A HIGH RESOLUTION DOA ESTIMATING METHOD WITHOUT ESTIMATING THE NUMBER OF SOURCES

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Abstract—The performance of high resolution subspace-based algorithms are particularly sensitive to the prior information of the source number, the Signal-to-Noise Ratio (SNR) and the snapshot. Although the existing direction-of-arrival (DOA) estimation methods without estimating the source number could eliminate the awful impact brought by incorrect source number estimation, yet its performance would get deteriorated by small snapshots and low SNR. Methods which exploit noise and signal subspaces information simultaneously, such as SSMUSIC, could provide a high resolution performance in such nonideal circumstances. However, its performance would degrade severely when the prior information of the source number is incorrect. To provide a DOA estimation method without estimating the number of source, which has a high resolution performance in small sample and low SNR scenario, using all information spreads in eigenvalues and eigenvectors, this paper reconstructs a new spatial spectrum which is very similar to the SSMUSIC algorithm. In order to enhance the robustness of the new method, we provide an empirical method to modify the eigenvalues to prohibit the spreading of noise eigenvalues caused by snapshot deficient and low SNR. To verify the validity of the new method, comparisons with other algorithms are made in computer simulations and the measured data test.

1. INTRODUCTION

Direction-Of-Arrival (DOA) estimation is one of the most important research problems in various applications such as radar [1–3], sonar [4], communications [5–6], etc. Among various methods, subspace-based methods [7–10] have received widely attention because of their

relatively high resolution and computational simplicity. However, all subspace-based algorithms need the exact number of sources to separate signal subspace and noise subspace. In fact, the number of sources is unknown, which need to be provided by the source number estimation methods. Nevertheless, those source number estimation methods [11–14] also exist some shortcomings. If the models they use do not accord with the practical signal environment, the number of sources will be incorrectly estimated, making the performance of subspace-based algorithms deteriorate significantly. In order to eliminate the awful impact brought by source number estimation, on the one hand, we need to improve the accuracy of source number estimation method. On the other hand, it is necessary to develop DOA estimation method without estimating the source number [15–19].

ASPECT (Adaptive Signals Parameter Estimation and Classification Technique) algorithm [20] is based an adaptive rotation of an initial subspace to the point at which the initial subspace is rotated to until it coincides with the true signal subspace. In [15], the author connects the Pisarenko algorithm with the ASPECT principle to develop a new DOA estimation technique without estimating the source number. Due to that the Pisarenko algorithm only uses the eigenvector corresponding to the smallest noise eigenvalue, it is likely to produce many spurious peaks in its spatial spectrum, which brings about a high resolution as the multiple signal classification (MUSIC) algorithm [7]. After the spurious peaks removed by ASPECT projection, the real DOAs are obtained. Similar to [15], the method in [16] is also based on the ASPECT algorithm. These methods based on the ASPECT algorithm have a common drawback: it is likely to become invalid when the source number is larger than half of the number of sensors. Beamforming techniques can avoid estimating the number of sources, though it cannot provide high resolution as MUSIC. Through establishing some connections between MVDR (minimum variance distortionless response) beamformer [21] and MUSIC, some authors also find high resolution DOA estimation methods without estimating the source number [17–19]. However, these methods bring about new problems. The proposed algorithm in [17] requires generalized eigendecomposition for every direction. Although some methods to reduce computational complexity are recommended, the problem of high computational is still a burden. In the small snapshots and low Signal-to-Noise Ratio (SNR) circumstance, the proposed algorithm in [18, 19] is close to the Pisarenko algorithm, which is likely to produce many spurious peaks.

All these DOA estimation methods without estimating the source number could be classified to noise subspace method, for

the information they use is completely from noise subspace. In an ideal circumstance, they could get as high resolution as MUSIC. However, the resolution properties of these MUSIC-class algorithms are particularly sensitive to the SNR and the snapshots. Moreover, it is common to find scenarios of snapshot deficient and low SNR in practical engineering. In this case, the spatial spectrums [8–10] which exploit noise and signal subspaces information simultaneously have better resolution and higher robustness performance than MUSIC-class algorithms. At the same time, these methods require more accurate information of the source number than the MUSIC algorithm.

Facing these difficulties, the purpose of this paper is to provide a DOA estimation method without estimating the number of source, which has a high resolution and robust performance in the snapshot deficient and low SNR scenario. We exploit all the information obtained from eigendecomposition of the array covariance matrix to reconstruct a new spatial spectrum, which is very similar to the SSMUSIC (Signal Subspace Scaled MUSIC) algorithm [8]. In order to enhance the robustness of the new method, we provide an empirical method to modify the eigenvalues to prohibit the spreading of noise eigenvalues caused by snapshot deficient and low SNR.

The rest of this paper is organized as follows. The signal model is introduced, and several relevant algorithms are reviewed in Section 2. Section 3 presents the proposed method. To verify the validity of the proposed algorithm, computer simulations are conducted in Section 4 and test of practical engineering is made in Section 5. Finally, we make conclusions in Section 6.

2. SIGNAL MODEL AND RELEVANT ALGORITHMS

Consider a planer array with arbitrary geometry constituting of M sensors. Suppose that P $(1 \leq P \leq M - 1)$ independent narrowband signals $\{s_i(t)\}$ with center frequency f_0 are in the field far from the array and impinge on the array from distinct direction $\{\theta_i\}$. The received noisy signals can be expressed in a compact form as:

$$X(t) = \sum_{i=1}^{P} a(\theta_i)s_i(t) + n(t) = A(\theta)s(t) + n(t) \qquad (1)$$

where $X(t)$, $s(t)$, $n(t)$ are the vectors of the received signals, the incident signals and the additive noise, and A is $M \times P$ matrix $A(\theta) = [a(\theta_1), a(\theta_2), \ldots, a(\theta_M)]$. Here $a(\theta_i)$ is the steering vector of the array toward the direction θ_i and T denotes transpose.

Assume that signals and additive noises are stationary and ergodic zero mean complex valued random processes. In addition, the noises

are assumed to be uncorrelated with signals, uncorrelated between sensors, and to have identical variances σ^2 in each sensor. Under these assumptions, the covariance matrix R of array outputs is given by

$$R = E\left[x(t)x^H(t)\right] = AR_SA^H + \sigma^2 I \tag{2}$$

where $E[\bullet]$ denotes the statistical expectation, H denotes conjugate transpose, R_S is signal covariance matrix, I denotes the identity matrix. The eigendecomposition of matrix R yields

$$R = \sum_{i=1}^{M} \lambda_i u_i u_i^H \tag{3}$$

where λ_i and u_i are the ith eigenvalue and ith corresponding eigenvector, respectively. In the ideal environment, we have

$$\lambda_1 \geq \ldots \geq \lambda_P > \lambda_{P+1} = \cdots = \lambda_M = \sigma^2 \tag{4}$$

According to the multiplicity of each eigenvalue, R can be expressed as

$$R = [\ U_S \quad U_N\ ]\left[\begin{array}{cc} \Lambda_S & 0 \\ 0 & \sigma^2 I \end{array}\right][\ U_S \quad U_N\ ]^H \tag{5}$$

where Λ_S is a $P \times P$ diagonal matrix containing the P upper eigenvalues of R, U_S is an matrix that contains the eigenvectors corresponding to the upper eigenvalues, and U_N is an $(M-P) \times M$ matrix that contains the eigenvectors associated with the $M-P$ smallest eigenvalues. So the eigenvalues and eigenvectors of the covariance matrix R can be split into two sets that generate independent linear spaces: the signal subspace, generated by the columns of U_S, and the noise subspace, generated by the columns of U_N .

It is well known that the eigenvectors corresponding to the $M-P$ minimum eigenvalue are orthogonal to the columns of the matrix A. Namely, they are orthogonal to the steering vectors of the signals. All subspace-based methods are based on the property. Now we review some DOA estimation algorithms which are relevant to the proposed algorithm.

The spatial spectrum of the MUSIC algorithm is given as

$$P_{MUSIC}(\theta) = \frac{1}{A^H(\theta)U_N U_N^H A(\theta)} \tag{6}$$

Due to the orthogonality between signal subspace and noise subspace, a search for directions is made by looking for steering vectors that are as orthogonal to the noise subspace as possible. The estimation variance of MUSIC approaches the Cramer-Rao lower bound when SNR approaches infinity [22] .

In the standard form, the separation of signal subspace and noise subspace is based on the priori information offered by the source number estimation method. The noise subspace projection matrix $U_N U_N^H$ also could be expressed as:

$$U_N U_N^H = \sum_{i=P+1}^{M} u_i u_i^H; \tag{7}$$

Through the following equation,

$$\lim_{m \to \infty} \left(\frac{\lambda_M}{\lambda_i} \right)^m u_i u_i^H \cong \left\{ \begin{array}{ll} 0, & \text{for } i = 1, \ldots, P \\ u_i u_i^H, & \text{for } i = P+1, \ldots, M. \end{array} \right. \tag{8}$$

$U_N U_N^H$ can be approximately derived as

$$\lim_{m \to \infty} \sigma^{2m} R^{-m} \cong U_N U_N^H \tag{9}$$

Thus, the noise subspace projection matrix is obtained without knowing the source number. This is the principle involved in these so called "DOA estimation methods without estimating the source number" [18, 19]. However, All these methods could be classified to noise subspace methods for the information that they use is completely from noise subspace.

A different subspace-based method is presented in [8] under the name of SSMUSIC, which has shown better performance than traditional MUSIC in a finite sample regime. To deal with the subspace mismatches caused by small sample, the numerator in MUSIC is replaced by a signal subspace function in the SSMUSIC algorithm. Its spatial spectrum is given as

$$P_{SSMUSIC}(\theta) = \frac{A^H(\theta) \sum_{i=1}^{P} \frac{1}{\lambda_i - \sigma^2} u_i u_i^H A(\theta)}{A^H(\theta) U_N U_N^H A(\theta)} \tag{10}$$

In the sight of the subspace projection theory, a method under the name of SSM (Synthetic Spatial Spectrum) which is similar to SSMUSIC is proposed in [10]. Its spatial spectrum is given as

$$P_{SSM}(\theta) = \frac{A^H(\theta) \sum_{i=1}^{P} \frac{1}{\lambda_i} u_i u_i^H A(\theta)}{A^H(\theta) U_N U_N^H A(\theta)} \tag{11}$$

Although the weights of signal subspace projection are different in SSMUIC and SSM. However the performances of them are nearly the same, which can be observed in the latter simulation. In the other way, for the operation of SSMUIC and SSM, an accurate estimation of the source number is very critical.

3. PROPOSED ALGORITHMS

In this paper, we propose a new algorithm to realize DOA estimation without estimating the source number through spatial spectrum reconstruction. When the number of sources is unknown, although we cannot know the place of the least signal eigenvalue, yet λ_1 corresponds to the signal eigenvalue on all accounts. From the simulation result of SSMUSIC and SSM, the weights of signal subspace projection could be changed slightly. Thus, a new spatial spectrum function similar to SSMUSIC is given as

$$P_{proposed}(\theta) = \frac{A^H(\theta)\left(\frac{1}{\lambda_1 - \sigma^2}u_1u_1^H + \sum_{i=2}^{P}\frac{1}{\lambda_i}u_iu_i^H\right)A(\theta)}{A^H(\theta)U_NU_N^HA(\theta)} \quad (12)$$

λ_M corresponds to the noise eigenvalue in any case. Under the assumption (4) in Section 2, we can easily derive an approximation of the noise subspace projection matrix from Equation (8):

$$U_NU_N^H \cong \lim_{m\to\infty}\sum_{i=2}^{M}\left(\frac{\lambda_M}{\lambda_i}\right)^m u_iu_i \quad (13)$$

According to the assumption (4) and Equation (13), we obtain

$$\frac{1}{\lambda_1 - \sigma^2}u_1u_1^H \cong \frac{1}{\lambda_1 - \lambda_M}u_1u_1^H \quad (14)$$

$$\sum_{i=2}^{P}\frac{1}{\lambda_i}u_iu_i^H \cong \sum_{i=2}^{M}\frac{1}{\lambda_i}u_iu_i^H - \frac{1}{\lambda_M}\lim_{m\to\infty}\sum_{i=2}^{M}\left(\frac{\lambda_M}{\lambda_i}\right)^m u_iu_i \quad (15)$$

Therefore, an approximate representation of the proposed spatial spectrum could be obtained based on Equations (12), (14)–(15). As the power m increases, the distinction between these functions decreases. However, all the derivations of Equations (9), (13)–(15) are based on the assumption (4). In fact, the eigenvalues obtained in the real circumstance are hardly satisfied with the assumption. Especially in the snapshot deficient and low SNR case, the spreading of noise eigenvalues is quite significant whereas all the noise eigenvalues converge to the finite value σ^2 in the ideal environment.

In order to overcome the unfavorable impact caused by the spreading of noise eigenvalues, we modify the eigenvalues obtained from the eigendecomposition of the matrix $\hat{R}$. After loading a proper value λ'' to all eigenvalues, we can make the new eigenvalues λ' meet:

$$\lambda_1' \geq \cdots \geq' \lambda_P' > \lambda_{P+1}' \approx \cdots \approx \lambda_M' \quad (16)$$

Although an optimal loading value is too difficult to provide, yet we could obatin an appropriate value through an empirical method. We define the loading value as:

$$\lambda'' = k \frac{\sum_{i=1}^{M} \lambda_i}{M} \qquad (17)$$

If k is too small, the spreading of the noise eigenvalues cannot be suppressed. If k is too large, the difference between signal eigenvalue and noise eigenvalue is not obvious, which also deteriorate the performance of the proposed method. So we impose restrictions on the loading value λ'':

$$\lambda_M < \lambda'' < \lambda_1 \qquad (18)$$

In [11], the floor of the normalized MUSIC spatial spectrum and the number of peaks are used as indicators of estimating the source number. Similarly, two important features are observed in simulation, which help us to find an appropriate k. Firstly, the floor of its normalized spatial spectrum is rather low when k is appropriate. Secondly, the floor of the spatial spectrum rises and its resolution improves as k decreases. According to the two features, an appropriate k could be obtained through testing the performance of the spatial spectrum as k decreases. The following is the detailed process.

When k select a large value, the algorithm would get a low resolution. Only the signals which are far apart could be resolved, appearing peaks in the corresponding signal directions. While the others which are close to each other could not be distinguished, only a peak would be formed in the middle of the directions. As k decreases, when the improving resolution of the algorithm makes the signals which have not been previously distinguished resolve, the spatial spectrum in the corresponding signal directions becomes sharp and form peaks. At the same time, the floor of the spatial spectrum rises as k decreases, increasing the possibility of producing spurious peaks. During this variation, the locations of those initial peaks is a important feature to distinguish whether those new peaks are real or not. If the locations where are far away from the initial peaks produce peaks, those peaks can be regard as spurious peaks. To better distinguish real peaks and spurious peaks, a moderate floor threshold value L is selected. When the spatial spectrum floor exceeds the threshold and the spurious peaks occur, the test of lowering k should be stopped immediately. Then, considering the spatial spectrum floor and the resolution of closely signals, select an appropriate k and its corresponding spatial spectrum.

For simplicity, a large m is selected before finding k. Summarizing the whole analysis, we implement the proposed method as follows.

Step 1) Select a large value for m and select a moderate value for threshold L. Based on Equations (17)–(18), a set of k corresponding to different level is provided.

Step 2) Compute the normalized spatial spectrum based on Equations (12)–(15) as k decreases. When the spatial spectrum floor exceeds the threshold and spurious peak occurs, stop decreasing k.

Step 3) Compare the spatial spectrum floor and its resolution of closely signals, select an appropriate k and its corresponding spatial spectrum.

4. COMPUTER SIMULATIONS

To illustrate the proposed method is suitable in an arbitrary array, we use an arc array in the simulation. Assuming a uniform circular array with 40 sensors, its radius is 150 meters. The arc array is consisted by 16 sequential sensors of the uniform circular array. The center frequency of signals is 10 MHz. The Rayleigh resolution limit in this case is about $5°$. Due to that the performances of those conventional DOA estimation methods without knowing source number is no better than that of MUSIC, and many tests about the awful impact brought by incorrect source number estimation have appeared in [11, 15, 17, 19], the computer simulation only compares the proposed algorithm, SSMUIC, SSM and MUSIC (assuming the number of sources is estimated accurately). To calculate the resolution probability and the estimation Root-Mean-Square-Error (RMSE), every single experiment has computed 500 times. Regarding to two closely spaced signals in a single experiment, if the estimated DOAs satisfy:

$$\left|\hat{\theta}_1 - \theta_1\right| + \left|\hat{\theta}_2 - \theta_2\right| < \left|\hat{\theta}_1 - \hat{\theta}_2\right| \tag{19}$$

we define the trial of angle separation is successful. On this basis, the RMSE is calculated as:

$$RMSE(\theta) = \sqrt{\frac{1}{K} \sum_{i=1}^{K} \left(\hat{\theta}^i - \theta\right)^2} \tag{20}$$

where K is the number of successful trials in the Mento-Carlo simulation, θ is the true DOA and $\hat{\theta}^i$ represents the estimated DOA of the ith trial.

4.1. The Implement of the Proposed Algorithm

Three uncorrelated narrowband sources with the same SNR of 10 dB are assumed to impinge from $66°$, $67.8°$ and $85°$. The snapshots is 40.

$m = 10$. A set of k is provided as $[10, 2.5, 0.5, 0.25, 0.08]$ which is based on Equation (18). It is observed from Fig. 1 that the resolution of two closely spaced signals improves as k decreases. At the same time, the floor of the spatial spectrum rises which increases the possibility of producing spurious peaks. The threshold value L is selected as $-15\,\mathrm{dB}$. When k is 0.08, the floor of the spectrum has exceeded the threshold seriously so that many spurious peaks occur. So k should not be smaller than 0.08. To obtain a moderate floor of the spectrum and resolution, we prefer k as 0.25.

4.2. Comparisons with SSMUIC, SSM and MUSIC

Figure 2 shows the spatial spectrums of these algorithms. In this simulation, the DOAs are fixed at $66°$, $67.8°$ and $85°$, the SNR is

(a) the whole spatial spectrums   (b) performances of two closely spaced signals
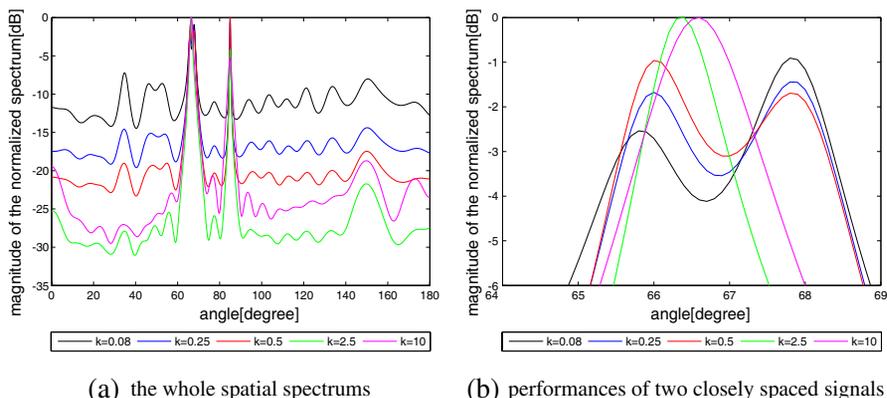
Figure 1. The normalized spatial spectrums of different loading value.
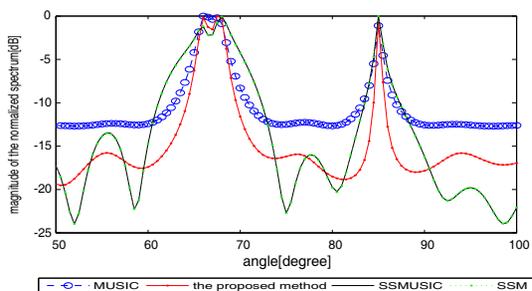
Figure 2. Direction finding comparison, $k = 0.25$.

10 dB and the snapshots is 40. It is observed that only the MUSIC algorithm cannot resolve 66° and 67.8° , which appears a peak around 66°. The performance of SSMUIC and SSM are nearly the same. It is notable that the floor of the proposed method is much lower than that of MUSIC when a proper loading value is selected.

Figure 3 shows the resolution performances of two closely spaced signals against separation angle. In this simulation, one DOA is fixed at 66°, while the other varied with separation angle. Assume the probability of 90% as a resolution threshold. The proposed method begins to distinguish two signals at 1.5° whereas the MUSIC algorithm

(a) Resolution probability   (b) RMSE
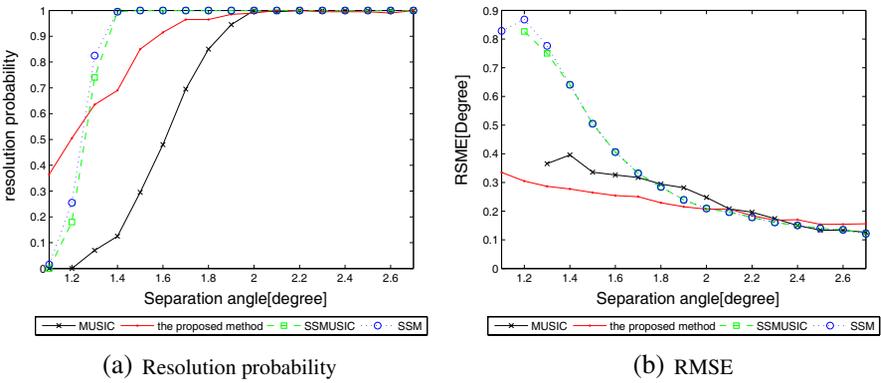
Figure 3. The resolution performances against separation angle with the SNR fixed at 10 dB and the snapshots fixed at 40, $k = 0.25$.

(a) Resolution probability   (b) RMSE
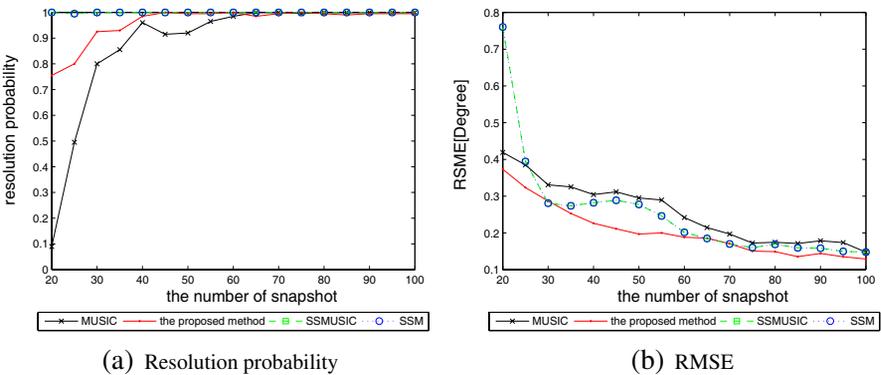
Figure 4. The resolution performances against snapshot with the angle separation fixed at 1.8° and the SNR fixed at 10 dB, $k = 0.25$.

starts at $1.8°$. Although the resolution of the proposed method is weaker than SSMUSIC and SSM, which distinguish two signals at $1.3°$. However, its resolution is evidently better than the MUSIC algorithm. Besides, the RSME of the proposed method is the smallest in the four methods, which indicates its estimating result is quite reliable.

Figures 4 and 5 show the resolution performances of two closely spaced signals against snapshot and SNR, respectively. It is observed that the performances of all the algorithms are ameliorated as the SNR and the snapshot increase. However, the performance of MUSIC is rather poor in the snapshot deficient and low SNR scenario while the proposed method, SSM and SSMUSIC perform quite robust.

(a) Resolution probability

(b) RMSE
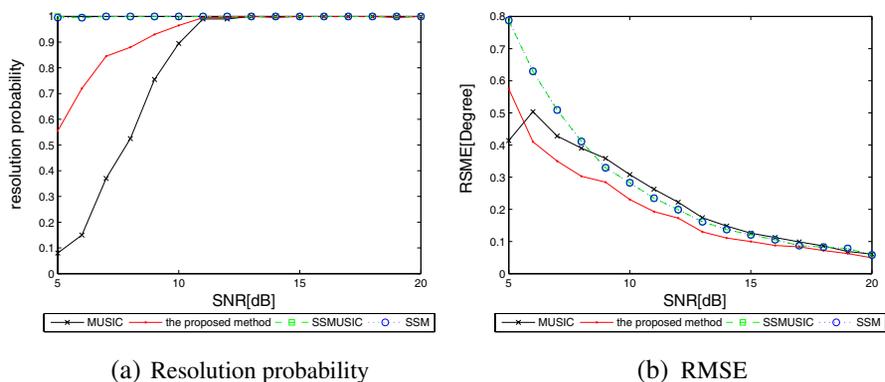
Figure 5. The resolution performances against SNR with the angle separation fixed at $1.8°$ and the snapshot fixed at 40, $k = 0.25$.

(a) Spatial spectrums of different loading value

(b) Comparisons with SSMUIC, SSM
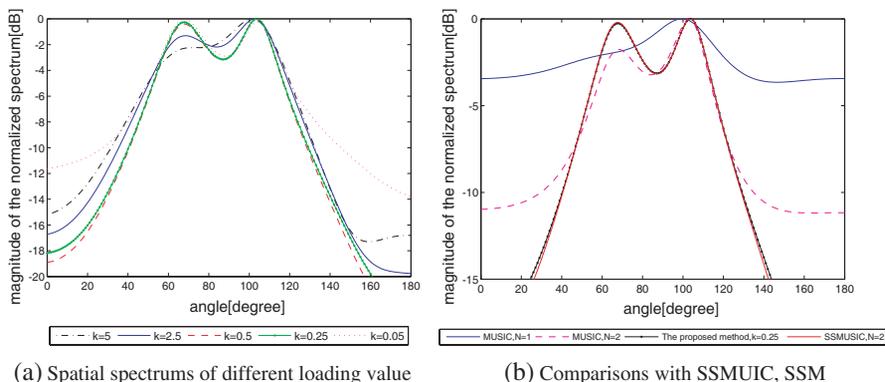
Figure 6. Range cell $= 11$, current velocity $= 18.13\,\text{cm/s}$, $m = 10$.

5. TEST OF THE MEASURED DATA

In this paper, the measured data comes from the HF ground wave
radar [11, 23, 24] experiments by Wuhan University. The receiving
array consists of eight sensors, which are uniformly spaced in two rows.
The radar operates at 7.5 MHz. The Rayleigh resolution limit of this
array is about 42°. The actual snapshot obtained after sliding window
preprocessing is 21. Due to the complexity of estimating the number
of ocean current, the traditional method introduced in [11, 23] needs to
test the spectra of MUSIC while the source number gradually increases
from 1 to 7.

   A set of k is provided as [10, 2.5, 0.5, 0.25, 0.05] which is based on
Equation (18) . After many tests of the measured datas, the threshold
value L in the measured data is selected as -12 dB. It is observed from
Fig. 6 that the floor of the spectrum and resolution become moderate
when k decreases to 0.25. From the performances of MUSIC while the
source number increases from 1 to 2, it is easily to infer that the real
number of sources is 2. While the source number is 2 and $k = 0.25$,
the results of the proposed method, MUSIC and SSMUSIC are nearly
the same.

   From this example, we know the DOAs can be directly estimated
by the proposed method whereas MUSIC and SSMUSIC need to know
source number firstly.

6. CONCLUSION

In this paper, we propose a simple but high resolution DOA estimating
method, the validity of which has been proven in computer simulations
and the measured data test. It not only has a higher and more reliable
performance than MUSIC, but also avoids estimating the sources
number before the estimation of DOA. To prohibit the spreading
of noise eigenvalues caused by snapshot deficient and low SNR and
enhance the robustness of the method, an empirical method to modify
the eigenvalues is proposed. Although the validity of this empirical
method has been proven in simulation, a further study to simplify
the proposed method is essential in the future work. For further
improvement, it can be combined with other techniques such as
Beamspace [25, 26], ASPECT [20] to get better and more robust
performance.

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