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SCATTERING OF ELECTROMAGNETIC SPHERICAL WAVE BY A PERFECTLY CONDUCTING DISK

K. Hongo[1], A. D. U. Jafri[2, *], and Q. A. Naqvi[2]

[1]Consultant Physicist, 3-34-24, Nakashizu, Sakura City, Chiba, Japan

[2]Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan

Abstract—The scattering of electromagnetic spherical wave by a perfectly conducting circular disk is studied by using the method of Kobayashi Potential (abbreviated as KP method). The formulation of the problem yields the dual integral equations (DIE). The spherical wave is produced by an arbitrarily oriented dipole. The unknowns are the induced surface current (or magnetic field) and the tangential components of the electric field on the disk. The solution for the surface current is expanded in terms of a set of functions which satisfy one of a pair (equations for the magnetic field) of Maxwell equations and the required edge condition on the surface of the disk. At this stage we have used the vector Hankel transform. Applying the projection solves the rest of the pair of equations. Thus the problem reduces to the matrix equations for the expansion coefficients. The matrix elements are given in terms of the infinite integrals with a single variable and these may be transformed into infinite series that are convenient for numerical computation. The far field patterns of the scattered wave are computed and compared with those computed based on the physical optics approximation. The agreement between them is fairly good.

1. INTRODUCTION

The problem of scattering of electromagnetic wave by a circular disk of perfect conductor has attracted researchers much attentions theoretically and practically. The circular disk/aperture is canonical structure in the field of scattering. It has wide range of application in radars, reflectors, and antennas etc.. Many methods for analysis of electromagnetic scattering have been developed. High frequency

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* Corresponding author: Allah Ditta Ulfat Jafri (ulfatjafri@yahoo.com).

approximation techniques are approximate methods and are applied when size of the object is large in terms of wavelength. Among them simple and relatively accurate methods are the Physical Optics (PO) [1–4], Physical Theory of Diffraction (PTD) and Increment Length of Diffraction Coefficient (ILDC) [5–8], Geometrical Theory of Diffraction (GTD) [9–13], and the Method of Equivalent Current (MEC) [14, 15]. The method of moment (MoM) developed by Harrington is very useful and is considered to be numerically exact [16]. This method was initially improved by Kim and Thiele [17], and later enhanced by Kaye et al. [18] which is known as hybrid-iterative method. This method employs the magnetic field integral equation for the induced currents to solve the scattering problems. The hybrid-iterative method has been applied to the disk problem successfully by Li et al. for plane wave incidence [19]. The works on the circular aperture and the related problems, conducted before 1953, were reviewed by Bouwkamp [20, 21]. And these methods can also be applied to the present problem of spherical wave scattering by a circular disk.

Since the surface of the disk is the limiting case of the oblate spheroid, the direct approach is to find a series expansion for the field in terms of suitable characteristic functions. Meixner and Andrejewski [22, 23] and Flammer [24] used three rectangular components of the Hertz vectors that describe the incident and scattered fields which are expanded in terms of appropriate oblate spheroidal wave functions. But their work was limited to plane wave incidence. Bjrkberg and Kristensson [25, 26] solved the elliptic disk problem using the null field approach in which the incident and the scattered fields are expanded in terms of spherical vector waves. While Kristensson and Waterman [27, 28] applied the T-matrix approach to study the scattering by circular disk. Balaban et al. [29] used the coupled dual integral equation technique to solve the disk problem. However there is another set of characteristic functions which satisfy the boundary conditions and edge condition. These are constructed by applying the properties of the Weber-Schafheitlin's discontinuous integrals. This idea was first proposed by Kobayashi [30] to solve the electrostatic problem of the electrified conducting disk and this method was named by Sneddon [31] as the Kobayashi Potential (KP). The KP method is like eigen function expansion and also is similar to the Method of Moments (MoM) in its spectral domain, but the formulation is different. The MoM is based on an integral equation, whereas the KP method has dual integral equations. In addition, advantages of the KP method over the MoM are that these characteristic functions satisfy a proper edge condition as well as the required boundary conditions and it produces an accurate and faster convergent solution. The disadvantage of this method is that the tractable geometries are limited to special shapes, such as circular and rectangular plates and apertures. The KP method has already been successfully applied to many rectangular or related objects [32–35]. By using these characteristic functions, exact solutions were derived by Nomura and Katsura [32] for plane wave incidence and by Inawashiro [35] for spherical wave incidence. The work by Inawashiro was restricted to the case where the dipole is directed to parallel to the disk and is located on the axis passing through the center of the disk. They used two rectangular components of the Hertz vector plus auxiliary scalar wave function which describe the surface field of the disk to satisfy the edge conditions. The summaries of the works by the above two groups, Meixner and Andrejewski, and Nomura and Katsura are reproduced in the handbook by Bowman et al. [36].

In a paper [34], Hongo and Naqvi improved the treatment by Nomura and Katsura for plane wave excitation. It is the purpose of the present paper to improve and extend the work of Inawashiro [35] for spherical wave excitation using KP method. First we derived the expressions for the incident field produced by an arbitrarily oriented dipole. Next we introduced two longitudinal components of the vector potentials of electric and magnetic types to express the scattered wave. The expressions have the form of Fourier-Hankel transform. By applying the boundary conditions we derive the dual integral equations (DIE), one set is for induced electric current densities and another is for the tangential components of the electric field. The equations may be written in the form of the vector Hankel transform given by Chew and Kong [37]. The expressions for the current densities are expanded in terms of a set of the functions with expansion coefficients. These functions are constructed by applying the discontinuous properties of the Weber-Schafheitlin's integrals [38–41] and it is readily shown that these functions satisfy the Maxwell equations on the surface of the disk and the required edge conditions. Applying the inverse vector Hankel transform derives corresponding spectral functions of the current densities. The derived results are substituted into the equation for the electric field in DIE and we derive the solutions of the expansion coefficients by using the projection. Thus the problem reduces to the matrix equation for the expansion coefficients of the current densities. The matrix elements are given in the form of an infinite integrals. These integrals can be transformed into infinite series which is convenient for numerical computation. Numerical computation is made using FORTRAN to obtain the far field patterns.

2. EXPRESSIONS FOR INCIDENT WAVE

2.1. Derivation of Dyadic Green's Function in Free Space in Terms of Cylindrical Coordinates

First we discuss the derivation of the dyadic Green's function in free space, which is the solution of inhomogeneous Maxwell's equations expressed by

$$\begin{aligned} &\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \qquad \nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \mathbf{J}, \\ &\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon} = -\frac{\nabla \cdot \mathbf{J}}{j\omega\epsilon}, \quad \nabla \cdot \mathbf{H} = 0; \quad \mathbf{J} = \mathbf{I}\delta(\mathbf{r} - \mathbf{r_0}) \end{aligned} \tag{1}$$

where $\mathbf{r}$ and $\mathbf{r_0}$ denote the position vectors of the observation and source points, respectively, and ρ_e is the electric charge density. In (1), we assume a homogeneous medium. The solution of (1) can be written in the form [42, p. 197].

$$\begin{aligned} \mathbf{E} \ = \ & \frac{1}{j\omega\epsilon}\left\{\left(\frac{\partial}{\partial z}\nabla_t - \mathbf{i}_z\nabla_t^2\right)\left(\frac{\partial}{\partial z_0}\nabla_{t0} - \mathbf{i}_z\nabla_{t0}^2\right)S(\mathbf{r},\mathbf{r_0})\right. \\ & \left. + k^2(\nabla \times \mathbf{i}_z)(\nabla_0 \times \mathbf{i}_{z0})S(\mathbf{r},\mathbf{r_0})\right\} \cdot \mathbf{I} \end{aligned} \tag{2a}$$

$$\begin{aligned} \mathbf{H} \ = \ & \left\{\left(\frac{\partial}{\partial z}\nabla_t - \mathbf{i}_z\nabla_t^2\right)(\nabla_{t0} \times \mathbf{i}_{z0})S(\mathbf{r},\mathbf{r_0})\right. \\ & \left. -(\nabla \times \mathbf{i}_z)\left(\frac{\partial}{\partial z_0}\nabla_{t0} - \mathbf{i}_z\nabla_{t0}^2\right)S(\mathbf{r},\mathbf{r_0})\right\} \cdot \mathbf{I} \end{aligned} \tag{2b}$$

where $S(\mathbf{r}, \mathbf{r_0})$ is derived from the scalar Green's function $G(\mathbf{r}, \mathbf{r_0})$ through the relation

$$G(\mathbf{r},\mathbf{r_0}) \ = \ \left(\frac{\partial^2}{\partial z^2} + k^2\right)S(\mathbf{r},\mathbf{r_0}) = -\nabla_t^2 S(\mathbf{r},\mathbf{r_0}) \tag{3a}$$

$$\nabla^2 G + k^2 G \ = \ -\delta(\mathbf{r} - \mathbf{r_0}) \tag{3b}$$

In free space, (2a) is replaced by dyadic Green's function [43, 44]

$$\tilde{G}(\mathbf{r},\mathbf{r_0}) = \frac{1}{4\pi}\left[\tilde{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right]\frac{\exp(-jkR)}{R} \tag{4a}$$

where $\tilde{\mathbf{I}}$ is unit dyadic and $R = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{\frac{1}{2}}$. The equivalence of (2a) and (4) in free space may be verified readily.

In circular cylindrical coordinates, function $G(\mathbf{r}, \mathbf{r_0})$ satisfies the wave equation

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial G}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2 G}{\partial\phi^2}+\frac{\partial^2 G}{\partial z^2}+k^2 G=-\frac{\delta(\rho-\rho_0)\delta(\phi-\phi_0)\delta(z-z_0)}{\rho} \quad \text{(4b)}$$

By using the theory of Fourier series for the angular variable ϕ and Hankel transform for the variable ρ defined by

$$F(\alpha)=\int_0^\infty f(\rho)J_m(\alpha\rho)\rho d\rho, \quad f(\rho)=\int_0^\infty F(\alpha)J_m(\alpha\rho)\alpha d\alpha \quad (5)$$

we can represent $G(\mathbf{r}, \mathbf{r_0})$ and $S(\mathbf{r}, \mathbf{r_0})$ in the form

$$G(\mathbf{r},\mathbf{r_0})=\frac{1}{4\pi ja}\sum_{m=0}^{\infty}\epsilon_m\cos m(\phi-\phi_0)\int_0^\infty\frac{J_m(\alpha\rho_{0a})J_m(\alpha\rho_a)}{\sqrt{\kappa^2-\alpha^2}} \exp(-jh_a|z_a-z_{0a}|)\alpha d\alpha \quad \text{(6a)}$$

$$S(\mathbf{r},\mathbf{r_0})=\frac{a}{4\pi j}\sum_{m=0}^{\infty}\epsilon_m\cos m(\phi-\phi_0)\int_0^\infty\frac{J_m(\alpha\rho_{0a})J_m(\alpha\rho_a)}{\alpha^2\sqrt{\kappa^2-\alpha^2}} \exp(-jh_a|z_a-z_{0a}|)\alpha d\alpha \quad \text{(6b)}$$

where we have normalized the variables and parameters by the radius of the disk

$$\rho_a=\frac{\rho}{a},\ \rho_{0a}=\frac{\rho_0}{a},\ z_a=\frac{z}{a},\ z_{0a}=\frac{z_0}{a},\ h_a=ha,\ \kappa=ka,\ h=\sqrt{\kappa^2-\alpha^2}$$

Here "a" is the radius of the disk as shown in the Fig. 1. (ρ_0, z_0) are the coordinates of the position of the dipole/source point and (ρ, z) are the coordinates of the observation point. Here k is the wave number. The electromagnetic field due to the dipole is derived by substituting (6) into (2a) and (2b) and we write the value of the tangential components of the electromagnetic field for a disk problem and hole problem derived from S.

$$\begin{pmatrix}\mathbf{E}^i\\ \mathbf{H}^i\end{pmatrix}=\sum_{m=0}^{\infty}\left\{\mathbf{i}_\rho\left[\begin{pmatrix}E^i_{\rho c,m}\\ H^i_{\rho c,m}\end{pmatrix}\cos m\phi+\begin{pmatrix}E^i_{\rho s,m}\\ H^i_{\rho s,m}\end{pmatrix}\sin m\phi\right] +\mathbf{i}_\phi\left[\begin{pmatrix}E^i_{\phi c,m}\\ H^i_{\phi c,m}\end{pmatrix}\cos m\phi+\begin{pmatrix}E^i_{\phi s,m}\\ H^i_{\phi s,m}\end{pmatrix}\sin m\phi\right]\right\} \quad (7)$$

Also the the field produced by an arbitrarily oriented dipole is derived in [45]. And we can assume $\phi_0 = 0$ without a loss of generality because

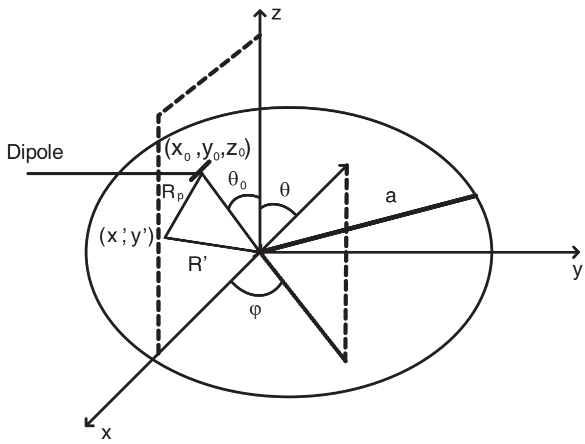

Figure 1. Scattering of an arbitrarily oriented dipole field by a circular disk.

of the symmetry of the problem. The expressions for the Fourier components defined in (7) at $z = 0$ are given as follows.

(i): ρ-Directed Field

$$E^i_{\rho c,m} = -\frac{Z_0}{4\pi\kappa a^2}\epsilon_m \int_o^\infty \frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}\Big[h_a^2 J'_m(\alpha\rho_{0a})J'_m(\alpha\rho_a) + \kappa^2\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\Big]\alpha d\alpha \tag{8a}$$

$$E^i_{\phi s,m} = \frac{Z_0}{2\pi\kappa a^2}\int_o^\infty \frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}\Big[h_a^2 J'_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a) + \kappa^2\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})J'_m(\alpha\rho_a)\Big]\alpha d\alpha \tag{8b}$$

$$H^i_{\rho s,m} = -\frac{1}{2\pi a^2}\int_o^\infty \Big[J'_m(\alpha\rho_a)\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a}) + \frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)J'_m(\alpha\rho_{0a})\Big]\exp(-jh_a z_{0a})\alpha d\alpha \tag{9a}$$

$$H^i_{\phi c,m} = \frac{\epsilon_m}{4\pi a^2}\int_o^\infty \Big[\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a}) + J'_m(\alpha\rho_a)J'_m(\alpha\rho_{0a})\Big]\exp(-jh_a z_{0a})\alpha d\alpha \tag{9b}$$

(ii): ϕ-Directed Field

$$E^i_{\rho s,m} = -\frac{Z_0}{2\pi\kappa a^2}\int_o^\infty \frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}\Big[h_a^2\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})J'_m(\alpha\rho_a) + \kappa^2 J'_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\Big]\alpha d\alpha \tag{10a}$$

$$E^i_{\phi c,m} = -\frac{Z_0}{4\pi\kappa a^2}\epsilon_m\int_o^\infty \frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}\Big[\kappa^2 J'_m(\alpha\rho_{0a})J'_m(\alpha\rho_a) + h_a^2\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\Big]\alpha d\alpha \tag{10b}$$

$$H^i_{\rho c,m} = -\frac{\epsilon_m}{4\pi a^2}\int_o^\infty\Big[J'_m(\alpha\rho_a)J'_m(\alpha\rho_{0a}) + \frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big]\exp(-jh_a z_{0a})\alpha d\alpha \tag{11a}$$

$$H^i_{\phi s,m} = \frac{1}{2\pi a^2}\int_o^\infty\Big[\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)J'_m(\alpha\rho_{0a}) + \frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})J'_m(\alpha\rho_a)\Big]\exp(-jh_a z_{0a})\alpha d\alpha \tag{11b}$$

(iii): z-Directed Field

$$E^i_{\rho c,m} = \frac{Z_0}{4\pi j\kappa a^2}\epsilon_m\int_o^\infty J_m(\alpha\rho_{0a})J'_m(\alpha\rho_a)\exp(-jh_a z_{0a})\alpha^2 d\alpha \tag{12a}$$

$$E^i_{\phi s,m} = -\frac{Z_0}{2\pi j\kappa a^2}\int_o^\infty J_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\exp(-jh_a z_{0a})\alpha^2 d\alpha \tag{12b}$$

$$H^i_{\rho s,m} = \frac{1}{2\pi j a^2}\int_o^\infty \frac{1}{\sqrt{\kappa^2-\alpha^2}}J_m(\alpha\rho_{0a})\frac{m}{\alpha\rho_a}J_m(\alpha\rho_a)\exp(-jh_a z_{0a})\alpha^2 d\alpha \tag{13a}$$

$$H^i_{\phi c,m} = \frac{1}{4\pi j a^2}\epsilon_m\int_o^\infty \frac{1}{\sqrt{\kappa^2-\alpha^2}}J_m(\alpha\rho_{0a})J'_m(\alpha\rho_a)\exp(-jh_a z_{0a})\alpha^2 d\alpha \tag{13b}$$

where $\epsilon_m = 1$ for $m = 0$ and $\epsilon_m = 2$ for $m \geq 1$.

2.2. Relation to Plane Wave Incidence

It may be expected that the dipole solution reduce to that of plane wave incidence when the dipole recedes to infinity. When $\kappa R_a \gg 1$, we can derive the asymptotic expressions for dipole field given in (8) to (13). We consider only the electric field components. First we consider the integral having the form

$$I = Q_0 \int_o^\infty P(\alpha) J_m(\alpha\rho_{0a}) \exp(-jh_a z_a) d\alpha \tag{14a}$$

By using the integral representation of the Bessel function

$$J_m(\alpha\rho_{0a}) = \frac{j^m}{\pi} \int_{-\delta}^{\pi-\delta} \exp(-jmt - j\alpha\rho_{0a}\cos t) dt \tag{14b}$$

with $0 < \delta < \pi/2$, I becomes

$$I = \frac{Q_0}{\pi} j^m \int_o^\infty \int_{-\delta}^{\pi-\delta} P(\alpha) \exp\Big(-jmt - j\alpha\rho_{0a}\cos t - jh_a z_a\Big) dt d\alpha \tag{14c}$$

We impose the transformation of the variables

$$\alpha = \kappa\sin\beta, \quad h_a = \kappa\cos\beta, \quad z_a = R_a\cos\theta_0, \quad \rho_{0a} = R_a\sin\theta_0 \tag{14d}$$

and apply the stationary phase method of integration, then we have

$$I = 2Q_0 j^{m+1} \frac{\cos\theta_0}{\sin\theta_0} P(\kappa\sin\theta_0) \frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{14e}$$

Therefore, Equations (8), (10) and (12) are simplified as follows

(i): $\boldsymbol{\rho}$-Directed Field

$$E^i_{\rho c,m} = -\frac{Z_0\kappa}{2\pi a^2} \epsilon_m j^m \cos^2\theta_0 J'_m(\kappa\rho_a\sin\theta_0) \frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{15a}$$

$$E^i_{\phi s,m} = \frac{Z_0\kappa}{\pi a^2} j^m \cos^2\theta_0 \frac{m}{\kappa\rho_a\sin\theta_0} J_m(\kappa\rho_a\sin\theta_0) \frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{15b}$$

(ii): $\boldsymbol{\phi}$-Directed Field

$$E^i_{\rho s,m} = -\frac{Z_0\kappa}{\pi a^2} j^m \sin\theta_0 \frac{m}{\kappa\rho_a\sin\theta_0} J_m(\kappa\rho_a\sin\theta_0) \frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{16a}$$

$$E^i_{\phi s,m} = -\frac{Z_0\kappa}{2\pi a^2} \epsilon_m j^m \sin\theta_0 J'_m(\kappa\rho_a\sin\theta_0) \frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{16b}$$

(iii): z-Directed Field

$$E^i_{\rho c,m} = \frac{Z_0\kappa}{2\pi a^2}\epsilon_m j^m \cos\theta_0 \sin\theta_0 J'_m(\kappa\rho_a\sin\theta_0)\frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{17a}$$

$$E^i_{\phi s,m} = -\frac{Z_0\kappa}{\pi a^2} j^m \cos\theta_0 \sin\theta_0 \frac{m}{\kappa\rho_a\sin\theta_0} J_m(\kappa\rho_a\sin\theta_0)\frac{\exp(-j\kappa R_{0a})}{R_{0a}} \tag{17b}$$

It may be readily verified that z-component of the magnetic dipole located at (ρ_0, 0, z_0) produces the same field as that of ϕ-directed electric dipole located at the same place. And when z-directed electric and magnetic dipoles recedes to infinite they produce the same electromagnetic field as those of perpendicular and parallel polarized plane wave incidence provided that some factor proportional to $R_{0a}\exp(j\kappa R_{0a})$, more explicitly, we use the replacement

$$Z_0\frac{\kappa}{\pi a^2}\sin\theta_0\frac{\exp(-j\kappa R_{0a})}{R_{0a}} \to jE_1, \quad \frac{Z_0\kappa}{\pi a^2}\sin\theta_0\frac{\exp(-j\kappa R_{0a})}{R_{0a}} \to -E_2 \tag{18}$$

3. THE EXPRESSIONS FOR THE FIELDS SCATTERED BY A DISK

We now discuss our analytical method for predicting the field scattered by a perfectly conducting disk on the plane $z = 0$ when it is excited by a dipole current.

3.1. Spectrum Functions of the Current Density on the Disk

Since E^d_ρ and E^d_ϕ are continuous on the plane $z = 0$, we assume the vector potentials corresponding to the diffracted field are expressed in the form

$$A^d_z(\rho,\phi,z) = \pm\mu_0 a\kappa Y_0 \sum_{m=0}^{\infty}\int_o^\infty \Big[\tilde{f}_{cm}(\xi)\cos m\phi + \tilde{f}_{sm}(\xi)\sin m\phi\Big] J_m(\rho_a\xi)\exp\Big[\mp\sqrt{\xi^2-\kappa^2}z_a\Big]\xi^{-1}d\xi \tag{19a}$$

$$F^d_z(\rho,\phi,z) = \epsilon_0 a \sum_{m=0}^{\infty}\int_o^\infty \Big[\tilde{g}_{cm}(\xi)\cos m\phi + \tilde{g}_{sm}(\xi)\sin m\phi\Big] J_m(\rho_a\xi)\exp\Big[\mp\sqrt{\xi^2-\kappa^2}z_a\Big]\xi^{-1}d\xi \tag{19b}$$

where the upper and lower signs refer to the region $z > 0$ and $z < 0$, respectively. In the above equations $\tilde{f}(\xi)$ and $\tilde{g}(\xi)$ are the unknown spectrum functions and they are to be determined so that they satisfy all the required boundary conditions and edge conditions. Equations (19a) and (19b) are of the form of the Hankel transform for $z = 0$. First we consider the surface field at the plane $z = 0$ to derive the dual integral equations for seeking the solution for the spectrum functions. By using the relations between the vector potentials (19) and the electromagnetic field, we obtain the expressions for the field components.

$$\begin{bmatrix} E^d_{\rho c,m}(\rho_a) \\ E^d_{\phi s,m}(\rho_a) \end{bmatrix} = \int_o^\infty [H^-(\xi\rho_a)] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{f}_{cm}(\xi)\xi^{-1} \\ \tilde{g}_{sm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi \tag{20a}$$

$$\begin{bmatrix} E^d_{\rho s,m}(\rho_a) \\ E^d_{\phi c,m}(\rho_a) \end{bmatrix} = \int_o^\infty [H^+(\xi\rho_a)] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{f}_{sm}(\xi)\xi^{-1} \\ \tilde{g}_{cm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi \tag{20b}$$

where $E^d_{\rho c,m}(\rho_a) \sim E^d_{\phi s,m}(\rho_a)$ are the Fourier components of scattered electric field components E_ρ and E_ϕ. Similarly, the components of the electric current density having the relations with magnetic field components by $K_\rho = -2H_\phi$ and $K_\phi = 2H_\rho$ are given by

$$\begin{aligned} \begin{bmatrix} K_{\rho c,m}(\rho_a) \\ K_{\phi s,m}(\rho_a) \end{bmatrix} &= 2Y_0 \int_o^\infty [H^-(\xi\rho_a)] \begin{bmatrix} \kappa\tilde{f}_{cm}(\xi)\xi^{-1} \\ j\sqrt{\xi^2-\kappa^2}\tilde{g}_{sm}(\xi)(\kappa\xi)^{-1} \end{bmatrix} \xi d\xi \\ &= \int_o^\infty \Big[H^-(\xi\rho_a)\Big] \begin{bmatrix} \tilde{K}_{\rho c,m}(\xi) \\ \tilde{K}_{\phi s,m}(\xi) \end{bmatrix} \xi d\xi = 0 \quad \rho_a > 1 \end{aligned} \tag{21a}$$

$$\begin{aligned} \begin{bmatrix} K_{\rho s,m}(\rho_a) \\ K_{\phi c,m}(\rho_a) \end{bmatrix} &= 2Y_0 \int_o^\infty [H^+(\xi\rho_a)] \begin{bmatrix} \kappa\tilde{f}_{sm}(\xi)\xi^{-1} \\ j\sqrt{\xi^2-\kappa^2}\tilde{g}_{cm}(\xi)(\kappa\xi)^{-1} \end{bmatrix} \xi d\xi \\ &= \int_o^\infty [H^+(\xi\rho_a)] \begin{bmatrix} \tilde{K}_{\rho s,m}(\xi) \\ \tilde{K}_{\phi c,m}(\xi) \end{bmatrix} \xi d\xi = 0 \quad \rho_a > 1 \end{aligned} \tag{21b}$$

From these equations we have the relations

$$\begin{aligned} \tilde{f}_{cm}(\xi) &= \frac{Z_0}{2\kappa}\tilde{K}_{\rho c,m}(\xi)\xi & \tilde{f}_{sm}(\xi) &= \frac{Z_0}{2\kappa}\tilde{K}_{\rho s,m}(\xi)\xi \\ \tilde{g}_{cm}(\xi) &= \frac{\kappa Z_0}{j2\sqrt{\xi^2-\kappa^2}}\tilde{K}_{\phi c,m}(\xi)\xi & \tilde{g}_{sm}(\xi) &= \frac{\kappa Z_0}{j2\sqrt{\xi^2-\kappa^2}}\tilde{K}_{\phi s,m}(\xi)\xi \end{aligned} \tag{21c}$$

The kernel matrices $[H^+(\xi\rho_a)]$ and $[H^-(\xi\rho_a)]$ are defined by

$$\Big[H^\pm(\xi\rho_a)\Big] = \begin{bmatrix} J'_m(\xi\rho_a) & \pm\frac{m}{\xi\rho_a}J_m(\xi\rho_a) \\ \pm\frac{m}{\xi\rho_a}J_m(\xi\rho_a) & J'_m(\xi\rho_a) \end{bmatrix} \tag{22}$$

To solve (21) we expand $K(\rho_a)$ in terms of the functions which satisfy the Maxwell's equations and edge conditions. These functions can be found by taking into account the discontinuity property of the Weber-Schafheitlin's integrals. Once the expressions for $K(\rho_a)$ are established, the corresponding spectrum functions can be derived by applying the vector Hankel transform introduced by Chew and Kong [37]. That is, from (21a) and (21b) we have

$$\begin{aligned}\begin{bmatrix}\tilde{K}_{\rho c,m}(\xi)\\ \tilde{K}_{\phi s,m}(\xi)\end{bmatrix} &= \int_o^\infty \left[H^-(\xi\rho_a)\right]\begin{bmatrix}K_{\rho c,m}(\rho_a)\\ K_{\phi s,m}(\rho_a)\end{bmatrix}\rho_a d\rho_a,\\ \begin{bmatrix}\tilde{K}_{\rho s,m}(\xi)\\ \tilde{K}_{\phi c,m}(\xi)\end{bmatrix} &= \int_o^\infty \left[H^+(\xi\rho_a)\right]\begin{bmatrix}K_{\rho s,m}(\rho_a)\\ K_{\phi c,m}(\rho_a)\end{bmatrix}\rho_a d\rho_a\end{aligned} \tag{23}$$

where $K_{\rho c,m} \sim K_{\phi s,m}$ are the Fourier components of the surface current on the disk.

$$\begin{aligned}K_\rho(\rho_a,\phi) &= \sum_{m=0}^{\infty}\Big[K_{\rho c,m}(\rho_a)\cos m\phi + K_{\rho s,m}(\rho_a)\sin m\phi\Big]\\ K_\phi(\rho_a,\phi) &= \sum_{m=0}^{\infty}\Big[K_{\phi c,m}(\rho_a)\cos m\phi + K_{\phi s,m}(\rho_a)\sin m\phi\Big]\end{aligned} \tag{24a}$$

It is noted that (K_ρ, K_ϕ) satisfy the vector Helmholtz equation $\nabla^2\mathbf{K} + k^2\mathbf{K} = 0$ in circular cylindrical coordinates on the plane $z = 0$ since $\mathbf{K}$ and $\mathbf{H}$ are related by $\mathbf{K} = \hat{\mathbf{n}} \times \mathbf{H}$ and in our problem $\hat{n}$ is $\hat{z}$. Furthermore (K_ρ, K_ϕ) have the properties $K_\rho \sim (1-\rho_a^2)^{\frac{1}{2}}$ and $K_\phi \sim (1-\rho_a^2)^{-\frac{1}{2}}$ near the edge of the disk. By taking into account these facts, we set

$$\begin{aligned}K_{\rho c,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[A_{mn}F_{mn}^-(\rho_a) - B_{mn}G_{mn}^+(\rho_a)\Big],\\ K_{\rho s,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[C_{mn}F_{mn}^-(\rho_a) + D_{mn}G_{mn}^+(\rho_a)\Big]\\ K_{\phi s,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[-A_{mn}F_{mn}^+(\rho_a) + B_{mn}G_{mn}^-(\rho_a)\Big],\\ K_{\phi c,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[C_{mn}F_{mn}^+(\rho_a) + D_{mn}G_{mn}^-(\rho_a)\Big]\end{aligned} \tag{24b}$$

where

$$F^{\pm}_{mn}(\rho_a) = \int_o^\infty \Big[J_{m-1}(\eta\rho_a)J_{m+2n-\frac{1}{2}}(\eta) \pm J_{m+1}(\eta\rho_a)J_{m+2n+\frac{3}{2}}(\eta)\Big] \eta^{\frac{1}{2}}d\eta \quad m \geq 1 \tag{25a}$$

$$F^{+}_{0n}(\rho_a) = 2\int_o^\infty J_1(\eta\rho_a)J_{2n+\frac{3}{2}}(\eta)\eta^{\frac{1}{2}}d\eta$$

$$G^{\pm}_{mn}(\rho_a) = \int_o^\infty \Big[J_{m-1}(\eta\rho_a)J_{m+2n+\frac{1}{2}}(\eta) \pm J_{m+1}(\eta\rho_a)J_{m+2n+\frac{5}{2}}(\eta)\Big] \eta^{-\frac{1}{2}}d\eta \quad m \geq 1 \tag{25b}$$

$$G^{+}_{0n}(\rho_a) = 2\int_o^\infty J_1(\eta\rho_a)J_{2n+\frac{5}{2}}(\eta)\eta^{-\frac{1}{2}}d\eta$$

and these functions can be represented in terms of the hypergeometric functions and their explicit expressions are given in the paper [34]. It may be readily verified that $F^{\pm}_{mn}(\rho_a) = G^{\pm}_{mn}(\rho_a) = 0$ for $\rho_a \geq 1$, and $F^{+}_{mn}(\rho_a) \sim (1-\rho_a^2)^{-\frac{1}{2}}$, $F^{-}_{mn}(\rho_a) \sim (1-\rho_a^2)^{\frac{1}{2}}$, $G^{+}_{mn}(\rho_a) \sim (1-\rho_a^2)^{\frac{1}{2}}$, and $G^{-}_{mn}(\rho_a) \sim (1-\rho_a^2)^{-\frac{1}{2}}$ near the edge $\rho_a \simeq 1$. To derive the spectrum functions $\tilde{f}(\xi)$ and $\tilde{g}(\xi)$ of the vector potentials we first determine the spectrum functions of the current densities $\tilde{\mathbf{K}}(\rho_a)$, since they are related each other. We substitute (24) into (23) and perform the integration, then the spectrum functions of the current density are determined. The result is

$$\begin{aligned}
\tilde{K}_{\rho c,m}(\xi) &= \sum_{n=0}^{\infty}\Big[A_{mn}\Xi^{+}_{mn}(\xi) - B_{mn}\Gamma^{-}_{mn}(\xi)\Big],\\
\tilde{K}_{\phi s,m}(\xi) &= \sum_{n=0}^{\infty}\Big[-A_{mn}\Xi^{-}_{mn}(\xi) + B_{mn}\Gamma^{+}_{mn}(\xi)\Big]\\
\tilde{K}_{\rho s,m}(\xi) &= \sum_{n=0}^{\infty}\Big[C_{mn}\Xi^{+}_{mn}(\xi) + D_{mn}\Gamma^{-}_{mn}(\xi)\Big]\\
\tilde{K}_{\phi c,m}(\xi) &= \sum_{n=0}^{\infty}\Big[C_{mn}\Xi^{-}_{mn}(\xi) + D_{mn}\Gamma^{+}_{mn}(\xi)\Big]
\end{aligned} \tag{26a}$$

for $m \geq 1$ and

$$\tilde{K}_{\rho c,0}(\xi)=2\sum_{n=0}^{\infty}B_{0n}J_{2n+\frac{5}{2}}(\xi)\xi^{-\frac{3}{2}} \quad \tilde{K}_{\phi c,0}(\xi)=-2\sum_{n=0}^{\infty}C_{0n}J_{2n+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}} \tag{26b}$$

for $m = 0$. In the above equations the functions $\Xi^{\pm}_{mn}(\xi)$ and $\Gamma^{\pm}_{mn}(\xi)$ are defined by

$$\begin{aligned}\Xi^{\pm}_{mn}(\xi) &= \left[J_{m+2n-\frac{1}{2}}(\xi) \pm J_{m+2n+\frac{3}{2}}(\xi)\right]\xi^{-\frac{1}{2}},\\ \Gamma^{\pm}_{mn}(\xi) &= \left[J_{m+2n+\frac{1}{2}}(\xi) \pm J_{m+2n+\frac{5}{2}}(\xi)\right]\xi^{-\frac{3}{2}}, \quad m \geq 1\end{aligned} \tag{26c}$$

In deriving (26a) and (26b) we used the formula of the Hankel transform given by

$$\int_o^{\infty} J_m(\alpha\rho)J_m(\beta\rho)\rho d\rho = \frac{\delta(\alpha-\beta)}{\alpha} \tag{27}$$

where $\delta(x)$ is the Dirac's delta function. We see from (21c) the spectral functions $\tilde{f}_{cm}(\xi) \sim \tilde{g}_{sm}(\xi)$ are represented in terms of $\tilde{K}_{\rho c}(\xi) \sim \tilde{K}_{\phi s}(\xi)$.

3.2. Derivation of the Expansion Coefficients

The equations for the expansion coefficients can be obtained by applying the remaining boundary condition that the tangential components of the electric field vanish on the disk. If we substitute (21c) into (20a) and (20b), we have the relations along with incident electric field, and we projected them into the functional space with elements $v^m_n(\rho_a^2)$ for E_ρ and $u^m_n(\rho_a^2)$ for E_ϕ, where $v^m_n(\rho_a^2)$ and $u^m_n(\rho_a^2)$ are the Jacobi's polynomials given in Appendix of the paper [34]. Then we obtain the matrix equations for the expansion coefficients $A_{mn} \sim D_{mn}$. The result is given by

$$\begin{aligned}\sum_{n=0}^{\infty}\left[A_{mn}Z^{(1,1)}_{mp,n} - B_{mn}Z^{(1,2)}_{mp,n}\right] &= H^{(1)}_{m,p},\\ \sum_{n=0}^{\infty}\left[A_{mn}Z^{(2,1)}_{mp,n} - B_{mn}Z^{(2,2)}_{mp,n}\right] &= H^{(2)}_{m,p}\end{aligned} \tag{28a}$$

$$\begin{aligned}\sum_{n=0}^{\infty}\left[C_{mn}Z^{(1,1)}_{mp,n} + D_{mn}Z^{(1,2)}_{mp,n}\right] &= K^{(1)}_{m,p},\\ \sum_{n=0}^{\infty}\left[C_{mn}Z^{(2,1)}_{mp,n} + D_{mn}Z^{(2,2)}_{mp,n}\right] &= K^{(2)}_{m,p}\\ m = 1,2,3,\ldots; \quad p &= 0,1,2,3,\ldots\end{aligned} \tag{28b}$$

$$\sum_{n=0}^{\infty} B_{0n} Z_{0p,n}^{(1,2)} = H_{0,p}^{(1)}, \quad \sum_{n=0}^{\infty} C_{0n} Z_{0p,n}^{(2,1)} = K_{0,p}^{(2)} \quad p = 0, 1, 2, 3, \ldots \tag{28c}$$

These equations may be conveniently represented in terms of the matrices.

$$\left[Z_m^{(1,1)}\right]\left[A_m\right]-\left[Z_m^{(1,2)}\right]\left[B_m\right]=\left[H_m^{(1)}\right], \quad \left[Z_m^{(2,1)}\right]\left[A_m\right]-\left[Z_m^{(2,2)}\right]\left[B_m\right]=\left[H_m^{(2)}\right] \tag{29a}$$

$$\left[Z_m^{(1,1)}\right]\left[C_m\right]+\left[Z_m^{(1,2)}\right]\left[D_m\right]=\left[K_m^{(1)}\right], \quad \left[Z_m^{(2,1)}\right]\left[C_m\right]+\left[Z_m^{(2,2)}\right]\left[D_m\right]=\left[K_m^{(1)}\right] \tag{29b}$$

The matrix elements $Z_m^{(1,1)} \sim Z_m^{(2,2)}$ are defined in [34]. The functions in the right hand side of the above equations which differ from plane wave incidence are given as follows.

(i): ρ-Directed Dipole

$$H_{m,p}^{(1)} = -\frac{j}{2\pi\kappa a^2}\epsilon_m \int_o^{\infty} \sqrt{\kappa^2-\alpha^2} J_m'(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p+\frac{1}{2}}(\alpha)-\left(\alpha_p^m+3\right)$$

$$J_{m+2p+\frac{5}{2}}(\alpha)\Big]\times\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha - j\frac{\kappa}{2\pi a^2}\epsilon_m\cdot m\int_o^{\infty}\frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}$$

$$\frac{m}{\alpha\rho_{0a}} J_m(\alpha\rho_{0a})\left[J_{m+2p+\frac{1}{2}}(\alpha)+J_{m+2p+\frac{5}{2}}(\alpha)\right]\alpha^{-\frac{1}{2}}d\alpha \tag{30a}$$

$$H_{m,p}^{(2)} = -\frac{j}{\pi\kappa a^2}\cdot m\int_o^{\infty}\sqrt{\kappa^2-\alpha^2} J_m'(\alpha\rho_{0a})\left[J_{m+2p-\frac{1}{2}}(\alpha)+J_{m+2p+\frac{3}{2}}(\alpha)\right]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha - \frac{j\kappa}{\pi a^2}\int_o^{\infty}\frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})$$

$$\left[\alpha_p^m J_{m+2p-\frac{1}{2}}(\alpha)-(\alpha_p^m+1)J_{m+2p+\frac{3}{2}}(\alpha)\right]\alpha^{\frac{1}{2}}d\alpha \tag{30b}$$

$$H_{0,p}^{(1)} = \frac{j}{2\pi\kappa a^2}\int_o^{\infty}\sqrt{\kappa^2-\alpha^2} J_1(\alpha\rho_{0a})\left[-pJ_{2p+\frac{1}{2}}(\alpha)+(p+1.5)J_{2p+\frac{5}{2}}(\alpha)\right]$$

$$\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha \tag{30c}$$

(ii): ϕ-Directed Dipole

$$K_{m,p}^{(1)} = -\frac{j}{\pi\kappa a^2}\int_o^\infty \sqrt{\kappa^2-\alpha^2}\,\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p+\frac{1}{2}}(\alpha)-\big(\alpha_p^m+3\big)$$

$$J_{m+2p+\frac{5}{2}}(\alpha)\Big]\times\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha-\frac{j\kappa}{\pi a^2}\cdot m\int_o^\infty\frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}$$

$$J_m'(\alpha\rho_{0a})\Big[J_{m+2p+\frac{1}{2}}(\alpha)+J_{m+2p+\frac{5}{2}}(\alpha)\Big]\alpha^{-\frac{1}{2}}d\alpha \qquad (31a)$$

$$K_{m,p}^{(2)} = -\frac{j}{\pi\kappa a^2}\cdot m\int_o^\infty\sqrt{\kappa^2-\alpha^2}\,\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big[J_{m+2p-\frac{1}{2}}(\alpha)+J_{m+2p+\frac{3}{2}}(\alpha)\Big]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha-\frac{j\kappa}{\pi a^2}\int_o^\infty\frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}J_m'(\alpha\rho_{0a})$$

$$\Big[\alpha_p^m J_{m+2p-\frac{1}{2}}(\alpha)-(\alpha_p^m+1)J_{m+2p+\frac{3}{2}}(\alpha)\Big]\alpha^{\frac{1}{2}}d\alpha \qquad (31b)$$

$$K_{0p}^{(2)} = \frac{j\kappa}{2\pi a^2}\int_o^\infty\frac{\exp(-jh_a z_{0a})}{\sqrt{\kappa^2-\alpha^2}}J_1(\alpha\rho_{0a})\Big[-pJ_{2p-\frac{1}{2}}(\alpha)+(p+0.5)$$

$$J_{2p+\frac{3}{2}}(\alpha)\Big]\alpha^{\frac{1}{2}}d\alpha \qquad (31c)$$

(iii): z-Directed Dipole

$$H_{m,p}^{(1)} = \frac{1}{\pi\kappa a^2}\int_o^\infty J_m(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p+\frac{1}{2}}(\alpha)-\big(\alpha_p^m+3\big)J_{m+2p+\frac{5}{2}}(\alpha)\Big]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \qquad (32a)$$

$$H_{m,p}^{(2)} = \frac{1}{\pi\kappa a^2}\cdot m\int_o^\infty J_m(\alpha\rho_{0a})\Big[J_{m+2p-\frac{1}{2}}(\alpha)+J_{m+2p+\frac{3}{2}}(\alpha)\Big]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{3}{2}}d\alpha \qquad (32b)$$

$$H_{0,p}^{(1)} = \frac{1}{2\pi\kappa a^2}\int_o^\infty J_0(\alpha\rho_{0a})\Big[-pJ_{2p+\frac{1}{2}}(\alpha)+(p+1.5)J_{2p+\frac{5}{2}}(\alpha)\Big]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \qquad (32c)$$

3.3. Far Field Expressions

The far field expressions can be derived by applying the stationary phase method of integration. Vector potential given in (19) can be written in the form

$$I_{nt} = \int_{o}^{\infty} \tilde{P}(\xi) J_m(\rho_a \xi) \exp\left[-\sqrt{\xi^2 - \kappa^2} z_a\right] \xi^{-1} d\xi \tag{33a}$$

and after application of method, the following result is produced

$$I_{nt} = \exp\left(j\frac{m+1}{2}\pi\right) \frac{\exp(-j\kappa R_a)}{\kappa R_a} \tilde{P}(\kappa \sin\theta) \frac{\cos\theta}{\sin^2\theta} \tag{33b}$$

This formula is applied to the vector potential given in (19) and then the far electromagnetic field is obtained from vector potential via the relation

$$E_\theta = -j\omega A_\theta = j\omega \sin\theta A_z, \quad E_\phi = jZ_0 \omega \sin\theta F_z \tag{34a}$$

and these are written as

$$E_\theta = Z_0 \frac{\exp(-jkR)}{kR} D_\theta(\theta, \phi), \quad E_\phi = Z_0 \frac{\exp(-jkR)}{kR} D_\phi(\theta, \phi) \tag{34b}$$

where

$$\begin{aligned} D_\theta(\theta, \phi) =& -k^2 a^2 \cos\theta \sum_{n=0}^{\infty} B_{0n} J_{2n+\frac{5}{2}}(\kappa \sin\theta)(\kappa \sin\theta)^{-\frac{3}{2}} + \frac{j}{2} k^2 a^2 \cos\theta \\ & \sum_{m=1}^{\infty} j^{m+1} \sum_{n=0}^{\infty} \Big\{ \Big[A_{mn} \Xi_{mn}^{+}(\kappa \sin\theta) - B_{mn} \Gamma_{mn}^{-}(\kappa \sin\theta) \Big] \cos m\phi \\ & + \Big[C_{mn} \Xi_{mn}^{+}(\kappa \sin\theta) + D_{mn} \Gamma_{mn}^{-}(\kappa \sin\theta) \Big] \sin m\phi \Big\} \end{aligned} \tag{34c}$$

$$\begin{aligned} D_\phi(\theta, \phi) =& k^2 a^2 \sum_{n=0}^{\infty} C_{0n} J_{2n+\frac{3}{2}}(\kappa \sin\theta)(\kappa \sin\theta)^{-\frac{1}{2}} + \frac{j}{2} k^2 a^2 \\ & \sum_{m=1}^{\infty} j^{m+1} \sum_{n=0}^{\infty} \Big\{ \Big[C_{mn} \Xi_{mn}^{-}(\kappa \sin\theta) + D_{mn} \Gamma_{mn}^{+}(\kappa \sin\theta) \Big] \cos m\phi \\ & + \Big[-A_{mn} \Xi_{mn}^{-}(\kappa \sin\theta) + B_{mn} \Gamma_{mn}^{+}(\kappa \sin\theta) \Big] \sin m\phi \Big\} \end{aligned} \tag{34d}$$

3.4. Physical Optics Approximate Solution

Since we assume that the dipole is located at $\phi_0 = 0$, ρ-directed and ϕ-directed dipoles are the x-dierected and y-directed. In this section we consider an approximate solution for the spherical field scattered by a disk. The current density induced on the disk and far field radiated from the current density are obtained as follows.

(i): $\boldsymbol{x}$-Directed Dipole

$$J_x = -\frac{I_x}{2\pi}\left[jk+\frac{1}{R_p}\right]\frac{z_0}{R_p^2}\exp(-jkR_p) \tag{35a}$$

$$A_x = -\frac{\mu I_x}{8\pi^2 R_0}\exp(-jkR_0)\int_S\left[jk+\frac{1}{R_p}\right]\frac{z_0}{R_p^2}\exp(-jkR_p) \times\exp\Big[jk\sin\theta\Big(x'\cos\phi+y'\sin\phi\Big)\Big]dx'dy' \tag{35b}$$

where I_x is the strength of the dipole current; (x', y') are the rectangular coordinates of the point on the disk; (θ, ϕ) are the spherical angular coordinates of the observation point; R_0 is the distance of the observation point from the center of the disk; R_p is the distance between the source point and the point on the disk. R_0 and R_p are given by

$$R_0 = \sqrt{x^2+y^2+z^2}, \qquad R_p = \sqrt{(x'-x_0)^2+(y'-y_0)^2+z_0^2} \tag{35c}$$

(ii): $\boldsymbol{y}$-Directed Dipole

$$J_y = -\frac{I_y}{2\pi}\left[jk+\frac{1}{R_p}\right]\frac{z_0}{R_p^2}\exp(-jkR_p) \tag{36a}$$

$$A_y = -\frac{\mu I_y}{8\pi^2 R_0}\exp(-jkR_0)\int_S\left[jk+\frac{1}{R_p}\right]\frac{z_0}{R_p^2}\exp(-jkR_p) \times\exp\Big[jk\sin\theta\Big(x'\cos\phi+y'\sin\phi\Big)\Big]dx'dy' \tag{36b}$$

(iii): $\boldsymbol{z}$-Directed Dipole

$$J_x = -\frac{I_z}{2\pi}\left[jk+\frac{1}{R_p}\right]\frac{x'-x_0}{R_p^2}\exp(-jkR_p) \tag{37a}$$

$$J_y = -\frac{I_z}{2\pi}\left[jk+\frac{1}{R_p}\right]\frac{y'-y_0}{R_p^2}\exp(-jkR_p) \tag{37b}$$

$$A_x = -\frac{\mu I_z}{8\pi^2 R_0}\exp(-jkR_0)\int_S\left[jk+\frac{1}{R_p}\right]\frac{x'-x_0}{R_p^2}\exp(-jkR_p) \times\exp\Big[jk\sin\theta\Big(x'\cos\phi+y'\sin\phi\Big)\Big]dx'dy' \tag{37c}$$

$$A_y = -\frac{\mu I_z}{8\pi^2 R_0} \exp(-jkR_0) \int_S \left[jk + \frac{1}{R_p}\right] \frac{y' - y_0}{R_p^2} \exp(-jkR_p) \times \exp\left[jk \sin\theta \left(x' \cos\phi + y' \sin\phi\right)\right] dx' dy' \tag{37d}$$

In contrast to the case of plane wave incidence, the numerical computation of the PO results are not easy. We need double numerical integration. Far field is obtained from the relation $E_\theta = -j\omega A_\theta$ and $E_\phi = -j\omega A_\phi$ with $A_\theta = A_x \cos\theta \cos\phi + A_y \cos\theta \sin\phi$ and $A_\phi = -A_x \sin\phi + A_y \cos\phi$.

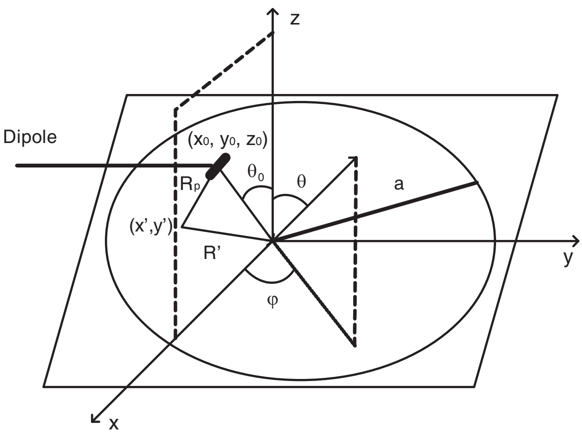

Figure 2. Scattering of an arbitrarily oriented dipole field by a circular hole.

4. THE EXPRESSIONS FOR THE FIELDS DIFFRACTED BY A CIRCULAR HOLE IN A PERFECTLY CONDUCTING PLATE

Figure 2 is a complementary problem of dipole field scattering by a perfectly conducting disk discussed in the previous section.

4.1. Dual Integral Equations for the Spectrum Functions

The diffracted field can be derived from the vector potentials which have the same form as (19). The boundary conditions are that the tangential components of the diffracted electric field $\mathbf{E}_t^d$ vanish in the $z = 0$ plane for $\rho_a \geq 1$, and the tangential components of the total magnetic field $\mathbf{H}_t^{\text{total}}$ are continuous on the hole. The former condition gives

$$\begin{bmatrix} E^d_{\rho c,m}(\rho_a) \\ E^d_{\phi s,m}(\rho_a) \end{bmatrix} = \int_o^\infty \left[H^-(\xi\rho_a)\right] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{f}_{cm}(\xi)\xi^{-1} \\ \tilde{g}_{sm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi$$

$$= \int_o^\infty \left[H^-(\xi\rho_a)\right] \begin{bmatrix} \tilde{E}_{\rho c,m}(\xi) \\ \tilde{E}_{\phi s,m}(\xi) \end{bmatrix} \xi d\xi = 0 \quad \rho_a \geq 1 \quad (38a)$$

$$\begin{bmatrix} E^d_{\rho s,m}(\rho_a) \\ E^d_{\phi c,m}(\rho_a) \end{bmatrix} = \int_o^\infty \left[H^+(\xi\rho_a)\right] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{f}_{sm}(\xi)\xi^{-1} \\ \tilde{g}_{cm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi$$

$$= \int_o^\infty \left[H^+(\xi\rho_a)\right] \begin{bmatrix} \tilde{E}_{\rho s,m}(\xi) \\ \tilde{E}_{\phi c,m}(\xi) \end{bmatrix} \xi d\xi = 0 \quad \rho_a \geq 1 \quad (38b)$$

From the above equations we have the relations between the spectrum functions $\tilde{f}_{cm}(\xi) \sim \tilde{g}_{sm}(\xi)$ and the spectrum functions of the surface field $\tilde{\mathbf{E}}(\xi)$ as given below

$$\begin{array}{ll} \tilde{f}_{cm}(\xi)\xi^{-1} = \frac{1}{j\sqrt{\xi^2-\kappa^2}}\tilde{E}_{\rho c,m}(\xi) & \tilde{g}_{sm}(\xi)\xi^{-1} = \tilde{E}_{\phi s,m}(\xi) \\ \tilde{f}_{sm}(\xi)\xi^{-1} = \frac{1}{j\sqrt{\xi^2-\kappa^2}}\tilde{E}_{\rho s,m}(\xi) & \tilde{g}_{cm}(\xi)\xi^{-1} = \tilde{E}_{\phi c,m}(\xi) \end{array} \quad (38c)$$

And from the continuity of the tangential components of the magnetic field on the aperture we have

$$Y_0 \int_o^\infty \left[H^-(\xi\rho_a)\right] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{g}_{cm}(\xi)(\kappa\xi)^{-1} \\ -\kappa\tilde{f}_{sm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi + \begin{bmatrix} H^i_{\rho c,m}(\rho_a) \\ H^i_{\phi s,m}(\rho_a) \end{bmatrix} = 0$$

$$0 \leq \rho_a \leq 1 \quad (39a)$$

$$Y_0 \int_o^\infty \left[H^+(\xi\rho_a)\right] \begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\tilde{g}_{sm}(\xi)(\kappa\xi)^{-1} \\ -\kappa\tilde{f}_{cm}(\xi)\xi^{-1} \end{bmatrix} \xi d\xi + \begin{bmatrix} H^i_{\rho s,m}(\rho_a) \\ H^i_{\phi c,m}(\rho_a) \end{bmatrix} = 0$$

$$0 \leq \rho_a \leq 1 \quad (39b)$$

where the kernel functions $H^{\pm}(\xi\rho_a)$ are defined by (22). The functions $H^i_{\rho c,m}$ and $H^i_{\rho s,m}$ denote the coefficients of $\cos m\phi$ and $\sin m\phi$ parts of the incident wave H^i_ρ, respectively, and same is true for $H^i_{\phi c,m}$ and $H^i_{\phi s,m}$. The explicit expressions of these functions are given in (9), (11) and (13) for ρ-directed, ϕ-directed and z-directed dipole respectively.

The aperture electric field can be expanded in a manner similar to the disk problem given in (24a) and (24b). It is noted that $(E_\rho,\, E_\phi)$ satisfy the vector Helmholtz equation $\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$ in circular cylindrical coordinates. Furthermore (E_ρ, E_ϕ) have the property $E_\rho \sim (1-\rho_a^2)^{-\frac{1}{2}}$ and $E_\phi \sim (1-\rho_a^2)^{\frac{1}{2}}$ near the edge of the hole. By taking into these facts, we set

$$\begin{aligned}
E_{\rho c,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[C_{mn}F_{mn}^{+}(\rho_a) + D_{mn}G_{mn}^{-}(\rho_a)\Big],\\
E_{\rho s,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[A_{mn}F_{mn}^{+}(\rho_a) - B_{mn}G_{mn}^{-}(\rho_a)\Big]\\
E_{\phi s,m}(\rho_a) &= -\sum_{n=0}^{\infty}\Big[C_{mn}F_{mn}^{-}(\rho_a) + D_{mn}G_{mn}^{+}(\rho_a)\Big],\\
E_{\phi c,m}(\rho_a) &= \sum_{n=0}^{\infty}\Big[A_{mn}F_{mn}^{-}(\rho_a) - B_{mn}G_{mn}^{+}(\rho_a)\Big]
\end{aligned} \tag{40}$$

where $A_{mn} \sim D_{mn}$ are the expansion coefficients and are to be determined from the remaining boundary condition (39). We have allocated these coefficients so that they satisfy the same Equation (28). The functions $F_{mn}^{\pm}(\rho_a)$ and $G_{mn}^{\pm}(\rho_a)$ are defined in (25a), (25b). Then the corresponding spectrum functions can be derived by applying the vector Hankel transform given by

$$\begin{aligned}
\begin{bmatrix}\tilde{E}_{\rho c,m}(\xi)\\ \tilde{E}_{\phi s,m}(\xi)\end{bmatrix} &= \int_o^{\infty}\Big[H^{-}(\xi\rho_a)\Big]\begin{bmatrix}E_{\rho c,m}(\rho_a)\\ E_{\phi s,m}(\rho_a)\end{bmatrix}\rho_a d\rho_a,\\
\begin{bmatrix}\tilde{E}_{\rho s,m}(\xi)\\ \tilde{E}_{\phi c,m}(\xi)\end{bmatrix} &= \int_o^{\infty}\Big[H^{+}(\xi\rho_a)\Big]\begin{bmatrix}E_{\rho s,m}(\rho_a)\\ E_{\phi c,m}(\rho_a)\end{bmatrix}\rho_a d\rho_a
\end{aligned} \tag{41}$$

We substitute (40) into (41) and perform the integration, then the spectrum functions of the aperture electric field are determined. The result is

$$\begin{aligned}
\tilde{E}_{\rho c,m}(\xi) &= -\sum_{n=0}^{\infty}[C_{mn}\Xi_{mn}^{-}(\xi) + D_{mn}\Gamma_{mn}^{+}(\xi)]\\
\tilde{E}_{\phi s,m}(\xi) &= \sum_{n=0}^{\infty}[C_{mn}\Xi_{mn}^{+}(\xi) + D_{mn}\Gamma_{mn}^{-}(\xi)]\\
\tilde{E}_{\rho s,m}(\xi) &= \sum_{n=0}^{\infty}[A_{mn}\Xi_{mn}^{-}(\xi) - B_{mn}\Gamma_{mn}^{+}(\xi)]\\
\tilde{E}_{\phi c,m}(\xi) &= \sum_{n=0}^{\infty}[A_{mn}\Xi_{mn}^{+}(\xi) - B_{mn}\Gamma_{mn}^{-}(\xi)]
\end{aligned} \tag{42a}$$

for $m \geq 1$ and

$$\tilde{E}_{\rho c,0}(\xi) = -2\sum_{n=0}^{\infty} C_{0n} J_{2n+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}} \quad \tilde{E}_{\phi c,0}(\xi) = 2\sum_{n=0}^{\infty} B_{0n} J_{2n+\frac{5}{2}}(\xi)\xi^{-\frac{3}{2}} \tag{42b}$$

for $m = 0$, where $\Xi^{\pm}_{mn}$ and $\Gamma^{\pm}_{mn}$ are defined by (26c). From (38c) we can express the spectral functions $\tilde{f}_{cm}(\xi) \sim \tilde{g}_{sm}(\xi)$ of the vector potentials in terms of those of the aperture distribution. This means that the surface magnetic field can be expressed in terms of the spectrum functions of the surface electric field.

4.2. Derivation of the Expansion Coefficients

We substitute (38c) into (39a) and (39b) and the both sides of the resulting equations are projected into the functional space with elements $v^m_n(\rho^2_a)$ for H_ρ and $u^m_n(\rho^2_a)$ for H_ϕ. The functions $v^m_n(\rho^2_a)$ and $u^m_n(\rho^2_a)$ are the Jacobi's polynomials and the explicit expressions and their some properties are defined in [34]. Then we have the matrix equations for the expansion coefficients. The equations have the same form as those for disk problem given by (28) except that $H_{m,p}$ and $K_{m,p}$ in the right hand sides are interchanged. The expressions for $H_{m,p}$ and $K_{m,p}$ for the hole problem are given as follows.

(i): $\boldsymbol{\rho}$-Directed Dipole

$$\begin{aligned} K^{(1)}_{m,p} = & \frac{jZ_0}{2\pi a^2}\int_o^{\infty}\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big[\alpha^m_p J_{m+2p+\frac{1}{2}}(\alpha)-\left(\alpha^m_p+3\right)J_{m+2p+\frac{5}{2}}(\alpha)\Big] \\ & \times\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha-\frac{jZ_0}{2\pi a^2}m\int_o^{\infty}J'_m(\alpha\rho_{0a})\Big[J_{m+2p+\frac{1}{2}}(\alpha) \\ & +J_{m+2p+\frac{5}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha \end{aligned} \tag{43a}$$

$$\begin{aligned} K^{(2)}_{m,p} = & \frac{jZ_0}{2\pi a^2}\cdot m\int_o^{\infty}\frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big[J_{m+2p-\frac{1}{2}}(\alpha)+J_{m+2p+\frac{3}{2}}(\alpha)\Big] \\ & \exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha-\frac{jZ_0}{2\pi a^2}\int_o^{\infty}J'_m(\alpha\rho_{0a})\Big[\alpha^m_p J_{m+2p-\frac{1}{2}}(\alpha) \\ & -(\alpha^m_p+1)J_{m+2p+\frac{3}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \end{aligned} \tag{43b}$$

$$K^{(2)}_{0,p} = -\frac{jZ_0}{8\pi a^2}\int_o^{\infty}J_1(\alpha\rho_{0a})\Big[-pJ_{2p-\frac{1}{2}}(\alpha)+(p+0.5)J_{m+2p+\frac{3}{2}}(\alpha)\Big]$$

$$\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \tag{43c}$$

(ii): ϕ-Directed Dipole

$$\begin{aligned} H_{m,p}^{(1)} &= -\frac{jZ_0}{2\pi a^2}\int_o^\infty J'_m(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p+\frac{1}{2}}(\alpha) - \left(\alpha_p^m + 3\right) J_{m+2p+\frac{5}{2}}(\alpha)\Big] \\ &\quad \exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha + \frac{jZ_0}{2\pi a^2}\cdot m\int_o^\infty \frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a}) \\ &\quad \Big[J_{m+2p+\frac{1}{2}}(\alpha) + J_{m+2p+\frac{5}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha \end{aligned} \tag{44a}$$

$$\begin{aligned} H_{m,p}^{(2)} &= -\frac{jZ_0}{2\pi a^2}\cdot m\int_o^\infty J'_m(\alpha\rho_{0a})\Big[J_{m+2p-\frac{1}{2}}(\alpha) + J_{m+2p+\frac{3}{2}}(\alpha)\Big] \\ &\quad \exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha + \frac{jZ_0}{2\pi a^2}\int_o^\infty \frac{m}{\alpha\rho_{0a}}J_m(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p-\frac{1}{2}}(\alpha) \\ &\quad -(\alpha_p^m + 1)J_{m+2p+\frac{3}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \end{aligned} \tag{44b}$$

$$\begin{aligned} H_{0p}^{(1)} &= \frac{jZ_0}{8\pi a^2}\int_o^\infty J_1(\alpha\rho_{0a})\Big[-pJ_{2p+\frac{1}{2}}(\alpha) + (p+1.5)J_{2p+\frac{5}{2}}(\alpha)\Big] \\ &\quad \exp(-jh_a z_{0a})\alpha^{-\frac{1}{2}}d\alpha \end{aligned} \tag{44c}$$

(iii): z-Directed Dipole

$$\begin{aligned} K_{m,p}^{(1)} &= \frac{Z_0}{2\pi a^2}m\int_o^\infty \frac{1}{\sqrt{\alpha^2-\kappa^2}}J_m(\alpha\rho_{0a})\Big[J_{m+2p+\frac{1}{2}}(\alpha) + J_{m+2p+\frac{5}{2}}(\alpha)\Big] \\ &\quad \exp(-jh_a z_{0a})\alpha^{\frac{1}{2}}d\alpha \end{aligned} \tag{45a}$$

$$\begin{aligned} K_{m,p}^{(2)} &= \frac{Z_0}{2\pi a^2}\int_o^\infty \frac{1}{\sqrt{\alpha^2-\kappa^2}}J_m(\alpha\rho_{0a})\Big[\alpha_p^m J_{m+2p-\frac{1}{2}}(\alpha) \\ &\quad -(\alpha_p^m + 1)J_{m+2p+\frac{3}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{\frac{3}{2}}d\alpha \end{aligned} \tag{45b}$$

$$\begin{aligned} K_{0p}^{(2)} &= -\frac{Z_0}{8\pi a^2}\int_o^\infty \frac{1}{\sqrt{\alpha^2-\kappa^2}}J_0(\alpha\rho_{0a})\Big[-pJ_{2p-\frac{1}{2}}(\alpha) \\ &\quad +(p+0.5)J_{2p+\frac{3}{2}}(\alpha)\Big]\exp(-jh_a z_{0a})\alpha^{\frac{3}{2}}d\alpha \end{aligned} \tag{45c}$$

4.3. Far Field Expressions

Far field expression is obtained from (19) by applying the stationary phase method of integration. In a far region we have the relations $E_\theta \sim -j\omega A_\theta = j\omega \sin\theta A_z$ and $H_\theta \sim -j\omega F_\theta$ or $E_\phi = -j\omega \sin\theta F_z$, and we can write

$$E_\theta = \frac{\exp(-jkR)}{kR} D_\theta(\theta,\phi), \quad E_\phi = \frac{\exp(-jkR)}{kR} D_\phi(\theta,\phi) \tag{46a}$$

where

$$\begin{aligned} D_\theta(\theta,\phi) \;=\; & -2k^2a^2 \sum_{n=0}^{\infty} C_{0n} J_{2n+\frac{3}{2}}(\kappa\sin\theta)(\kappa\sin\theta)^{-\frac{1}{2}} \\ & +k^2a^2 \sum_{m=1}^{\infty} j^m \sum_{n=0}^{\infty} \Big\{ \Big[C_{mn}\Xi^-_{mn}(\kappa\sin\theta) + D_{mn}\Gamma^+_{mn}(\kappa\sin\theta) \Big] \\ & \cos m\phi + \Big[A_{mn}\Xi^-_{mn}(\kappa\sin\theta) - B_{mn}\Gamma^+_{mn}(\kappa\sin\theta) \Big] \sin m\phi \Big\} \end{aligned} \tag{46b}$$

$$\begin{aligned} D_\phi(\theta,\phi) \;=\; & 2k^2a^2\cos\theta \sum_{n=0}^{\infty} B_{0n} J_{2n+\frac{5}{2}}(\kappa\sin\theta)(\kappa\sin\theta)^{-\frac{3}{2}} \\ & +k^2a^2\cos\theta \sum_{m=0}^{\infty} j^m \sum_{n=0}^{\infty} \Big\{ \Big[A_{mn}\Xi^+_{mn}(\kappa\sin\theta) - B_{mn}\Gamma^-_{mn}(\kappa\sin\theta) \Big] \\ & \cos m\phi - \Big[C_{mn}\Xi^+_{mn}(\kappa\sin\theta) + D_{mn}\Gamma^-_{mn}(\kappa\sin\theta) \Big] \sin m\phi \Big\} \end{aligned} \tag{46c}$$

5. NUMERICAL COMPUTATION

With the formulation developed in the previous sections, we have performed the numerical computation for far field patterns. To perform the computation we must determine the expansion coefficients $A_{mn} \sim D_{mn}$ for given values of radius $\kappa = ka$ and incident angle θ_0. These are derived from (29) by using the standard numerical code of matrix equation for complex coefficients. The matrix elements contain the functions $G(\alpha, \beta)$, $G_2(\alpha, \beta)$, and $K(\alpha, \beta)$ and these infinite integrals are transformed into infinite series and these series are convenient for numerical computation. How to derive these expressions are firstly discussed by Nomura and Katsura [32], but we used a slightly different method as discussed in [34]. To verify the validity of these expressions numerically, we compare the results of series expressions with those by direct numerical integration. The agreement is complete. The required maximum size M of matrix equations to determine the expansion coefficients depends on the values of κ and we chose $M \simeq 1.6\kappa + 5$ and it is found to be sufficient in the present computation.

5.1. Radiation Pattern

The theoretical expressions for the far field are given by (34) for the disk and (46) for aperture problem. Fig. 3 to Fig. 11 show the far field patterns of circular disk in the ϕ-cut plane $\phi = 0, \pi$ in the presence of ρ-directed, ϕ-directed and z-directed dipole. The normalized radii are $\kappa = ka = 3$ and $ka = 5$, respectively. The dipole is placed at $\rho_0 = 5$. The plane of incidence is xz-plane ($\phi_0 = 0$, π). In all these figures, the normal incidence is for $\theta = 0$. In these figures the results obtained using the physical optics (PO) method are also included for comparison. The PO expressions are given by (35)–(37). It is observed from the comparison that the PO and KP results agree well for normal incidence ($\theta = 0$) but the degree of discrepancy increases as the angle of incidence becomes large and radius of disk decreases. It is due to the fact that the PO approximation inaccuracy increases for shadow region

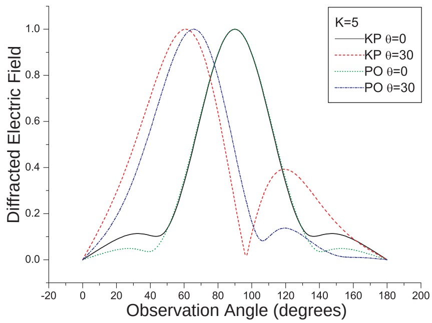

Figure 3. Diffracted far field patterns of disk for ρ-directed dipole.

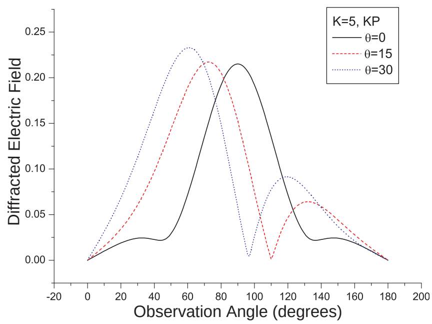

Figure 4. Diffracted far field patterns of disk for ρ-directed dipole.

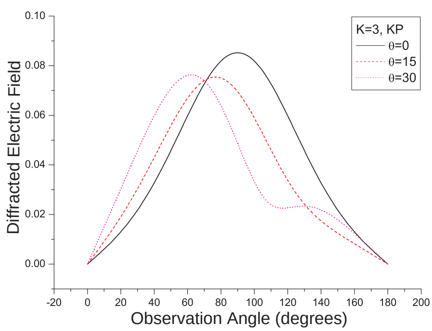

Figure 5. Diffracted far field patterns of disk for ρ-directed dipole.

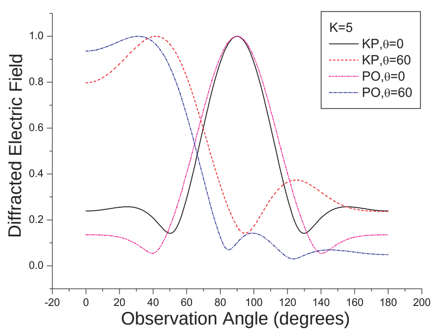

Figure 6. Diffracted far field patterns of disk for ϕ-directed dipole.

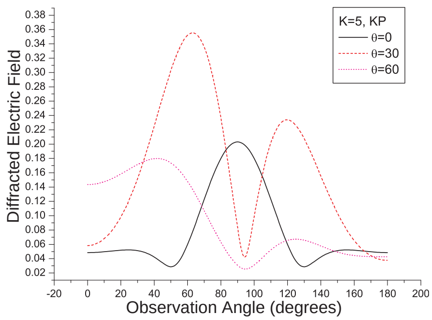

Figure 7. Diffracted far field patterns of disk for ϕ-directed dipole.

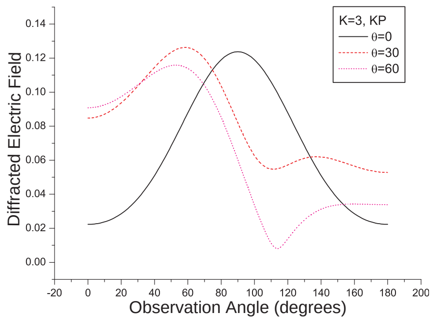

Figure 8. Diffracted far field patterns of disk for ϕ-directed dipole.

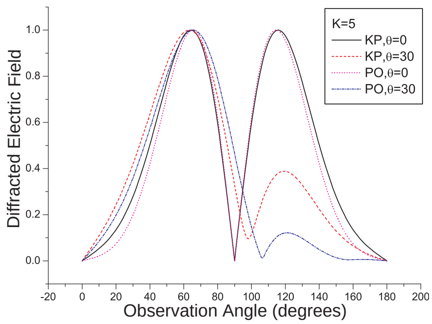

Figure 9. Diffracted far field patterns of disk for z-directed dipole.

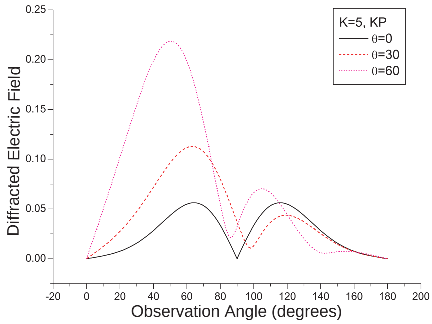

Figure 10. Diffracted far field patterns of disk for z-directed dipole.

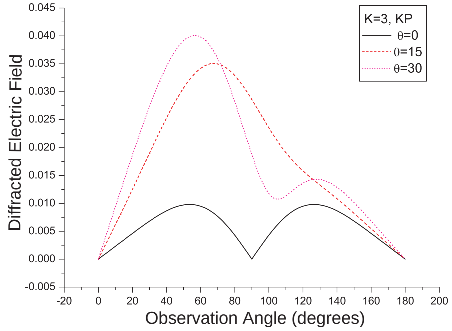

Figure 11. Diffracted far field patterns of disk for z-directed dipole.

contribution. Secondly PO gives good results for bigger objects(in terms of wavelength).

6. CONCLUSION

We have formulated the spherical wave field produced by an arbitrarily oriented dipole scattered by a perfectly conducting circular disk and its complementary circular hole in a perfectly conducting infinite plane. We derived dual integral equations for the induced current and the tangential components of the electric field on the disk. The equations for the current densities are solved by applying the discontinuous properties of the Weber-Schafheitlin's integrals and the vector Hankel transform. It is readily found that the solution satisfies Maxwell's equations and edge conditions. Therefore it may be considered as the eigen function expansion. The equations for the electric field are solved by applying the projection. We use the functional space of the Jacobi's polynomials. Thus the problem reduces to the matrix equations and their elements are given by infinite integrals of a single variables. These integrals are transformed into infinite series in terms of the normalized radius. Numerical computation for the far field patterns has been carried out for different values of κ and incident angles θ.

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