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# "NATURAL" DEFINITION OF THE MODAL IMPEDANCES IN NON-HOMOGENEOUS DIELECTRIC LOADED RECTANGULAR WAVEGUIDE

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**Abstract**—In this paper, the definition of the modal impedances of the electromagnetic field in a nonhomogeneously filled waveguide is discussed. The presence of TM modal impedances, which are functions of the transverse coordinate, does not permit us to obtain a unique $Z$ matrix of these guides. Hence, the evaluation of the scattering matrix can be involved. The introduction of a "natural" EM expansion overcomes this problem leading to the definition of a unique modal impedance and a unique $Z$ matrix. This approach is applied to the simulation of the effect of a block of dielectric in an empty waveguide by "cascading" the $S$ matrices of the existing junctions. Finally, this "natural" EM expansion is applied to the junction between an empty waveguide and a completely filled waveguide, obtaining an equivalent circuit which better represents the physics of this problem, and to the optical fibers.

## 1. INTRODUCTION

In recent years, dielectric loaded cavities have been thoroughly analyzed mainly for their application of dielectric resonators as microwave filters in many scenarios. Several techniques have been used to deal with this problem such as mode matching [1, 2], the finite-element method [3, 4], the finite-difference time-domain method [5], the coupled mode method [6] or the expansion of the EM fields with respect to the complete set of the closed cavity containing the resonator [7]. These techniques analyze the dielectric loaded cavity as a 3-D block and the scattering parameters are always referred to the ports in the empty waveguide. In other words, the EM fields are expanded in the

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3-D space containing the dielectric resonator and a part of the empty waveguide. Only in [7], there is an attempt to refer the scattering parameters to the port of the dielectric-loaded guide.

Another interesting field of investigation is the reconstruction of dielectric properties required in many areas of geophysical prospecting and subsurface imaging, where it helps in the geological investigation of different kinds of rocks and soils [8]. In fact, for a reliable interpretation of ground-penetrating radar (GPR) data for this kind of applications in the $L$ frequency band, an accurate knowledge of the dielectric properties of rock materials is required. Similarly, the dielectric constants are required in theoretical models that calculate propagation constants and radar backscatter coefficients from a vegetation medium such as a forest stand. Another example of geophysical application is the estimation of the mass balance of ice sheets and glaciers on Earth which can be evaluated with the help of the synthetic aperture radar (SAR) and GPR if the dielectric properties of the ice are known [9–11]. Analogously, the use of GPR techniques at high frequencies for investigations on ornamental stones or masonries, either in quarries or on historical buildings, requires an accurate knowledge of the material complex permittivity.

One very common method for determining the permittivity of materials is to use a transmission line approach, where the material under test is placed either in a section of coaxial airline or in a section of rectangular waveguide in order to measure the reflection and transmission coefficient data in the frequency band of interest. The dielectric properties of the material are then determined from the scattering data using either an analytical or an optimization approach [12–16]. Even in this case, the EM field analysis needed for permittivity determination is performed in terms of a 3-D block, as in the above scenario.

A simpler approach to the solution of the 3-D problem arising from the dielectric loaded cavities could be obtained by "cascading" the generalized scattering matrix (GSM) representing:

- the junction between the empty waveguide and the partially filled waveguide (placed at $z = 0$);
- the partially filled waveguide of length $L$;
- the junction between the partially filled waveguide and the empty waveguide (placed at $z = L$);

as shown in Fig. 1.

The problem is the evaluation of the GSM, which can be involved. In fact, it is well-known that GSM is obtained by the $Z$ (or $Y$) matrix representing the junction and it can be influenced by the

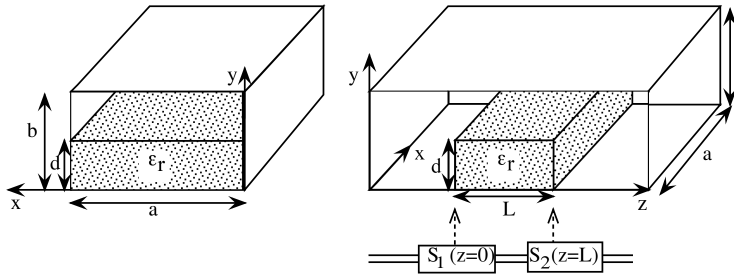


**Figure 1.** A waveguide filled with a nonhomogeneous dielectric.

definition of the reference impedance used in defining the scattering (or pseudo-scattering) waves, as shown in [17]. In fact, any choice of reference impedance is equally valid [18] but it has repercussions in the evaluation of the equivalent circuit of the junction. In fact, as Marcuvitz states in his excellent book [19], "*No general criterion exists to determine which of the equivalent networks is most appropriate. [...] Usually, there is a "natural" one distinguished by having minimum number of impedance elements. [...] In special cases, however, the same criteria of a minimum number of network parameters, simple frequency dependence, etc., can be employed to determine the best network representation.*" Moreover, a correct definition of the normalization quantities of voltages and currents permits to satisfy the *causality* condition as in [20] or the preservation of reciprocity for lossy medium as in [21]. For lossless TEM, TE and TM guides it can be shown that *causality* is satisfied if there is a *unique* modal impedance relating electric and magnetic modal fields. This latter condition is satisfied if the waveguide is filled with a homogeneous medium, because the modal impedances are perfectly defined. If the medium filling the guide is nonhomogenous, the modal impedances may be a function of the part of the dielectric we are analyzing. Hence, a proper definition of the $Z$ (or $Y$) matrix could be very difficult.

The main aim of this paper is the introduction of a "natural" definition of voltage and current related to the $Z$ matrix in order to easily obtain a GSM for each junction of the problem shown in Fig. 1. To that end, at first, a "natural" development of the EM field is proposed to overcome the difficulty in defining the modal impedances, then the definition of the voltages and currents related by the $Z$ (or $Y$) matrix is introduced to obtain the GSM of a junction between an empty waveguide and a partially filled waveguide. Obviously, the "natural" development of the EM field can be extended to a more complex nonhomogeneously filled waveguide, like optical fibers. Just as an example of application and to validate the new definition, the obtained GSM will be applied to the solution of the problem mentioned in the first part of the introduction, i.e., a waveguide filled with a block of lossless dielectric. Results will be compared with those obtained with commercial software. The solution of an elementary electromagnetic problem (the discontinuity between an empty and a completely filled waveguide) will also be briefly discussed with the help of this "natural" development of the EM fields.

## 2. THEORY

In a homogeneously filled waveguide, the voltage and current are the modal amplitude of transverse electric and magnetic modal fields, $\overline{e}_{t,n}(x, y)$ and $\overline{h}_{t,n}(x, y)$, defined as follows [19]:

$$\overline{E}_t(x, y, z) = \sum_{n=1}^{\infty} V_n(z)\overline{e}_{t,n}(x, y) \tag{1}$$

$$\overline{H}_t(x, y, z) = \sum_{n=1}^{\infty} I_n(z)\overline{h}_{t,n}(x, y) \tag{2}$$

It is well-known that, in a partially filled waveguide such as the one shown in Fig. 1, $TE_y$ and $TM_y$ modes (or LSE and LSM modes) are used to satisfy the boundary conditions at the discontinuity interface [22]. Hence, we will use these families of modes in the following discussion.

Under the hypothesis of using (1)–(2) also for the partially filled waveguide of Fig. 1, the fields have a $\sin(k_{x,m}x)$ or $\cos(k_{x,m}x)$ $k_{x,m} = m\pi/a$ variation in the $x$ direction, while they have different characteristics in the dielectric $\varepsilon_r$ and in the air ($y$ direction). Hence, by substituting (1)–(2) in the Maxwell's equations, and, under the hypothesis of propagation in $\pm z$-direction ($e^{\mp\gamma_n z}$), it can be shown that for forward and backward waves ($V_n(z) = V_n^+(z) + V_n^-(z)$, $I_n(z) = I_n^+(z) + I_n^-(z)$, $V_n^\pm(z) = \pm Z_{0,n}(y)I_n^\pm(z)$):

$$Z_{0,n}(y) = \begin{cases} \dfrac{\gamma_n^2 - k_{x,m}^2}{j\omega\varepsilon_0\varepsilon_r(y)\gamma_n} & TM_y \\ \\ \dfrac{j\omega\mu_0\gamma_n}{\gamma_n^2 - k_{x,m}^2} & TE_y \end{cases} \qquad \varepsilon_r(y) = \begin{cases} \varepsilon_r & 0 \le y \le d \\ 1 & d < y \le b \end{cases} \tag{3}$$

Hence, the modal impedances are a function of the transverse $y$ direction under $TM_y$ polarization and we need to define two voltages and two currents, one for each dielectric ($\varepsilon_r$ and air), in the partially filled waveguide. In these conditions, the definition of the $Z$ matrix would be quite difficult. For example, the $Z_{11}$ element relative to the n-th mode of the partially filled waveguide of length $L$ would be

$$Z_{11} = Z_{0,n}(y)\coth(\gamma_n L) \tag{4}$$

and we need two $Z$ matrices, one for each dielectric filling the unit cell.

To overcome this problem, following the arbitrariness in the choice of the voltage and the current, we try to define a "natural" modal impedance which is the same for the two dielectric media, in the sense specified by Marcuvitz [19]. Hence, we replace the usual expansions (1), (2) with the following, "natural", ones

$$\overline{E}_t(x,y,z) = \sum_{n=1}^{\infty} V_n(z)\frac{\overline{e}_{t,n}(x,y)}{g_n(y)} \tag{5}$$

$$\overline{H}_t(x,y,z) = \sum_{n=1}^{\infty} I_n(z)\overline{h}_{t,n}(x,y)\,g_n(y) \tag{6}$$

$$g_n(y) = \begin{cases} \sqrt{\varepsilon_r(y)} & TM_y \\ 1 & TE_y \end{cases} \tag{7}$$

In this case the definition of the same $Z$ matrix for the two dielectric media is ensured. In fact, by substituting (5)–(6) in the Maxwell's equation

$$H_x(x,y,z) = \frac{\frac{\partial E_y(x,y,z)}{\partial z} - \frac{\partial E_z(x,y,z)}{\partial y}}{j\omega\mu_0} \tag{8}$$

$$E_z(x,y,z) = \frac{\frac{\partial H_y(x,y,z)}{\partial x} - \frac{\partial H_x(x,y,z)}{\partial y}}{j\omega\varepsilon(y)} \tag{9}$$

after some mathematical manipulations and recalling that the permittivity $\varepsilon(y)$ is a piecewise-defined function of the $y$ transverse coordinate in (9), we obtain the following relationship for $TM_y$ modes:

$$\sum_{n=1}^{\infty}\left[I_n(z)g_n(y)\left(\gamma_n^2 - k_{x,m}^2\right)h_{x,n}(x,y) \pm V_n(z)\frac{j\omega\varepsilon(y)\gamma_n}{g_n(y)}e_{y,n}(x,y)\right] = 0 \tag{10}$$

with $\gamma_n$ being the propagation constant along $z$ for the $n$-th mode.

Waveguide theory tells us that we can choose $e_{y,n}(x,y) = -h_{x,n}(x,y)$ [19, 23] and, from (7), (10), we can define the new ("natural") modal impedance for $TM_y$ modes as:

$$Z_{0,n} = \frac{\gamma_n^2 - k_{x,m}^2}{j\omega\varepsilon_0\gamma_n} \tag{11}$$

which is no longer a function of $y$. Similar manipulations can be done to obtain the modal impedance for $TE_y$ modes yielding

$$Z_{0,n} = \frac{j\omega\mu_0\gamma_n}{\gamma_n^2 - k_{x,m}^2} \tag{12}$$

which does not change with respect to the case of homogeneously filled waveguides.

With these new "natural" definitions of modal impedances (11) and (12), any element of the $Z$ matrix relative to the $n$-th mode of the partially filled waveguide of length $L$, like (4), is no longer a function of $y$ and voltage and current are functions of the $z$ propagation coordinate alone. Hence, we can correctly define a unique $Z$ matrix for the partially filled waveguide, using (5)–(6).

## 3. ANALYSIS OF A DIELECTRIC LOADED RECTANGULAR CAVITY

Having established the capability to define the "natural" modal impedances of a partially filled waveguide as discussed above, we can analyze the derivation of the $Z$ matrix relative to the junction between the empty waveguide and the partially filled waveguide placed at $z = 0$ in Fig. 1 by applying the multimode equivalent network approach [24–26]. For this discontinuity, the spectra of the two regions are needed (empty and partially filled waveguides). Both are well-known [22] and we report only the $TM_y$ main component for the partially filled waveguide which must obey the "natural" definitions (5)–(6):

$$e_{y,n}(x,y) = A\sin(k_{x,m}x)\begin{cases} \dfrac{1}{\sqrt{\varepsilon_r}}\dfrac{\cos(k_{yr}y)}{\cos(k_{yr}d)} & \forall\gamma_n \qquad 0 \le y \le d \\ \begin{cases} \dfrac{\cosh[k_{y0}(y-b)]}{\cosh[k_{y0}(b-d)]} & \gamma_n \in \Im \\ \dfrac{\cos[k_{y0}(y-b)]}{\cos[k_{y0}(b-d)]} & \gamma_n \in \Re \end{cases} & \qquad\qquad d \le y \le b \end{cases}$$

$$k_{y0} = \sqrt{k_0^2 + \gamma_n^2 - k_{x,m}^2}\,, \quad k_{yr} = \sqrt{k_0^2\varepsilon_r + \gamma_n^2 - k_{x,m}^2}$$

where $A$ is a normalization constant, not reported here for the sake of brevity. The other components can be obtained from

$$\begin{aligned} e_{x,n}(x,y) &= \frac{1}{k_{x,m}^2 - \gamma_n^2}\frac{\partial^2 e_{y,n}(x,y)}{\partial x\partial y} \qquad h_{x,n}(x,y) = -e_{y,n}(x,y) \\ e_{z,n}(x,y) &= \mp\frac{\gamma_n}{k_{x,m}^2 - \gamma_n^2}\frac{\partial e_{y,n}(x,y)}{\partial y} \qquad h_{z,n}(x,y) = \mp\frac{1}{\gamma_n}\frac{\partial e_{y,n}(x,y)}{\partial x} \end{aligned} \tag{13}$$

where the longitudinal components have been defined as:

$$E_z(x,y,z) = \sum_{n=1}^{\infty} V_n(z)\frac{e_{z,n}(x,y)}{g_n(y)}$$
$$H_z(x,y,z) = \sum_{n=1}^{\infty} I_n(z) h_{z,n}(x,y)\, g_n(y)$$

We can now proceed with the integral equation formulation of the discontinuity placed at $z = 0$. The approach is very similar to the one reported in [25]. Hence, only a few steps will be discussed. The first step consists in the introduction of the "accessible" and "localized" modes concept [27]. The accessible modes are the first modes excited by the discontinuity (all the propagating modes plus the first few non-propagating modes). These modes are responsible for the interaction between adjacent discontinuities. The localized modes are the infinite remaining modes localized in the neighborhood of the discontinuity. By separating the accessible from the localized modes and by (5)–(6), we can write the continuity of the magnetic field at the surface discontinuity placed at $z = 0$ as:

$$\sum_{n=1}^{N_1} I_n^{(1)}(0^-)\overline{h}_{t,n}^{(1)}(x,y)\, g_n^{(1)}(y) - \sum_{n=N_1+1}^{\infty} V_n^{(1)}(0^-)Y_{0,n}^{(1)}\overline{h}_{t,n}^{(1)}(x,y)\, g_n^{(1)}(y)$$
$$= \sum_{n=1}^{N_2} I_n^{(2)}(0^+)\overline{h}_{t,n}^{(2)}(x,y) g_n^{(2)}(y) + \sum_{n=N_2+1}^{\infty} V_n^{(2)}(0^+)Y_{0,n}^{(2)}\overline{h}_{t,n}^{(2)}(x,y) g_n^{(2)}(y) \quad (14)$$

where the index $n$ covers both $TE_y$ and $TM_y$ modes, the superscripts (1) and (2) refer to regions to the left and to the right of the discontinuity, respectively. $Y_{0,n}^{(1)}, Y_{0,n}^{(2)}$ are the modal admittances as in (11)–(12) and $N_1$, $N_2$ are the number of the accessible modes in regions 1 (empty waveguide) and 2 (partially filled waveguide), respectively. $g_n^{(1,2)}(y)$ are defined as in (7). The $x$-functions in (14) are relative to the $(1, 0)$ order mode, with $k_{x,m} = \pi/a$, because the structure does not vary along $x$. In (14) we have introduced the following modal voltage amplitudes,

$$V_n^{(j)} = \int_0^a \int_0^b \left[\widehat{z} \times \overline{E}_t(x,y)\right] \cdot \overline{h}_{t,n}^{(j)*}(x,y) g_n^{(j)}(y) dx dy \quad (15)$$

with $z$ being the direction of propagation and $j = 1, 2$.

The unknown quantity of the problem, namely the transverse electric field in the aperture $\left[\widehat{z} \times \overline{E}_t(x,y)\right]$, can now be expanded in terms of proper sets of vectorial expanding functions $\overline{M}_k^{(j)}$ weighted by the amplitudes of the accessible modes

$$\widehat{z} \times \overline{E}_t(x, y) = \sum_{k=1}^{N_1} I_k^{(1)}(0^-)\overline{M}_k^{(1)}(x, y) - \sum_{k=1}^{N_2} I_k^{(2)}(0^+)\overline{M}_k^{(2)}(x, y) \quad (16)$$

In fact, we can expect the resulting electric field to be dependent on the amplitudes of the exciting modes. Expressions (15) and (16) can now be used in (14), then equating term by term, we can define a set of integral equations, where the unknown quantities are the expanding vector functions $\overline{M}_k^{(j)}$. These integral equations can now be solved by using the method of moments. The unknown vector functions $\overline{M}_k^{(j)}$ are expanded as linear combinations of orthonormal modes of the two regions 1 (the empty waveguide) and 2 (the partially filled waveguide) [22, 25]. The final step for the solution of the resulting equations is the application of Galerkin's procedure that leads to an equations system similar to the one reported in [25]. Finally, recalling (15) and (16), we can write

$$\begin{bmatrix} \mathbf{V}^{(1)}(0^-)\big|_{N_1} \\ \mathbf{V}^{(2)}(0^+)\big|_{N_2} \end{bmatrix} = \mathbf{Z} \begin{bmatrix} \mathbf{I}^{(1)}(0^-)\big|_{N_1} \\ \mathbf{I}^{(2)}(0^+)\big|_{N_2} \end{bmatrix}$$

where we have defined

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{(1,1)}\big|_{N_1 X N_1} & \mathbf{Z}^{(1,2)}\big|_{N_1 X N_2} \\ \\ \mathbf{Z}^{(2,1)}\big|_{N_2 X N_1} & \mathbf{Z}^{(2,2)}\big|_{N_2 X N_2} \end{bmatrix} \quad (17)$$

and the elements of the $Z$ matrix (17) are

$$Z_{n,k}^{(i,j)} = \int_0^a \int_0^b \overline{M}_k^{(j)}(x, y) \cdot \overline{h}_{t,n}^{(i)*}(x, y) g_n^{(i)}(y) dx dy \quad i, j = 1, 2 \quad (18)$$

and they are the same for the two dielectric media of the partially filled waveguide. The S matrix of the discontinuity can be obtained by normalizing (17) with (11)–(12) and with $\mathbf{S} = (\bar{\mathbf{Z}} + \mathbf{I})^{-1}(\bar{\mathbf{Z}} - \mathbf{I})$.

The length $L$ of the partially filled waveguide (Fig. 1) can be taken into account by adding a proper phase shift in some blocks of the $S$-matrix. The $S$ matrix of the discontinuity placed at $z = L$ (inverse with respect to the previous matrix) can be obtained by simply changing input ports with output ports relative to the S matrix of the discontinuity placed at $z = 0$. The entire structure can be analyzed by "cascading" the S matrices of each block, connected with the lines relative to the accessible modes.

## 4. NUMERICAL RESULTS AND APPLICATIONS

Having obtained all the $S$ matrices of the discontinuities, we can "cascade" them to simulate the properties of the overall structure and to validate the proposed approach. Results are shown in Fig. 2 where the reflection coefficient of the fundamental $TM_{y,10}$ mode obtained with this approach and with commercial software (CST) is reported for a WR90 waveguide ($a = 22.86\,\text{mm}$, $b = 10.16\,\text{mm}$) in the $X$-band. The dielectric block dimensions $(a, d, L)$ are $22.86\,\text{mm} \times 5.08\,\text{mm} \times 15\,\text{mm}$ and $\varepsilon_r = 2.2$. The resonance is due to the length of the partially filled waveguide which is about $\lambda_g/2$ at 9.6 GHz. The continuity of the electric and magnetic fields at $z = 0$ is discussed in Fig. 3, where the y-variation of the components intensities at $z = 0^-$ (suffix 1) and $z = 0^+$ (suffix 2) are shown. The x-variation is sin/cos $(\frac{\pi}{a}x)$.

As the reader can see, the $E_y$ and $D_z$ components show singularities at the dielectric wedge, unlike the $E_x$ component and all the magnetic fields, as is well-known [22] (chapter 1.5). In fact, the electric components normal to the dielectric wedge in $z = 0, y = d$ ($E_y$ and $D_z$) exhibit a singularity of the type $r^{\nu-1}$ with $r$ being the distance from the wedge and $\nu$ being the solution of the equation

$$\nu = \frac{2}{\pi}\cos^{-1}\left[\frac{\varepsilon_r - 1}{2(\varepsilon_r + 1)}\right] \quad 0 \le \nu \le 2/3 \tag{19}$$

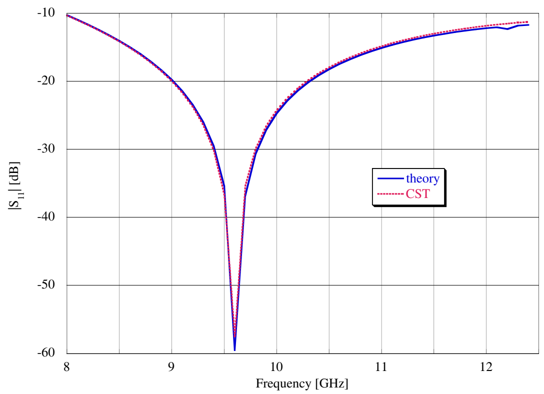


**Figure 2.** $|S_{11}|$ in the $X$-band for the fundamental mode of an empty WR90 waveguide partially filled with a dielectric block ($\varepsilon_r$=2.2) of height 5.08 mm and length 15 mm. Theoretical results obtained with the "natural" definitions (5)–(6) are compared with those obtained with commercial software CST.

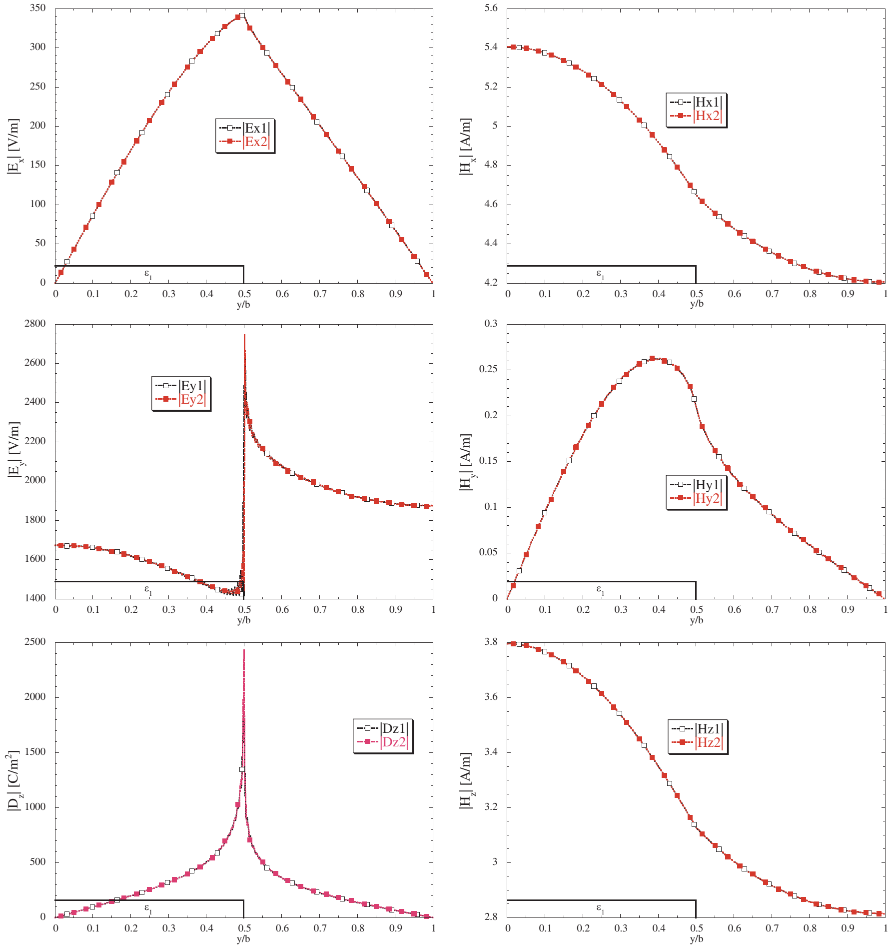


**Figure 3.** Continuity of the electric and magnetic fields at $z = 0$: comparison between components intensities at $z = 0^-$ (suffix 1) and $z = 0^+$ (suffix 2) for $TM_{y,10}$ incidence. The geometry of the partially filled waveguide is shown at the bottom.

For the analyzed discontinuity, $\nu - 1 = -0.12$. The ringing effect in $E_y$ is due to the truncation (about 1000 terms) of the infinite series in (14). All the non-singular fields at $z = 0^-$ and $z = 0^+$ are in very good agreement, with the curves overlapping. Curves relative to singular fields at $z = 0^-$ and $z = 0^+$ are very similar.

An interesting application of the "natural" definitions (5)–(6) regards the analysis of the junction between an empty waveguide and

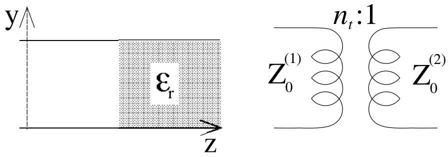


**Figure 4.** The transformer representing the junction between an empty waveguide and a completely filled waveguide.

a completely filled one. It is well-known that for such discontinuity it is not possible to define a $Z/Y$ matrix, but only a voltage-current transmission matrix, or $T$ [22]. In fact, only the fundamental mode $TM_{y,10}$ (which coincides with the $TE_{z,10}$ mode) is coupled at the discontinuity surface. Using the classic definition (1)–(2), with

$$e_{y,10}\left(x,y\right)=\sqrt{\frac{2}{ab}}\sin\left(\frac{\pi}{a}x\right) \tag{20}$$

the same for both the guides (empty and completely filled), the continuity of the tangential EM fields at $z=0$ results in

$$V^{(1)}e^{(1)}_{y,10}\left(x,y\right)=V^{(2)}e^{(2)}_{y,10}\left(x,y\right)\ ,\ I^{(1)}h^{(1)}_{x,10}\left(x,y\right)=I^{(2)}h^{(2)}_{x,10}\left(x,y\right)$$

Hence, the $T$ matrix is a unit matrix and the discontinuity is not "seen" by the transformer representing the $T$ matrix, shown in Fig. 4, having it a transformer ratio $n_t=1$. Only the transformation of $T$ in $S$ leads to the definition of the reflection coefficient as:

$$S'_{11}=\frac{Z_0^{(2)'}-Z_0^{(1)'}}{Z_0^{(1)'}+Z_0^{(2)'}} \tag{21}$$

with

$$Z_0^{(1)'}=\frac{\gamma_{1,0}^{(1)^2}-\left(\frac{\pi}{a}\right)^2}{j\omega\varepsilon_0\gamma_{1,0}^{(1)}}\qquad Z_0^{(2)'}=\frac{\gamma_{1,0}^{(2)^2}-\left(\frac{\pi}{a}\right)^2}{j\omega\varepsilon_0\varepsilon_r\gamma_{1,0}^{(2)}}$$

With the "natural" choice (5)–(6), the $e^{(1)}_{y,10}$ component is equal to (20) for the empty guide, while

$$e^{(2)}_{y,10}\left(x,y\right)=\sqrt{\frac{2}{ab}}\frac{1}{\sqrt{\varepsilon_r}}\sin\left(\frac{\pi}{a}x\right) \tag{22}$$

for the completely filled one. Now, the continuity of the tangential EM fields at $z=0$ leads to

$$V^{(1)}e^{(1)}_{y,10}\left(x,y\right)=V^{(2)}\frac{e^{(2)}_{y,10}\left(x,y\right)}{\sqrt{\varepsilon_r}}\ ,\ I^{(1)}h^{(1)}_{x,10}\left(x,y\right)=I^{(2)}h^{(2)}_{x,10}\left(x,y\right)\sqrt{\varepsilon_r}$$

and the $T$ matrix becomes:

$$[T] = \begin{bmatrix} \frac{1}{\sqrt{\varepsilon_r}} & 0 \\ 0 & \sqrt{\varepsilon_r} \end{bmatrix} \tag{23}$$

Hence, the equivalent circuit of the discontinuity is a transformer with ratio $n_t = \frac{1}{\sqrt{\varepsilon_r}}$, as shown in Fig. 4, which seems more "natural" to describe such discontinuity. In fact, it is not possible to understand why this discontinuity should have a unit transformer as in the previous case, although the reflection coefficient is non-zero. The transformation of T in S and the definition (11) lead to:

$$S_{11} = \frac{\frac{Z_0^{(2)}}{\varepsilon_r} - Z_0^{(1)}}{Z_0^{(1)} + \frac{Z_0^{(2)}}{\varepsilon_r}} \tag{24}$$

which, obviously, equals (21), because $\frac{Z_0^{(2)}}{\varepsilon_r} = Z_0^{(2)'}$.

To sum up, the "natural" choice (5)–(6) seems to better represent the equivalent circuit relative to the junction between an empty waveguide and a completely filled one.

Another application of the "natural" definition of the modal impedances is relative to optical fibers. It is well-known that hybrid modes are used to describe the electromagnetic field. By applying usual definition (1)–(2), transverse components can be expressed as shown in [28] and the modal impedance, defined as the ratio $V_n^{\pm}(z)/I_n^{\pm}(z)$, is:

$$Z_{0,n}(r) = \pm\frac{\beta}{\omega\varepsilon_0}\begin{cases} \frac{1}{\varepsilon_{r,co}} & 0 \leq r \leq a \\ \frac{1}{\varepsilon_{r,cl}} & r > a \end{cases} \tag{25}$$

with $\varepsilon_{r,co}, \varepsilon_{r,cl}$ being the core and the cladding relative dielectric constants. If we apply the "natural" definitions (5)–(6) for the modal fields , it can be easily shown that the "natural" modal impedance is:

$$Z'_{0,n}(r) = \pm\frac{\beta}{\omega\varepsilon_0} = \pm\frac{\eta_0}{\sqrt{\varepsilon_{r,e}}} \tag{26}$$

which corresponds to the modal impedance of an electromagnetic wave propagating in an equivalent medium representing the fiber, characterized by the effective dielectric constant $\varepsilon_{r,e} = \frac{\beta}{k_0}$ [29].

## 5. CONCLUSIONS

In this paper, a "natural" expansion of the EM transverse fields has been proposed to define "natural" modal impedances in dielectric loaded waveguide, which have the same value in the two dielectric media constituting the guide. This expansion was then used to simulate the effect of a block of dielectric in an empty waveguide by "cascading" the S matrices of the existing junctions. Finally, the simple problem of the junction between an empty waveguide and a completely filled waveguide has been discussed, showing that the "natural" modal impedances can better represent the equivalent circuit of this problem.

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