

ANALYTICAL CALCULATION OF BAND GAP OF A 1D PLANAR TERNARY PHOTONIC CRYSTAL BY SIMULATING WITH A SYMMETRIC LOSSLESS TRANSMISSION LINE

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Abstract—We simulate a 1D ternary photonic crystal (TPC) employed as a clad of a photonic crystal waveguide (PCW) which consists three different lossless dielectric layers as a unit-cell. Calculating input impedance at each layer interface and using a lossless reciprocal transmission line as a model, we can predict angle intervals in which reflection occurs due to photonic crystal effect. Comparing this method with transfer matrix method and band structure shows perfect agreement.

1. INTRODUCTION

A transmission line guides energy from one place to another. Optical fibers, waveguides, telephone lines and power cables are all electromagnetic transmission lines. Photonic crystals are new kinds of materials that prohibit transmission of light in specific ranges of frequencies called photonic band gap (PBG). The PBG underlies the operation of such widespread optical components as multilayer coatings [1, 2], Bragg reflectors and waveguides [3–7], planar photonic crystal waveguide devices [8, 9], planar photonic crystal polarization splitter [10] and as optical filters by using a defect [11]. The PCWs have been the subject of extensive research recently because of their ability to control the propagation of light in a manner different from total internal reflection. The TPC can have several applications like refractometric sensing elements.

Received 20 October 2011, Accepted 30 November 2011, Scheduled 7 December 2011

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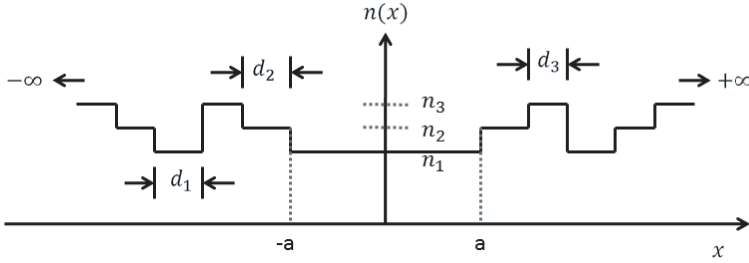


Figure 1. A TPC waveguide index profile.

Multilayer dielectric structures have been analyzed by different methods, like transmission line matrix method [12] which is used as a modeling tool for computation of the dispersion relation of photonic crystals (PCs) [13] and closed form analytical solution which has been presented before for 1D planar binary PCW's with infinite number of cladding layers [14, 15].

In this paper, we present exact analytical solutions for a 1D planar TPC waveguide by using transmission line concept.

2. REFLECTION OF A SEMI-INFINITE PERIODIC DIELECTRIC STRUCTURE

A profile of a 1D planar TPC waveguide with infinite cladding is shown in Figure 1. Such a waveguide may be viewed as a dielectric region of thickness $2a$ and refractive index n_1 sandwiched between two semi-infinite periodic structures (regions $x < -a$ and $a < x$). Thus, propagation of electromagnetic waves in this waveguide essentially amounts to multiple reflections between the two semi-infinite regions. These regions consist of periods of three different lossless dielectric layers with $n_1 < n_2 < n_3$ indices and $d_1 = d_2 = d_3$ thicknesses.

In the transmission lines theory, reflection coefficient is [16–18]:

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1}. \quad (1)$$

Input impedance Z_{in} (the impedance at the beginning of crystal or at the beginning of equivalent transmission line) at $x = a$ is equal to input impedance seen at $x = a + d_1 + d_2 + d_3$. Using relations between input impedance at boundaries, input impedances at first, second and third boundaries in the right hand clad are:

$$Z_{in}(x = a) = Z_2 \frac{Z'_{in} + iZ_2 \tan(\beta_2 d_2)}{Z_2 + iZ'_{in} \tan(\beta_2 d_2)}, \quad (2)$$

$$Z'_{in}(x = a + d_2) = Z_3 \frac{Z''_{in} + iZ_3 \tan(\beta_3 d_3)}{Z_3 + iZ''_{in} \tan(\beta_3 d_3)}, \quad (3)$$

$$Z''_{in}(x = a + d_2 + d_3) = Z_1 \frac{Z_{in} + iZ_1 \tan(\beta_1 d_1)}{Z_1 + iZ_{in} \tan(\beta_1 d_1)}, \quad (4)$$

where Z_i , n_i and β_i are wave impedances, refractive index propagation constants in the i th layer of a unit cell for $i = 1, 2, 3$, respectively. If we solve these three equations for Z_{in} , simplifying and rearranging the terms, we obtain

$$a_0 Z_{in}^2 + b_0 Z_{in} + c_0 = 0, \quad (5)$$

where the coefficients of Z_{in} are

$$a_0 = i \tan(\beta_2 d_2) Z_3 Z_1 + i \tan(\beta_1 d_1) Z_3 Z_2 + i \tan(\beta_3 d_3) Z_1 Z_2 + i Z_3^2 \tan(\beta_1 d_1) \tan(\beta_2 d_2) \tan(\beta_3 d_3), \quad (6)$$

$$b_0 = Z_2^2 Z_3 \tan(\beta_2 d_2) \tan(\beta_1 d_1) + Z_2^2 Z_1 \tan(\beta_2 d_2) \tan(\beta_3 d_3) + Z_3^2 Z_2 \tan(\beta_3 d_3) \tan(\beta_1 d_1) - Z_1^2 Z_3 \tan(\beta_2 d_2) \tan(\beta_1 d_1) - Z_3^2 Z_1 \tan(\beta_2 d_2) \tan(\beta_3 d_3) - Z_1^2 Z_2 \tan(\beta_3 d_3) \tan(\beta_1 d_1), \quad (7)$$

$$c_0 = -i Z_1^2 Z_2 Z_3 \tan(\beta_1 d_1) - i Z_3^2 Z_2 Z_1 \tan(\beta_3 d_3) - i Z_2^2 Z_1 Z_3 \tan(\beta_2 d_2) + i Z_2^2 Z_1^2 \tan(\beta_1 d_1) \tan(\beta_2 d_2) \tan(\beta_3 d_3). \quad (8)$$

Introducing $\bar{\beta} = n_1 \sin(\theta_1)$ as tangential effective index, which is constant in all layers and also

$$\beta_1 = n_1 k_0 \cos(\theta_1) = k_0 u, \quad U = \beta_1 d_1 = k_0 u d_1, \quad (9)$$

$$\beta_2 = k_0 v, \quad V = \beta_2 d_2 = k_0 v d_2, \quad (10)$$

$$\beta_3 = k_0 w, \quad W = \beta_3 d_3 = k_0 w d_3. \quad (11)$$

The input impedance in each layer is

$$Z_1 = \begin{cases} \frac{Z_0 \cos(\theta_1)}{n_1} = \frac{Z_0 u}{n_1^2} & \parallel \\ \frac{Z_0}{\cos(\theta_1) n_1} = \frac{Z_0}{u} & \perp \end{cases}, \quad Z_2 = \begin{cases} \frac{Z_0 \cos(\theta_2)}{n_2} = \frac{Z_0 v}{n_2^2} & \parallel \\ \frac{Z_0}{\cos(\theta_2) n_2} = \frac{Z_0}{v} & \perp \end{cases}, \quad (12)$$

$$Z_3 = \begin{cases} \frac{Z_0 \cos(\theta_3)}{n_3} = \frac{Z_0 w}{n_3^2} & \parallel \\ \frac{Z_0}{\cos(\theta_3) n_3} = \frac{Z_0}{w} & \perp \end{cases}.$$

where \perp and \parallel , denote to perpendicular and parallel polarizations, respectively. Introducing the parameters

$$u = \sqrt{n_1^2 - \bar{\beta}^2}, \quad v = \sqrt{n_2^2 - \bar{\beta}^2}, \quad w = \sqrt{n_3^2 - \bar{\beta}^2}, \quad (13)$$

The coefficients in Equations (6)–(8) for TE waves are

$$a_0 = -iZ_0^2 a_1 \Rightarrow a_1 = -\frac{\tan(V)}{uw} - \frac{\tan(U)}{vw} - \frac{\tan(W)}{uv} + \frac{\tan(U)\tan(V)\tan(W)}{w^2} \perp, \quad (14)$$

$$b_0 = -Z_0^3 b_1 \Rightarrow b_1 = -\frac{\tan(V)\tan(U)}{v^2 w} - \frac{\tan(V)\tan(W)}{uv^2} - \frac{\tan(W)\tan(U)}{vw^2} + \frac{\tan(V)\tan(U)}{u^2 w} + \frac{\tan(V)\tan(W)}{uw^2} + \frac{\tan(W)\tan(U)}{u^2 v} \perp, \quad (15)$$

$$c_0 = iZ_0^4 c_1 \Rightarrow c_1 = -\frac{\tan(U)}{u^2 vw} - \frac{\tan(W)}{uvw^2} - \frac{\tan(V)}{uv^2 w} + \frac{\tan(V)\tan(W)\tan(U)}{u^2 v^2} \perp. \quad (16)$$

and for TM waves are

$$a_0 = -iZ_0^2 \tilde{a}_1 \Rightarrow \tilde{a}_1 = -\frac{\tan(V)wu}{n_3^2 n_1^2} - \frac{\tan(U)wv}{n_3^2 n_2^2} - \frac{\tan(W)uv}{n_1^2 n_2^2} + \frac{w^2 \tan(V)\tan(W)\tan(U)}{n_3^4} \parallel, \quad (17)$$

$$b_0 = -Z_0^3 \tilde{b}_1 \Rightarrow \tilde{b}_1 = -\frac{\tan(V)\tan(U)wv^2}{n_3^2 n_2^4} - \frac{\tan(V)\tan(W)uw^2}{n_2^4 n_1^2} - \frac{\tan(U)\tan(W)vw^2}{n_2^2 n_3^4} + \frac{\tan(V)\tan(U)uw^2}{n_3^2 n_1^4} + \frac{\tan(U)\tan(W)vu^2}{n_2^2 n_1^4} + \frac{\tan(V)\tan(W)uw^2}{n_1^2 n_3^4} \parallel, \quad (18)$$

$$c_0 = iZ_0^4 \tilde{c}_1 \Rightarrow \tilde{c}_1 = -\frac{vwu^2 \tan(U)}{n_1^4 n_2^2 n_3^2} - \frac{vuw^2 \tan(W)}{n_1^2 n_2^2 n_3^4} - \frac{uwv^2 \tan(U)}{n_1^2 n_2^4 n_3^2} + \frac{v^2 u^2 \tan(U)\tan(V)\tan(W)}{n_1^4 n_2^4} \parallel. \quad (19)$$

Hence, the input impedances for both TE and TM waves are

$$\begin{cases} Z_{in} = -iZ_0 \frac{b_1 + \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \perp \\ Z_{in} = -iZ_0 \frac{\tilde{b}_1 + \sqrt{\tilde{b}_1^2 - 4\tilde{a}_1 \tilde{c}_1}}{2\tilde{a}_1} \parallel \end{cases}. \quad (20)$$

3. CALCULATING BAND GAP FROM INPUT IMPEDANCE

The input impedance method which has been discussed in the previous section is an exact method to investigate planar multilayer structures. Let us consider a 1D dielectric PCW has a geometry as shown in Figure 1 with properties of material structure $n_1 = 1$, $n_2 = 1.45$ and $n_3 = 3.5$; $d_1 = d_2 = d_3 = a = 1 \mu\text{m}$. Variations of the real and imaginary parts of Z_{in} of this system versus two wavelengths $\lambda = 1.5 \mu\text{m}$ and $\lambda = 1.55 \mu\text{m}$ for the case of perpendicular polarization and the ‘-’ solution of (5) which is normalized with Z_0 , is depicted in Figure 2.

In this figure, we see that in some regions (approximately 0–24, 42–62 and 71–90 degrees in part (a) and 0–7, 35–55 and 65–90 degrees in part (b)), the real part of Z_{in} is zero. Therefore, the reflection coefficient which is defined in (1) must have an absolute value equal to one.

4. CALCULATION OF BAND GAP FROM TRANSFER MATRIX METHOD

One of the methods to investigate a 1D photonic crystal is transfer matrix method [19–21] which is applicable to calculate transmission and reflection of these structures. We use it to obtain the reflectance of this system and compare it with input impedance. Using transfer

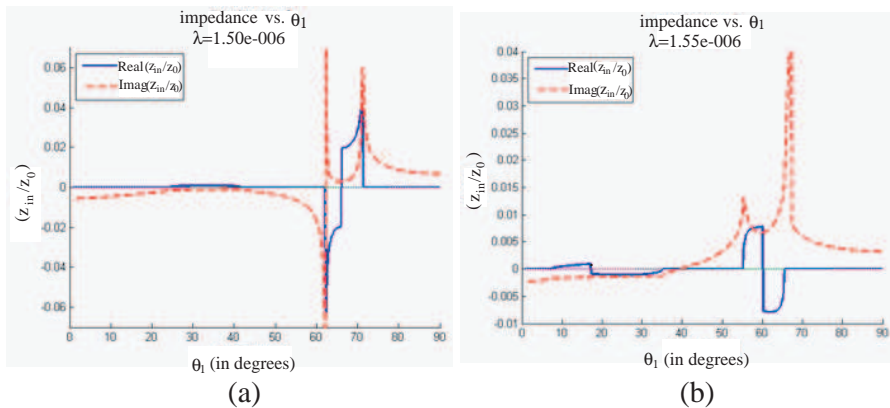


Figure 2. Real and imaginary parts of normalized input impedance of a periodic dielectric structure at two wavelengths. (a) $\lambda = 1.5 \mu\text{m}$. And (b) $\lambda = 1.55 \mu\text{m}$.

matrix method, we show the reflectance of the previous system vs. incident angle in Figure 3. For each specific wavelength, there are some angle intervals for which band gaps occur.

We can see that for some ranges of angles, real part of (Z_{in}/Z_0) is zero in Figure 2. If we compare these regions in Figure 2 with their correspondent's in Figure 3, we can observe that exactly anywhere the real part of input impedance is zero, the band gap occurs.

5. COMPARING WITH REFLECTANCE AND BAND STRUCTURE

In order to confirm our claim, we compare $|\text{Re}(Z_{in})|$ with reflectance and band gap structure. First, we compare $|\text{Re}(Z_{in})|$ with reflectance.

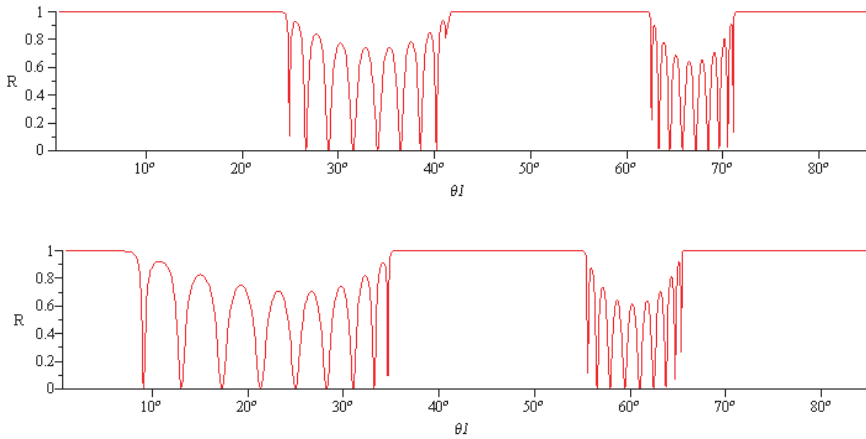


Figure 3. Reflectances of system of Figure 2 at two wavelengths.

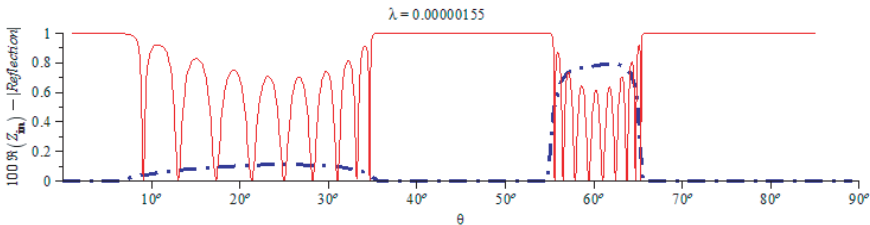


Figure 4. Comparing reflectance (solid line) and real part of normalized input impedance ($\times 100$) (dash dotted lines) of system of Figure 2(b).

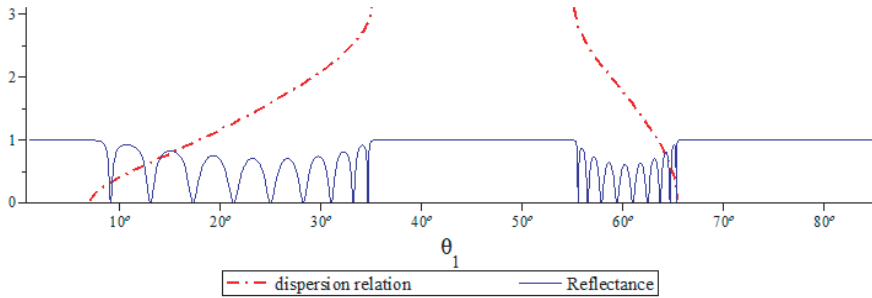


Figure 5. Comparing Reflectance (solid line) and band structures (dash dotted lines) of system of Figure 2(b)

$|\text{Re}(Z_{in})|$ and reflectance of system of Figure 2(b) are shown in Figure 4 simultaneously. Here, it is well displayed that everywhere the real part of input impedance is zero, the reflectance of that system is one and band gap occurs.

Second, in Figure 5, we have shown a couple of comparative plots of band structures and reflectivity of a semi-infinite periodic dielectric structure of Figure 2(b). It can be observed that they are completely compatible.

6. CONCLUSION

Photonic crystal waveguides can guide waves without total internal reflection which has some losses due to transmission of wave in a dielectric medium. In this paper, we use air as the core of a 1D planar TPC waveguide and simulate it with a lossless reciprocal transmission line. Using the input impedance of the system, we calculated angle regions of incident wave in which band gaps occur. Thus, wherever real part of input impedance is zero, transfer matrix method and band structure confirm these regions as PBG precisely.

ACKNOWLEDGMENT

This work has been financially supported by the Payame Noor University (PNU) I. R. of Iran under the grant No. 1390/3/0/14/185.

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