Progress In Electromagnetics Research B, Vol. 38, 367–385, 2012

EFFICIENT CIRCULAR ARRAY SYNTHESIS WITH A MEMETIC DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract—In this article, we introduce an improved optimization based technique for the synthesis of circular antenna array. The main objective is to achieve minimum side lobe levels, maximum directivity and null control for the non-uniform, planar circular antenna array. The design procedure utilizes an improved variant of a prominent and efficient metaheuristic algorithm of current interest, namely the Differential Evolution (DE). An efficient classical local search technique called Solis Wet's algorithm is incorporated with the competitive Differential Evolution. While the competitive DE is used for the global exploration, Solis Wet's algorithm is employed for local search. Combining the capability of both techniques the hybrid algorithm exhibits improved performance for circular array design problem. Three examples of circular array design problems are considered to illustrate the effectiveness of the hybrid algorithm cDESW (Competiteve Differential Evolution with Solis Wet's technique). The design results obtained using cDESW has comfortably outperformed the results obtained by other state-of-the-art metaheuristics like CLPSO, JADE.

Received 18 November 2011, Accepted 5 February 2012, Scheduled 10 February 2012
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1. INTRODUCTION

For the purpose of long distance communication antennas having very directive radiation characteristics are required. A single antenna may be unable to meet this requirement. The solution to this problem is achieved using antenna array, which can be formed by combining many individual antenna elements in certain electrical and geometrical configurations. Antenna arrays have been widely used in diverse applications including radar, sonar, radios, and third generation wireless communication systems [1–3].

An inefficient array design may result in wastage of power proving fatal for power limited wireless devices, whereas an efficient antenna array is very much useful in high power transmission, reduced power consumption and enhanced spectral efficiency. A very directive pattern is obtained if fields from the array elements add constructively in some desired directions and add destructively and cancel each other in the remaining space.

While designing an antenna array the primary objective is to determine the positions of array elements that jointly produce a radiation pattern that resembles the desired pattern as nearly as possible [4]. As the number of applications grow in wireless communications, the demand for new antenna design increases. Alongside, various features and performance metrics are desired like miniaturization, null control, pattern control, multifunction capabilities, etc.. In recent past the researchers working on electromagnetic optimization problems paid great attention on the design of uniformly and non-uniformly spaced linear arrays. Nowadays metaheuristic algorithms are used to solve antenna design problems. The reason for the use of these metaheuristic algorithms based on simulations of some natural phenomena lies in the computational drawbacks of existing numerical methods. Also the classical derivative-based optimization techniques are prone to getting trapped in local optima and are strongly sensitive to initialization, for complex optimization problems the gradient based methods fail to obtain significant solution. Due to these inherent shortcomings of the classical methods, many modern metaheuristics were tried to achieve optimized Side Lobe Level (SLL) and null control from the designed arrays [5–10]. The researches are now going on the design of antenna arrays with other geometrical shapes that help to obtain desired radiation pattern.

Some features of circular antenna array have made it advantageous over other configurations of antenna arrays. It has all azimuth scan capability and the array pattern is flexible as observed in [11–14]. For the advantages provided by the circular antenna array it has found wide application in sonar, radar, mobile, and commercial satellite communication systems [11, 13, 15, 16] and design of circular antenna array is becoming important gradually. The work of Panduro et al. [17] is the first metaheuristic approach towards the design of circular arrays. They applied the real-coded Genetic Algorithm (GA) for designing circular arrays with maximal side lobe level reduction coupled with the constraint of a fixed beam width. Later Particle swarm optimization was applied by Shihab et al. in [18] for the design of circular array. Panduro et al. [19] compared three powerful population-based optimization algorithms — PSO, GA, and Differential Evolution (DE) on the design problem of scanned circular arrays. Gurel and Ergul applied GA in [20] to design a circular array where each element was log-periodic antenna. Some other applications of metaheuristics for the design of circular antenna array is found in [21–23] that involve the use of PSO, invasive weed optimization (IWO) and Biogeography based optimization (BBO). In [19] a design problem is considered with number of antenna elements equal to 12 and for a uniform separation of $d = 0.5\lambda$, optimizing excitation current amplitudes and phase perturbations, with an objective of studying the behavior of array factor for the scanning range of 0° to 360° in angular steps of 30°. Huang et al. introduced a time-modulated circular array using uniform amplitude excitation for obtaining ultra low side lobe level in [24].

Amongst all Evolutionary Algorithms (EAs) described in various articles, Differential Evolution (DE) [25–27] has emerged as one of the most powerful tools for solving the real world optimization problems. Differential Evolution has been successfully applied to solve problems in electromagnetics as found in literature [28–30]. In this context we present here a new powerful variant of DE denoted by cDESW, for designing non-uniform circular arrays with optimized performance with respect to SLL, directivity, and null control in a scanning range $[0^\circ, 360^\circ]$. In the proposed algorithm, a competitive variant of DE is used for global exploration and the classic Solis Wet's algorithm [31] is used as the local search process. For the DE algorithm, we have developed a hybrid mutation strategy by hybridizing a modified "DE/current-to-best/2" mutation strategy with a modified "DE/rand/1"mutation strategy. We have discussed the mutation strategy later in sufficient details.

The rest of the paper is organized in the following way. In Section 2, we have given a brief overview of the classical DE algorithm. Section 3 provides a comprehensive overview of the proposed cDESW algorithm and also describes the modifications used over classical DE to improve the efficiency. A formulation of the array pattern synthesis as an optimization task has been discussed in Section 4. Simulation settings have been discussed and the results have been presented in Section 5. Section 6 finally concludes the paper and unfolds a few important future research issues.

2. CLASSICAL DE

An iteration of the classical DE algorithm consists of the four basic steps — initialization of a population of search variable vectors, mutation, crossover or recombination, and finally selection. The last three steps are repeated generation after generation until a stopping criterion is satisfied.

2.1. Initialization of the Population

If DE searches for the global optima within the continuous search space of dimensionality D then it begins with an initial population of target vectors $\vec{X}_i = \{x_i^1, x_i^2, \ldots, x_i^D\}$ where $i = 1, 2, 3 \ldots NP$ (NP is the population size). The individuals of the initial population are randomly generated from a uniform distribution within the search space which has maximum & minimum bounds as follows: $\vec{X}_{\max} = \{x_{\max}^1, x_{\max}^2, \ldots, x_{\max}^D\}$ and $\vec{X}_{\min} = \{x_{\min}^1, x_{\min}^2, \ldots x_{\min}^D\}$. The jth component of the ith individual is initialized as follows:

$$x_{i,0}^j = x_{\min}^j + \mathrm{rand}_i^j(0,1)(x_{\max}^j - x_{\min}^j); \quad j \in [1, D] \tag{1}$$

Here rand_i^j (0, 1) is a uniformly distributed random number in (0, 1) and it is instantiated independently for the j-th component of the i-th individual.

2.2. Mutation

After the initialization, DE evolves the population by three operations: mutation, crossover & selection. In each generation DE creates a *mutant vector* (also known as *donor vector*) corresponding to each individual or target vector of the current population. Three very common mutation strategies are described as follows:

a) DE/rand/1: $$\vec{V}_{i,G} = \vec{X}_{r1,G} + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r3,G}\right) \tag{2}$$

b) DE/best/1: $$\vec{V}_{i,G} = \vec{X}_{\mathrm{best},G} + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r2,G}\right) \tag{3}$$

c) DE/current-to-best/1:
$$\vec{V}_{i,G} = \vec{X}_{i,G} + F_{\mathrm{best}} \cdot \left(\vec{X}_{\mathrm{best},G} - \vec{X}_{i,G}\right) + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r2,G}\right) \tag{4}$$

The indices $r1$, $r2$ and $r3$ are mutually exclusive random integers in the range $[1, NP]$, they are also different from i. These indices are generated once for each mutant vector. $\vec{X}_{\text{best},G}$ is the target vector which has the best fitness value in the population at generation G. The scaling factor F and F_{best} control the amplification of the corresponding differentiation vector.

2.3. Crossover

In this operation, the donor vector mixes its components with the target vector $\vec{X}_{i,G}$ under this operation to form the trial vector $\vec{U}_{i,G} = \{u^1_{i,G}, u^2_{i,G}, \ldots, u^D_{i,G}\}$. Here we shall briefly outline the binomial crossover scheme that has been used in the proposed algorithm. Under this scheme the trial vector is created as follows:

$$u^j_{i,G} = \begin{cases} v^j_{i,G} & \text{if} \quad \text{rand}(0,1) \leq CR \quad \text{or} \quad j = j_{\text{rand}} \\ x^j_{i,G} & \text{otherwise} \end{cases} \tag{5}$$

where CR is a user-specified parameter (*Crossover Rate*) in the range $[0, 1)$ and $j_{\text{rand}} \in [1, 2, \ldots, D]$ is a randomly chosen index which ensures that the trial vector $\vec{U}_{i,G}$ will differ from its corresponding target vector $\vec{X}_{i,G}$ by at least one component.

2.4. Selection

To keep the population size constant over subsequent generations, the next step of the algorithm calls for *selection* to determine whether the target or the trial vector survives to the next generation i.e., at $G = G + 1$. For maximization problem, if the objective function value of the trial vector is not less than that of the corresponding target vector, then the trial vector is selected for the next generation; and if it is not so, then the trial vector is selected for the next generation. Obviously, for minimization problem the condition for selection is just the opposite. The selection operation works as follows (for maximization problem):

$$\vec{X}_{i,G+1} = \begin{cases} \vec{U}_{i,G} & \text{if } f(\vec{U}_{i,G}) \geq f(\vec{X}_{i,G}) \\ \vec{X}_{i,G} & \text{otherwise} \end{cases} \tag{6}$$

where $f(\cdot)$ is the objective function to be optimized.

3. PROPOSED ALGORITHM

In this section we have discussed the proposed cDESW algorithm in sufficient details. In this multi-population based algorithm, we have developed a competitive variant of DE which is accompanied by a local search method. Furthermore, this algorithm employs a hybrid mutation strategy for DE to enhance the searching ability and to circumvent stagnation of the population at any local optima. The combination of global exploration (modified DE) and local exploitation (Solis and Wet's technique) makes the algorithm better than the classical DE.

3.1. The Modified DE Algorithm

3.1.1. Competitive Variant of DE

For a heuristic search process, it is useful to exploit the neighborhood because it is similar to information exchange between the neighbors which leads to better solutions. So, here we have incorporated a competition between the neighbors. Also the success rate is measured at each generation which helps in determining the new individual generation process for the next generation. Actually, depending on the success rate, either the current individual or its nearest neighbor is used for mutant vector formation. If the corresponding trial vector is chosen for next generation then the corresponding success rate is increased by 1 and if it is not chosen then the corresponding success rate is decreased by 1. If both the success rates for current individual and its nearest neighbor are equal then the current individual is used. At the time of initialization, all the success rates were set to 0. Using this method, we can get far better solutions with less function evaluations. Also the population does not converge to any local minima too quickly because we set the competition with nearest neighbor. Here, the nearest neighbor is selected on the basis of Euclidean distance between the current individual and the other individuals in the corresponding subpopulation.

3.1.2. Hybrid Mutation Strategy

As mentioned earlier, depending on the success rate, either the current individual or the nearest neighbor of the current individual is used for the mutant vector generation process. Let the chosen individual be $\vec{X}_{c,G}$. In DE, greedy strategies like DE/current-to-best/n and DE/best/n benefit from their fast convergence property by guiding the search process with the best solution so far discovered, thereby converging faster to that point. But, as a result of such fast convergence tendency, in many cases, the population may lose its diversity and global exploration abilities within a relatively small number of generations, thereafter getting trapped to some locally optimal point in the search space. Taking into consideration these facts and to overcome the limitations of fast but less reliable convergence, we have developed a hybrid mutation strategy.

For constructing the final mutant vector, two mutant vectors generated by two different mutation strategies are combined with a weight factor ω. This way we developed a hybrid mutation strategy to prevent the algorithm from converging too quickly and at the same time exploring the whole search space to produce high quality results.

For the first mutation strategy, we have used a modified "DE/current-to-best/2" mutation strategy. For this modified mutation strategy, best individual of each subpopulation is stored in a memory archive; this memory archive is updated at each generation to store the new best individuals and delete the previous best individuals. During the mutation process, the nearest memory individual is used instead of the global best individual. The mutation process can be expressed as follows:

$$\vec{V}_{\text{mut},1} = \vec{X}_{c,G} + F_{\text{best}} \cdot (\vec{X}_{m,G} - \vec{X}_{c,G}) + F \cdot (\vec{X}_{r1,G} - \vec{X}_{r2,G}) \qquad (7)$$

where $\vec{X}_{m,G}$ is the nearest best individual as mentioned above. $\vec{X}_{r1,G}$ and $\vec{X}_{r2,G}$ are two distinct vectors randomly chosen from the subpopulation. For the second mutation strategy, we have used "DE/current/1" mutation strategy. The mutation process can be expressed as follows:

$$\vec{V}_{\text{mut},2} = \vec{X}_{c,G} + F \cdot (\vec{X}_{r1,G} - \vec{X}_{r2,G}) \qquad (8)$$

where $\vec{X}_{r1,G}$ and $\vec{X}_{r2,G}$ are two distinct vectors randomly chosen from the subpopulation independently of first mutation process. Now, the final mutant vector is a weighted sum of two above mentioned mutant vectors. If the weight factor for $\vec{V}_{\text{mut},1}$ is ω then the final mutant vector is

$$\vec{V}_{\text{mut}} = \omega \cdot \vec{V}_{\text{mut},1} + (1 - \omega) \cdot \vec{V}_{\text{mut},2} \qquad (9)$$

3.2. Local Search

3.2.1. Local Search Method

As mentioned earlier, we have used Solis and Wet's algorithm as the LS method. The algorithm is a randomized hill climber. It starts from a initial point i. Then a deviation d is chosen from a normal distribution whose mean is 0 and variance is a parameter σ which is adaptive in nature. If $i + d$ is better or $i - d$ is better than the current individual in terms of fitness value then the current individual is replaced by the better one and a success is recorded. Otherwise a failure is recorded. After some consecutive success (we set this value to 1) σ is increased (we set this increment as 1.5 times) to progress quicker. Similarly, after some consecutive failure (we set this value to 1) σ is decreased (we set this decrement as 1/3 times) to focus the search. Also a bias term is included to guide the search in the direction which gives success. As σ is adaptive, the step size of the search method is also adaptive which makes the algorithm very effective in exploiting the nearby region of the solution.

In our algorithm, the solutions got after executing the local search are also recorded. If the success exceeds the failure by at least S (we set it to 20) then the solution is recorded as a good solution and the corresponding σ value is also recoded. If the success does not exceed the failure by at least S then the solution is recorded as bad solution. When the local search algorithm runs again, the chosen individuals are compared with the previous good and bad solutions. If the current individual is a previous good solution then the search process starts with the previous value of σ. If the current individual is a previous bad solution then local search is not applied to the individual. We have incorporated this memory system to avoid unnecessary function evaluations which in turn increases the efficiency of the algorithm. In our algorithm, during the local search, the whole local search method is applied over the best individuals of all subpopulations.

4. FORMULATION OF THE DESIGN PROBLEM

We consider N antenna elements that are spaced on a circle of radius r in the x-y plane. The antenna elements constitute a circular antenna

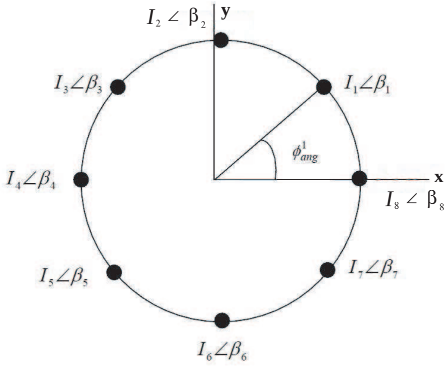

Figure 1. Geometry of circular antenna array.

array, which is shown in Fig. 1. The formulation of the array factor requires the following

- I_n is the normalized amplitude excitation.
- β_n is the phase excitation of the nth element.
- ϕ is the angle of incidence of the plane wave.

When the wavefront of incident plane wave is perpendicular to x-y plane (i.e., $\theta = 90°$), the array factor of the circular array can be written by

$$\mathrm{AF}(\phi)=\sum_{n=1}^{N} I_n \exp\left[j\left\{kr\left(\cos\left(\phi-\phi_{\mathrm{ang}}^n\right)-\cos\left(\phi_0-\phi_{\mathrm{ang}}^n\right)\right)+\beta_n\right\}\right] \quad (10)$$

where, $\phi_{\mathrm{ang}}^n = 2\pi(n-1)/N$ is the angular position of the nth element on the x-y plane, $kr = Nd$ where k is the wave number, d is the angular spacing between elements and r is the radius of the circle defined by the circular array. ϕ_0 is the direction of maximum radiation.

The main objective is to suppress side-lobe level, maximize directivity and achieve null control. To fulfill the objective the amplitude and phase excitations of the antenna elements are varied. The range of variation of normalized amplitude excitation is [0, 1]. The range of phase excitation is $[-180°, 180°]$.

A symmetrical excitation of the circular antenna array is considered. The following relations hold for the array elements.

$$\begin{aligned} I_{n/2+1}\angle\beta_{n/2+1} &= \mathrm{conj}(I_1\angle\beta_1), \\ I_{n/2+2}\angle\beta_{n/2+2} &= \mathrm{conj}(I_1\angle\beta_1), \ldots \\ I_n\angle\beta_n &= \mathrm{conj}(I_{n/2}\angle\beta_{n/2}) \end{aligned}$$

The objective function is given by,

$$\begin{aligned} F \;=\; & \left|\mathrm{AF}\left(\phi_{\mathrm{sll}}, \vec{I}, \vec{\beta}, \phi_0\right)\right| / \left|\mathrm{AF}\left(\phi_{\max}, \vec{I}, \vec{\beta}, \phi_0\right)\right| + 1/\mathrm{DIR}\left(\phi_0, \vec{I}, \vec{\beta}\right) \\ & + \left|\phi_0-\phi_{\mathrm{des}}\right| + \sum_{k=1}^{nl} \mathrm{AF}(\phi_k, \vec{I}, \vec{\beta}, \phi_0) \end{aligned} \quad (11)$$

The goal can be achieved by minimizing the objective function F. The four components of the objective function perform different tasks. The first component attempts to minimize the side lobe level. ϕ_{sll} denotes the angle at which maximum side lobe level is obtained. As the objective function is minimized the side lobe level is minimized also. The job of the second component is to maximize the directivity of the array pattern. Directivity is a useful figure of merit when different array patterns are compared. $\mathrm{DIR}\,(\phi_0, \vec{I}, \vec{\beta})$ denotes the directivity in the direction indicated by ϕ_0. As directivity increases the objective function is minimized. The desired maxima is denoted by ϕ_{des}. The third component strives to drive the maxima of the array pattern close to ϕ_{des}. The fourth component deals with the nulls. If proper null control is not achieved then the objective function is penalized due to the fourth component. nl is the number of null control directions and ϕ_k is the kth null control direction.

5. SIMULATIONS RESULT

In this paper we have considered three design problems. The optimal array pattern is found using cDESW, an improved version of Differential Evolution. The results obtained are compared to the result obtained by other powerful Evolutionary Algorithms like JADE [26] and CLPSO [32]. The comparison clearly reveals that our algorithm produces better results than JADE or CLPSO. For our algorithm, NP was kept fixed at 60 throughout the search process. We divided the whole population into 6 subpopulations, each containing 10 individuals. Weight factor ω for the first mutation scheme in hybrid mutation strategy was set to 0.7. The scaling factor F of DE algorithm was generated in each iteration for each individual, randomly between 0.3 and 0.7. The crossover probability CR was set to 0.9.

5.1. Case 1. 12 Element Array with No Null Control

In this instance we consider a 12 element array. We need to find the optimal pattern with desired maximum at 180°. No null control is required. We obtain the current excitation and phase of the antenna elements that produce the desired pattern.

Table 1 shows the SLL and directivity (in dB) for JADE, CLPSO, and cDESW for the median of 25 independent runs of each algorithm on the problem of case 1. Table 2 presents mean objective function values and corresponding standard deviations over 25 independent trials of the algorithms on the design problem considered in case 1.

The obtained array pattern is shown in Fig. 2. We can see

Table 1. Results for median of 25 trials (case 1).

Algorithm	SLL (dB)	Directivity (dB)
CLPSO	−17.6751	10.0341
JADE	−19.5412	10.9585
cDESW	**−22.2907**	**11.3350**

Table 2. Mean objective function values and standard deviations over 25 independent trials for case 1.

Algorithm	Mean Obj. function	Std. Deviation
CLPSO	0.2241	0.0432
JADE	0.2076	0.0311
cDESW	**0.1672**	**0.0203**

Table 3. Current amplitude excitation and phase excitation of elements (case 1).

I_1	I_2	I_3	I_4	I_5	I_6
0.9622	0.4127	0.4036	0.1541	0.3970	0.5509

β_1	β_2	β_3	β_4	β_5	β_6
−30.50	26.41	−90.30	−39.02	86.52	−18.27

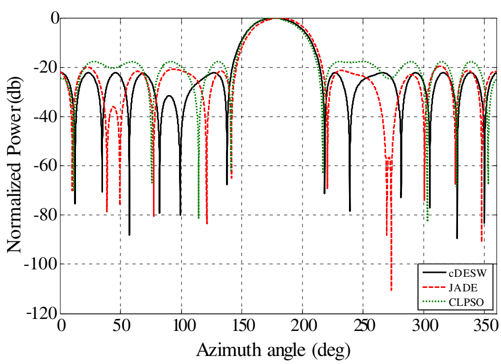

Figure 2. Array pattern plot for 12 element array without null.

from the results that cDESW finds an array pattern with the best minimized side-lobes and greatest directivity. The figure also confirms that cDESW has outperformed the competitor algorithms.

The convergence graphs of the three algorithms for 12 element array with no null control case study are represented in Fig. 3. "FEs" means the number function evaluations. The amplitude excitation and phase excitation of the first six elements are listed in Table 3. As the circular array is symmetric we can find the amplitude excitation and phase excitation of the other elements from these by calculating the complex conjugate values.

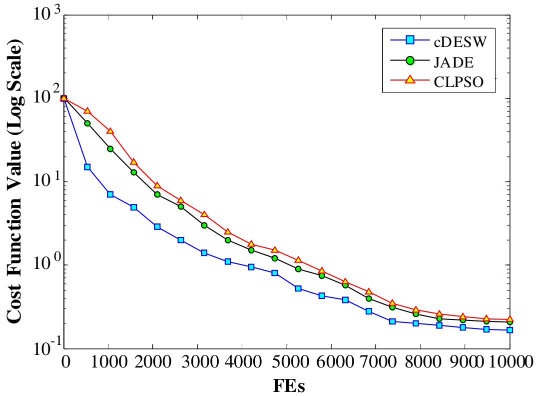

Figure 3. Convergence graph for case 1.

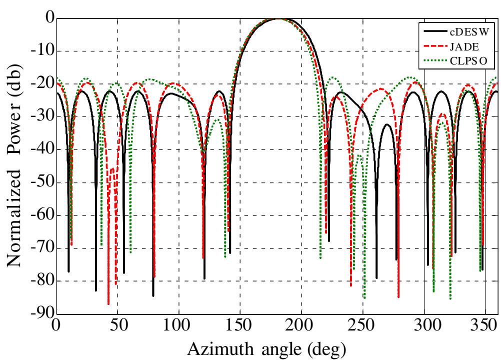

Figure 4. Array pattern plot for 12 element array with null at 120°.

5.2. Case 2. 12 Element Array with Null at 120°

In this instance we consider a 12 element array. We need to find the optimal pattern with desired maximum at 180°. Null is present at 120°. We obtain the current excitation and phase of the antenna elements that produce the desired pattern.

Tables 4 and 5 are similar in spirit to Tables 1 and 2 except for the fact that now the values are reported for the design problem of case 2.

The obtained array pattern is shown in Fig. 4. We can see from the results that cDESW finds an array pattern with the best minimized side-lobes and greatest directivity. The figure also confirms that cDESW has outperformed the competitor algorithms. cDESW suppresses the null at 120° to -79.64 dB which is better than the suppressions obtained in competitor algorithms.

Table 4. Results for median of 25 trials (case 2).

Algorithm	SLL (dB)	Directivity (dB)	AF at 120° (dB)
CLPSO	−17.9262	9.9843	−70.0000
JADE	−19.5453	10.7466	−74.3922
cDESW	**−22.2563**	**11.0971**	**−79.6419**

Table 5. Mean objective function values and standard deviations over 25 independent trials for case 2.

Algorithm	Mean Obj. function	Std. Deviation
PSO	0.2247	0.0489
DE	0.2083	0.0322
cDESW	**0.1674**	**0.0211**

Table 6. Current amplitude excitation and phase excitation of elements (case 2).

I_1	I_2	I_3	I_4	I_5	I_6
0.9744	0.5572	0.4017	0.1517	0.4097	0.4204

β_1	β_2	β_3	β_4	β_5	β_6
−30.56	18.44	−86.38	38.45	89.85	−26.28

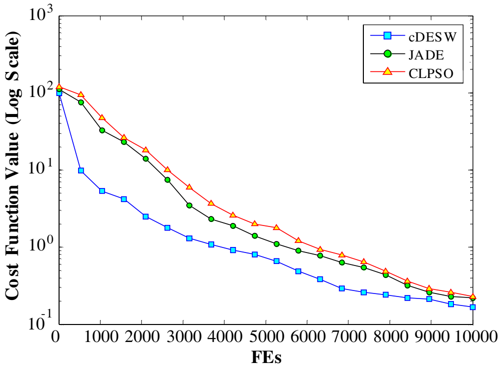

Figure 5. Convergence graph for case 2.

The current amplitude excitation and phase excitation of the first six elements are listed in Table 6. As the circular array is symmetric we can find the current amplitude excitation and phase excitation of the other elements from these as mentioned earlier.

The convergence graphs of the three algorithms for 12 element array with null at 120° case study are given in Fig. 5.

5.3. Case 3. 24 Element Array with No Null Control

In this instance we consider a 24 element array. We need to find the optimal pattern with desired maximum at 180°. No null control is required. We obtain the current excitation and phase of the antenna elements that produce the desired pattern.

Tables 7 and 8 correspond to Tables 1 and 2 and contain values for the design problem of case 3.

The obtained array pattern is shown in Fig. 6. We can see

Table 7. Results for median of 25 trials (case 3).

Algorithm	SLL (dB)	Directivity (dB)
CLPSO	−16.2118	12.9819
JADE	−17.3565	13.2127
cDESW	**−19.8348**	**13.6155**

Table 8. Mean objective function values and standard deviations over 25 independent trials for case 3.

Algorithm	Mean Obj. function	Std. Deviation
CLPSO	0.4012	0.0683
JADE	0.3785	0.0654
cDESW	**0.2246**	**0.0453**

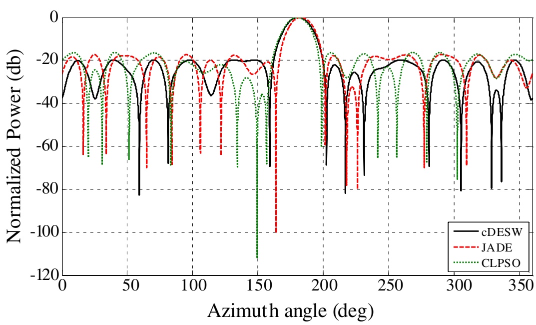

Figure 6. Array pattern plot for 24 element array without null.

Table 9. Current amplitude excitation and phase excitation of elements (case 3).

I_1	I_2	I_3	I_4	I_5	I_6
0.9996	0.6381	0.1080	0.3891	0.0002	0.3399
β_1	β_2	β_3	β_4	β_5	β_6
9.6527	15.40	71.84	−68.25	87.89	47.06

I_7	I_8	I_9	I_{10}	I_{11}	I_{12}
0.1095	0.2934	0.1014	0.4145	0.0000	0.5600
β_7	β_8	β_9	β_{10}	β_{11}	β_{12}
24.05	−53.48	−179.9	60.14	14.38	−12.50

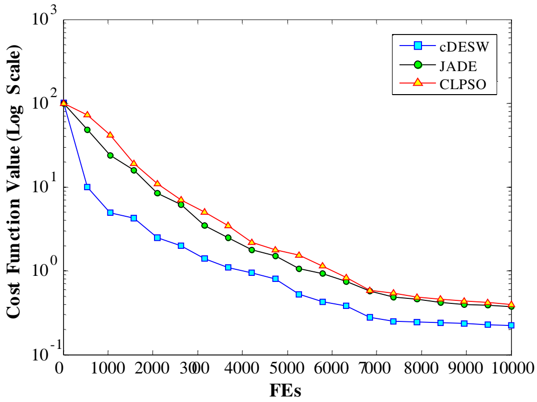

Figure 7. Convergence graph for Case 3.

from the results that cDESW finds an array pattern with the best minimized side-lobes and greatest directivity. The figure also confirms that cDESW has outperformed the competitor algorithms.

The convergence graphs of the three algorithms for 24 element array with no null control case study are given in Fig. 7. The current amplitude excitation and phase excitation of the first twelve elements are listed in Table 9. As the circular array is symmetric we can find the current amplitude excitation and phase excitation of the other elements from these values as mentioned earlier.

To give an idea of runtime of the simulation process, we have presented the comparison of average CPU time required per run in cDESW, JADE and CLPSO based design methods in Table 10. We performed the simulation in the following experimental environment:

Table 10. Comparison of average CPU time required per run.

Problem	Average CPU Time Required Per Run (in Seconds)		
	cDESW	**JADE**	**CLPSO**
Case 1	**2.12**	2.49	2.88
Case 2	**2.29**	2.71	3.02
Case 3	**3.98**	4.73	5.65

- CPU: 2.4 GHz Intel®Core™2.
- RAM: 2 GB DDR2.
- Language: MATLAB 7.

6. CONCLUSION

Due to the large application in different areas of electromagnetics, designing circular antenna arrays with minimum SLL, maximum directivity is an important problem. This also offers several challenges due to its complex formulation and dependence on various parameters like the current excitation amplitude of antenna elements, phase, number of elements, etc.. In this paper, for solving the complex problem of circular antenna array design we introduce an improved variant of a well known metaheuristic algorithm called Differential Evolution (DE) which incorporates Solis Wet's local search technique along with the global exploration of competitive Differential Evolution, and the superiority of the proposed technique over other existing stochastic optimizers is illustrated through simulation in the context of three-instances of the circular antenna array design problem. The design problem was formulated as an optimization task on the basis of a cost function that takes care of the average side lobe levels, the null control, directivity. The experimental results indicate that the proposed cDESW algorithm has outperformed CLPSO and JADE over all the three instances of circular array design problems based on metrics such as average final accuracy, best obtained design figures of merit (like SLL, directivity, null control). Thus we can say that the proposed cDESW algorithm can be used efficiently for designing circular arrays.

Future research will focus on exploring the design of other array geometries and concentric circular arrays with cDESW and its improved variant if possible. Also treating the four different components of the cost function given in (11) as a multi-objective optimization problem may prove to be a significant avenue of future investigation, but some problem-specific expert's knowledge may have to be incorporated then for pointing out the best suitable solution from the Pareto-optimal set produced by a multi-objective optimizer to implement the configuration in practical applications.

REFERENCES

1. Godara, L. C. (ed.), *Handbook of Antennas in Wireless Communications*, CRC, Boca Raton, FL, 2002.
2. Chandran, S. (ed.), *Adaptive Antenna Arrays: Trends and Applications*, Springer, 2004.
3. Tsoulos, G. V. (ed.), *Adaptive Antennas for Wireless Communications*, IEEE Press, Piscataway, NJ, 2001.
4. Rahmat-Samii, Y. and E. Michielssen, *Electromagnetic Optimization by Genetic Algorithms*, Wiley, New York, 1999.
5. Udina, A., N. M. Martin, and L. C. Jain, "Linear antenna array optimization by genetic means," *Third International Conference on Knowledge-based Intelligent Information Engineering Systems Adelaide*, Australia, Sept. 1999.
6. Cengiz, Y. and H. Tokat, "Linear antenna array design with use of genetic, memetic and tabu search optimization algorithms," *Progress In Electromagnetics Research C*, Vol. 1, 63–72, 2008.
7. Weng, W-C., F. Yang, and A. Z. Elsherbeni, "Linear antenna array synthesis using Taguchi's method: A novel optimization technique in electromagnetics," *IEEE Transactions on Antennas and Propagation*, Vol. 55, No. 3, 723–730, Mar. 2007.
8. Ares-Pena, F. J., A. Rodriguez-Gonzalez, E. Villanueva-Lopez, and S. R. Rengarajan, "Genetic algorithms in the design and optimization of antenna array patterns," *IEEE Transactions on Antennas and Propagation*, Vol. 47, 506–510, Mar. 1999.
9. Tian, Y. B. and J. Qian, "Improve the performance of a linear array by changing the spaces among array elements in terms of genetic algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 53, 2226–2230, Jul. 2005.
10. Khodier, M. M. and C. G. Christodoulou, "Linear array geometry synthesis with minimum side lobe level and null control using particle swarm optimization," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 8, Aug. 2005.
11. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular

arrays," *Progress In Electromagnetics Research*, Vol. 65, 187–200, 2006.

12. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Optimum normalized-Gaussian tapering window for side lobe reduction in uniform concentric circular arrays," *Progress In Electromagnetics Research*, Vol. 69, 35–46, 2007.
13. Dessouky, M., H. Sharshar, and Y. Albagory, "A novel tapered beamforming window for uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 2077–2089, 2006.
14. Chen, T. B., Y. L. Dong, Y. C. Jiao, and F. S. Zhang, "Synthesis of circular antenna array using crossed particle swarm optimization algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 13, 1785–1795, 2006.
15. Fallahi, R. and M. Roshandel, "Effect of mutual coupling and configuration of concentric circular array antenna on the signal-to-interference performance in CDMA systems," *Progress In Electromagnetics Research*, Vol. 76, 427–447, 2007.
16. Zhang, J., W. Wu, and D. G. Fang, "360° scanning multi-beam antenna based on homogeneous ellipsoidal lens fed by circular array," *Electronics Letters*, 298–300, Institution of Engineering and Technology, 2011.
17. Panduro, M., A. L. Mendez, R. Dominguez, and G. Romero, "Design of non-uniform circular antenna arrays for side lobe reduction using the method of genetic algorithms," *Int. J. of Electron. and Commun., AEU*, Vol. 60, 713–717, 2006.
18. Shihab, M., Y. Najjar, N. Dib, and M. Khodier, "Design of non-uniform circular antenna arrays using particle swarm optimization," *Journal of Electrical Engineering*, Vol. 59, No. 4, 216–220, 2008.
19. Panduro, M. A., C. A. Brizuela, L. I. Balderas, and D. A. Acosta, "A comparison of genetic algorithms, particle swarm optimization and the differential evolution method for the design of scannable circular antenna arrays," *Progress In Electromagnetics Research B*, Vol. 13, 171–186, 2009.
20. Gurel, L. and O. Ergul, "Design and simulation of circular arrays of trapezoidal-tooth log-periodic antennas via genetic optimization," *Progress In Electromagnetics Research*, Vol. 85, 243–260, 2008.
21. Barkat, O. and A. Benghalia, "Synthesis of superconducting circular antennas placed on circular array using a particle swarm optimisation and the full-wave method," *Progress In*

Electromagnetics Research B, Vol. 22, 103–119, 2010.

22. Singh, U. and T. S. Kamal, "Design of non-uniform circular antenna arrays using biogeography-based optimization," *Microwaves, Antennas & Propagation, IET*, 1365–1370, 2011.
23. Roy, G. G., S. Das, P. Chakraborty, and P. N. Suganthan, "Design of non-uniform circular antenna arrays using a modified invasive weed optimization algorithm," *IEEE Transactions on Antennas and Propagation*, 110–118, 2011.
24. Huang, M., S. Yang, G. Li, and Z. Nie, "Synthesis of low and equal-ripple sidelobe patterns in time-modulated circular array antennas," *J. Infrared Milli. Terahz. Waves*, Vol. 30, No. 8, 802–812, 2009.
25. Storn, R. and K. Price, "Differential evolution — A simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Vol. 11, No. 4, 341–359, 1997.
26. Zhang J. and A. C. Sanderson, "JADE: Adaptive differential evolution with optional external archive," *IEEE Transactions on Evolutionary Computation*, Vol. 13, No. 5, 945–958, Oct. 2009.
27. Das, S. and P. N. Suganthan, "Differential evolution — A survey of the state-of-the-art," *IEEE Transactions on Evolutionary Computation*, Vol. 15, No. 1, 4–31, 2011.
28. Qing, A., "Electromagnetic inverse scattering of multiple two-dimensional perfectly conducting objects by the differential evolution strategy," *IEEE Transactions on Antennas and Propagation*, Vol. 51, 1251–1262, 2003.
29. Huang, M., S. Yang, W. Xiong, and Z.-P. Nie, "Design and optimization of spherical lens antennas including practical feed models," *Progress In Electromagnetics Research*, Vol. 120, 355–370, 2011.
30. Rocca, P., G. Oliveri, and G. Massa, "Differential evolution as applied to electromagnetics," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 1, 38–49,2011.
31. Solis, F. J. and R. J. Wets, "Minimization by random search techniques," *Mathematical Operations Research*, Vol. 6, 19–30, 1981.
32. Liang, J. J., A. K. Qin, P. N. Suganthan, and S. Baskar, "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Transactions on Evolutionary Computation*, Vol. 10, 281–295, 2006.