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THE ELLIPTIC GAUSSIAN BEAM SCATTERING ON PHASED ANTENNA ARRAY WITH RECTANGULAR WAVEGUIDES

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Abstract—The diffraction problem of a three-dimensional elliptic Gaussian beam on a aperture array of rectangular holes is solved. The both normal and oblique incidences of the beam are considered and the results are presented in the form of the three-dimensional patterns. The pattern lobe distortion and conditions at which the side lobes appear are studied. The conditions under which the shift of the reflected and transmitted field patterns appears are studied. The existence of higher spatial Floquet harmonics in the case of oblique beam incidence is observed.

1. INTRODUCTION

To date scattering features of a plane electromagnetic wave on periodic structures are studied quite well. However in the real devices, the field exists in the form of the beams with certain distribution of the field intensity on their cross-section (for example, Gaussian beams). In the case of the bounded beam scattering on semi-transparent two-dimensional periodic structures not only form distortion of the reflected and transmitted beams occurs, but also diverse intensity modulations on its cross-section appear. Therefore the analysis of the transformation and shift of the beam profiles scattered on various types of two-dimensional periodic structures is important.

The plane phased antenna arrays (PAA's), which are made of waveguides with certain cross-sections, are widely applied in a radar-location, radio communication, and radio astronomy. They are also effectively used as irradiators in hybrid reflector antennas. The

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radiating or reflecting surface of such antenna (effective radiating plane) can be considered as a two-dimensional periodic structure which basic cell consists of a waveguide unit. For successful application of waveguide-type two-dimensional periodic diffraction arrays in PAA's it is necessary to consider their scattering electromagnetic characteristics in the case when the incident field has a form of spatial beam.

The problems of the beams diffraction on various periodic structures were investigated by a number of authors [1–16]. In [15] the results of the scattering electromagnetic characteristics of the linearly polarized Gaussian beam with circular cross-section are presented in the case of normal beam incidence on the PAA. The array is made of a plane perfectly conducting screen of a finite thickness with rectangular cross-section waveguide channels. It is shown that the form of field pattern of the reflected and transmitted beams significantly changes in comparison with the form of field pattern of the incident beam. The effect of the narrowing of the transmitted field pattern appears, and the reflected field pattern undergoes some distortions.

The special interest is to the electromagnetic characteristics of the reflected and transmitted Gaussian beams with elliptic cross-section in the case of both normal and oblique beam incidences on the phased antenna array with waveguide type units. The field pattern, magnitude and phase distribution of transmitted and reflected fields of an elliptic beam can differ considerably from corresponding characteristics of fields in the case of circular cross-section beam scattering.

In the case of oblique three-dimensional Gaussian beam incidence on the two-dimensional periodic phased antenna array it is important to consider not only the scattered field pattern of main diffraction lobe, but also to elucidate the effect of side diffraction lobes to the spatial electromagnetic characteristics of the reflected and transmitted fields. It is important to know, under which conditions (incident beam angle, frequency, array parameters) and at which angles of observation the side-lobes arise. The knowledge about their role in changing spatial configuration of an electromagnetic field distribution of a scattered three-dimensional beam is also sufficiently significant.

The purpose of this work is to study the field pattern transformation of Gaussian linearly polarized beam with elliptic cross-section in the case when the beam normally and obliquely impinges on the phased antenna array in the form of rectangular waveguides. The main goal is to evaluate the conditions when the side–lobes occur in the scattered field pattern.

2. PROBLEM FORMULATION

The linearly polarized Gaussian beam obliquely impinges on the two-dimensional periodical structure from the half space $z > 0$ (Fig. 1). The apertures array is located in the plane xOy. The centers of the elementary cells are located at the nodes of the oblique coordinate system, which is placed in the plane of the aperture screen. The node positions are determined with the angle χ. It is required to find the electromagnetic field scattered by an array in space. The transverse electric field component distribution of the incident beam in the plane $z_p = 0$ is given in the following form.

$$\vec{E}_t^i\left(x_p, y_p, 0\right) = \frac{4\pi}{\sqrt{S}} \exp\left\{-\left(\frac{x_p}{w_1}\right)^2 - \left(\frac{y_p}{w_2}\right)^2\right\} \cdot \left(\vec{e}_{xp} \cos\alpha_0 - \vec{e}_{yp} \sin\alpha_o\right) \quad (1)$$

where S is the area of the screen's unit cell; w_1, w_2 are parameters which define the effective size of the beam in the place $z_p = 0$; $\vec{e}_{xp}$, $\vec{e}_{yp}$ are unit vectors of the coordinate system $x_p y_p z_p$. The polarization angle α_0 is defined in the coordinate system $x_p y_p z_p$, which is associated with the beam (Fig. 2).

The transverse component of the electric field of the incident beam is represented as the sum of the two beams with different

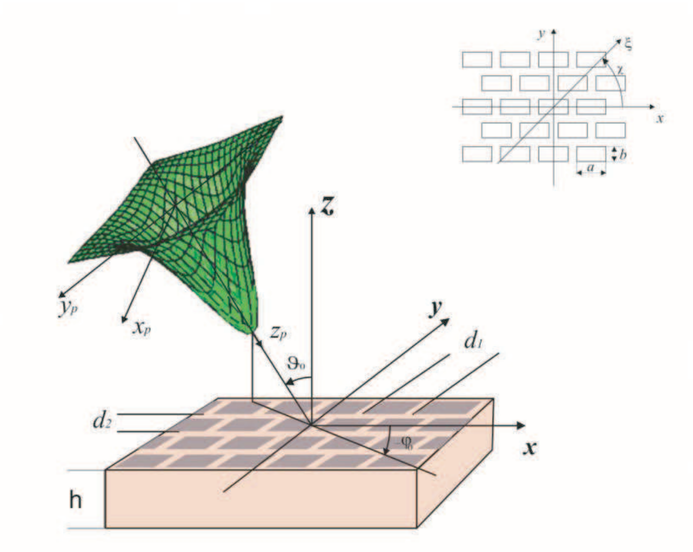


Figure 1. The Gaussian beam incidence on two-dimensional periodical structure.

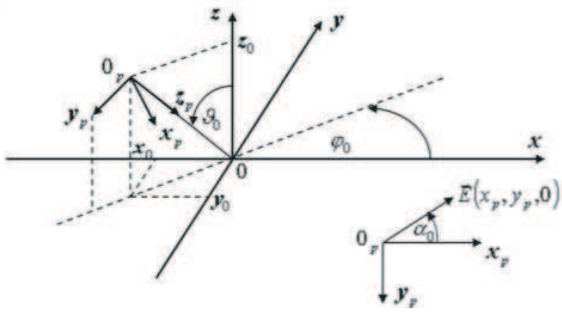


Figure 2. Coordinate systems associated with array (xyz) and beam ($x_p y_p z_p$).

polarizations (TE and TM polarized beams). Each of these beams can be represented as an expansion in the form of Fourier integral related to the plane TE and TM polarized waves, respectively:

$$\begin{aligned}\vec{E}_t^i(x,y,z) &= \frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_1(\xi,\zeta)\frac{\xi\vec{e}_x-\zeta\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}e^{ik(x\zeta+y\xi-\gamma z)}d\xi d\zeta \\ &+\frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_2(\xi,\zeta)\frac{\zeta\vec{e}_x+\xi\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}e^{ik(x\zeta+y\xi-\gamma z)}d\xi d\zeta \quad (2)\end{aligned}$$

where $G_{1,2}(\xi,\zeta)$ are the incident beam spectral functions; $k = 2\pi/\lambda$, $\gamma = \sqrt{1-\xi^2-\zeta^2}$. The integration variables ξ, ζ have the following meanings: $\xi = \sin\vartheta\cos\varphi$, $\zeta = \sin\vartheta\sin\varphi$, where ϑ, φ are the angles of incidence of a separate spatial TE or TM polarized harmonics with the amplitude $G_1(\xi,\zeta)$ and $G_2(\xi,\zeta)$, respectively. The angles ϑ, φ are determined similarly as the incidence angles ϑ_0, φ_0 in the range of their real values.

Transverse electric components of the reflected field can be also represented as the sum of the transverse field components of two TE and TM polarized beams. Each of them is expanded in the form of Fourier integrals related to the TE and TM polarized plane waves:

$$\begin{aligned}\vec{E}_t^r(x,y,z) &= \frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} R_1(\xi,\zeta)e^{ik(x\zeta+y\xi+\gamma z)}\frac{\xi\vec{e}_x-\zeta\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}d\zeta d\xi \\ &+\frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} R_2(\xi,\zeta)e^{ik(x\zeta+y\xi+\gamma z)}\frac{\zeta\vec{e}_x+\xi\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}d\zeta d\xi \quad (3)\end{aligned}$$

where $R_1(\xi,\zeta)$ and $R_2(\xi,\zeta)$ are unknown spectral functions, and indexes 1 and 2 correspond to the TE and TM beams, respectively.

A way of solving this problem is described in our previous paper [15] in detail. It consists in finding a relation between the unknown spectral functions $R_1(\xi,\zeta)$, $R_2(\xi,\zeta)$ and the certain elements of generalized scattering matrix and the known spectral features $G_1(\xi,\zeta)$, $G_2(\xi,\zeta)$. This relation has the next form:

$$\begin{aligned} R_1(\xi,\zeta) &= \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\left\{G_1(\hat{\xi},\hat{\zeta})_{TE}r_{qs}^{(1)}(\hat{\xi},\hat{\zeta})+G_2(\hat{\xi},\hat{\zeta})_{TM}r_{qs}^{(1)}(\hat{\xi},\hat{\zeta})\right\} \\ R_2(\xi,\zeta) &= \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\left\{G_1(\hat{\xi},\hat{\zeta})_{TE}r_{qs}^{(2)}(\hat{\xi},\hat{\zeta})+G_2(\hat{\xi},\hat{\zeta})_{TM}r_{qs}^{(2)}(\hat{\xi},\hat{\zeta})\right\} \end{aligned} \tag{4}$$

where $\hat{\xi} = \xi + s/\kappa_2 - q\cot(\chi)/\kappa_1$, $\hat{\zeta} = \zeta + q/\kappa_1$, $\kappa_1 = d_1/\lambda$, $\kappa_2 = d_2/\lambda$, $\xi = \sin\vartheta\sin\varphi$, $\varsigma = \sin\vartheta\cos\varphi$, and $r_{qs}^{(1,2)}$ are certain elements of the generalized scattering matrix of the two-dimensional periodical structure. The latter ones are found via solution of the key diffraction problem related to the spectra of the TE and TM linearly polarized plane electromagnetic waves. The upper indexes 1 and 2 correspond to the TE and TM waves, respectively. We can choose the beam and array parameter thus that in the double sums (4) would be to consider enough only one term of a series. The given approach is true as the absolute value of functions $G_1(\hat{\xi},\hat{\zeta})$ and $G_2(\hat{\xi},\hat{\zeta})$ for Gaussian beams is distinct from zero only in a small interval of values of angels ϑ,ϕ in the case when only one spatial Floquet harmonic propagates $q = s = 0$. At $q \neq 0$, $s \neq 0$ the absolute values of spectral functions tend to zero $\left|G_1(\hat{\xi},\hat{\zeta})\right| \to 0$, $\left|G_2(\hat{\xi},\hat{\zeta})\right| \to 0$ in all intervals of argument values. Then, the spectral functions of the scattered beam can be calculated using:

$$\begin{aligned} R_1(\vartheta,\phi) &\approx \left\{G_1(\vartheta,\phi)_{TE}r_{00}^{(1)}(\vartheta,\phi) + G_2(\vartheta,\phi)_{TM}r_{00}^{(1)}(\vartheta,\phi)\right\}, \\ R_2(\vartheta,\phi) &\approx \left\{G_1(\vartheta,\phi)_{TE}r_{00}^{(2)}(\vartheta,\phi) + G_2(\vartheta,\phi)_{TM}r_{00}^{(2)}(\vartheta,\phi)\right\}, \end{aligned} \tag{5}$$

where ${}_{TE}r_{00}^{(1)}(\vartheta,\phi)$, ${}_{TM}r_{00}^{(1)}(\vartheta,\phi)$, ${}_{TE}r_{00}^{(2)}(\vartheta,\phi)$, ${}_{TM}r_{00}^{(2)}(\vartheta,\phi)$ are amplitudes of zero spatial Floquet harmonics, which are found via solution of the key diffraction problem of the TE and TM linearly polarized plane electromagnetic waves on two-dimensional periodic structure. If the relation between sizes of the periods of an array, a wavelength and an incident angel of beam are that there are several spatial Floquet harmonics in space (diffraction beams of the higher usages) it is necessary to use exact formulas (4). It is enough to consider 50 spatial harmonics for convergence of the double sums for the chosen parameters of a beam and array.

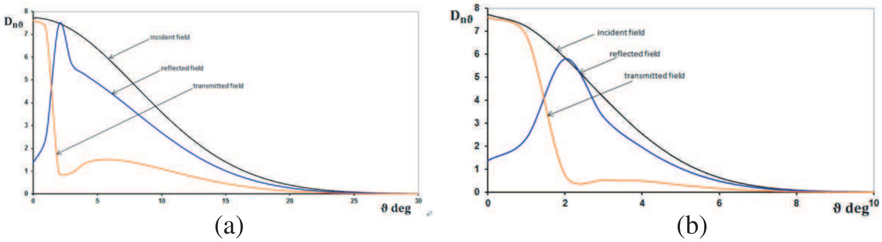


Figure 3. The field pattern. (a) $w_1 = 10\,\mathrm{mm}$, $w_2 = 30\,\mathrm{mm}$. (b) $w_1 = 30\,\mathrm{mm}$, $w_2 = 10\,\mathrm{mm}$, $f = 48.2\,\mathrm{GHz}$.

The expressions for the patterns across the field and the intensity of the reflected beam in the far-field region are obtained as follows [15]:

$$D_{n\varphi} = |R_1(\vartheta,\varphi)|\cos\vartheta,\; D_{n\vartheta} = |R_2(\vartheta,\varphi)|\,,\; D = (D_{n\varphi})^2 + (D_{n\vartheta})^2 \qquad (6)$$

3. NUMERICAL RESULTS

The structure under study is a plane, perfectly conducting screen of finite thickness h, in which the rectangular holes are periodically perforated in two not orthogonal directions (Fig. 1). The rectangular holes of the finite thickness screen are rectangular waveguides $(a \times b)$ in which only the fundamental mode can propagate. For such structure the form of generalized scattering matrix is known [16]. The rectangular mesh screen parameters are: $a = 5\,\mathrm{mm}$, $b = 1\,\mathrm{mm}$, $S = (6 \times 6)\,\mathrm{mm}^2$, $h = 9\,\mathrm{mm}$, $\chi = 90\,\mathrm{deg}$. Polarization angle is $\alpha_0 = 0\,\mathrm{deg}$. For the chosen beam polarization there is the most efficient excitation of the waveguide fundamental mode in the waveguide channels of the two-dimensional periodic structure. When the condition $w_1 < w_2$ for elliptic beam cross-section size is satisfied, the semi-major axis of the ellipse is parallel to the x-axis, and minor axis is parallel to the y-axis. In the case of $w_1 > w_2$, the semi-major axis of the ellipse is parallel to the y-axis, and minor axis is parallel to the x-axis. The calculations are provided at the frequencies where the most dramatic changes in the forms of patterns of the reflected and transmitted beams appear. In addition, at these frequencies, the reflection coefficient of the beam reaches its minimal values. In Fig. 3 the field patterns of incident, reflected and transmitted beams at plane $\varphi = 90\,\mathrm{deg}$ are presented.

Also in Fig. 4 for clearness sake, the three-dimensional incident, reflected and transmitted field distribution diagrams are presented in the case of the normal incidence of elliptic beam.

The obtained results show that the patterns of the reflected

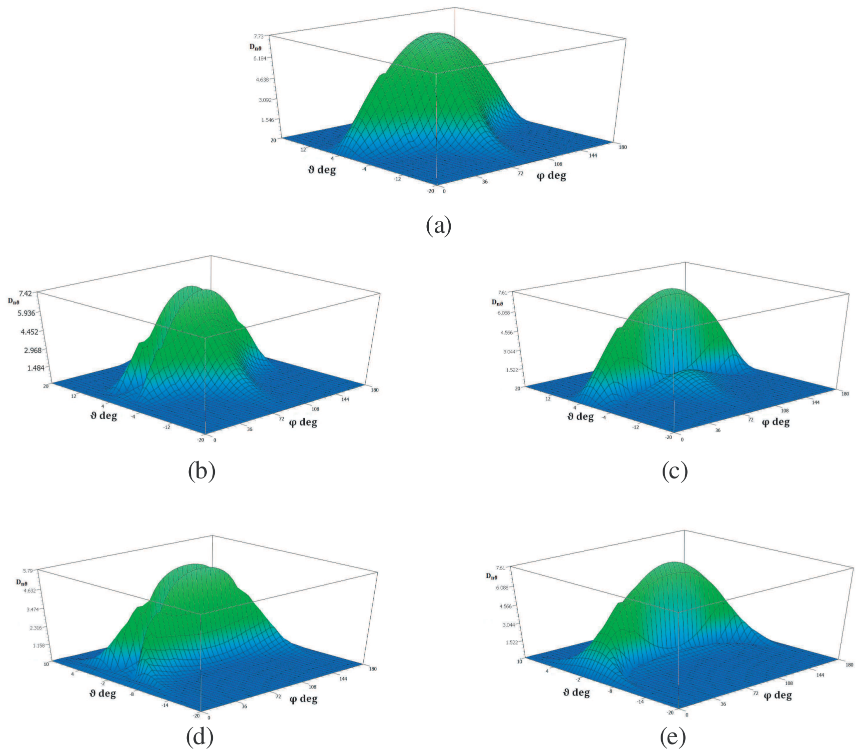


Figure 4. The pattern of (a) incident field $w_1 = 30\,\mathrm{mm}$, $w_2 = 10\,\mathrm{mm}$ (b), (d) reflected and (c), (e) transmitted fields of elliptic beam (b), (c) $w_1 = 10\,\mathrm{mm}$, $w_2 = 30\,\mathrm{mm}$ and (d), (e) $w_1 = 30\mathrm{mm}$, $w_2 = 10\,\mathrm{mm}$ $f = 48.2\,\mathrm{GHz}$.

and transmitted beams change, as compared with the pattern of the incident beam. At certain frequencies the patterns become narrower and the focusing of the transmitted field appears whereas the pattern of the reflected field undergoes some distortion. The effect of the transmitted pattern narrowing can be explained from the following consideration. As noted above, at the frequency $f = 48.2\,\mathrm{GHz}$, the wavelength of the fundamental mode in the waveguide channels is approximately equals to the thickness of the screen. In this case, a sharp increasing of the field magnitude of the fundamental mode in the waveguide channels appears. It results in the rising of efficiency of the wave interaction between adjacent waveguide channels in the entire array. In addition, the resonance frequency $f = 48.2\,\mathrm{GHz}$ lies nearly the "mixing point" when the magnitude of the surface harmonics

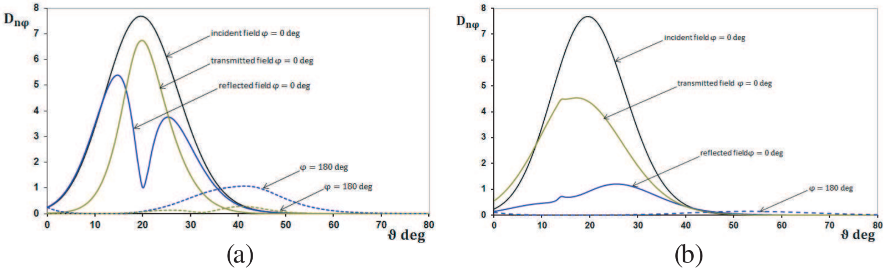


Figure 5. The field patterns (a) $f = 49.5\,\text{GHz}$ and (b) $f = 40.0\,\text{GHz}$.

increases rapidly and the higher spatial harmonics begin to propagate. The degree of the interaction between the waveguide channels and the effective radiating surface of the screen also increases, compared with the transverse dimensions of the incident beam. This leads to a narrowing of the pattern of the scattered field, compared to the pattern of the incident beam. The effect of the pattern narrowing is the most pronounced in the planes $\varphi = \pm 90\,\text{deg}$, since the electric field vector of the fundamental mode lies in the plane parallel to the plane, $\varphi = \pm 90\,\text{deg}$ and the interaction of the waveguide channels, which operates in the single-mode regime on the fundamental mode, is provided strongly in this plane.

The dependences of pattern form of the transmitted and reflected fields versus the size of the cross-section of the incident beam are investigated. Thus the reflected and transmitted field patterns distortion is the most underlined in the case when the cross-section of beam is narrowing. It is established that the focusing effect of the transmitted field occurs at the frequencies, where the higher spatial Floquet harmonics begin to propagate. The focusing effect occurs when the screen thickness is approximately equals to the one wavelength of the fundamental mode in the waveguide.

The results of numerical studies of the scattering characteristics of an elliptic beam which obliquely impinges on the two-dimensional periodic screen of finite thickness with rectangular holes at an angle $\vartheta_0 = 20\,\text{deg}$ are presented in Figs. 5, 6. Geometric parameters of the screen are the same as in previous cases. The elliptic beam with polarization angle $\alpha_0 = 0\,\text{deg}$ incidents on structure at plane $\varphi = 0\,\text{deg}$ and has spatial sizes $w_1 = 30\,\text{mm}$, $w_2 = 10\,\text{mm}$. The calculations are performed at the frequencies where the transmission of the electromagnetic field through the screen reaches its maximum. The fields patterns in the planes $\varphi = 0\,\text{deg}$ and $\varphi = 180\,\text{deg}$ are presented in Fig. 5. Also the narrowing of the transmitted field pattern and

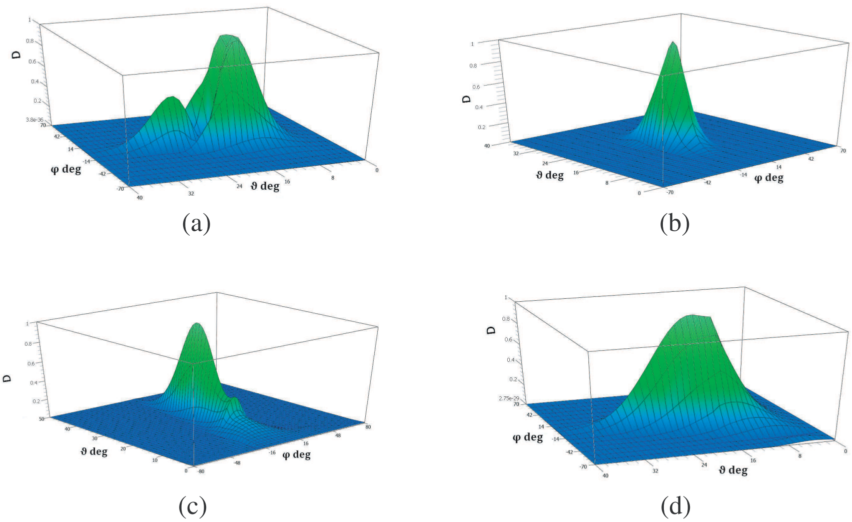


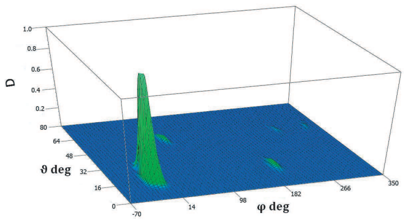
Figure 6. The pattern of (a), (c) reflected and (b), (d) transmitted fields for (a), (b) $f = 49.5\,\mathrm{GHz}$ and (c), (d) $f = 40.0\,\mathrm{GHz}$.

the distortion of the reflected field pattern can be observed versus incident field pattern. The three-dimensional normalized reflected and transmitted intensity patterns are also depicted in Fig. 6.

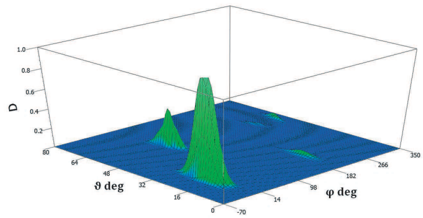
From the analysis of the data presented in Figs. 5 and 6 one can see that in the case of oblique incidence of an elliptic beam, the effects of the pattern narrowing and the maximum shifting of the reflected and transmitted fields in far-field zone are also observed. These effects appear due to the amplitude-phase distribution variation on the screen surface on its both sides. The maximum shift of the field lobe in the pattern occurs at the frequency of total transmission of the electromagnetic field through the screen. There is another parameter, which can affect on the value of beam shifting and narrowing. It is the relation between the axes sizes of an ellipse in the cross-section of the beam. These effects are the most pronounced when the major axis of the ellipse is parallel to the y-axis.

From view point of practical applications it is very important to know the overall picture of the beam scattering at oblique incidence of beam on the two-dimensional periodic structure at frequencies at which the higher spatial Floquet harmonics appear. In this case the side lobes in the reflected and transmitted beam pattern appear.

In Fig. 7 the three-dimensional normalized reflected and transmitted intensity patterns in the case of oblique incidence of elliptic



(a)



(b)

Figure 7. The pattern of (a) reflected and (b) transmitted beam, $f = 74.8\,\text{GHz}$.

beam on screen are presented.

One can see that at frequency at which the higher spatial Floquet harmonics appear for different angles ϑ and φ there are additional diffraction lobes in the case of oblique beam incidence. It is evident that the level of the side diffraction lobes may be sufficiently high as compared with that ones of the main lobes, which lead to a redistribution of scattered, beam power between the main and the diffraction lobes.

In the case of an arbitrary incidence of a plane linearly polarized *TE* and *TM* waves on phased antenna array with rectangular wave guides, the transverse component of the reflected or transmitted electric fields can be represented in the next form [15]:

$$\begin{pmatrix} {}_{TE}\vec{E}_t^r(x,y,z) \\ {}_{TM}\vec{E}_t^r(x,y,z) \end{pmatrix} = \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} \begin{pmatrix} {}_{TE}r_{qs}^{(1)} \\ {}_{TM}r_{qs}^{(1)} \end{pmatrix} \vec{\Psi}_{qs}^{(1)} e^{i\Gamma_{qs}z} + \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} \begin{pmatrix} {}_{TE}r_{qs}^{(2)} \\ {}_{TM}r_{qs}^{(2)} \end{pmatrix} \vec{\Psi}_{qs}^{(2)} e^{i\Gamma_{qs}z},\ z > 0, \quad (7)$$

where $\Gamma_{qs} = \sqrt{k^2 - \kappa_x^2 - \kappa_y^2}$ — is the propagation constant of a single spatial harmonic, $k = 2\pi/\lambda, \kappa_x = \sin\vartheta\cos\varphi - 2\pi q/d_x,\ \ \kappa_y = \sin\vartheta\sin\varphi - 2\pi s/d_y + 2\pi q/d_x cot\chi,\ d_x,\ d_y$ — are basic cell sizes along x-axis and y-axis, $\vec{\Psi}_{qs}^{(1,2)}$ is the orthonormal system of the vector spatial harmonics, $r_{qs}^{(1,2)}$ are certain elements of the generalized scattering matrix of the array of rectangular cross-section wave guides. The latter ones are found via solution of the key diffraction problem related on the spectra of the *TE* and *TM* linearly polarized plane electromagnetic waves. The upper indexes 1, 2 correspond to the *TE* and *TM* waves, respectively. The position and quantity of side lobes of the scattered field on a theta-phi plane are defined from conditions $Re\,(\Gamma_{qs}) > 0$, $Im\,(\Gamma_{qs}) = 0$ for the case of beam incidence on PAA's. The intensity of scattered field of the main and side lobes can be defined under the formula:

$$W = \frac{\lambda^2}{S} \int\limits_{\varphi_1}^{\varphi_2} \int\limits_{\vartheta_1}^{\vartheta_2} \sin\vartheta \left\{ \cos^2\vartheta \, |R_1(\vartheta,\varphi)|^2 + |R_2(\vartheta,\varphi)|^2 \right\} d\vartheta d\varphi, \tag{8}$$

where ϑ_1, ϑ_2 and φ_1, φ_2 are angles, that define lobes position in a theta-phi plane.

4. CONCLUSION

The case of normal incidence of linearly polarized Gaussian beam of elliptic cross-section on the phased antenna array of rectangular waveguides is investigated. The field patterns in a far-field zone of the reflected and transmitted beams are calculated. The effect of narrowing of the field pattern of the transmitted beam is found out. The physical explanation of this effect is given and the analysis of dependence of narrowing degree of the pattern on the beam parameters is carried out. At oblique incidence of Gaussian beam with the elliptic form of cross-section the effect of maximum shifting in the field pattern of the reflected and transmitted beams is studied in a far-field zone versus the structure and beam parameters.

Conditions at which the field pattern of the scattered beam is characterized by irregularity and the shift of pattern maximum is pronounced are defined. The regularities of parameters influence of an incident beam on character of transformation and size of shift of maxima of pattern of the reflected and transmitted beams are established. Frequency band at which the reflected and transmitted beams have not got Gaussian form is evaluated. Character of transformation of beams at various frequencies is investigated. The conditions of occurrence of side lobes in the three-dimensional patterns of the reflected and transmitted beams are found out. The effect of side lobes, the knowledge of side lobes position in a theta-phi plane is useful in radio location and wide band antennas building.

REFERENCES

1. Tamir, T. and H. L. Bertoni, "Lateral displacement of optical beams at multilayered and periodic structures," *Journal of the Optical Society of America*, Vol. 61, No. 10, 1397–1413, October 1971.

2. Gribovsky, A. V., "A new method of controlling the radiation in waveguide phased array," *Telecommunications and Radio Engineering*, Vol. 61, No. 1, 58–66, 2004.
3. Gribovsky, A. V. and S. L. Prosvirin, "Scattering of a wave beam by a reflected array made of rectangular waveguides," *Journal of Communications Technology and Electronics*, Vol. 42, No. 9, 961–972, 1997.
4. Tretyakova, S. S., O. A. Tretyakov, and V. P. Shestopalov, "Wave beam diffraction on plane periodical structures," *Radiotekhnika* & *Electronical*, Vol. 17, 1366–1373, July 1972 (in Russian).
5. Shin, S. Y. and L. B. Felsen, "Lateral shift of totally reflected gaussian beams," *Radio Sci.*, Vol. 12, 551–564, July 1977.
6. Litvinenko, L. N. and S. L. Prosvirnin, "Spectral scattering operators in problems of wave diffraction on flat screens," *Nauk. Dumka*, 1984.
7. Godin, O. A., "Diffraction theory of lateral shift of bounded wave beams during reflection," *Zh. Tekh. Fiz.*, Vol. 54, No. 4, 2094–2104, 1984; Vol. 55, No. 2, 17–25, 1985 (in Russian).
8. Riesz, R. P. and R. Simon, "Reflection of gaussian beam from a dielectric slab," *J. Opt. Soc. Am. A*, Vol. 2, 1809–1817, 1985.
9. Maciel, J. and L. Felsen, "Gaussian beam analysis of propagation from an extended plane aperture distribution through dielectric layers," *IEEE Trans. Antennas Propagat.*, Vol. 38, 1607–1624, October 1990.
10. Shulga, S. N., "Two-dimensional wave beam scattering on an anisotropic half-space with anisotropic inclusion," *Optics and Spectroscopy*, Vol. 87, 503–509, March 1999 (in Russian).
11. Tuz, V. R., "Three-dimensional Gaussian beam scattering from a periodic sequence of bi-isotropic and material layers," *Progress In Electromagnetics Research B*, Vol. 7, 53–73, 2008.
12. Wu. Z. and L. Guo, "Electromagnetic scattering from a multilayered cylinder arbitrarily located in a gaussian beam, a new recursive algorithms," *Progress In Electromagnetics Research*, Vol. 18, 317–333, 1998.
13. Shen, T., W. Dou, and Z. Sun, "Gaussian beam scattering from a semicircular channel in a conducting plane," *Progress In Electromagnetics Research*, Vol. 16, 67–85, 1997.
14. Dong, W. T., L. Gao., and C. W. Qiu, "Goos-Hänchen shift at the surface of chiral negative refractive media," *Progress In Electromagnetics Research*, Vol. 90, 255–268, 2009.

15. Gribovsky, A. V. and O. A. Yeliseyev, "Gaussian beam scattering on two-dimensional periodic aperture array," *Progress In Electromagnetics Research B*, Vol. 13, 283–296, 2011.
16. Gribovsky, A. V., S. L. Prosvirin, and I. I. Reznik, "Reflective phased array antenna of rectangular waveguides of finite depth," *Radio Physics and Radio Astronomy*, Vol. 2, No. 1, 52–60, 1997 (in Russian).