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A CALIBRATION PROCEDURE FOR TWO-PORT VNA WITH THREE MEASUREMENT CHANNELS BASED ON T-MATRIX

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Abstract—A simplified calibration procedure using T-matrix concept is presented for two-port vector network analyzer (VNA) with three measurement channels. Compared with Short-Open-Load-Thru (SOLT) calibration method based on 10-term error model where 10 error terms must be solved and saved at each frequency, the proposed method need define fewer characteristic variables. Moreover, a length-unknown $50\,\Omega$ line can be used instead of a random single-port standard, for example, substituting Load standard with $50\,\Omega$ line in SOLT calibration procedure. Via the simplified calibration procedure, the scattering parameters of a two-port device under test (DUT) can be finally obtained. Experimental verification is carried out, and good agreement is observed.

1. INTRODUCTION

Numerous techniques were studied in the calibration procedure for two-port VNA [1–3]. The choice of calibration techniques depends on the hardware topology of VNA and required measurement accuracy. Two-port VNAs are generally built on four or three measurement channel concepts [1]. In SOLT calibration procedure for three-channel VNA, 10-term error model is often selected for the high precision [4, 5]. But all 10 error terms must be solved and saved at each frequency. The whole calculation procedure is relatively trivial.

In this letter, simple formulas involved in calibration procedure are rederived in concept of T-matrix for two-port VNA. Compared with the traditional SOLT method, fewer characteristic variables need be calculated by the measurement of standards. Moreover, a

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length-unknown 50 Ω line can be used instead of a random single-port standard in SOLT method. A good agreement of the scattering parameters can be observed, and the results prove the correctness of the method proposed in this paper.

2. THEORY

Two-port VNA with three measurement channels is extensively used in the actual engineering due to lower cost as shown in Fig. 1. As the excitation port is changed from one to another, VNA's working state will switch between the forward (port 1) and reverse (port 2) excitation cases in Fig. 2. T-matrixes in Fig. 2 are satisfied as

$$\begin{bmatrix} a_i \\ b_i \end{bmatrix} = [T_{ij}] \begin{bmatrix} a_j \\ b_j \end{bmatrix} \tag{1}$$

From the definition of the T-parameters, we can easily obtain the equation as follow

$$[T_{ij}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} [T_{ji}]^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

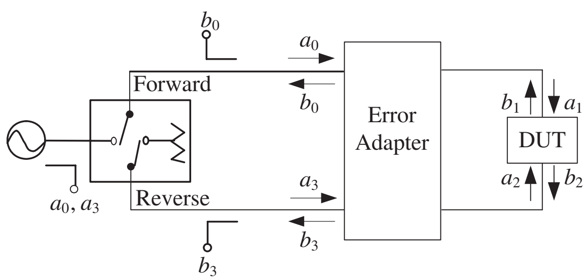

Figure 1. Block diagram of a VNA with three measurement channels.

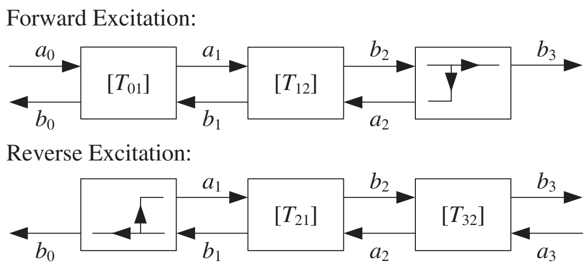

Figure 2. Forward and reverse excitation cases using T-matrix concept.

Before the measurement of DUT with VNA, standards should be measured to determine the relationship between reading values and actual values of the network parameters. In this letter, variable S_{mij}^{X} is defined as the raw measured S-parameter of X then the calculation procedure is given below

1) When port 1 is in turn connected to S ($b_1 = \Gamma_S a_1$) and O ($b_1 = \Gamma_O a_1$) standards, we can get

$$\begin{bmatrix} 1 & 1 \\ S_{m11}^{S} & S_{m11}^{O} \end{bmatrix} = [T_{01}] \begin{bmatrix} x & x' \\ \Gamma_S x & \Gamma_O x' \end{bmatrix} \tag{3}$$

where x and x' are normalized waves. In the following analysis we'll define and substitute a characteristic variable $m = x\prime/x$. The variable m can be easily calculated by the measurement of L ($\Gamma_L = 0$) standard.

$$m = \frac{(\Gamma_L - \Gamma_S)(S_{m11}^{O} - S_{m11}^{L})}{(\Gamma_O - \Gamma_L)(S_{m11}^{L} - S_{m11}^{S})} \tag{4}$$

2) Measuring T ($b_2 = a_1, b_1 = a_2$) standard, we can obtain the following equations in forward excitation case.

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} x & mx \\ \Gamma_S x & \Gamma_O mx \end{bmatrix} r_1 \tag{5}$$

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = S_{m21}^{T} \begin{bmatrix} T_F \\ L_F \end{bmatrix} \tag{6}$$

where the vector $r_1 = \begin{bmatrix} 1 & 1 \\ S_{m11}^{S} & S_{m11}^{O} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ S_{m11}^{T} \end{bmatrix}$. Variables T_F and L_F are inherent characteristic coefficients and expressed from Equations (5) and (6) by

$$\begin{bmatrix} T_F \\ L_F \end{bmatrix} = \frac{1}{S_{m21}^{T}} \begin{bmatrix} x & mx \\ \Gamma_S x & \Gamma_O mx \end{bmatrix} r_1 \tag{7}$$

3) Measuring DUT and connecting the source to port 1, we get the equation as

$$\begin{bmatrix} x & mx \\ \Gamma_S x & \Gamma_O mx \end{bmatrix} R_1 = [T_{12}] \frac{S_{m21}^{\text{DUT}}}{S_{m21}^{T}} \begin{bmatrix} x & mx \\ \Gamma_S x & \Gamma_O mx \end{bmatrix} r_1 \tag{8}$$

where the vector $R_1 = \begin{bmatrix} 1 & 1 \\ S_{m11}^{S} & S_{m11}^{O} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ S_{m11}^{\text{DUT}} \end{bmatrix}$. The similar equation can be obtained in reverse excitation case and finally from Equations (2) and (8) we can conclude the formula as

$$\begin{cases} \begin{bmatrix} 1 & m \\ \Gamma_s & \Gamma_o m \end{bmatrix} R_1 = [T_{12}] \dfrac{S_{m21}^{\text{DUT}}}{S_{m21}^{T}} \begin{bmatrix} 1 & m \\ \Gamma_s & \Gamma_o m \end{bmatrix} r_1 \\ \begin{bmatrix} \Gamma_s & \Gamma_o n \\ 1 & n \end{bmatrix} r_2 = [T_{12}] \dfrac{S_{m12}^{T}}{S_{m12}^{\text{DUT}}} \begin{bmatrix} \Gamma_s & \Gamma_o n \\ 1 & n \end{bmatrix} R_2 \end{cases} \tag{9}$$

where n similar to m is calculated in reverse excitation case. Finally, the T-matrix of DUT can be solved by Equation (9).

$$[T_{12}] = \left\{ \begin{bmatrix} 1 & m \\ \Gamma_s & \Gamma_o m \end{bmatrix} R_1 \begin{bmatrix} \Gamma_s & \Gamma_o n \\ 1 & n \end{bmatrix} r_2 \right\} \left\{ \frac{S_{m21}^{\mathrm{DUT}}}{S_{m21}^{T}} \begin{bmatrix} 1 & m \\ \Gamma_s & \Gamma_o m \end{bmatrix} r_1 \frac{S_{m12}^{T}}{S_{m12}^{\mathrm{DUT}}} \begin{bmatrix} \Gamma_s & \Gamma_o n \\ 1 & n \end{bmatrix} R_2 \right\}^{-1} \tag{10}$$

The actual scattering parameters of DUT can be obtained by the above T-matrix. It is obvious that variables m and n need be calculated for formula (10) at each frequency. Compared with the traditional SOLT calibration based on 10-term error model, in which 10 error terms must be calculated, the proposed SOLT calibration in concept of T-matrix requires only two variables m and n computed. Not only is the number of the unknown variables reduced, but also the formula is simplified. So we called the proposed SOLT calibration based on T-matrix concept as simplified SOLT calibration.

In this letter we choose the $50\,\Omega$ line instead of L standard to solve m and n. When $50\,\Omega$ line is connected between port 1 and port 2, final equation similar to Equation (8) can be depicted below.

$$\begin{bmatrix} x & mx \\ \Gamma_s x & \Gamma_o mx \end{bmatrix} r_1' = \begin{bmatrix} e & \\ & e^{-1} \end{bmatrix} \frac{S_{m21}^{\mathrm{LINE}}}{S_{m21}^{T}} \begin{bmatrix} x & mx \\ \Gamma_s x & \Gamma_o mx \end{bmatrix} r_1 \tag{11}$$

where the vector $r' = \begin{bmatrix} 1 & 1 \\ S_{m11}^{S} & S_{m11}^{O} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ S_{m11}^{\mathrm{LINE}} \end{bmatrix}$ and variable e is the T-parameter of an ideal lossless transmission line. By rearrangement of Equation (11), we can obtain the following equation as

$$\begin{bmatrix} a - be & c - de \\ \Gamma_s ae - \Gamma_s b & \Gamma_o ce - \Gamma_o d \end{bmatrix} \begin{bmatrix} 1 \\ m \end{bmatrix} = 0 \tag{12}$$

where we define the variables as $a = S_{m21}^{T} r_{11}'$, $b = S_{m21}^{\mathrm{LINE}} r_{11}$, $c = S_{m21}^{T} r_{12}'$ and $d = S_{m21}^{\mathrm{LINE}} r_{12}$. Since the solution of Equation (12) is non-zero, we have

$$\begin{vmatrix} a - be & c - de \\ \Gamma_s ae - \Gamma_s b & \Gamma_o ce - \Gamma_o d \end{vmatrix} = 0 \tag{13}$$

Solving Equation (13), the solution of T-parameter e is computed.

$$e = \frac{\pm\sqrt{(\Gamma_o - \Gamma_s)^2 (ac + bd)^2 - 4(\Gamma_s ad - \Gamma_o bc)(\Gamma_s bc - \Gamma_o ad)}}{2(\Gamma_s ad - \Gamma_o bc)} \tag{14}$$

Because the electric length of microstrip line is less than a half wave length in measurement frequency, the angle of e is between 0 and π. A simple rough knowledge of the unknown phase shift ($\leqslant \pi$) allows us to solve the e sign ambiguity [6]. The characteristic variable m is obtained.

$$m = -\frac{a - be}{c - de} \tag{15}$$

By the measurements of SO standards at port 2 and $50\,\Omega$ line connected between port 1 and port 2, the solution of variable n is similar to variable m. Variables m and n are inherent parameters of the measurement system which is composed of the 2-port VNA and some calibration standards. Finally, variables m and n can be solved. So the characteristic variables m and n can be calculated by the measurement of load standard or $50\,\Omega$ line. This proposed calibration method using short, open, thru and $50\,\Omega$ line is called as SOT-Line calibration.

3. EXPERIMENTAL RESULTS

To verify the correction of the calibration algorithm, an unknown two-port DUT is measured by both simplified SOLT and SOT-Line methods. In SOT-Line method, we choose a $50\,\Omega$ microstrip line instead of L standard. The $50\,\Omega$ microstrip line with conductor width of 1.5 mm and length of 3.95 cm is fabricated on Rogers 5880 substrate (ε_r 2.2, dielectric height 0.508 mm, 1/2 oz copper thickness 0.017 mm). First, the characteristic variables m and n are calculated respectively by the measurement of SOLT standards and SOT-Line standards using Agilent N5230A VNA in Figs. 3 and 4. Finally, S-parameters of DUT are obtained from Equation (10) and shown in Figs. 5–6. In this method, $50\,\Omega$ microstrip line is common and shown in Fig. 3. These figures can prove that the presented method is effective and practicable.

The difference between simplified SOLT and SOT-$50\,\Omega$ Line results from not ideal propagation character of line. Finally, a good agreement of the corrected S-parameters by simplified SOLT algorithm and traditional SOLT algorithm is observed and shows the precision of the calibration algorithm. This method can be still used by multiline SOT-Line calibration if the measurement frequency is above 1 GHz. Transmission lines with different lengths can be chosen for broadband measurement [7]. In this letter, the scattering parameters associated with the $50\,\Omega$ line are not ideal due to relatively low machining accuracy. Such a nonideal $50\,\Omega$ line then leads to the deviations of m and n, and consequently, the deviations of calibrated S-parameters of DUT. In the commercial VNAs, the high-precision transmission line can be applied for high accuracy.

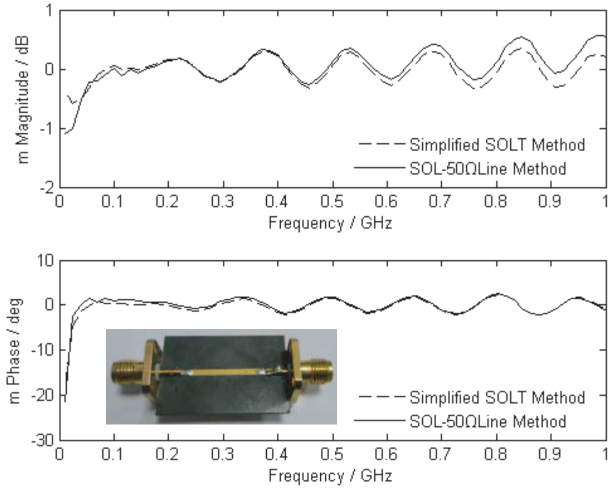

Figure 3. Comparison of characteristic variable m.

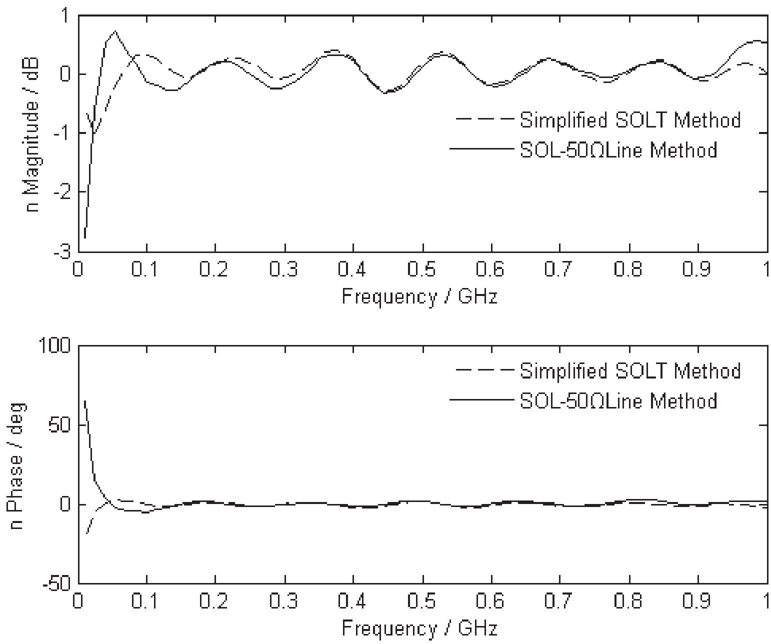

Figure 4. Comparison of characteristic variable n.

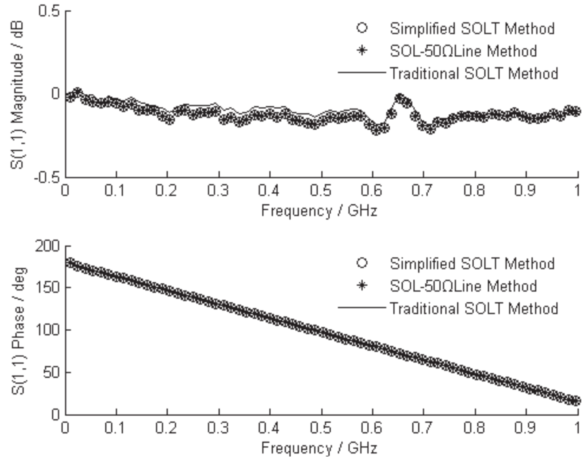

Figure 5. Comparison of S_{11}.

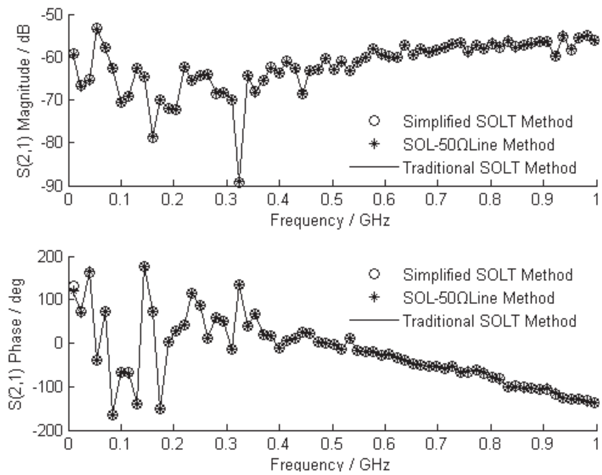

Figure 6. Comparison of S_{21}.

4. CONCLUSION

In this paper, a simplified calibration procedure using T-matrix concept is applied to a two-port VNA with three measurement channels. Simple formulas involved in calibration procedure are rederived for the two-port VNA. Only two characteristic variables need be calculated by the measurement of standards. Moreover, a length-unknown $50\,\Omega$ line can be used instead of a random single-port standard in simplified SOLT method. Detailed calibration steps are given, and the final formula of T-parameters is derived. The accurate scattering parameters of a two-port DUT are obtained by the method proposed in this paper. The result of experimental verification attests the precision of the calibration algorithm.

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