Progress In Electromagnetics Research, Vol. 123, 67–81, 2012

ELECTROMAGNETIC FIELD TRANSMITTED BY DIELECTRIC PLANO CONVEX LENS PLACED IN CHIRAL MEDIUM

A. Ghaffar[1,*], M. Y. Naz[1], and Q. A. Naqvi[2]

[1]Department of Physics, University of Agriculture, Faisalabad, Pakistan

[2]Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan

Abstract—In this paper, a theoretical investigation of electromagnetic field transmission through dielectric plano convex lens placed in chiral medium is analyzed. The chiral medium is described electromagnetically by the constitutive relations $\mathbf{D} = \epsilon(\mathbf{E} + \gamma\nabla \times \mathbf{E})$ and $\mathbf{B} = \mu(\mathbf{H} + \gamma\nabla \times \mathbf{H})$. Transmission's coefficients for chiral-dielectric and dielectric-chiral interfaces are derived analytically. The analytical field expressions for right circularly polarized (RCP) and left circularly polarized (LCP) waves are obtained using Maslov's method. Numerical computations are made for the field patterns around the caustic region using Mathcad software to observe the effect of chirality parameter.

1. INTRODUCTION

In 19th century the chiral medium was first explored due to its optical rotation phenomenon. After this discovery, it was experimentally and theoretically proved that the right circularly polarized (RCP) and the left circularly polarized (LCP) waves in chiral medium have different refraction indices due to different phase velocities [1]. Their different polarization rotations give different mode of propagations [2]. Many researchers have studied the interaction of electromagnetic waves with chiral slabs and other possible structures of chiral materials [3–17].

Focussing of electromagnetic waves from focussing systems into dielectric media is a subject of considerable current interest due to its applications in hyperthermia, microscopy, and optical data storage. The focussing of dielectric focussing systems into chiral medium has

Received 26 October 2011, Accepted 5 December 2011, Scheduled 13 December 2011
* Corresponding author: Abdul Ghaffar (chabdulghaffar@yahoo.com).

attained great attention from last decade by many researchers. In this study, analysis of focussing of electromagnetic waves from dielectric plano convex lens placed in chiral medium is discussed in detail. Specifically, transmission coefficients of chiral- dielectric and dielectric-chiral are obtained using boundary conditions. The analytical field expressions in the caustic region are obtained using Maslov's method. The analytical expressions are computed and presented for the normal incidence, different lens and different chirality parameters. According to Maslov's method, the field expression near the caustic can be constructed by using the geometrical optics information, though we must perform the integration in the spectrum domain in order to predict the field in the space domain [18].

The physical interpretation of the mathematics of Maslov's method and its relation to other asymptotic ray theory methods have been discussed by Ziolkowski and Dechamps [19]. This method has been successfully applied to predict the field in the focal region of spherical dielectric interface, plano-convex antenna, inhomogeneous slab and focussing of lens into uniaxial crystal by Hongo, Ghaffar and co-workers [20–23].

2. GEOMETRICAL OPTICS APPROXIMATION AND MASLOV'S METHOD IN DIELECTRIC MEDIUM

Geometrical-optics field expression is given by [20–23]

$$\mathbf{E}^r(x,y,z) = \mathbf{E}_T(\xi,\eta)\,[J(t)]^{-\frac{1}{2}} \exp\Big[-jk\Big(S_0(\xi,\eta)+t\Big)\Big] \tag{1}$$

In above expressions $\mathbf{E}_T(\xi,\eta)$ is initial value of amplitude and $J(t) = D(t)/D(0)$. where $D(t) = \frac{\partial(x,y,z)}{\partial(\xi,\eta,t)}$ is the Jacobian of coordinate transformation from ray coordinates (ξ,η,t) to rectangular coordinates (x,y,z), $S_0(\xi,\eta)$ is initial phase and t is parameter along the ray. GO approximation for waveform modelling is attractive in electromagnetics because it provides insight into how a wave front responds to a given structure. In this technique, user can monitor the phase of the electromagnetic wave as it propagates through the medium. GO is concerned only with the relatively high frequency component of the waveform, provided the ray tube does not vanish. However, there exist regions $J(t) = 0$ where ray tube shrinks to zero, called caustics and GO fails there [9]. This drawback of GO is overcome by Maslov's method. It uses a combination of spatial domain and wave vector domain and gives rise to a hybrid space. This eliminates the possibility of occurrence of singularity around the caustic. General expression of

Maslov's method for field calculation is given as [21–23]:

$$\mathbf{E}^r(\mathbf{r}) = \sqrt{\frac{k}{j2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{E}_T(\xi, \eta) \left[\frac{D(t)}{D(0)} \frac{\partial(q_x, q_y)}{\partial(x, y)} \right]^{-\frac{1}{2}}$$
$$\exp\left\{-jk\left[S_0+t-x(q_x, q_y, z)q_x-y(q_x, q_y, z)q_y+q_x x+q_y y\right]\right\} dq_x dq_y \quad (2)$$

Equation (2) is derived by applying the stationary phase method to the conventional Fourier-transform representation for $\mathbf{E}^r(\mathbf{r})$. Thus the integrand of the inverse Fourier transform of the wave function is derived through the information of the GO solution. In above equation [11]

$$J(t)\frac{\partial(q_x, q_y)}{\partial(x, y)} = \frac{1}{D(0)} \frac{\partial(q_x, q_y, z)}{\partial(x, y, t)} = \frac{1}{D(0)} \left(\frac{\partial q_x}{\partial \xi} \frac{\partial q_y}{\partial \eta} - \frac{\partial q_y}{\partial \xi} \frac{\partial q_x}{\partial \eta} \right) \frac{\partial z}{\partial t}$$

The ray expression of the refracted field is derived from the solutions for Hamiltons equations [18] as

$$x(q_x, q_y, z) = \xi + q_x t, \quad y(q_x, q_y, z) = \eta + q_y t, \quad z(q_x, q_y, z) = \zeta + q_z t$$

It may be noted that (q_x, q_y, q_z) are components of ray.

3. GO FIELD OF PLANO CONVEX LENS PLACED IN CHIRAL MEDIUM

Consider the geometry which contains a plano convex lens placed in chiral medium as shown in Figure 1. Profile of the plano convex lens

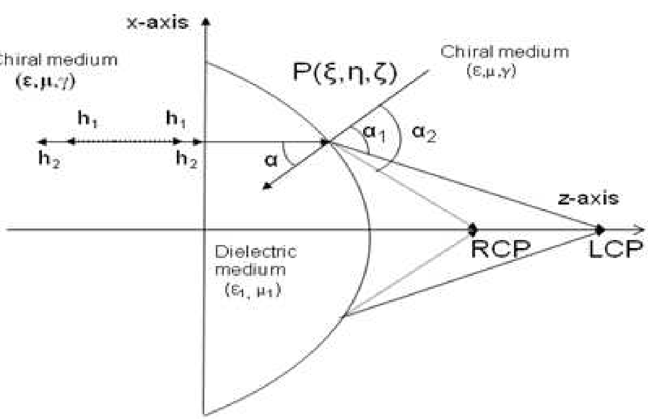

Figure 1. Plano convex lens placed in chiral medium.

is defined as [21]

$$\zeta = g(\rho) = \frac{n}{n+1} f - \frac{1}{\sqrt{n^2-1}} \sqrt{\rho^2 + \frac{n-1}{n+1} f^2}, \qquad \rho = \sqrt{\xi^2 + \eta^2} \quad (3)$$

where (ξ, η, ζ) are the Cartesian coordinates of the point on the plano convex lens and n is the refractive index of plano-convex lens and f is the focal length. The chiral medium is defined by the following relations [4–6]

$$\mathbf{D} = \epsilon(\mathbf{E} + \gamma \nabla \times \mathbf{E}) \quad (4)$$

$$\mathbf{B} = \mu(\mathbf{H} + \gamma \nabla \times \mathbf{H}) \quad (5)$$

where γ is the chirality parameter which is assumed to be positive in this paper, ϵ and μ are the permittivity and permeability of the chiral medium, respectively. Some natural and optically chiral media can be considered as a homogeneous medium.

Let us denote the incident fields propagating in the chiral medium in the RC polarization by $\mathbf{E_1}$ and $\mathbf{H_1}$ and in the LC polarization by $\mathbf{E_2}$ and $\mathbf{H_2}$. Let us denote the reflected fields propagating downwards in the chiral medium in the RC polarization by $\mathbf{E_3}$ and $\mathbf{H_3}$ and in the LC polarization by $\mathbf{E_4}$ and $\mathbf{H_4}$. Let us also denote the transmitted fields propagating upwards in the dielectric medium with an elliptic polarization by $\mathbf{E_5}$ and $\mathbf{H_5}$. The various fields are given as follows [5]

$$\mathbf{E_1} = E_1 \mathbf{i}_x \exp(jn_1kz) \quad (6)$$

$$\mathbf{H_1} = -jZ^{-1}\mathbf{E_1} \quad (7)$$

$$\mathbf{E_2} = E_2 \mathbf{i}_x \exp(jn_2kz) \quad (8)$$

$$\mathbf{H_2} = jZ^{-1}\mathbf{E_2} \quad (9)$$

$$\mathbf{E_3} = -E_3 \mathbf{i}_x \exp(-jn_1kz) \quad (10)$$

$$\mathbf{H_3} = -jZ^{-1}\mathbf{E_3} \quad (11)$$

$$\mathbf{E_4} = -E_4 \mathbf{i}_x \exp(-jn_2kz) \quad (12)$$

$$\mathbf{H_4} = -jZ^{-1}\mathbf{E_4} \quad (13)$$

$$\mathbf{E_5} = E_5 \mathbf{i}_x \exp(jk\zeta) \quad (14)$$

$$\mathbf{H_5} = -j\eta^{-1} E_5 \mathbf{i}_x \exp(jk\zeta) \quad (15)$$

where $\eta = \sqrt{(\mu_1/\epsilon_1)}$, $Z = \sqrt{(\mu/\epsilon)}/\sqrt{(1 + (\mu/\epsilon)\gamma^2)}$ and ϵ_1, μ_1 are the permittivity and permeability of the dielectric medium. The RCP and LCP with reflective indices n_1, and n_2 are given by [3].

$$n_1 = \frac{1}{1 - k\gamma}, \quad n_2 = \frac{1}{1 + k\gamma} \quad (16)$$

where $k^2 = \omega^2 \mu \epsilon$. For $\gamma > 0$ the right-circularly polarized wave is the slower mode, whereas for $\gamma < 0$ the left-circularly polarized wave is the slower mode.

4. TRANSMISSION COEFFICIENTS AT CHIRAL-DIELECTRIC INTERFACE

The transmissions coefficients for chiral-dielectric interface are derived analytically [3]

$$(\mathbf{E_1}+\mathbf{E_2}+\mathbf{E_3}+\mathbf{E_4})\times\mathbf{i}_z = \mathbf{E_5}\times\mathbf{i}_z \tag{17a}$$

$$(\mathbf{H_1}+\mathbf{H_2}+\mathbf{H_3}+\mathbf{H_4})\times\mathbf{i}_z = \mathbf{H_5}\times\mathbf{i}_z \tag{17b}$$

We consider perfect transmission which occurs at $\eta = Z$. This condition of totally transmission occurs for $\mu = \mu_1$ when $\epsilon_1 = \epsilon + \mu\gamma^2$. We neglect the reflection coefficients and obtained transmissions coefficients using above conditions as

$$E_{5\|} = \frac{2\eta}{\eta+Z}(E_1+E_2), \quad E_{5\perp} = \frac{2\eta}{\eta+Z}(E_1+E_2) \tag{18}$$

This transmitted wave into dielectric lens propagate towards the curved surface of lens at $z = \zeta$. We apply again boundary conditions at dielectric-chiral interface.

5. TRANSMISSION COEFFICIENTS AT DIELECTRIC-CHIRAL INTERFACE

When a plane wave again incident on curved dielectric-chiral interface, it is partially transmitted into chiral medium and partially reflected as shown in Figure 3. We ignore the reflection, the transmitted wave into chiral medium will split into two waves designated as RCP and LCP waves making angles of α_1 and α_2 with the normal, respectively. The transmission's coefficients for dielectric-chiral interface are derived analytically as [4]

$$\begin{pmatrix} E_R \\ E_L \end{pmatrix} = \begin{pmatrix} T_{c11} & T_{c12} \\ T_{c21} & T_{c22} \end{pmatrix} \begin{pmatrix} E_{5\perp} \\ E_{5\|} \end{pmatrix} \tag{19}$$

where

$$T_{c11} = \frac{A_i}{A_1}\frac{-2j\cos\alpha(g\cos\alpha+\cos\alpha_2)}{(1+g^2)\cos\alpha(g\cos\alpha_1+\cos\alpha_2)+2g(\cos\alpha_1\cos\alpha_2+\cos^2\alpha)}$$

$$T_{c12} = \frac{A_i}{A_1}\frac{2\cos\alpha(\cos\alpha+g\cos\alpha_2)}{(1+g^2)\cos\alpha(g\cos\alpha_1+\cos\alpha_2)+2g(\cos\alpha_1\cos\alpha_2+\cos^2\alpha)}$$

$$T_{c21} = \frac{A_i}{A_2}\frac{2j\cos\alpha(g\cos\alpha+\cos\alpha_1)}{(1+g^2)\cos\alpha(g\cos\alpha_1+\cos\alpha_2)+2g(\cos\alpha_1\cos\alpha_2+\cos^2\alpha)}$$

$$T_{c22} = \frac{A_i}{A_2}\frac{2\cos\alpha(\cos\alpha+g\cos\alpha_1)}{(1+g^2)\cos\alpha(g\cos\alpha_1+\cos\alpha_2)+2g(\cos\alpha_1\cos\alpha_2+\cos^2\alpha)}$$

$$\begin{aligned} A_i &= \exp(jk\zeta) \\ A_2 &= \exp(jn_1k\zeta) \\ A_3 &= \exp(jn_2k\zeta) \\ g &= \sqrt{(\mu_1/\epsilon_1)\gamma^2 + (\epsilon\mu_1/\mu\epsilon_1)} \end{aligned}$$

GO field expressions for plano convex lens placed in chiral medium may be obtained in similar manner as in dielectric medium by combining the contributions of both RCP and LCP waves. Unit normal $\mathbf{N}$ of the surface is given by [20]

$$\mathbf{N} = \sin\alpha\cos\beta\mathbf{i}_x + \sin\alpha\sin\beta\mathbf{i}_y + \cos\alpha\mathbf{i}_z \tag{20}$$

where (α, β) are angular polar coordinates of the point (ξ, η, ζ) defined by

$$\begin{aligned} \xi &= \rho\cos\beta \\ \eta &= \rho\sin\beta \\ \zeta &= g(\rho) \\ \rho &= \frac{(n-1)f\tan\alpha}{\sqrt{1-(n^2-1)\tan^2\alpha}} \\ \sin\alpha &= -\frac{g'(\rho)}{\sqrt{1+(g'(\rho))^2}} \\ \cos\alpha &= \frac{1}{\sqrt{1+(g'(\rho))^2}} \\ \tan\beta &= \frac{\eta}{\xi} \end{aligned}$$

The ray vector of the refracted ray by plano convex lens may be obtained using the relation $\mathbf{q} = n\mathbf{p}^i + \sqrt{n_1^2 - n^2 + n^2(\mathbf{p}\cdot\mathbf{N})^2}\mathbf{N} - n(\mathbf{p}\cdot\mathbf{N})\mathbf{N}$, which is derived from Snell's law with n is the refractive indices of the lens. The ray vectors of the rays refracted by the plano convex lens are given by

$$\begin{aligned} \mathbf{n_1 q} &= K_1(\alpha)\sin\alpha\cos\beta\mathbf{i}_x + K_1(\alpha)\sin\alpha\sin\beta\mathbf{i}_y + (n + K_1(\alpha)\cos\alpha)\mathbf{i}_z \\ &= Q_{1t}\cos\beta\mathbf{i}_x + Q_{1t}\sin\beta\mathbf{i}_y + Q_{1z}\mathbf{i}_z \qquad (21\text{a}) \\ \mathbf{n_2 q} &= K_2(\alpha)\sin\alpha\cos\beta\mathbf{i}_x + K_2(\alpha)\sin\alpha\sin\beta\mathbf{i}_y + (n + K_2(\alpha)\cos\alpha)\mathbf{i}_z \\ &= Q_{2t}\cos\beta\mathbf{i}_x + Q_{2t}\sin\beta\mathbf{i}_y + Q_{2z}\mathbf{i}_z \qquad (21\text{b}) \end{aligned}$$

where

$$K_1(\alpha) = \sqrt{n_1^2 - n^2\sin^2\alpha} - n\cos\alpha, \quad K_2(\alpha) = \sqrt{n_2^2 - n^2\sin^2\alpha} - n\cos\alpha$$

The plano convex lens will refract into chiral medium consist of two wave, RCP and LCP, each making an angle α_1 and α_2 with normal to lens surface, where

$$\alpha_1 = \sin^{-1}\left(\frac{k_1}{n_1}\sin\alpha\right), \qquad \alpha_2 = \sin^{-1}\left(\frac{k_1}{n_2}\sin\alpha\right)$$

Geometrical-optics solution for RCP and LCP are derived as

$$\mathbf{E}_R(x,y,z) = \mathbf{E}_R^t(\xi,\eta)\left[J_1(t_1)\right]^{-\frac{1}{2}}\exp\Big[jn_1k\Big(S_0(\xi,\eta)+t\Big)\Big] \tag{22}$$

$$\mathbf{E}_L(x,y,z) = \mathbf{E}_L^t(\xi,\eta)\left[J_2(t_2)\right]^{-\frac{1}{2}}\exp\Big[jn_2k\Big(S_0(\xi,\eta)+t\Big)\Big] \tag{23}$$

where $J_1(t_1)$ and $J_2(t_2)$ are the Jacobian of coordinate transformation from ray coordinates (ξ,η,t_1) and (ξ,η,t_2) to rectangular coordinates (x,y,z) for RCP and LCP fields

$$J_1(t_1) = \frac{D(t_1)}{D(0)} = \frac{1}{D(0)}\frac{\partial(x,y,z)}{\partial(\xi,\eta,t_1)} = \left(P\frac{U_1}{E_1}t_1+1\right)\left(\frac{Q_{1t}(\alpha)}{\rho}t_1+1\right)$$

$$J_2(t_2) = \frac{D(t_2)}{D(0)} = \frac{1}{D(0)}\frac{\partial(x,y,z)}{\partial(\xi,\eta,t_2)} = \left(P\frac{U_2}{E_2}t_2+1\right)\left(\frac{Q_{2t}(\alpha)}{\rho}t_2+1\right)$$

where

$$\begin{aligned}
P &= \frac{\left(\sqrt{n^2-1}\right)\frac{n-1}{n+1}f^2}{\left[n^2\xi^2+(n^2-1)^2f^2\right]\left[\xi^2+\frac{n^2-1}{n+1}f^2\right]^{\frac{1}{2}}}\\
\rho &= \frac{(n-1)f\tan\alpha}{\sqrt{1-(n^2-1)\tan^2\alpha}}\\
U_1 &= Q_{1t}\frac{\partial Q_{1z}}{\partial\alpha} - Q_{1z}\frac{\partial Q_{1t}(\alpha)}{\partial\alpha}\\
E_1 &= Q_{1z}+Q_{1t}\tan\alpha\\
U_2 &= Q_{2t}\frac{\partial Q_{2z}}{\partial\alpha} - Q_{2z}\frac{\partial Q_{2t}}{\partial\alpha}\\
E_2 &= Q_{2z}+Q_{2t}\tan\alpha\\
\frac{\partial Q_{1z}}{\partial\alpha} &= \frac{-\sin\alpha\left(n_1^2+n^2\cos2\alpha\right)}{\sqrt{n_1^2-n^2\sin^2\alpha}}+n\sin^2\alpha\\
\frac{\partial Q_{1t}}{\partial\alpha} &= \frac{\left(n_1^2-2n^2\sin^2\alpha\right)\cos\alpha}{\sqrt{n_1^2-n^2\sin^2\alpha}}-n\cos2\alpha\\
\frac{\partial Q_{2z}}{\partial\alpha} &= \frac{-\sin\alpha\left(n_2^2+n^2\cos2\alpha\right)}{\sqrt{n_2^2-n^2\sin^2\alpha}}+n\sin^2\alpha
\end{aligned}$$

$$\frac{\partial Q_{2t}}{\partial \alpha} = \frac{\left(n_2^2 - 2n^2 \sin^2 \alpha\right) \cos \alpha}{\sqrt{n_2^2 - n^2 \sin^2 \alpha}} - n \cos 2\alpha$$

and $\mathbf{E}_R^t$ and $\mathbf{E}_L^t$ are the vector amplitudes of the refracted rays at the refraction points. It is readily seen that the GO field expression becomes infinity at the points F_1 and F_2 as are expected. According to Maslov's method, the expressions for the field that is valid near the caustic for RCP and LCP fields are given by [10, 11].

$$\begin{aligned}
&\mathbf{E}_R(x, y, z) = \frac{n_1 k}{2\pi} \int_0^T \int_0^{2\pi} \mathbf{E}_R^t(\xi, \eta) \left[\frac{\partial Q_{1t}(\alpha)}{\partial \alpha} \frac{U_1 Q_{1t}}{P Q_{1z}}\right]^{\frac{1}{2}} \\
&\exp\Big[j n_1 k\Big(K_1(\alpha) r \sin\alpha \sin\theta_0 \cos(\phi_0 - \beta)\Big)\Big] \\
&\exp\Big[j n_1 k\Big((n + K_1(\alpha)\cos\alpha) z - K_1(\alpha)(\rho \sin\alpha + \zeta \cos\alpha)\Big)\Big] d\alpha d\beta \qquad (24a)
\end{aligned}$$

$$\begin{aligned}
&\mathbf{E}_L(x, y, z) = \frac{n_2 k}{2\pi} \int_0^T \int_0^{2\pi} \mathbf{E}_L^t(\xi, \eta) \left[\frac{\partial Q_{2t}(\alpha)}{\partial \alpha} \frac{U_2 Q_{2t}}{P Q_{2z}}\right]^{\frac{1}{2}} \\
&\exp\Big[j n_2 k\Big(K_2(\alpha) r \sin\alpha \sin\theta_0 \cos(\phi_0 - \beta)\Big)\Big] \\
&\exp\Big[j n_2 k\Big((n + K_2(\alpha)\cos\alpha) z - K_2(\alpha)(\rho \sin\alpha + \zeta \cos\alpha)\Big)\Big] d\alpha d\beta \qquad (24b)
\end{aligned}$$

Subtended angle T of lens is given by

$$T = \arctan\left(\frac{1}{\sqrt{n-1}} \frac{a}{\sqrt{(n+1)a^2 + (n-1)f^2}}\right)$$

The transmitted field for RCP field by lens at dielectric-chiral interface is related with the incident field by the relation [21].

$$\mathbf{E}_R^t = \tilde{\mathbf{T}} \cdot \mathbf{E_1} = \left[T_{R\|} i_\|^t i_\|^i + T_{R\perp} i_\perp^t i_\perp^i\right] \cdot \mathbf{E_1} \qquad (25)$$

where $\tilde{T}$ denotes the dyadic transmission coefficients. The subscripts $\|, \perp$ denotes components with respect to the plane of incidence, and subscripts t, i represents transmitted and incident waves, respectively

$$\begin{aligned}
\mathbf{E}_R^t \;=\; &\left(T_{R\perp} \sin^2\beta + \left[n \sin^2\alpha + \cos\alpha \sqrt{n_1^2 - n^2 \sin^2\alpha}\right] T_{R\|} \cos^2\beta\right) \mathbf{i}_x \\
&- \sin\beta \cos\beta \left[T_{R\perp} + \left(n \sin^2\alpha + \cos\alpha \sqrt{n_1^2 - n^2 \sin^2\alpha}\right) T_{R\|}\right] \mathbf{i}_y \\
&+ T_{R\|}\left(n \cos\alpha - \sqrt{n_1^2 - n^2 \sin^2\alpha}\right) \sin\alpha \cos\beta \mathbf{i}_z \qquad (26)
\end{aligned}$$

where

$$T_{R\parallel} = \frac{2n(E_1 - E_2)}{\eta + Z} T_{c11}, \quad T_{R\perp} = \frac{2n(E_1 + E_2)}{\eta + Z} T_{c12}$$

The integration with respect to β may be carried out using the integral representation of Bessel function. The results are expressed as

$$E_{Rx} = \frac{n_1 k}{2}\Big[P_R(r,\theta_0) - R_R(r,\theta_0)\cos 2\phi_0)\Big] \tag{27}$$

$$E_{Ry} = \frac{n_1 k}{2}[Q_R(r,\theta_0)\sin 2\phi_0] \tag{28}$$

$$E_{Rz} = jn_1 k[R_R(r,\theta_0)\sin\phi_0 \tag{29}$$

where

$$P_R(r,\theta_0) = \int_0^T \left[T_{R\perp} + \left(n\sin^2\alpha + \cos\alpha\sqrt{n_1^2 - n^2\sin^2\alpha}\right) T_{R\parallel}\right]$$
$$J_0(n_1 k K_1(\alpha) r \sin\theta_0 \sin\alpha)\left[\frac{\partial Q_{1t}(\alpha)}{\partial\alpha}\frac{U_1 Q_{1t}}{PQ_{1z}}\right]^{\frac{1}{2}}$$
$$\exp\Big[jn_1 k\Big((n+K(\alpha)\cos\alpha)z - K_1(\alpha)(\rho\sin\alpha + \zeta\cos\alpha)\Big)\Big]\,d\alpha$$

$$Q_R(r,\theta_0) = \int_0^T \left[\left(T_{R\parallel}(n\cos\alpha - \sqrt{n_1^2 - n^2\sin^2\alpha})\right)\sin\alpha\right]$$
$$J_1(n_1 k K_1(\alpha) r \sin\theta_0 \sin\alpha)\left[\frac{\partial Q_{1t}(\alpha)}{\partial\alpha}\frac{U_1 Q_{1t}}{PQ_{1z}}\right]^{\frac{1}{2}}$$
$$\exp\Big[jn_1 k\Big((n+K_1(\alpha)\cos\alpha)z - K_1(\alpha)(\rho\sin\alpha + \zeta\cos\alpha)\Big)\Big]\,d\alpha$$

$$R_R(r,\theta_0) = \int_0^T \left[\left(-T_{R\perp} + (n\sin^2\alpha + \cos\alpha\sqrt{n_1^2 - n^2\sin^2\alpha})T_{R\parallel}\right)\right.$$
$$J_2(n_1 k K_1(\alpha) r \sin\theta_0 \sin\alpha)\left[\frac{\partial Q_{1t}(\alpha)}{\partial\alpha}\frac{U_1 Q_{1t}}{PQ_{1z}}\right]^{\frac{1}{2}}$$
$$\exp\Big[jn_1 k\Big((n+K_1(\alpha)\cos\alpha)z - K_1(\alpha)(\rho\sin\alpha + \zeta\cos\alpha)\Big)\Big]\,d\alpha$$

The transmitted field for LCP field by lens at dielectric-chiral interface is related with the incident field by the relation [21]

$$\mathbf{E}_L^t = \tilde{\mathbf{T}} \cdot \mathbf{E_2} = \left[T_{R\parallel} i_\parallel^t i_\parallel^i + T_{R\perp} i_\perp^t i_\perp^i\right] \cdot \mathbf{E_2} \tag{30a}$$

$$\begin{aligned}\mathbf{E}_L^t &= \left(T_{L\perp}\sin^2\beta+\left[n\sin^2\alpha+\cos\alpha\sqrt{n_1^2-n^2\sin^2\alpha}\right]T_{L\|}\cos^2\beta\right)\mathbf{i}_x\\ &\quad-\sin\beta\cos\beta\left[T_{L\perp}+\left(n\sin^2\alpha+\cos\alpha\sqrt{n_1^2-n^2\sin^2\alpha}\right)T_{L\|}\right]\mathbf{i}_y\\ &\quad+T_{L\|}\left(n\cos\alpha-\sqrt{n_1^2-n^2\sin^2\alpha}\right)\sin\alpha\cos\beta\mathbf{i}_z \qquad (30\text{b})\end{aligned}$$

where

$$T_{L\|}=\frac{2n(E_1-E_2)}{\eta+Z}T_{c21},\quad T_{L\perp}=\frac{2n(E_1+E_2)}{\eta+Z}T_{c22}$$

The integration with respect to β again carried out and the results are

$$E_{Lx} = \frac{n_2k}{2}\Big[P_L(r,\theta_0)-R_L(r,\theta_0)\cos2\phi_0)\Big] \qquad (31)$$

$$E_{Ly} = \frac{n_2k}{2}\left[Q_L(r,\theta_0)\sin2\phi_0\right] \qquad (32)$$

$$E_{Lz} = jn_2k[R_L(r,\theta_0)\sin\phi_0 \qquad (33)$$

where

$$\begin{aligned}P_L(r,\theta_0)=&\int_0^T\left[T_{L\perp}+\left(n\sin^2\alpha+\cos\alpha\sqrt{n_2^2-n^2\sin^2\alpha}\right)T_{L\|}\right]\\ &J_0(n_2kK_2(\alpha)r\sin\theta_0\sin\alpha)\left[\frac{\partial Q_{2t}(\alpha)}{\partial\alpha}\frac{U_2Q_{2t}}{PQ_{2z}}\right]^{\frac{1}{2}}\\ &\exp\Big[jn_2k\Big((n+K_2(\alpha)\cos\alpha)z-K_2(\alpha)(\rho\sin\alpha+\zeta\cos\alpha)\Big)\Big]d\alpha\\ Q_L(r,\theta_0)=&\int_0^T\left[\left(T_{L\|}\left(n\cos\alpha-\sqrt{n_2^2-n^2\sin^2\alpha}\right)\right)\sin\alpha\right]\\ &J_1(n_2kK_2(\alpha)r\sin\theta_0\sin\alpha)\left[\frac{\partial Q_{2t}(\alpha)}{\partial\alpha}\frac{U_2Q_{2t}}{PQ_{2z}}\right]^{\frac{1}{2}}\\ &\exp\Big[jn_2k\Big((n+K_2(\alpha)\cos\alpha)z-K_2(\alpha)(\rho\sin\alpha+\zeta\cos\alpha)\Big)\Big]d\alpha\\ R_L(r,\theta_0)=&\int_0^T\left[\left(-T_{L\perp}+\left(n\sin^2\alpha+\cos\alpha\sqrt{1-n^2\sin^2\alpha}\right)T_{L\|}\right)\right.\\ &J_2(n_2kK_2(\alpha)r\sin\theta_0\sin\alpha)\left[\frac{\partial Q_{2t}(\alpha)}{\partial\alpha}\frac{U_2Q_{2t}}{PQ_{2z}}\right]^{\frac{1}{2}}\\ &\exp\Big[jn_2k\Big((n+K_2(\alpha)\cos\alpha)z-K_2(\alpha)(\rho\sin\alpha+\zeta\cos\alpha)\Big)\Big]d\alpha\end{aligned}$$

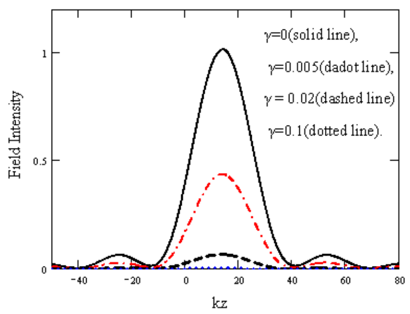

Figure 2. Field intensity around focal point along z-axis for RCP at $ka = 10$, $kf = 10$.

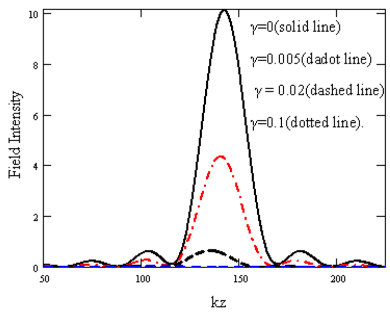

Figure 3. Field intensity around focal point along z-axis for RCP at $ka = 100$, $kf = 100$.

6. RESULTS AND DISCUSSION

In this section, above analytical Expressions (27)–(29) and (31)–(33) are used in calculating refracted electromagnetic field of dielectric plano convex lens placed in chiral medium for RCP and LCP by solving numerically. All the equations give similar results but we will present here results obtained from Equations (27)and (31). This problem has been studied by Hongo using Maslov's method for focussing of plano convex lens placed in free space [21]. Our expressions reduces to Hongo results at $\gamma = 0$ which verify validity of our expressions. Incident electric fields are assumed to be plane waves RCP and LCP. The dielectric lens has refractive index 2.5. Field intensity of LCP and RCP waves are obtained for different chirality parameters $\gamma = 0.0$, 0.005, 0.02 and 0.1.

Figure 2 represents the response for case of field intensity distribution around focal region of RCP field along z-axis at $ka = 10$ and $kf = 10$, $\gamma = 0$ (solid line), $\gamma = 0.005$ (dadot line), $\gamma = 0.02$ (dashed line) and $\gamma = 0.1$ (dotted line). Figure 3 represents the response for case of field intensity distribution around focal point of RCP along z-axis at $\gamma = 0$ (solid line), $\gamma = 0.005$ (dadot line), $\gamma = 0.02$ (dashed line) and $\gamma = 0.1$ (dotted line), $ka = 100$ and $kf = 100$. Figure 4 represents the response for case of field intensity distribution around focal region of RCP field along z-axis at $\gamma = 0$ (solid line), $\gamma = 0.005$ (dadot line), $\gamma = 0.02$ (dashed line) and $\gamma = 0.1$ (dotted line), $ka = 100$ and $kf = 200$. Figure 5 represents the response for case of field intensity distribution around focal region of RCP field along z-axis at $\gamma = 0$ (solid line), $\gamma = 0.005$ (dadot line), $\gamma = 0.02$ (dashed line) and $\gamma = 0.1$ (dotted line), $ka = 100$ and $kf = 100$.

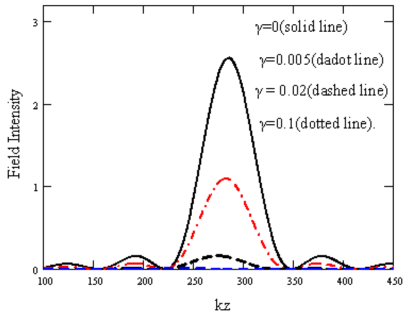

Figure 4. Field intensity around focal point along z-axis for RCP at $ka = 100$, $kf = 200$.

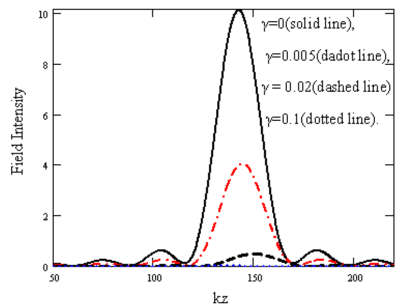

Figure 5. Field intensity around focal point along z-axis for LCP waves at $ka = 100$, $kf = 100$.

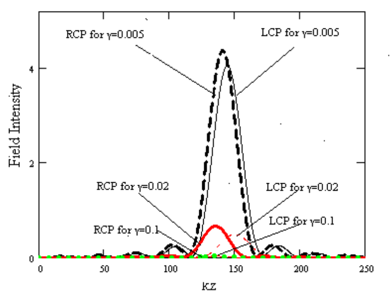

Figure 6. Comparison of field intensity around focal point at $ka = 100$, $kf = 100$ between RCP (Solid) and LCP (dotted) waves.

Figure 6 represents the response for cases of comparison field intensity distribution around focal region of LCP wave (dadot line) and RCP wave (solid line) along z-axis with variations of chirality parameters $\gamma = 0.005$, $\gamma = 0.02$ and $\gamma = 0.1$, $ka = 100$ and $kf = 100$ respectively.

We have imposed the condition for impedance matching for $\eta = Z$. At this condition there are no reflected waves in chiral medium, and all the field is transmitted into dielectric plano convex lens. This occurs for $\mu = \mu_1$ when $\epsilon_1 = \epsilon + \mu_1\gamma^2$. It is concluded from Figure 3 to Figure 6. that field intensity decreases as chirality parameters increase.The variation in field behaviors also observed by replacing different dielectric lens. The comparison of LCP wave and RCP wave show that focal region of RCP wave is displaced toward lens and focal region of LCP wave is displaced away from lens. It is also observed that field intensity of LCP wave is smaller than RCP wave.

7. CONCLUSION

Explicit analytical expressions have been obtained around the caustic region of plano convex lens placed in chiral medium. Effect of chiral-dielectric and dielectric-chiral interfaces have been studied using Maslov's method in the focal region of plano convex lens. The effect of chirality on the amplitude of transmitted field by lens in caustic region are shown and discussed. It is observed that field intensity in the caustic region of a plano convex lens decreases by increasing the chirality parameter. This study is helpful for remote sensing as a sensor and its practical interest is emerging from an engineering applications viewpoint. In remote sensing, the vegetation layer can be design as chiral lens. These results obtained can be used to analyze radar data from such vegetation layers and to obtain the physical properties of the layers.

ACKNOWLEDGMENT

Authors would like to thank Higher Education Commission (HEC) for startup research grant under IPFP.

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