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DESIGN OF DUAL PATTERN CONCENTRIC RING ARRAY ANTENNA USING DIFFERENTIAL EVOLUTION ALGORITHM WITH NOVEL EVOLUTIONARY OPERATORS

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Abstract—Concentric ring antenna arrays with the ability to produce dual pattern have many applications in communications and radar engineering. In this paper, we represent a new method for design of an optimized reconfigurable concentric ring array with dual pattern of desired specifications. Here, our goal is to find a suitable common element excitation amplitude distribution and two different element excitation phase distributions for two desired radiation patterns. For this purpose, we have proposed a novel objective function which is completely different from the traditional objective functions usually used in antenna design problems. For the optimization procedure, we have developed a modified Differential Evolution (DE) algorithm, denoted as DE_rBM_2SX, which employs new kinds of crossover and mutation operators to overcome some drawbacks of the classical DE on single-objective fitness landscapes. We consider three types of dual pattern — pencil beam+pencil beam, pencil beam+flat-top beam, flat-top beam+flat-top beam. The simulation results obtained by applying our proposed method clearly indicate that our method is very convenient to obtain radiation patterns of desired specifications. We compare results of the modified DE algorithm with those of another state-of-the-art improved variant of DE, called JADE and a state-of-the-art variant of the Particle Swarm Optimization (PSO) algorithm called Comprehensive Learning Particle Swarm Optimizer (CLPSO). Such comparisons reflect that the proposed algorithm is more efficient than JADE or CLPSO in finding optimum configuration of the dual pattern concentric ring array antenna.

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1. INTRODUCTION

In the field of communication and radar applications, it is often required to generate multiple radiation patterns from a single antenna array. A very common approach to generate multiple radiation patterns from one single antenna array is to switch between excitation phase distributions of the array elements while sharing common amplitude distributions. This approach simplifies the hardware implementation of the feed network because it is technically easier to design a feed network if element excitations for different patterns differ only in phase than if they differ both in phase and amplitude. Several approaches of generating phase-only multiple pattern antenna arrays have been described in [1–4].

Diaz et al. [1] proposed a method to design phase-differentiated multiple pattern antenna arrays based on simulated annealing algorithm. Durr et al. proposed a modified Woodward-Lawson technique based phase only pattern synthesis method to generate multiple radiations of fixed amplitude distribution [2]. A method of projection to synthesize reconfigurable array antennas with asymmetrical pencil and flat-top beam patterns using common amplitude and varying phase distributions was proposed by Bucci et al. [3]. Vaitheeswaran [4] proposed a non-uniformly spaced array approach for multiple beam generation.

Electromagnetic design optimization problems usually involve several parameters that are non-linearly related to the objective functions [1–8]. In order to solve these problems efficiently, Evolutionary Algorithms (EAs) [9–11] have been considered and successfully applied to such problems [12–22]. Differential Evolution (DE) [9, 10, 23, 24] has emerged as one of the most powerful real parameter optimizers of current interest. In this article, we propose to apply a modified version of DE to design a concentric ring array [25–30] which will produce desired dual pattern. Actually the optimization algorithm is used to find the optimum values of the element excitation amplitudes and phases so that the desired patterns are produced. In most of antenna array design problems, the objective function is defined using a Heaviside step function. This method inhibits the optimization algorithm to obtain better results than the desired one. For example, if we use a Heaviside step function with the side lobe levels (SLL) then the optimization algorithm will be unable to find a configuration for which the SLL is less than the maximum allowed level. To overcome this problem, we also propose a completely different method for defining the objective function. We compare our results with the results obtained using two state-of-the-art real parameter optimizers — JADE [10] and CLPSO [11]. The results clearly reveal that our method of designing the concentric ring array is superior to the other two methods.

2. FORMULATION OF THE ANTENNA DESIGN PROBLEM

As we have mentioned already that, in case of phase differentiated dual pattern antenna array, both of the patterns share the same element excitation amplitude. So, we have to search for a common amplitude distribution for both the patterns. Then we have to find two optimum phase distributions for the two desired patterns. The desired dual patterns are generated by switching between two optimum phase distributions. In this paper, we have considered three different dual pattern configurations — pencil+pencil beam pair, pencil+flat-top beam pair and flat-top+flat-top beam pair. Each configuration has different desired specifications such as SLL, First Null Beam Width (FNBW). Now, let us consider the general characteristics of a concentric ring array antenna as shown in Figure 1.

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The normalized array factor for the above shown concentric ring

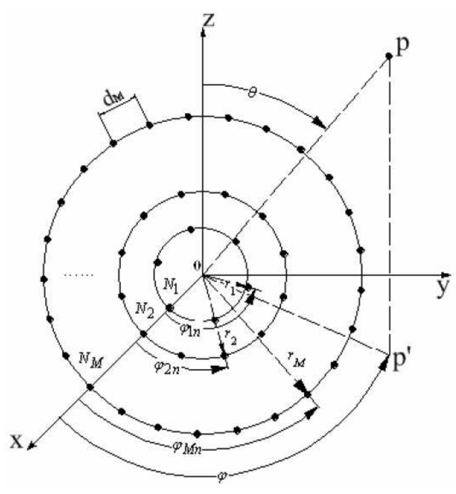

Figure 1. Concentric ring array antenna in X-Y plane.

array can be expressed as follows:

$$AF(\theta,\varphi)=\frac{1}{AF_{\max}}\sum_{m=1}^{M}\sum_{n=1}^{N_m}I_m e^{j[kr_m\sin\theta\cos(\varphi-\varphi_{mn})+\phi_m]} \tag{1}$$

The normalized absolute power pattern $P(\theta,\varphi)$ in dB can be expressed as follows:

$$P(\theta,\varphi)=10\log_{10}|AF(\theta,\varphi)|^2=20\log_{10}|AF(\theta,\varphi)| \tag{2}$$

where $AF_{\max}$ is the maximum value of the magnitude of the array factor, M = number of concentric rings, N_m = number of isotropic elements in the m-th ring, I_m = amplitude of excitation of all the elements in the m-th ring, d_m = inter element arc spacing of m-th ring, $r_m = N_m d_m/2\pi$ is the radius of the m-th ring, $\varphi_{mn} = 2n\pi/N_m$ is the angular position of the mn-th element ($1 \leq n \leq N_m$), θ, φ = polar, azimuth angle, $k = 2\pi/\lambda$ is wave number, ϕ_m = excitation phase of elements on m-th ring.

3. DESIGN OBJECTIVES

In this article, we have represented a unique method for defining the objective function. As mentioned earlier, our method is quite different from the conventional methods used for antenna design optimization problems. The conventional methods generally use the following difference:

$$T=(Ch_O-Ch_D) \tag{3}$$

where Ch_O is the obtained characteristic and Ch_D is the desired characteristic.

The objective is to:

Minimize: $F = \sum T^n H(T)$ where the summation means consideration of all the characteristics to be optimized. Usually, n is kept 1 or 2. The Heaviside step function $H(T)$ can be expressed as follows:

$$H(T)=\begin{cases}0 & \text{if} \quad T<0\\ 1 & \text{if} \quad T\geq 0\end{cases} \tag{4}$$

As can be seen, if the obtained characteristic becomes less than the desired one then T becomes 0. So, we can not obtain a characteristic which is less than the desired one. This is disadvantageous in some cases such as for minimizing SLL and Ripple. Since for all values of $T < 0$ the contribution to the fitness function is 0, this method is also inappropriate for obtaining the perfect desired FNBW.

Our method can be represented as follows:

$$\text{Let } T_1 = SLL_O - SLL_M,\ T_2 = FNBW_O - FNBW_D,$$
$$T_3 = Ripple_O - Ripple_M$$

where subscript O means obtained, D means desired and M means maximum allowed. However, we need to calculate the ripple deviation only in case of flat-top beam within a predefined range of θ ($\theta_{\min} \leq \theta \leq \theta_{\max}$). The pseudocode for calculating the fitness function is given below:

Psuedo-code for objective function calculation:

```
Temp F_i = (T_1^i)^2 H(T_1^i) + (T_2^i)^2 + (T_3^i)^2 // where i ∈ {1, 2}; 1 for first beam and 2 for the second
beam. As mentioned previously, in case of pencil beam we have to consider only T_1 & T_2. //
if T_1^i < 0
    F_i = -(T_1^i)^2 H(-T_1^i)
else
    F_i = TempF_i
end
if T_3^i ≤ Ripple_M ∀_i // for pencil beam+pencil beam configuration this step is omitted//
    if TempF_i ≤ ζ_max ∀_i // where ζ_max is summation of square of both maximum tolerable
        deviation of FNBW and Ripple_M (we set it to 1) //
        Fitness = Σ F_i + (Ripple_O^i)^2
    else
        Fitness = Σ TempF_i
    end
else
    Fitness = C × Σ (Ripple_O^i)^2 // where C is a large constant (we set it to 10^6) //
end
```

Our objective is to minimize this fitness function. As can be seen from the pseudocode, $FNBW$ is strictly forced to lie in some tolerable range from the desired value, SLL and $Ripple$ are tried to minimize as farther as possible even beyond the maximum allowed value.

4. CLASSICAL DIFFERENTIAL EVOLUTION ALGORITHM

An iteration of the classical DE algorithm consists of the four basic steps — initialization of a population of search variable vectors, mutation, crossover or recombination, and finally selection. The last three steps are repeated generation after generation until a stopping criterion is satisfied.

1) Initialization of the Population: If DE searches for the global optima within the continuous search space of dimensionality D then it begins with an initial population of target vectors $\vec{X}_i =$

$\{x_i^1, x_i^2, \ldots, x_i^D\}$ where $i = 1, 2, 3 \ldots NP$ (NP is the population size). The individuals of the initial population are randomly generated from a uniform distribution within the search space which has maximum & minimum bounds as follows: $\vec{X}_{\max} = \{x_{\max}^1, x_{\max}^2, \ldots, x_{\max}^D\}$ and $\vec{X}_{\min} = \{x_{\min}^1, x_{\min}^2, \ldots, x_{\min}^D\}$. The jth component of the ith individual is initialized as follows:

$$x_{i,0}^j = x_{\min}^j + rand_i^j(0,1)\left(x_{\max}^j - x_{\min}^j\right); \quad j \in [1, D] \tag{5}$$

Here $rand_i^j(0,1)$ is a uniformly distributed random number in $(0,1)$ and it is instantiated independently for the j-th component of the i-th individual.

2) Mutation of Operation: After the initialization, DE evolves the population by three operations: mutation, crossover & selection. This process is usually labeled as *DE/x/y/z,* where x denotes the method of selection of base vectors for mutation, y denotes the number of differential vectors used to construct the mutant vector, and z denotes the crossover type (bin: for binomial, exp: for exponential).

In each generation DE creates a *mutant vector* (also known as *donor vector*) corresponding to each individual or target vector of the current population. Three very common mutation strategies are described as follows:

$$a)\ DE/rand/1: \ \vec{V}_{i,G} = \vec{X}_{r1,G} + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r3,G}\right) \tag{6}$$

$$b)\ DE/best/1: \ \vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r2,G}\right) \tag{7}$$

$$c)\ DE/current\text{-}to\text{-}best/1: \ \vec{V}_{i,G} = \vec{X}_{i,G} + F_{best} \cdot \left(\vec{X}_{best,G} - \vec{X}_{i,G}\right) + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r2,G}\right) \tag{8}$$

The indices r_1, r_2 and r_3 are mutually exclusive random integers in the range $[1, NP]$, they are also different from i. These indices are generated once for each mutant vector. $\vec{X}_{best,G}$ is the target vector which has the best fitness value in the population at generation G. The scaling factor F and F_{best} control the amplification of the corresponding differentiation vector.

3) Crossover Operation: To enhance the potential diversity of the population, a crossover operation comes into play after generating the donor vector through mutation. The donor vector mixes its components with the target vector $\vec{X}_{i,G}$ under this operation to form the trial vector $\vec{U}_{i,G} = \{u_{i,G}^1, u_{i,G}^2, \ldots, u_{i,G}^D\}$. The DE family of algorithms uses mainly two kinds of crossover methods — *exponential* (or two-point modulo) and *binomial* (or uniform). Here we shall briefly outline the binomial crossover scheme that has been used in the proposed algorithm. Under this scheme the trial vector is created as follows:

$$u_{i,G}^{j} = \begin{cases} v_{i,G}^{j}, & \text{if } rand(0,1) \leq CR \quad \text{or} \quad j = j_{rand}, \\ x_{i,G}^{j}, & \text{otherwise,} \end{cases} \tag{9}$$

where CR is a user-specified parameter (*Crossover Rate*) in the range $[0, 1)$ and $j_{rand} \in [1, 2, \ldots, D]$ is a randomly chosen index which ensures that the trial vector $\vec{U}_{i,G}$ will differ from its corresponding target vector $\vec{X}_{i,G}$ by at least one component.

4) *Selection Operation:* To keep the population size constant over subsequent generations, the next step of the algorithm calls for *selection* to determine whether the target or the trial vector survives to the next generation, i.e., at $G = G + 1$. For maximization problem, if the objective function value of the trial vector is not less than that of the corresponding target vector, then the trial vector is selected for the next generation; and if it is not so, then the trial vector is selected for the next generation. Obviously, for minimization problem the condition for selection is just the opposite.

The selection operation works as follows (for maximization problem):

$$\vec{X}_{i,G+1} = \begin{cases} \vec{U}_{i,G}, & \text{if } f\left(\vec{U}_{i,G}\right) \geq f\left(\vec{X}_{i,G}\right), \\ \vec{X}_{i,G}, & \text{otherwise,} \end{cases} \tag{10}$$

where $f(\cdot)$ is the objective function to be optimized.

5. THE DE_RBM_2SX ALGORITHM

In our algorithm, denoted by DE_rBM_2SX, we have done two modifications over the classical DE — a random best mutation strategy and a new crossover strategy involving two parent vectors of similar fitness values. The comparison of the results represented later will show that these modifications significantly improve the performance of DE.

1) Random Best Mutation Strategy: Among several mutation strategies in DE, strategies like “DE/current-to-best” and “DE/best” are greedy in nature, i.e., they converge vey quickly by guiding the algorithm to the best position so far discovered. But, as a result of such exploitative tendency, in many cases, the population may lose its diversity and global exploration abilities within a relatively small number of generations, thereafter getting trapped to some locally optimal point in the search space [24]. Taking into consideration these facts and to overcome the limitations of fast but less reliable convergence performance of DE/current-to-best/1 scheme, in this article, we propose a less greedy and more explorative variant of the DE/current-to-best/1 mutation strategy by using randomly selected individual from M top ranking individuals for each target vector. The new scheme can be expressed as:

$$\vec{V}_{i,G} = \vec{X}_{i,G} + F_{best} \cdot \left(\vec{X}_{Mrand,i,G} - \vec{X}_{i,G}\right) + F \cdot \left(\vec{X}_{r1,G} - \vec{X}_{r2,G}\right) \quad (11)$$

where $\vec{X}_{Mrand,i,G}$ is the randomly selected individual from the M top ranking individuals for the i-th individual. Other notations bear the same meaning as mentioned previously in Section 4 (2). Under this scheme, the target solutions are not always attracted towards the same best position found so far by the entire population and this feature is helpful in avoiding premature convergence at local optima.

2) New Crossover Scheme: Crossover operation is very important for any Evolutionary Algorithm because by this procedure, information gathered by the current generation is transferred to the next generation. In the classic crossover operation, the mutant vector exchanges its components with the current individual to produce the trial vector. In our new crossover scheme, the mutant vector exchanges its component with two parent vectors to produce the trial vector. One of the parent vectors is the current vector and the other one is the vector whose fitness value is the closest one to the current vector. This process allows different components transfer of different vectors of same fitness value to the next generation vector. As a consequence the exploration efficiency of the algorithm increases. As there are two parent vectors, there are also two crossover probabilities (CR). The pseudocode of the proposed crossover scheme is given below.

Psuedo-code for proposed crossover scheme:

Let X_i be the current individual and X_{close} is the vector whose fitness value is the closest one to X_i. The mutant vector is V_{mut} and the trial vector is U_i.

Let *CR1* and *CR2* be two crossover probabilities set by user. Usually *CR1* is in the range (0,0.5] and *CR2* is in the range (0.5,1).

Randomval=rand(0,1)
if Randomval<CR1 & $j \neq j_{rand}$ *then*
$U_{i,j} = X_{i,j}$
elseif Randomval>CR2 & $j \neq j_{rand}$ *then*
$U_{i,j} = X_{close,j}$
else
$U_{i,j} = V_{mut,j}$
end

Here subscript *j* means the *j*th component of the corresponding vector. All other notations bear the same meaning as mentioned in the classical crossover operation (Section 3.3).

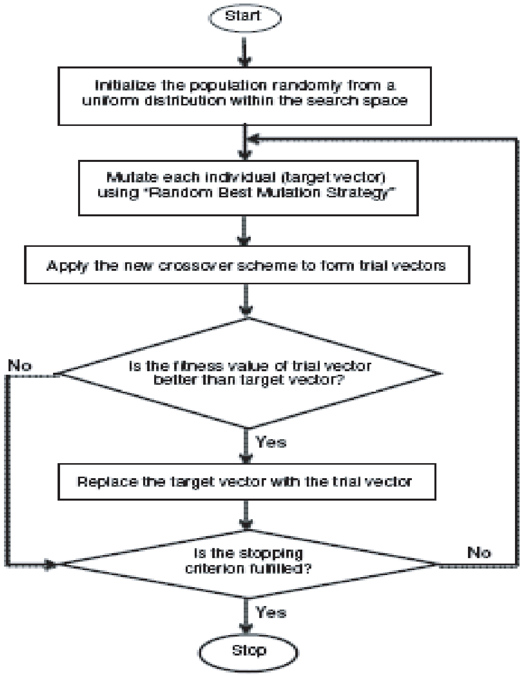

Figure 2. Flow chart of the algorithm.

The flow chart of the algorithm is presented in Figure 2.

6. DESIGN EXAMPLES

For the design examples, we consider a 10 ring concentric circular array and in each ring, the number of elements is $4 \times$ *ring number*, i.e., the number of elements in m-th ring is $4m$. The inter element spacing d_m is taken as $\lambda/2$. We have considered the radiation patterns only in $\varphi = 0°$ plane. For the flat-top beams, we calculated the ripple over $-15° \leq \theta \leq 15°$ range. For the optimization process, we set the number of population to 60 and executed the optimization algorithm for 300 iterations.

The desired characteristics and the obtained characteristics are represented in Table 1 for pencil beam+pencil beam, Table 2 for pencil beam+flat-top beam and Table 3 for flat-top beam+flat-top beam. These tables also represent the comparison of results obtained by optimizing with JADE and CLPSO. One thing to be noted here is that for the pencil beam+pencil beam and pencil beam+flat-top beam configuration, the element excitation phase distribution for

Table 1. Desired and obtained characteristics for pencil beam+pencil beam configuration.

Beam Type	Method	Directivity (dB)	SLL (dB)		FNBW (degree)	
			Obtained	Max. Allowed	Obtained	Desired
Pencil beam 1	DE_rBM_2SX	**31.38**	**−38.0600**	−30	**30**	30
	JADE	27.19	−31.7640		28.8606	
	CLPSO	26.55	−30.4070		28.2017	
Pencil beam 2	DE_rBM_2SX	**30.26**	**−37.7417**	−35	**30**	30
	JADE	29.13	−35.4973		30	
	CLPSO	27.61	−34.9654		29	

Table 2. Desired and obtained characteristics for pencil beam+flat-top beam configuration.

Beam Type	Method	Directivity (dB)	SLL (dB)		FNBW (degree)		Ripple (dB)	
			Obtained	Max. Allowed	Obtained	Desired	Obtained	Max. Allowed
Pencil beam	DE_rBM_2SX	24.56	–25.8661	–25	28	28	-	-
	JADE	24.37	–25.6242		28		-	
	CLPSO	24.49	–25.7731		28		-	
Flattop beam	DE_rBM_2SX	22.47	–37.1051	–25	60	60	0.2447	1
	JADE	17.22	–24.3493		58		0.4936	
	CLPSO	18.28	–25.9573		56		0.7009	

Table 3. Desired and obtained characteristics for flat-top beam+flat-top beam configuration.

Beam Type	Method	Directivity (dB)	SLL(dB)		FNBW(degree)		Ripple(dB)	
			Obtained	Max. Allowed	Obtained	Desired	Obtained	Max. Allowed
Flattop beam1	DE_rBM_2SX	21.07	–33.2002	–20	50	50	0.2985	1
	JADE	15.33	–19.8655		50		0.3364	
	CLPSO	16.53	–21.0372		48		0.5681	
Flattop beam2	DE_rBM_2SX	20.24	–28.3807	–25	50	50	0.4731	1
	JADE	18.01	–25.3697		50		0.5031	
	PSO	17.86	–25.1934		48.7863		0.7025	

the first beam is set to 0 for all the elements for the ease of the optimization procedure because it easy to generate desired pattern by only optimizing the element excitation amplitudes in case of pencil beam.

Now we have represented the element excitation amplitude values and element excitation phase values obtained from DE_rBM_2SX for the above mentioned three configurations respectively in Table 4, Table 5 and Table 6.

Table 4. Element excitation amplitude and phase distribution for pencil beam+pencil beam.

Ring Number	**Pencil Beam 1**		**Pencil Beam 2**	
	Amplitude	Phase (degree)	Amplitude	Phase (degree)
1	0.9241	0	0.9241	17.1503
2	0.9506	0	0.9506	57.7023
3	0.7903	0	0.7903	9.0855
4	0.6338	0	0.6338	60.8599
5	0.7243	0	0.7243	15.0863
6	0.4018	0	0.4018	60.8409
7	0.3983	0	0.3983	15.3023
8	0.3447	0	0.3447	49.6725
9	0.0899	0	0.0899	−17.5906
10	0.2693	0	0.2693	42.0468

Table 5. Element excitation amplitude and phase distribution for pencil beam+flat-top beam.

Ring Number	**Pencil Beam**		**Flattop Beam**	
	Amplitude	Phase (degree)	Amplitude	Phase (degree)
1	0.9960	0	0.9960	−149.3854
2	0.8112	0	0.8112	−123.9871
3	0.6425	0	0.6425	−147.8514
4	0.4603	0	0.4603	−62.5924
5	0.4870	0	0.4870	−129.3249
6	0.5252	0	0.5252	7.2593
7	0.3486	0	0.3486	−111.0005
8	0.4299	0	0.4299	41.2533
9	0.1464	0	0.1464	−95.8414
10	0.1768	0	0.1768	69.0968

The radiation patterns obtained from DE_rBM_2SX, JADE and CLPSO is represented in Figure 3 for pencil beam+pencil beam configuration, Figure 4 for pencil beam+flat-top beam configuration and Figure 5 for flat-top beam+flat-top beam configuration.

To give an idea of runtime of the simulation process, we have represented the comparison of average CPU time required per run in DE_rBM_2SX, JADE and CLPSO based design methods in Table 7. We performed the simulation in the following experimental

Table 6. Element excitation amplitude and phase distribution for flat-top beam+flat-top beam.

Ring Number	Flattop Beam 1		Flattop Beam 2	
	Amplitude	Phase (degree)	Amplitude	Phase (degree)
1	0.9586	134.3824	0.9586	63.2547
2	0.9892	145.1088	0.9892	53.3510
3	0.4390	97.9985	0.4390	17.4993
4	0.7356	150.2150	0.7356	71.9028
5	0.4156	13.1893	0.4156	−48.5748
6	0.3139	135.7605	0.3139	96.4516
7	0.3414	−12.5025	0.3414	−53.4456
8	0.1186	28.1845	0.1186	−128.8574
9	0.0944	−15.1713	0.0944	−38.7117
10	0.1313	−2.7354	0.1313	−100.9348

Table 7. Comparison of average CPU time required per run.

Problem	Average CPU Time Required Per Run (in Seconds)		
	DE_rBM_2SX	JADE	CLPSO
Pencil beam+Pencil beam	**5.72**	9.29	12.31
Pencil beam+Flat-top beam	**6.29**	8.49	13.02
Flat-top beam+Flat-top beam	**6.88**	9.53	16.57

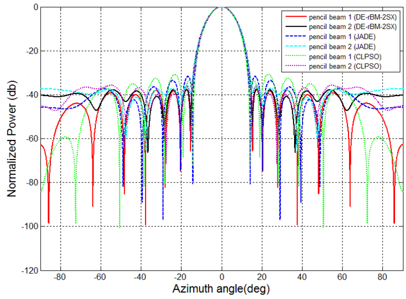

Figure 3. Normalized power patterns in dB for pencil beam+pencil beam configuration.

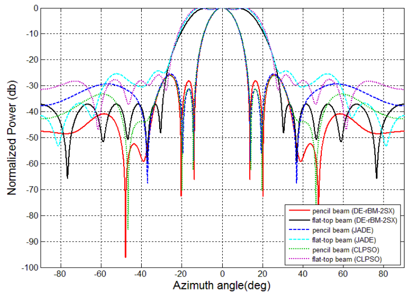

Figure 4. Normalized power patterns in dB for pencil beam+flat-top beam configuration.

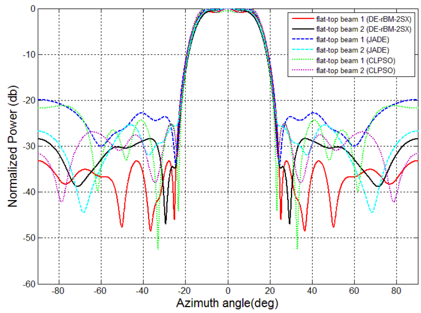

Figure 5. Normalized power patterns in dB for flat-top beam+flat-top beam configuration.

environment:

- CPU: 2.4 GHz Intel®Core$^{\mathrm{TM}}$2
- RAM: 2 GB DDR2
- Language: MATLAB 7

7. CONCLUSIONS

In this paper, we have discussed the designing of a dual pattern concentric ring array antenna so that the array can produce the desired beam pairs satisfying the required specifications. To produce the desired beam pairs, we have considered a common element excitation amplitude distribution and two different phase distributions for two beam pairs of different specifications. Hence the beam pairs are generated by altering the phase distributions only. This process makes the hardware implementations easy. We have proposed a novel fitness function approach for the evaluation of design configurations. As we can see from the simulation results, this approach helps the design optimization process to achieve far better results than the desired one. For the optimization purpose, we have developed a modified DE, denoted by DE_rBM_2SX, which overcomes some drawbacks of classical DE. A new mutation strategy and a new crossover strategy are incorporated in this modified DE to make it more efficient than classical DE. We have compared the results obtained from DE_rBM_2SX with the results obtained from JADE and CLPSO. The comparison shows that our method of designing is more promising than JADE and CLPSO in case of dual pattern concentric ring array design. However, we have only optimized the radiation patterns in $\varphi = 0°$ plane. But in some practical applications, we have to optimize the radiation pattern for the full range of φ, i.e., [0°–360°]. To avoid the extra computational burden and represent the process in a simple manner, we have not performed such simulation. Such tasks will be addressed in our future studies.

REFERENCES

1. Diaz, X., J. A. Rodriguez, F. Ares, and E. Moreno, "Design of phase-differentiated multiple-pattern antenna arrays," *Microwave Opt. Technol. Lett.*, Vol. 26, 52–53, Jul. 2000.
2. Durr, M., A. Trastoy, and F. Ares, "Multiple-pattern linear antenna arrays with single prefixed amplitude distributions: Modified Woodward-Lawson synthesis," *Electronics Letters*, Vol. 36, No. 16, 1345–1346, Aug. 2000.
3. Bucci, O. M., G. Mazzarella, and G. Panariello, "Reconfigurable arrays by phase only control," *IEEE Trans. Antennas Propag.*, Vol. 39, No. 7, 919–925, Jul. 1991.
4. Vaitheeswaran, S. M., "Dual beam synthesis using element position perturbations and the G3-GA algorithm," *Progress In Electromagnetics Research*, Vol. 87, 43–61, 2008.
5. Kumar, B. P. and G. R. Brenner, "Design of unequally spaced arrays for improved performance," *IEEE Trans. Antennas Propag.*, Vol. 47, No. 32, 511–523, Mar. 1999.
6. Haupt, R. L., "Optimized element spacing for low side lobe concentric ring arrays," *IEEE Trans. Antennas Propag.*, Vol. 56, No. 1, 266–268, Jan. 2008.
7. Chakrabarty, A., B. N. Das, and G. S. Sanyal, "Beam shaping using nonlinear phase distribution in a uniformly spaced array," *IEEE Trans. Antennas and Propag.*, Vol. 30, 1031–1034, 1982.
8. Trastoy, A. and F. Ares, "Phase-only control of antenna sum patterns," *Progress In Electromagnetics Research*, Vol. 30, 47–57, 2001.
9. Storn, R. and K. Price, "Differential evolution — A simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Vol. 11, No. 4, 341–359, 1997.
10. Zhang, J. and A. C. Sanderson, "JADE: Adaptive differential evolution with optional external archive," *IEEE Trans. Evol. Comput.*, Vol. 13, No. 5, 945–958, Oct. 2009.
11. Liang, J. J., A. K. Qin, P. N. Suganthan, and S. Baskar, "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Trans. on Evol. Compt.*, Vol. 10, 281–295, 2006.

12. Mahanti, G. K., A. Chakrabarty, and S. Das, "Phase-only and amplitude-phase synthesis of dual-pattern linear antenna arrays using floating-point genetic algorithms," *Progress In Electromagnetics Research*, Vol. 68, 247–259, 2007.
13. Haupt, L. R., "Phase-only adaptive nulling with a genetic algorithm," *IEEE Trans. Antennas Propag.*, Vol. 45, No. 6, 1009–1015, 1997.
14. Ares, F., J. A. Rodriguez, E. Villanueva, and S. R. Rengarajan, "Genetic algorithms in the design and optimization of antenna array patterns," *IEEE Trans. Antennas Propag.*, Vol. 47, 506–510, 1999.
15. Boeringer, D. W. and D. H. Werner, "Particle swarm optimization versus genetic algorithms for phased array synthesis," *IEEE Trans. Antennas Propag.*, Vol. 52, 771–779, 2004.
16. Grimaccia, F., M. Mussetta, and R. E. Zich, "Genetical swarm optimization: Self-adaptive hybrid evolutionary algorithm for electromagnetics," *IEEE Trans. Antennas Propagat.*, Vol. 55, 781–785, 2007.
17. Guney, K. and M. Onay, "Amplitude-only pattern nulling of linear antenna arrays with the use of bees algorithm," *Progress In Electromagnetics Research*, Vol. 70, 21–36, 2007.
18. Xu, Z., H. Li, and Q. Z. Liu, "Pattern synthesis of conformal antenna array by the hybrid genetic algorithm," *Progress In Electromagnetics Research*, Vol. 79, 75–90, 2008.
19. Khodier, M. M. and C. G. Christodoulou, "Linear array geometry synthesis with minimum sidelobe level and null control using particle swarm optimization," *IEEE Trans. Antennas Propagat.*, Vol. 53, 2674–2679, 2005.
20. Jin, N. and Y. Rahmat-Samii, "Advances in particle swarm optimization for antenna designs: Real-number, binary, singleobjective and multiobjective implementations," *IEEE Trans. Antennas Propagat.*, Vol. 55, 556–567, 2007.
21. Kurup, D. G., M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm," *IEEE Trans. Antennas Propagat.*, Vol. 51, 2210–2217, 2003.
22. Ho, S. L., S. Yang, G. Ni, et al., "A particle swarm optimization method with enhanced global search ability for design optimizations of electromagnetic devices," *IEEE Trans. Magnetics.*, Vol. 42, 1107–1110, 2006.

23. Das, S. and P. N. Suganthan, "Differential evolution — A survey of the state-of-the-art," *IEEE Transactions on Evolutionary Computation*, Vol. 15, No. 1, 4–31, 2011.
24. Lampinen, J. and I. Zelinka, "On stagnation of the differential evolution algorithm," *Proc. of MENDEL 2000, 6th International Mendel Conference on Soft Computing*, P. Ošmera (ed.), 76–83, Brno, Czech Republic, Jun. 7–9, 2000.
25. Stearns, C. and A. Stewart, "An investigation of concentric ring antennas with low sidelobes," *IEEE Trans. Antennas Propag.*, Vol. 13, No. 6, 856–863, Nov. 1965.
26. Goto, N. and D. K. Cheng, "On the synthesis of concentric-ring arrays," *IEEE Proc.*, Vol. 58, No. 5, 839–840, May 1970.
27. Biller, L. and G. Friedman, "Optimization of radiation patterns for an array of concentric ring sources," *IEEE Trans. Audio Electroacoust.*, Vol. 21, No. 1, 57–61, Feb. 1973.
28. Huebner, D., "Design and optimization of small concentric ring arrays," *Proc. IEEE Antennas Propagation Int. Symp.*, Vol. 16, 455–458, May 1978.
29. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular arrays," *Progress In Electromagnetics Research*, Vol. 65, 187–200, 2006.
30. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Optimum normalized-Gaussian tapering window for side lobe reduction in uniform concentric circular arrays," *Progress In Electromagnetics Research*, Vol. 69, 35–46, 2007.