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A SEMI-ANALYTICAL METHOD FOR THE DESIGN OF COIL-SYSTEMS FOR HOMOGENEOUS MAGNETO-STATIC FIELD GENERATION

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Abstract—This paper proposes a simple semi-analytical method for designing coil-systems for homogeneous magnetostatic field generation. The homogeneity of the magnetic field and the average magnitude of the magnetic flux density inside of the volume of interest are the objective functions chosen for the selection of the coil-system geometry (size and location), number of coils and the number of turns of each winding. The spatial distribution of the magnetostatic field is estimated superposing the magnetic induction numerically computed from the analytical expression of the magnetic field generated by each coil, obtained using the Biot-Savart's law and the current filament method. The homogeneous magnetic field is synthesized using an iterative algorithm based on TABU search with geometric constraints, which varies the design parameters of the windings to meet the requirements. The number of turns of each coil and gauge of wire used for the windings is adjusted automatically in order to achieve the target average magnitude of the magnetic induction under the constraints imposed by power consumption. This method was used to design a coil arrangement that can generate up to 10 mT within a volume $0.5\,\mathrm{m}\times0.5\,\mathrm{m}\times1\,\mathrm{m}$ with 99% of spatial homogeneity, with square loops of length less than or equal to 1.5 m, and with a power dissipated by Joule effect less than or equal to 1 W per coil. The synthesized magnetic field distribution was validated using Finite Element Method simulation, showing a good correspondence between the objective values and the simulated fields. This method is an alternative to design magnetic field exposure systems over large volumes such as those used in bioelectromagnetics applications.

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1. INTRODUCTION

Several applications require the generation of a very homogeneous and controlled magnetic field, such as magnetic fluid hyperthermia in cancer therapy [1], improvement of flight efficiency and even maneuverability at hypersonic speed [2], testing the magnetic properties and instruments of spacecraft [3], magnetic resonance imaging [4], calibration of magnetic fields [5], plasma physics experiments, particle accelerators, electromagnetic compatibility testing [6] and bioelectromagnetics research among others.

Many experiments performed to study the effect of the magnetic treatment on biological systems require devices that can generate electromagnetic fields, to apply a controlled and repeatable exposure dose to the samples involved in these experiments, but additionally allowing the experimenter to observe, manipulate and easily locate the sample. Therefore, the working volume should be conveniently accessible and the electrical design and construction should be as simple as possible. Thus, air-core coil arrangements has been traditionally opted to guarantee a certain spatial distribution of electromagnetic fields, when fed by an electric current. [7].

In this regard, the aforementioned applications require coil-systems designed specifically to generate a spatial distribution of magnetic field meeting the requirements of homogeneity, H, and average magnitude of the magnetic flux density, $\bar{B}$, within a certain volume, V, containing the sample treated. In order to design such a coil-systems, there are two main approaches, the local and the integral.

The local approach was developed for axially symmetric magnetic coil-systems such as Helmholtz and Maxwell coils. It is based on the assumption that in a space free from field sources, the magnetic field satisfies Laplace equation, and in an ideal axially symmetric system the field can be expressed as a series expansion about some point on the axis, taken as the origin in terms of zonal spherical harmonics [8]:

$$B_z(R,\theta) = \sum_{n=0}^{\infty} C_n R^n P_n(\cos\theta). \tag{1}$$

A similar expansion with different coefficients is valid for the radial component of the field $B_r(R,\theta)$. Here R and θ are the spherical coordinates of the point considered and $P_n(x)$ is a Legendre polynomial. The series converges within a sphere of radius $R < R_0$, where R_0 is the distance from the origin to the nearest current loop. The C_n coefficients depend on the system geometry and the current.

At $\theta = 0$ (along the longitudinal axis of the coil-system) only the field on the system axis is considered, and the series in (1) takes the form of a Maclaurin's series,

$$B_z(z) = \sum_{n=0}^{\infty} C_n z^n, \tag{2}$$

where the coefficients, C_n, are given by,

$$C_n = \frac{1}{n!} \left. \frac{d^n B_z(z)}{dz^n} \right|_{z=0}. \tag{3}$$

In practice symmetric systems are used, because all the odd coefficients in (1) are zero. Therefore, the local design approach is used to calculate the relationship between the electrical current intensity, I, the number of turns in the windings and their location on the axis, such that the even coefficients of (2) are canceled (C_2=0, C_4=0, C_6=0, ...), reducing the variability of the magnetic field around the center of the array and thus improving the homogeneity.

The local approach in the synthesis of magnetic field has been widely used to propose several coil-systems of circular or square windings such as those that have been reviewed in [9] and in [10], because of the simplicity of the analytical function that describes the magnetic flux density along their axis.

However, the fundamental flaw of the local approach is that the synthesis condition is restricted to the central point. The fields start to deviate from the required value in an uncontrolled way as the distance from the reference point increases. In practice, the requirements should be met for all the points within the working volume, but neither its size nor its shape are included in the synthesis conditions. If a given magnetic field distribution is required for a large volume, this approach is not recommended.

On the other hand, the integral approach requires that when designing a coil-system the magnetic field generated should minimally deviate from the desired one over the whole working volume. For this purpose, the strength of the field sources (current system) are chosen and located in space so that the magnetic field they create is as close as possible to the required one.

The determination of the unknown current density distribution placed outside the working volume and generating the desired magnetic field, is an inverse problem. This inverse problem can have more than one solution. The determination of the particular solution, that will be adopted for the design, constitutes a synthesis problem. This problem has been solved through local or global, deterministic or stochastic, optimization techniques [11].

In this regard, the integral method has at its foundation the numerical minimization of an objective function determined by the design goals, such as the mean-square deviation of the axial components of the magnetic field within the working volume, proposed by [8], or the magnetic field homogeneity, as has been used in [1, 12].

In addition, the integral approach is typically combined with numerical integration and simulation techniques (i.e., the Finite Elements Method) to calculate the approximated spatial distribution of magnetic induction, since for some geometries the exact analytical expression is complex to obtain or cannot be directly evaluated.

As expected, the main weaknesses of the integral approach to the synthesis of the magnetic field are the large calculation time and computational cost that are associated with the solution of a highly non-linear inverse problem where the objective function can be non-convex, stiff, non-differentiable, ill-conditioned, etc. [11].

As an alternative for the homogeneous magnetostatic field synthesis, this paper proposes a simple semi-analytical method for designing coil-systems. To this end, first the homogeneity of the magnetic field will be defined (Section 2), later to explain in detail the method (Section 3) and its implementation as an algorithm (Section 4), which will allow formulating and resolving a specific design example (Section 6) whose results will be discussed and validated (Sections 7 and 8) to conclude the matter.

2. MAGNETIC FIELD HOMOGENEITY

The homogeneity of the magnetic field, H, is a measure of the variability of the magnetic field within a defined region of space. Different approaches have been taken to quantify H. One of the most common is the one that defines the non-homogeneity of the field in terms of the variation in magnetic flux density at a given point in space within the volume of interest, regarding the value of magnetic induction in the central point of the coil system [12]. Hence, that definition for the magnetic field homogeneity considers that it is a point-dependent index, not providing global information about the homogeneity within the volume of interest, and consequently the aforementioned definition is not useful to describe the whole volume of interest.

Therefore, for the purposes of this paper, it was considered that a better definition of H is given by (4) [1], because it provides a single global index that summarizes the maximum change in the magnetic field magnitude within the entire working volume, V, with respect to its average value. That fact is important in biomagnetic experimentation because allows controlling the doses and treatments applied to the samples being investigated.

$$H\left(B_{\max}, B_{\min}, \bar{B}\right) = 1 - \frac{B_{\max} - B_{\min}}{\bar{B}}, \tag{4}$$

where $B_{\max}$ and $B_{\min}$ are the maximum and minimum values taken by the magnitude of the magnetic flux density within the working volume, respectively. Thus, H is measured as the maximum deviation of the magnitude of magnetic flux density in relation to the average value of the magnitude of $\vec{B}$, within V.

3. A SEMI-ANALYTICAL METHOD FOR HOMOGENEOUS MAGNETIC FIELD SYNTHESIS

The method is based on the assumption that it is possible to estimate, with sufficient accuracy within the working volume, the spatial distribution of the magnetic flux density generated by a coil-system formed by N independent coils, $\vec{B}$, by superposing the magnetic flux density generated by each coil, $\vec{B}_i$, as shown in (5).

$$\vec{B} = \sum_{i=1}^{N} \vec{B}_i. \tag{5}$$

An approximated analytical expression for each $\vec{B}_i$ is obtained using the Biot-Savart's law and simplifying the path of the ith current, I_i, considering that it follows a close trajectory formed by line segments, as given by (6).

$$\vec{B}_i = \frac{\mu_0}{4\pi} \oint \frac{I_i d\vec{l}_i \times \vec{r}}{r^3}, \tag{6}$$

where, μ_0 is the magnetic permeability of free space (considering that the working volume is filled with air), $d\vec{l}$ is a vector, whose magnitude is the length of the differential element of the wire, and whose direction coincides with the one of the current, and $\vec{r}$ is the displacement vector in the direction pointing from the wire element towards the point at which the field is being computed.

Considering that the symmetry of the windings favors homogeneity, it is recommendable, from an analytical and practical standpoint, the construction of coil arrangements with symmetric geometry in which each one of the coils maintains a regular and similar form within the coil set. Thus, in most applications, coil systems have either a rectangular or a circular cross section. In this regard, each $\vec{B}_i$ is analytically calculated through (6) at any point space, (x, y, z), for the

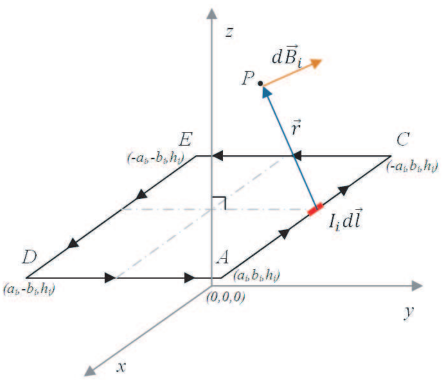


Figure 1. Simplified geometry of a rectangular current loop.

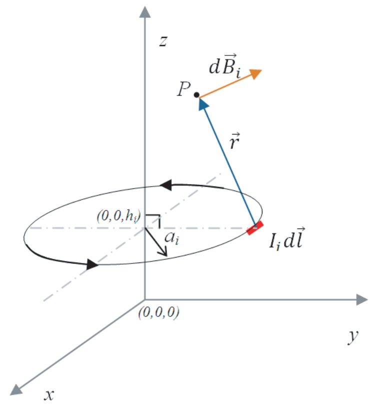


Figure 2. Simplified geometry of a circular current loop.

particular cases of a rectangular and a circular current loop, as shown in Figure 1 and Figure 2, respectively.

Additionally, the calculations are simplified considering that all the coils are thin, and therefore, assuming that all the turns of each winding contribute equally to the total magnetic flux density.

Consequently, the expression of the magnetic flux density for the rectangular coil geometry, is given by (7),

$$\vec{B}_i = n_i \left(B_{ix}, B_{iy}, B_{iz} \right), \tag{7}$$

where n_i is the number of turns of the ith coil and B_{ix}, B_{iy} and B_{iz} are the rectangular components of $\vec{B}_i$ in $\mathbb{R}^3$, which are given by (8), (13) and (18) respectively,

$$B_{ix} = \frac{\mu_0 I_i}{4\pi} \left(B^*_{ix1} + B^*_{ix2} + B^*_{ix3} + B^*_{ix4} \right), \tag{8}$$

where, B^*_{ix1}, B^*_{ix2}, B^*_{ix3} and B^*_{ix4} are given by (9), (10), (11) and (12) respectively.

$$B^*_{ix1} = \frac{z-h_i}{(x-a_i)^2+(z-h_i)^2} \frac{y+b}{\sqrt{(x-a_i)^2+(y+b_i)^2+(z-h_i)^2}}, \tag{9}$$

$$B^*_{ix2} = \frac{z-h_i}{(x-a_i)^2+(z-h_i)^2} \frac{-(y-b)}{\sqrt{(x-a_i)^2+(y-b_i)^2+(z-h_i)^2}}, \tag{10}$$

$$B^*_{ix3} = \frac{z-h_i}{(x+a_i)^2+(z-h_i)^2} \frac{y-b}{\sqrt{(x+a_i)^2+(y-b_i)^2+(z-h_i)^2}}, \tag{11}$$

$$B^*_{ix4} = \frac{z - h_i}{(x + a_i)^2 + (z - h_i)^2} \frac{-(y + b)}{\sqrt{(x + a_i)^2 + (y + b_i)^2 + (z - h_i)^2}}, \quad (12)$$

and,

$$B_{iy}(x, y, z) = \frac{\mu_0 I_i}{4\pi} \left(B^*_{iy1} + B^*_{iy2} + B^*_{iy3} + B^*_{iy4}\right), \quad (13)$$

where, B^*_{iy1}, B^*_{iy2}, B^*_{iy3} and B^*_{iy4} are given by (14), (15), (16) and (17) respectively.

$$B^*_{iy1} = \frac{z - h_i}{(y - b_i)^2 + (z - h_i)^2} \frac{x + a}{\sqrt{(x + a_i)^2 + (y - b_i)^2 + (z - h_i)^2}}, \quad (14)$$

$$B^*_{iy2} = \frac{z - h_i}{(y - b_i)^2 + (z - h_i)^2} \frac{x - a}{\sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - h_i)^2}}, \quad (15)$$

$$B^*_{iy3} = \frac{z - h_i}{(y + b_i)^2 + (z - h_i)^2} \frac{x - a}{\sqrt{(x - a_i)^2 + (y + b_i)^2 + (z - h_i)^2}}, \quad (16)$$

$$B^*_{iy4} = \frac{z - h_i}{(y + b_i)^2 + (z - h_i)^2} \frac{x + a}{\sqrt{(x + a_i)^2 + (y + b_i)^2 + (z - h_i)^2}}, \quad (17)$$

and finally,

$$B_{iz}(x, y, z) = \frac{\mu_0 I_i}{4\pi} \left(B^*_{iz1} + B^*_{iz2} + B^*_{iz3} + B^*_{iz4}\right), \quad (18)$$

where, B^*_{iz1}, B^*_{iz2}, B^*_{iz3} and B^*_{iz4} are given by (19), (20), (21) and (22) respectively.

$$B^*_{iz1} = \frac{(x - a_i)(y - b_i)}{\sqrt{(x - a_i)^2 + (y - b_i)^2 + (z - h_i)^2}} (\beta_1 + \beta_2), \quad (19)$$

$$B^*_{iz2} = \frac{(x - a_i)(y + b_i)}{\sqrt{(x - a_i)^2 + (y + b_i)^2 + (z - h_i)^2}} ({\beta_1}^* + \beta_2), \quad (20)$$

$$B^*_{iz3} = \frac{(x + a_i)(y - b_i)}{\sqrt{(x + a_i)^2 + (y - b_i)^2 + (z - h_i)^2}} (\beta_1 + {\beta_2}^*), \quad (21)$$

$$B^*_{iz4} = \frac{(x + a_i)(y + b_i)}{\sqrt{(x + a_i)^2 + (y + b_i)^2 + (z - h_i)^2}} ({\beta_1}^* + {\beta_2}^*), \quad (22)$$

where β_1, β_2, ${\beta_1}^*$ and ${\beta_2}^*$ are given by,

$$\beta_1 = \frac{1}{(y - b_i)^2 + (z - h_i)^2} \quad (23)$$

$$\beta_2 = \frac{1}{(x-a_i)^2 + (z-h_i)^2} \tag{24}$$

$$\beta_1{}^* = \frac{1}{(y+b_i)^2 + (z-h_i)^2} \tag{25}$$

$$\beta_2{}^* = \frac{1}{(x+a_i)^2 + (z-h_i)^2} \tag{26}$$

Furthermore, the spatial distribution of magnetic flux density generated by a circular current coil is given by [9, 13],

$$\vec{B}_i = n_i \left(B_{i\rho}, B_{i\varphi}, B_{iz}\right), \tag{27}$$

where n_i is the number of turns of the ith coil and $B_{i\rho}$, $B_{i\varphi}$ and B_{iz} are the vector components of $\vec{B}_i$ in the cylindrical coordinate system, which are given by (28), (29) and (30) respectively,

$$\vec{B}_{i\rho} = -\frac{\mu_0 I_i k\,(z-h_i)}{4\pi\rho\sqrt{a_i\rho}}\left(K(k) - \frac{2-k^2}{2\,(1-k^2)}E(k)\right), \tag{28}$$

$$\vec{B}_{i\varphi} = 0, \tag{29}$$

$$\vec{B}_{iz} = -\frac{\mu_0 I_i k}{4\pi\sqrt{a_i\rho}}\left(K(k) + \frac{k^2\,(a_i+\rho) - 2\rho}{2\rho\,(1-k^2)}E(k)\right). \tag{30}$$

In (28), (29) and (30), $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind respectively, and are defined by,

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}}, \qquad \text{and,} \tag{31}$$

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\alpha}\; d\alpha, \tag{32}$$

where, k is,

$$k = \sqrt{\frac{4a_i\rho}{(\rho+a_i)^2 + (z-h_i)^2}}. \tag{33}$$

Hence, by having analytical simplified expressions for $\vec{B}_i$, $\vec{B}$ can be numerically computed for any arrangement of rectangular or circular coaxial coils, and thus make the estimation of H within the working volume.

In this sense, the problem of synthesis requires iterative calculation of $\vec{B}$ and H, to find a geometry such that would meet the design requirements, optimizing the dimensions of the system. Consequently, $\vec{B}$ is numerically evaluated over a discrete set of points P. P is a set of uniform sample points distributed over the working volume, which generates a three-dimensional matrix of points (x_i, y_j, z_k) by varying a regular distance Δ the coordinates of the points.

As a result of the spatial sampling, an approximated discrete function of the sampled magnetic flux density in space is obtained as,

$$\vec{B}_{[i,j,k]}(\vec{r}) = \vec{B}(\vec{r}) \sum_{\vec{r}_{[i,j,k]} \in P} \delta_3 \left(\vec{r} - \vec{r}_{[i,j,k]}\right), \tag{34}$$

where $\delta_3(\vec{r})$, is the unit impulse function in three dimensions and,

$$\mathrm{P} = \left\{\vec{r}_{[i,j,k]} : \vec{r}_{[i,j,k]} = (\Delta i, \Delta j, \Delta k) \,/ i, j, k \in \mathbb{Z}\right\}, \ \mathrm{P} \subset V. \tag{35}$$

Thus, H is approximated to,

$$H = 1 - \frac{\max\left(\left|\vec{B}_{[i,j,k]}(\vec{r})\right|\right) - \min\left(\left|\vec{B}_{[i,j,k]}(\vec{r})\right|\right)}{\overline{\left|\vec{B}_{[i,j,k]}(\vec{r})\right|}} \tag{36}$$

Finally, the semi-analytical method for homogeneous magnetic field synthesis consists in an iterative evaluation of (36) following the procedure explained above, in order to find a suitable configuration that meets the design requirements. The iterative evaluation of H for different geometries, shall follow a logical procedure that allows the geometry parameters of the system to finish the convergence process to a solution that meets the optimization objectives. This procedure can be implemented through a heuristic search algorithm such as the TABU search.

4. HOMOGENEOUS MAGNETIC FIELD SYNTHESIS ALGORITHM

A simplified form of the algorithm for homogeneous magnetic field synthesis using rectangular coils is shown in Figure 3. The design configuration, X, is a vector set including the number of coil-windings, N, the number of turns of each winding, $n = \{n_1, n_2, \ldots, n_N\}$, and the size $a = \{a_1, a_2, \ldots, a_N\}$, $b = \{b_1, b_2, \ldots, b_N\}$ (considering a rectangular coil-system) and position, $h = \{h_1, h_2, \ldots, h_N\}$, of each coil-winding. As expected, H is iteratively evaluated for different design configurations to find a solution that meets the requirements of homogeneity. All the different design configurations form the neighborhood, X^*.

The search for a suitable design configuration was implemented using a TABU Search Algorithm, TSA. The TSA is a heuristic approach for solving optimization problems by using a guided, local

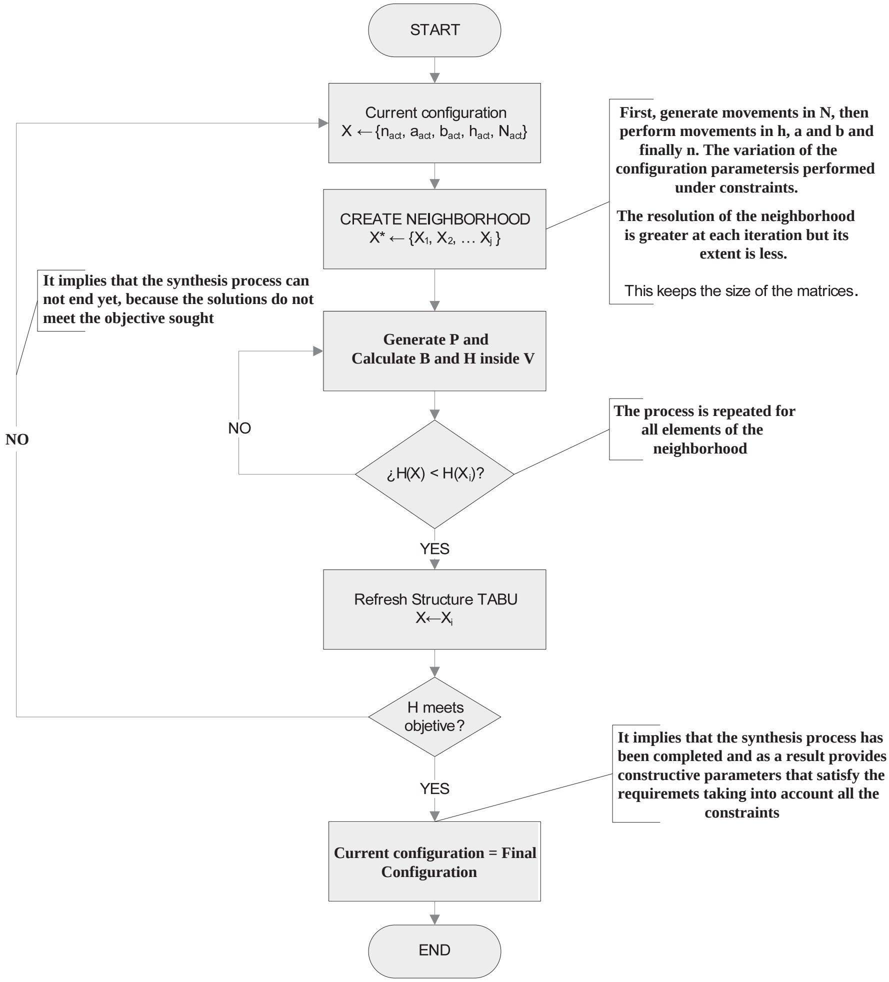


Figure 3. Simplified flow chart of the algorithm for homogeneous magnetic field synthesis (rectangular coil system).

search procedure to explore the entire solution space without becoming easily trapped in local optima. One characteristic of TABU search is that it finds good near-optimal solutions early in the optimization run. It does not require initial guesses, does not use derivatives, and is also independent of the complexity of the cost function considered [15].

In this particular case, the neighborhood is uniformly distributed over the possible values of each parameter, considering the constrains of maximum and minimum coil size, maximum number of turns of each coil and maximum number of coil windings. When the neighborhood is completely evaluated, the best design configuration found (the one with the highest H) is selected and the algorithm generates a new neighborhood around it, but with an increased resolution. The neighborhood size remains constant. This process is repeated until a solution is found or until the distance between the elements of the neighborhood cannot be reduced (in practice this limit is set by the manufacturing capacity).

The parameters n_{act}, a_{act}, b_{act}, h_{act} and N_{act} correspond to the best design configuration found. An initial design configuration shall be set. In this particular case, the initial design configuration was automatically selected as the one with all the windings of the same size, with the same number of turns and with their positions uniformly distributed along the longitudinal axis of the coil system.

5. DESIGNING THE COIL-SYSTEM UNDER POWER CONSTRAINTS

When designing magnetic coils, one important consideration is how to keep them cool while all the electric power is dissipated in the wire by the Joule effect. In some instances, no cooling is needed and in others, the coils systems shall be air cooled or water cooled. It really depends on the coil holder structure and airflow available [16]. Additionally, current and power constraints must be considered when designing magnetic coil systems, since there are limitations of the power supply that will be used the feed the coil system. The power dissipated in the ith coil by means of Joule effect, P_i, is given by:

$$P_i = {I_i}^2 R_i = {I_i}^2 \rho_{\mathrm{Cu}} \frac{l}{A} \approx {I_i}^2 \rho_{\mathrm{Cu}} n_i \frac{4 l_{turn}}{\pi d^2} \tag{37}$$

where R_i is the DC resistance of the ith coil, which is calculated approximately from the electrical resistivity of the wire (copper, Cu), ρ_{Cu}, the length of the wire, l and the cross-sectional area of the wire, A. The length of the wire is approximated as the average length of a single turn, l_{turn}, multiplied by the number of turns, n_i. A is calculated assuming a circular cross-section of diameter d.

Consequently, in order to choose a specific wire type for the coils, P_i is calculated for the typical wire sizes of the American Wire Gauge types [17]. Nevertheless, an additional alternative to reduce the power consumption of the coil system is to increase the number of turns of each coil winding while the current intensity is reduced proportionally in the same amount. In that sense, a brute-force search algorithm is used to find an appropriate combination of the number of coil winding turns and wire gauge, which allows to keep the power consumption below the constraint.

6. EXAMPLE OF A DESIGN PROBLEM

It is required to design a coil arrangement that can generate up to 10 mT within a working volume 0.5 m × 0.5 m × 1 m with 99% of spatial homogeneity, with a tolerance of up to 1% in the results of field homogeneity, in order to create magnetic field exposure system intended to be used to apply magnetic treatment to vegetable seeds to improve the germination rate and crops yield. The theoretical model that explains the influence of a stationary magnetic field on water relations in seeds, can be consulted in [18] and in a companion paper, it is presented some experimental evidence to support that hypothesis [19].

Since the working volume is a parallelepiped, square loops were selected. The maximum side length of the coils must be 1.5 m, the maximum longitudinal length is set at 1.5 m, and the maximum power consumed by Joule effect shall be 1 W per coil considering that the windings could be constructed with enameled copper wire using either type AWG 12, 10 or 8. The maximum number of coils is set at 7. Parameterized geometry of the coil system to be designed under the restrictions stated above, is shown in Figure 4.

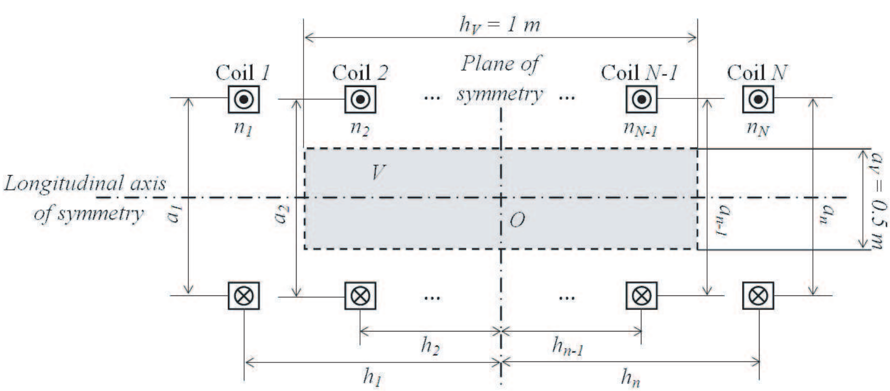


Figure 4. Geometry of the coil system for homogeneous magnetostatic field generation. The grey rectangle represents any vertical cross-section of the working volume V, parallel to the x or y axis.

It is important to note that, in order to solve the problem, the proposed method and algorithms were implemented using MATLAB^{TM} scripting.

7. DESIGN PROBLEM RESULTS

The algorithm converged to a solution satisfying the requirements of magnetic field homogeneity and magnetic flux density inside of the volume of interest, reaching $H = 98.7\%$, $\bar{B} = 10.00\,\text{mT}$, $B_{\text{max}} = 10.05\,\text{mT}$ and $B_{\text{min}} = 9.92\,\text{mT}$ for a six-coils (connected in series) system with the constructive parameters and characteristics shown in Table 1. The solution to the problem was found for $\Delta = 2\,\text{cm}$.

The magnitude of the magnetic flux density generated by the the designed coil system is shown in Figure 5. The calculations were performed using different input currents, proportional to the current required to achieve 10 mT ($I_{\text{max}} = 14.49\,\text{A}$). The results are in accordance with design specifications supplied to the synthesis algorithm.

Table 1. Constructive parameters and characteristics of the designed coil system.

Parameter	Coil 1	Coil 2	Coil 3	Coil 4	Coil 5	Coil 6
I_i (A)	14.49	14.49	14.49	14.49	14.49	14.49
R_i (Ω)	4.75	2.06	1.81	1.81	2.06	4.75
P_i (W)	996	433	379	379	433	996
a_i (m)	1	1	1	1	1	1
h_i (cm)	72.9	36.7	11.6	11.6	36.7	72.9
n_i (turns)	368	160	140	140	160	368

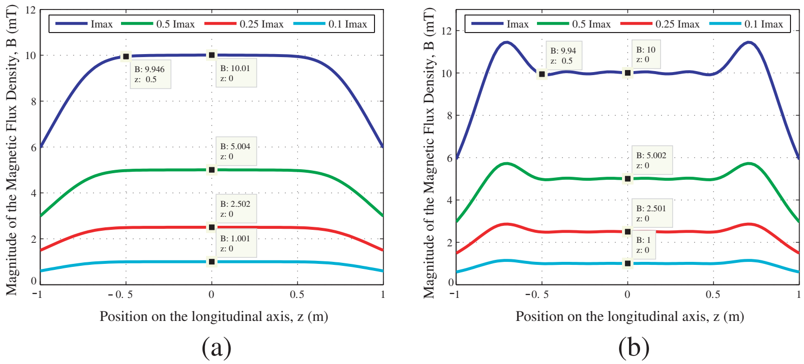


Figure 5. Magnitude of the magnetic flux density of the designed coil system. (a) $(x, y) = (0, 0)$. (b) $(x, y) = (\pm 0.25, \pm 0.25)\,\text{m}$.

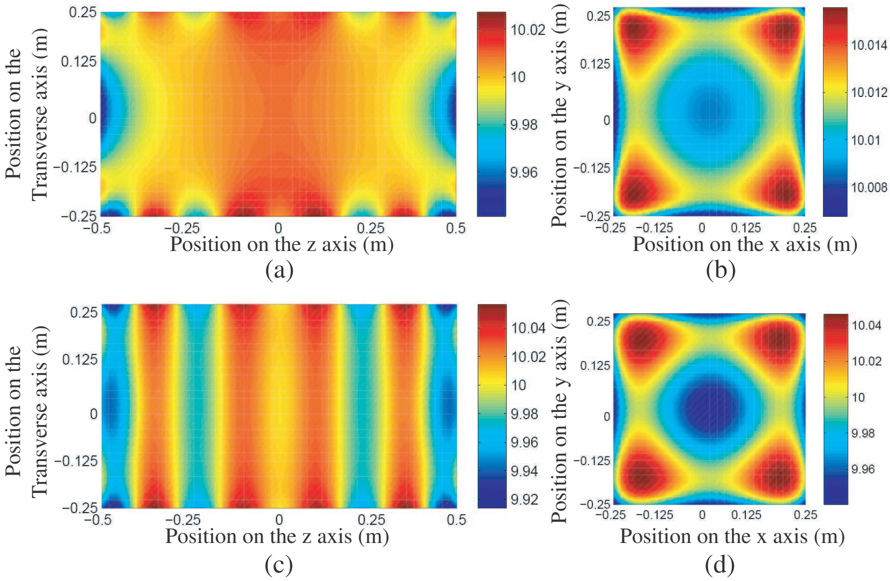


Figure 6. Magnitude of the magnetic flux density of the designed coil system in the external faces of V. Note: The colorbar scale indicates B in units of mT. (a) $x = 0$ or $y = 0$. (b) $z = 0$. (c) $c = \pm 0.25\,\text{m}$ or $y = \pm 0.25\,\text{m}$. (d) $z = \pm 0.5\,\text{m}$.

Figure 6 shows the magnetic flux density on the different faces of the volume, V. The results also show that the volume of interest is contained completely within the region of 99% homogeneity of the magnetic field.

8. VALIDATION OF RESULTS

The validation of the results of the synthesis of magnetic field was performed using COMSOL Multiphysics$^{\text{TM}}$ 4.2 which is a solver/simulation software package based on finite element method, FEM. The simulation model was implemented specifically with the AC/DC module. Figure 7 shows the 3D model of the coil system designed.

Within the domain defined for V, $B_{\max}$, $B_{\min}$ and $\bar{B}$ were calculated as derived values, obtaining 10.14 mT, 9.86 mT, 10.00 mT respectively, and in consequence the simulated value of H is 97.23%. The results are satisfactory, considering the differences inherent in the models used in the design process and simulation. Figures 8 and 9 shows good correspondence between the results obtained in calculating the magnitude of the magnetic flux density using both the proposed method and the FEM analysis within the volume of interest. The relative difference between the average magnetic flux density calculated with the proposed model within V and the one obtained by simulation using COMSOL Multiphysics was 0.026%. The average relative difference (measured in each grid point) between the magnetic flux density within V calculated with the proposed model and the one obtained by simulation using COMSOL Multiphysics was 0.1822%. The maximum relative difference between the simulated and synthesized magnetic flux density, within the volume of interest, was 3.30%.

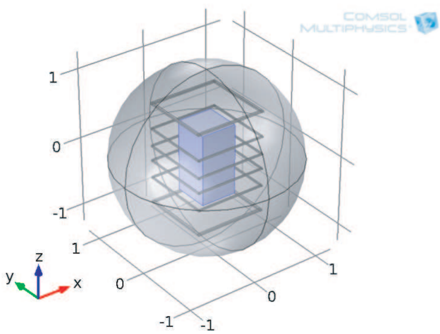
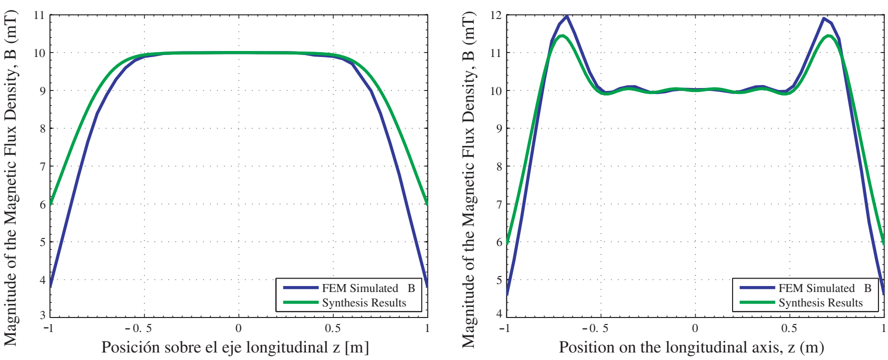


Figure 7. COMSOL multiphysics 3D model of the coil system designed.



(a) (b)

Figure 8. Comparison of the magnitude of the magnetic flux density synthesized and simulated. (a) $(x, y) = (0, 0)$. (b) $(x, y) = (\pm 0.25, \pm 0.25)$ m.

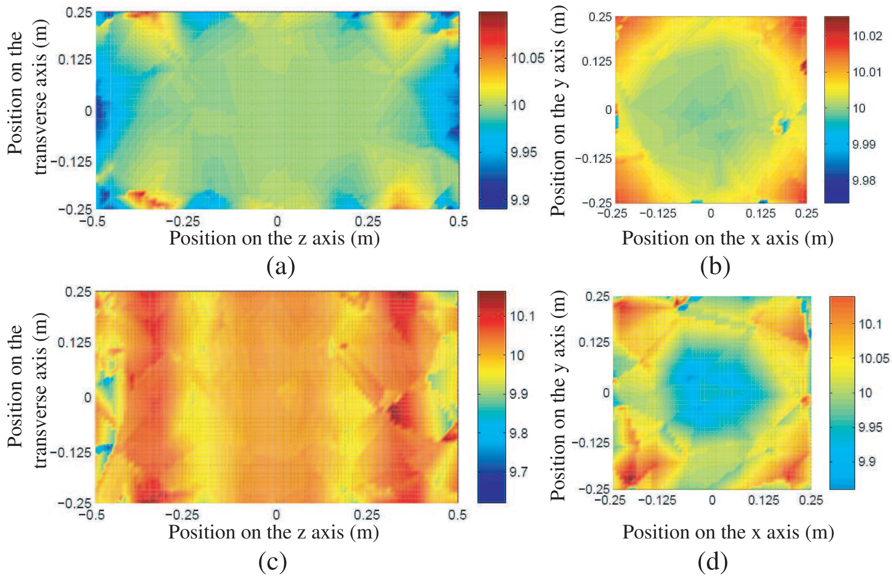


Figure 9. Magnitude of the magnetic flux density simulated using FEM. Note: The colorbar scale indicates B in units of mT. (a) $x = 0$ or $y = 0$. (b) $x = \pm 0.25\,\mathrm{m}$ or $y = \pm 0.25\,\mathrm{m}$. (c) $z = 0$. (d) $z = \pm 0.5\,\mathrm{m}$.

Results also show a greater differences in the magnitude of the magnetic flux density outside the working volume. This may be due to the thickness of the windings, which is considered in the model analyzed by FEM.

9. CONCLUSION

An alternative method for the design of coils systems for the generation of magnetostatic fields has been proposed in this work. The spatial distribution of the magnetostatic field is estimated superposing the magnetic induction numerically computed from the analytical expression of the magnetic field generated by each coil, obtained using the Biot-Savart's law and the current filament method. The estimated magnetic field using this method, coincides with the calculated using FEM, within the working volume.

Compared with other approaches used in the design of coils systems, the proposed method combines the speed of computation associated with the evaluation of the analytical expressions previously known (for the most common geometries used in the construction of windings) and the possibility of synthesizing the desired field within the entire volume of interest (inverse problem), based on a both the field homogeneity and average magnitude of the magnetic flux density. Additionally, given that the iterative computation algorithm takes into consideration the problem constraints, related to the practical limitations (maximum sizes and power consumption), the method is capable of calculate the fundamental constructive specifications and parameters that characterize a prototype coil system.

Another advantage of this method lies in the simplicity of analytical expressions that define the distribution of fields generated by individual coils, making the algorithm implementation process is very intuitive. Then, the results of the synthesis algorithm can be implemented with relative ease given the geometry of the coil arrangement either rectangular or circular. On the other hand, the main disadvantage of the approach is that it is possible that the simplifications considered in the synthesis method introduce significant errors when the volume of interest is near the periphery of the coils.

The proposed method has been probed as a alternative to aid engineers in the design process of coil-systems for homogeneous magnetostatic field generation, when specific requirements of magnetic field homogeneity and average magnitude of the magnetic flux density inside of the volume of interest are imposed. This method has a lot of potential applications in the design of magnetic field exposure systems over large volumes such as those used in bioelectromagnetics applications, due to the reliability of results ensure low uncertainty in the doses applied to the samples treated.

REFERENCES

1. Di Barba, P., F. Dughiero, and E. Sieni, "Magnetic field synthesis in the design of inductors for magnetic fluid hyperthermia," *IEEE Transactions on Magnetics*, Vol. 46, No. 8, 2931–2934, 2010.
2. Brooks, N. and T. Baldwin, "Methodology for universal synthesis of magnetic designs based on field specifications," *Proceedings of the Thirty-Fourth Southeastern Symposium on System Theory*, 113–117, 2002.
3. Pittman, M. E. and D. L. Waidelich, "Three and four coil systems for homogeneous magnetic fileds," NASA Technical Note, National Aeronautics and Space Administration, Washington, 1964.
4. Pulyer, Y. M. and M. I. Hrovat, "Generation of remote homogeneous magnetic fields," *IEEE Transactions on Magnetics*, Vol. 38, No. 3, 1553–1563, 2002.

5. Bongiraud, J. P., G. Cauffet, and C. Jeandey, "An optimization method for magnetic field generator," *2nd International Conference on Marine Electromagnetic*, Francia, 1999.
6. Anderson, T., "Design of a helmholtz coil for susceptibility testing using variational calculus and experimental verification," *IEEE International Symposium on Electromagnetic Compatibility*, Vol. 2, 601–604, 1999.
7. Cvetkovic, D. and I. Cosic, "Modelling and design of extremely low frequency uniform magnetic field exposure apparatus for in vivo bioelectromagnetic studies," *29th Annual International Conference of the IEEE on Engineering in Medicine and Biology Society*, 1675–1678, 2007.
8. Lugansky, L. B., "Field synthesis in solenoidal magnetic systems (review)," *Instruments and Experimental Techniques*, No. 4, 9–36, 1992.
9. Kirschvink, J., "Uniform magnetic fields and double-warpped coil systems: Improved techniques for the design of bioelectromagnetic experiments," *Bioelectromagnetics*, Vol. 13, No. 5, 401–411, 1992.
10. Wang, J., S. She, and S. Zhang, "An improved helmholtz coil and analysis of its magnetic field homogeneity," *Review of Scientific Instruments*, Vol. 76, No. 5, 2175–2179, 2002.
11. Borghi, C. A. and M. Fabbri, "A global optimization method for the solution of a magnetic field synthesis problem," *IEEE Transactions on Magnetics*, Vol. 32, No. 3, 1897–1904, 1996.
12. Sato, S., S. Sakaguichi, K. Futamata, and K. Katou, "Coil optimization for homogeneous magnetic field with small leakage field," *IEEE Transactions on Magnetics*, Vol. 36, No. 4, 649–653, 2000.
13. Kuns, K., "Calculation of magnetic field inside plasma chamber," UCLA Tech. Rep., 2007, http://plasmalab.pbworks.com/f/bfield.pdf.
14. Glover, F. and M. Laguna, *Tabu Search*, Kluwer Academic, Boston, 1997.
15. Turkmen, I. and K. Guney, "Tabu search tracker with adaptive neuro-fuzzy infeernce system fro multiple target tracking," *Progress In Electromagnetics Research*, Vol. 65, No. 10, 169–185, 2006.
16. Meyrath, T., "Electromagnet design basics for cold atom experiments," University of Texas, Austin, 2004, Availeable at: http://george.ph.utexas.edu/ meyrath/informal/.

17. Moore, J., C. Davis, M. Coplan, and S. Greer, *Building Scientific Apparatus* , Cambridge University Press, New York, 2009.
18. Garcia, F. and L. Arza, "Influence of a stationary magnetic field on water relations in lettuce seeds. Part I: Theoretical considerations," *Bioelectromagnetics*, Vol. 22, No. 8, 589–595, 2001.
19. Garcia, F., L. Arza, and I. Almanza, "Influence of a stationary magnetic field on water relations in lettuce seeds. Part II: Experimental results," *Bioelectromagnetics*, Vol. 22, No. 8, 596–602, 2001.