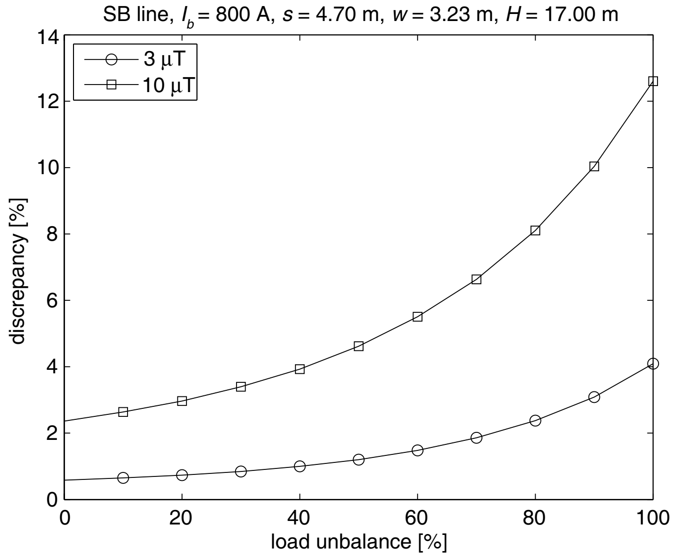

Figure 15. Discrepancy between numerical and analytical ROW width of a double circuit SB power line for different load unbalances ($I_a = (1-\alpha)I_b$, $I_b = 800\,\mathrm{A}$) and field exposure limits.

where the distance $\Delta r = |\frac{I_a - I_b}{I_a + I_b}|w$ takes into account the load unbalance.

The approximate formula is tested for different degrees of circuit current unbalance. Parameter α defines the amount of current on circuit a compared to circuit b, that is $I_a = (1-\alpha)I_b$ where I_b is set to 800 A rms. Figure 15 shows that the discrepancy between analytical and numerical ROW width values increases with the degree of unbalance and is acceptable up to 20–30%.

3.2.2. Low-reactance Phasing

In the case of evenly loaded circuits the ROW width for a double circuit line in LR phasing can be easily computed by inverting (14), yielding:

$$r^* = \sqrt[3]{\frac{\mu s I}{\pi} \frac{\sqrt{s^3 + 12w^2}}{b^*}}. \tag{22}$$

More complex is to obtain an analytical relationship for estimating the ROW distance for a LR line with unbalanced circuits. It can be shown that the approximate formula for computing the field rms value distribution (16) can be cast in the following 3-rd order equation to be solved in terms of r:

$$r^3 + pr + q = 0, \tag{23}$$

where:

$$p = -\frac{\mu \Delta I}{3\pi}\frac{s}{b^*}, \quad q = \frac{\mu I}{2\pi}\frac{s\sqrt{s^2 + 12w^2}}{b^*}, \tag{24}$$

and $I = \frac{I_a + I_b}{2}$, $\Delta I = \frac{|I_a - I_b|}{2}$. Even though a 3-rd order equation has in general three-complex solutions, it can be demonstrated in that case that a real solution exists since the discriminant $q^2 + p^3$ is positive. Other solutions, which are complex and conjugate, are discarded. Hence, the following approximate formula holds for assessing the ROW width of LR lines in unbalanced loading conditions:

$$r^* = \sqrt[3]{-q + \sqrt{q^2 + p^3}} + \sqrt[3]{-q - \sqrt{q^2 + p^3}}. \tag{25}$$

Figure 16 shows that this approximate formula is still accurate even with severe levels of unbalance. The overall discrepancy is below 4% in any loading condition; the best agreement is attained for evenly balanced circuits.

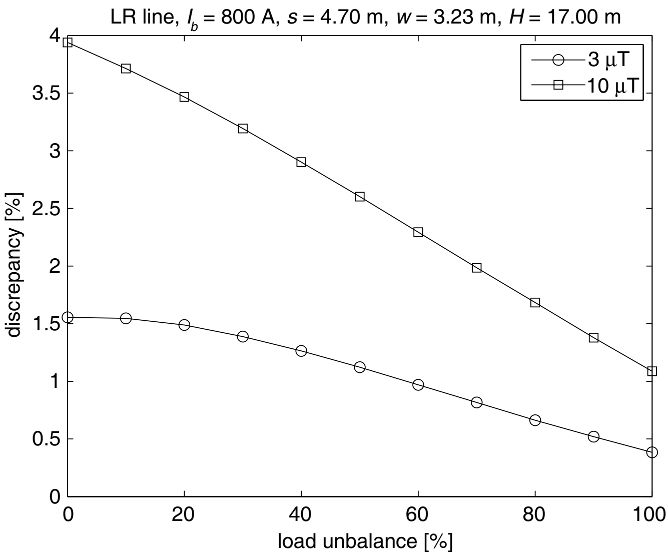

Figure 16. Discrepancy between numerical and analytical ROW width of a double circuit LR power line for different load unbalances α ($I_a = \alpha I_b$, $I_b = 800\,\mathrm{A}$) and field exposure limits.

4. CONCLUSION

It has been shown in this work that the magnetic field intensity generated by overhead power lines in single and double circuit configurations may be well approximated by analytical expressions which are a function of the line geometric parameters and the field point distance from the line center of gravity. In particular, original formulas are proposed for double circuit evenly and unevenly loaded lines in both super-bundle and low-reactance phasing.

It has been noted that contour lines of the rms magnetic flux density at a distance from the line greater than a few phase-spacings have an almost circular shape, whose radius can be determined by inverting analytical expressions for computing the field strength. These formulas have proved to be accurate in a variety of configurations and load conditions.

This approach leads to a simple yet accurate estimate of the ROW width for any overhead — and indeed also buried — power line configuration. The proposed relationships can be easily implemented on spreadsheets or Graphical Information Systems for environmental impact assessment.

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