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COMBINED VIBRATOR-SLOT STRUCTURES IN ELECTRODYNAMIC VOLUMES

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Abstract—The problem of excitation of electromagnetic fields by a material body of finite dimensions in presence of coupling hole between two arbitrary electrodynamic volumes is formulated. The problem is reduced to two-dimensional integral equations for the surface electric current on a material body and the equivalent magnetic current on a coupling hole. A physically correct transition from the initial integral equations to one-dimensional equations for the currents in a thin impedance vibrator which, in general case, may have irregular geometric parameters, and a narrow slot is justified. A solution of resulting equations system for the transverse slot in the broad wall of rectangular waveguide and a vibrator with variable surface impedance in it was found by a generalized method of induced electro-magneto-motive forces. The calculated and experimental plots of electrodynamic characteristics of a vibrator-slot structure in a rectangular waveguide are presented.

1. INTRODUCTION

At present, linear vibrator and slot radiators, i.e., radiators of electric and magnetic type, respectively, are widely used as separate receiver and transmitter structures, elements of antenna systems, and antenna-feeder devices, including combined vibrator-slot structures [1–4]. Widespread occurrence of such radiators is an objective prerequisite for theoretical analysis of their electrodynamic characteristics. During last decades, researchers have published results which make it possible to create a modern theory of thin vibrator and narrow

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slot radiators. This theory combines the fundamental asymptotic methods for determining the single radiator characteristics [5–7], hybrid analytic-numerical approaches [8–10], and direct numerical techniques for electrodynamic analysis of such radiators [11]. However, the electrodynamics of single linear electric and magnetic radiators is far from being completed. It may be explained by further development of modern antenna techniques and antenna-feeder devices which can be characterized by such features as multielement structures, integration and modification of structural units to minimize their mass and dimensions and to ensure electromagnetic compatibility of radio aids, application of metamaterials, formation of required spatial-energy and spatial-polarization distributions of electromagnetic fields in various nondissipative and dissipative media. To solve these tasks electric and magnetic radiators, based on various impedance structures with irregular geometric or electrophysical parameters, and on combined vibrator-slot structures, should be created [12–20].

Mathematical modeling of antenna-feeder devices requires multiparametric optimization of electrodynamic problem solution and, hence, effective computational resources and software. Therefore, in spite of rapid growth of computer potential, there exists necessity to develop new effective methods of electrodynamic analysis of antenna-feeder systems, being created with linear vibrator and slot structures with arbitrary geometric and electrophysical parameters, satisfying modern versatile requirements, and widening their application in various spheres. Efficiency of mathematical modeling is defined by rigor of corresponding boundary problem definition and solution, by performance of computational algorithm, requiring minimal possible RAM space, and directly depends upon analytical formulation of the models. Namely, the weightier is the analytical component of the method, the grater is its efficiency. In this connection, it should be noted that formation of analytical concepts of electrodynamic analysis extending the capabilities of physically correct mathematical models for new classes of boundary problems is always an important problem.

This paper presents the methodological basis of a new approach to solving the electrodynamic problems, associated with combined vibrator-slot structures, known as a generalized method of induced electro-magneto-motive forces (EMMF). This approach is based on the classical method of induced EMMF, i.e., basis functions approximating the currents along the vibrator and slot elements are obtained in advance as analytical solutions of key problems, formulated as integral equations for the currents by the asymptotic averaging method.

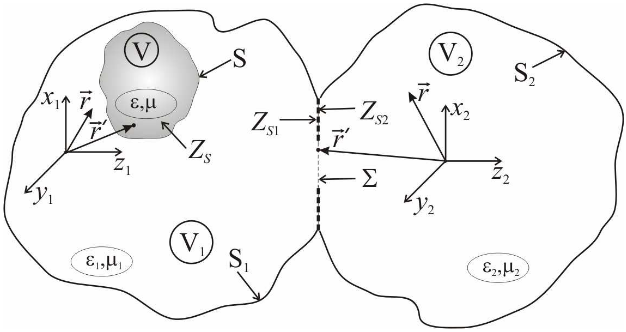

Figure 1. The problem geometry and notations.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

Let us formulate a general problem of electromagnetic waves excitation, scattering, and radiation by a material body of finite dimensions in presence of a coupling hole between two arbitrary electrodynamic volumes. The problem geometry and corresponding notations are represented in Figure 1. Consider an arbitrary volume, restricted by perfectly conducting, impedance, or partly impedance surface S_1, which may be infinitely distant. In the surface S_1 there exists a hole Σ, connecting the volume V_1 with another volume V_2. The boundary between the two volumes is infinitely thin. A material body, occupying volume V, restricted by smooth closed surface S, is situated in the volume V_1. The body has homogeneous material parameters: permittivity ε, permeability μ, and conductivity σ and there exists electromagnetic field of impressed sources $\{\vec{E}_0(\vec{r}),\, \vec{H}_0(\vec{r})\}$, depending upon time t as $e^{i\omega t}$ ($\vec{r}$ is the radius-vector of the observation point, $\omega = 2\pi f$ is a circular frequency, f is the frequency in Hz). The permittivity and permeability of the media in the volumes V_1 and V_2 are ε_1, μ_1 and ε_2, μ_2, respectively, which in the general case are step functions of coordinates. The field of impressed sources may be specified as an electromagnetic wave field, incident upon the body (scattering problem), or as a field of electromotive forces, applied to the body, nonzero only in a part of the volume V (radiation problem), or, in the general case, as combination of these fields. It is necessary to find the full electromagnetic fields $\{\vec{E}_{V_1}(\vec{r}),\, \vec{H}_{V_1}(\vec{r})\}$ and $\{\vec{E}_{V_2}(\vec{r}),\, \vec{H}_{V_2}(\vec{r})\}$ in the volumes V_1 and V_2, satisfying the Maxwell's equations and boundary conditions on the surfaces S, Σ, S_1 and S_2.

The problem, thus formulated, can be studied by solving equations for electromagnetic fields in a differential or an integral form. The application of integral equations has an advantage since the fields, derived by their solutions, are known to satisfy boundary conditions on the obstacle's surface automatically. By obstacle we mean a material body or coupling hole. Besides, integral equations are very effective, if boundary surfaces S, Σ, S_1 and S_2 are coordinate surfaces in different coordinate systems. For example, S_1 is a waveguide surface with cylindrical symmetry, while a body surface S may have another symmetry type. Therefore, the mathematical model of electromagnetic processes will be built, based on the integral equations of macroscopic electrodynamics, equivalent to the boundary problem in whole, i.e., to Maxwell's equations and boundary conditions on surfaces S, Σ, and on surfaces S_1 and S_2 of electrodynamics volumes.

To solve the above mentioned problem, it is convenient to express electromagnetic fields in volume V_1 and V_2 in terms of tangential field components on the surfaces S and Σ. In the Gaussian unit system CGS, these equations, known as the Kirchhoff-Kotler's surface integral equations [3, 4], may be written as

$$\begin{aligned}
\vec{E}_{V_1}(\vec{r}) &= \vec{E}_0(\vec{r}) + \frac{1}{4\pi i k\varepsilon_1}(\mathrm{graddiv} + k_1^2)\int_S \hat{G}^e_{V_1}(\vec{r},\vec{r}\,')[\vec{n}_1,\vec{H}_{V_1}(\vec{r}\,')]d\vec{r}\,' \\
&\quad - \frac{1}{4\pi}\mathrm{rot}\int_{S+\Sigma} \hat{G}^m_{V_1}(\vec{r},\vec{r}\,')[\vec{n}_{1,2},\vec{E}_{V_1}(\vec{r}\,')]d\vec{r}\,', \\
\vec{H}_{V_1}(\vec{r}) &= \vec{H}_0(\vec{r}) + \frac{1}{4\pi i k\mu_1}(\mathrm{graddiv} + k_1^2)\int_{S+\Sigma} \hat{G}^m_{V_1}(\vec{r},\vec{r}\,')[\vec{n}_{1,2},\vec{E}_{V_1}(\vec{r}\,')]d\vec{r}\,' \\
&\quad + \frac{1}{4\pi}\mathrm{rot}\int_S \hat{G}^e_{V_1}(\vec{r},\vec{r}\,')[\vec{n}_1,\vec{H}_{V_1}(\vec{r}\,')]d\vec{r}\,', \\
\vec{E}_{V_2}(\vec{r}) &= \frac{1}{4\pi}\mathrm{rot}\int_\Sigma \hat{G}^m_{V_2}(\vec{r},\vec{r}\,')[\vec{n}_2,\vec{E}_{V_2}(\vec{r}\,')]d\vec{r}\,', \quad \vec{H}_{V_2}(\vec{r}) \\
&= -\frac{1}{4\pi i k\mu_2}(\mathrm{graddiv} + k_2^2)\int_\Sigma \hat{G}^m_{V_2}(\vec{r},\vec{r}\,')[\vec{n}_2,\vec{E}_{V_2}(\vec{r}\,')]d\vec{r}\,'.
\end{aligned} \tag{1}$$

Here $k = 2\pi/\lambda$ is the wave number; λ is the wavelength in free space; $k_1 = k\sqrt{\varepsilon_1\mu_1}$, $k_2 = k\sqrt{\varepsilon_2\mu_2}$; $\vec{r}\,'$ is the position vector of the sources, lying on the surfaces S and Σ; $\vec{n}_1$ and $\vec{n}_2$ are unit vectors of outer normals to these surfaces; $\hat{G}^e(\vec{r},\vec{r}\,')$ and $\hat{G}^m(\vec{r},\vec{r}\,')$ are the electrical and magnetic tensor Green's functions for vector potential, corresponding to the volumes, and satisfying the vector Helmholtz's equation and corresponding boundary conditions on the surfaces S_1 and S_2. If the surface S_1 or S_2 is moved to infinity, the boundary conditions are transformed into the Sommerfeld radiation condition.

The fields on the left-hand side of Equation (1) may be interpreted, depending upon the position of observation point $\vec{r}$ where unknown field being defined. If a point $\vec{r}$ belongs to surface S of volume V or to the hole aperture Σ, fields $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ coincide with the fields in integrals on the right-hand side of Equation (1). Then, Equations (1) are the inhomogeneous linear Fredholm integral equations of the second kind, known to have a unique solution. Let us note once more that the Maxwell's equations are partial differential equations having infinite number of solutions, but only one solution satisfies the boundary conditions on the body's surface (or hole), and it coincides with solution of the integral Equation (1). If a point $\vec{r}$ lies outside the regions V and Σ, Equations (1) become equalities, defining the full electromagnetic field in the medium defined by the fields of impressed sources. Obviously, to find these fields, the integral equations should be solved in advance. Thus, the merit of integral equations method consists in the fact that the solution process is divided into two stages. On the first stage, the fields on the surfaces S and Σ are found as integral equation solutions for the given fields of impressed sources. On the second stage, the scattered (radiated) fields in any point of volumes V_1 and V_2 are defined by the fields found on the first stage.

Let us mention that formula (1) is often used, if field on a material body surface can be defined by some additional physical arguments. Thus, for good conducting bodies ($\sigma \to \infty$) induced current concentrates near a body surface. Then, neglecting skin-layer thickness, it is possible to use the Leontovich-Schukin approximate impedance boundary condition

$$[\vec{n}_1, \vec{E}_{V_1}(\vec{r})] = \bar{Z}_S(\vec{r})[\vec{n}_1, [\vec{n}_1, \vec{H}_{V_1}(\vec{r})]], \tag{2}$$

where $\bar{Z}_S(\vec{r}) = \bar{R}_S(\vec{r}) + i\bar{X}_S(\vec{r}) = Z_S(\vec{r})/Z_0$ is distributed surface impedance (normalized by characteristic impedance of free space $Z_0 = 120\pi$ Ohm). Note that impedance may vary along the body surface.

If an observation point is situated on an impedance body surface S then we arrive at a system of integro-differential equations

$$\begin{aligned} Z_S(\vec{r})\vec{J}^e(\vec{r}) + \frac{k}{\omega}\text{rot}\int\limits_{\Sigma} \hat{G}^m_{V_1}(\vec{r}, \vec{r}\,')\vec{J}^m(\vec{r}\,')d\vec{r}\,' \\ = \vec{E}_0(\vec{r}) + \frac{1}{i\omega\varepsilon_1}(\text{graddiv} + k_1^2)\int\limits_{S} \hat{G}^e_{V_1}(\vec{r}, \vec{r}\,')\vec{J}^e(\vec{r}\,')d\vec{r}\,' \end{aligned}$$

$$+\frac{1}{4\pi}\mathrm{rot}\int\limits_S \hat{G}^m_{V_1}(\vec{r},\vec{r}\,')Z_S(\vec{r}\,')[\vec{n}_1,\vec{J}^e(\vec{r}\,')]d\vec{r}\,', \tag{3a}$$

$$\vec{H}_0(\vec{r}) + \frac{1}{i\omega\mu_1}(\mathrm{graddiv} + k_1^2)\int\limits_\Sigma \hat{G}^m_{V_1}(\vec{r},\vec{r}\,')\vec{J}^m(\vec{r}\,')d\vec{r}\,'$$
$$+\frac{1}{i\omega\mu_2}(\mathrm{graddiv} + k_2^2)\int\limits_\Sigma \hat{G}^m_{V_2}(\vec{r},\vec{r}\,')\vec{J}^m(\vec{r}\,')d\vec{r}\,'$$
$$= \frac{1}{i\omega\varepsilon_1}(\mathrm{graddiv} + k_1^2)\int\limits_S \hat{G}^m_{V_1}(\vec{r},\vec{r}\,')\bar{Z}_S(\vec{r}\,')[\vec{n}_1,\vec{J}^e(\vec{r}\,')]d\vec{r}\,'$$
$$-\frac{k}{\omega}\mathrm{rot}\int\limits_S \hat{G}^e_{V_1}(\vec{r},\vec{r}\,')\vec{J}^e(\vec{r}\,')d\vec{r}\,', \tag{3b}$$

where $\vec{J}^e(\vec{r})$ is the density of electrical surface current at S, and $\vec{J}^m(\vec{r})$ is equivalent magnetic surface current at Σ

$$\vec{J}^e(\vec{r}) = \frac{c}{4\pi}[\vec{n}_1,\vec{H}(\vec{r})] \quad \vec{J}^m(\vec{r}) = \frac{c}{4\pi}[\vec{n}_2,\vec{E}(\vec{r})], \tag{4}$$

where $c \approx 2.998 \cdot 10^{10}$ cm/sec is the speed of light in vacuum. To derive (3b), we make use of continuity condition for tangential components of magnetic fields at the hole Σ.

Thus, the problem of electromagnetic waves excitation by the impedance body of finite dimensions and by the coupling hole between two electrodynamic volumes is formulated as a rigorous boundary value problem of macroscopic electrodynamics, reduced to the system of integral equations for surface currents. Solution of this system is an independent problem, significant in its own right, since it often presents considerable mathematical difficulties. If characteristic dimensions of an object are much greater than wavelength (high-frequency region), a solution is usually searched as series expansion in ascending power of inverse wave number. If dimensions of an object are less than wavelength (low-frequency or quasi-static region), representation of the unknown functions as series expansion in wave number powers reduces the problem to a sequence of electrostatic problems. Contrary to asymptotic cases, resonant region, where at least one dimension of an object is comparable with wavelength, is the most complex for analysis, and requires rigorous solution of field equations. It should be noted that, from the practical point of view, the resonant region is of exceptional interest for thin impedance vibrators and narrow slots.

3. INTEGRAL EQUATIONS FOR ELECTRICAL AND MAGNETIC CURRENTS IN THIN VIBRATORS AND NARROW SLOTS

Direct solution of Equations (3) for a material body V of complex shape and a coupling hole Σ of arbitrary geometry essential mathematical difficulties may be encountered. However, the problem may be considerably simplified for impedance cylinders, if cross section perimeter is less than their length and wavelength in a medium (thin vibrators) and for coupling hole, if one dimension of a slot satisfies the above conditions (narrow slots). Moreover, in this case, it is possible to extend the boundary condition (2) for cylindrical surfaces with arbitrary distribution of complex impedance regardless of the exciting field structure and the electro-physical characteristics of vibrator material.

Let us transform the integral equations system (3) to the form applicable to a thin vibrator, made of a bounded circular cylindrical wire (radius r and length $2L_v$), and for a narrow straight-line slot (width d and length $2L_{sl}$). In this case, the following inequalities hold

$$\frac{r}{2L_v} \ll 1, \quad \frac{r}{\lambda_1} \ll 1, \quad \frac{d}{2L_{sl}} \ll 1, \quad \frac{d}{\lambda_{1,2}} \ll 1, \tag{5}$$

where $\lambda_{1,2}$ are the wavelengths in the corresponding media. These inequalities permit to express the electric current density and equivalent magnetic current density, induced in the vibrator and in the slot, respectively, as

$$\vec{J}_v^e(\vec{r}) = \vec{e}_{s_1} J_v(s_1)\psi(\rho, \varphi), \quad \vec{J}_{sl}^m(\vec{r}) = \vec{e}_{s_2} J_{sl}(s_2)\chi(\xi), \tag{6}$$

where $\vec{e}_{s_1}$ and $\vec{e}_{s_2}$ are unit vectors along the axes of vibrator and slot, respectively; $\psi(\rho, \varphi)$ is a function of transverse ($\perp$) polar coordinate ρ, φ for the vibrator; $\chi(\xi)$ is a function of transverse coordinate ξ for the slot. The functions $\psi(\rho, \varphi)$ and $\chi(\xi)$ satisfy normalization conditions

$$\int\limits_{\perp} \psi(\rho, \varphi)\rho d\rho d\varphi = 1, \quad \int\limits_{\xi} \chi(\xi) d\xi = 1, \tag{7}$$

while unknown currents $J_v(s_1)$, $J_{sl}(s_2)$ obey the boundary conditions

$$J_v(\pm L_v) = 0, \quad J_{sl}(\pm L_{sl}) = 0. \tag{8}$$

Taking into account (7), (8), and relation $[\vec{n}_1, \vec{J}^e(\vec{r})] \ll 1$, readily derived from (5), and projecting Equations (3a) and (3b) on the vibrator and slot axes, respectively, we obtain a system of integro-differential equations for currents in a thin impedance vibrator and in a narrow slot. This system take into account the vibrator-slot interaction and may be written (the indexes e and m are omitted) as

$$\begin{aligned}
&\left(\frac{d^2}{ds_1^2}+k_1^2\right)\int\limits_{-L_v}^{L_v} J_v(s_1')G_{s_1}^{V_1}(s_1,s_1')ds_1' \\
&- ik\varepsilon_1\vec{e}_{s_1}\text{rot}\int\limits_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')G_{s_2}^{V_1}(s_1,s_2')ds_2' \\
=&- i\omega\varepsilon_1 E_{0s_1}(s_1)+i\omega\varepsilon_1 z_i(s_1)J_v(s_1), \\
&\frac{1}{\mu_1}\left(\frac{d^2}{ds_2^2}+k_1^2\right)\int\limits_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')G_{s_2}^{V_1}(s_2,s_2')ds_2' \\
&+\frac{1}{\mu_2}\left(\frac{d^2}{ds_2^2}+k_2^2\right)\int\limits_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')G_{s_2}^{V_2}(s_2,s_2')ds_2' \\
&+ik\vec{E}_{s_2}\text{rot}\int\limits_{-L_v}^{L_v} J_v(s_1')G_{s_1}^{V_1}(s_2,s_1')ds_1'=-i\omega H_{0s_2}(s_2),
\end{aligned} \tag{9}$$

where $z_i(s_1)$ is the internal impedance per unit length of the vibrator ([Ohm/m]); $(Z_S(\vec{r})=2\pi r z_i(\vec{r}))$, $E_{0s_1}(s_1)$ and $H_{0s_2}(s_2)$ are projections of impressed sources fields on the vibrator and the slot axes; $G_{s_1}^{V_1}(s_1,s_1')$ and $G_{s_2}^{V_{1(2)}}(s_2,s_2')$ are components of tensor Green's functions for the considered volumes, which may be written as

$$\begin{aligned}
G_{s_1}^{V_1}(s_1,s_1') &= \int\limits_{\perp} G_{s_1}^{V_1}(s_1,\rho,\varphi;s_1',\rho',\varphi')\psi(\rho',\varphi')\rho' d\rho' d\varphi', \\
G_{s_2}^{V_{1(2)}}(s_2,s_2') &= \int\limits_{\xi} G_{s_2}^{V_{1(2)}}(s_2,\xi;s_2',\xi')\chi(\xi')d\xi'.
\end{aligned}$$

For single vibrator or slot, as well as for the absence of electromagnetic interaction between them, system (9) splits into two independent equations:

$$\begin{aligned}
&\left(\frac{d^2}{ds_1^2}+k_1^2\right)\int\limits_{-L_v}^{L_v} J_v(s_1')G_{s_1}^{V_1}(s_1,s_1')ds_1' \\
&= -i\omega\varepsilon_1 E_{0s_1}(s_1)+i\omega\varepsilon_1 z_i(s_1)J_v(s_1),
\end{aligned} \tag{10}$$

$$\frac{1}{\mu_1}\left(\frac{d^2}{ds_2^2}+k_1^2\right)\int\limits_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')G_{s_2}^{V_1}(s_2,s_2')ds_2'$$
$$+\frac{1}{\mu_2}\left(\frac{d^2}{ds_2^2}+k_2^2\right)\int\limits_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')G_{s_2}^{V_2}(s_2,s_2')ds_2'=-i\omega H_{0s_2}(s_2). \quad (11)$$

In the general case, a vibrator or a slot may have curvilinear axial configuration. Then, if the radius of curvature of vibrator axis or slot center line is much greater than their lateral dimensions, the equations for the electrical current in the vibrator and for the magnetic current in the slot are reduced to:

$$\begin{aligned}
&\int\limits_{L_v}\left\{\left[\frac{\partial}{\partial s_1}\frac{\partial J_v(s_1')}{\partial s_1'}+k_1^2(\vec{e}_{s_1}\vec{e}_{s_1'})J_v(s_1')\right]G_v(s_1,s_1')\right.\\
&\quad\left.+J_v(s_1')\left[\frac{\partial^2}{\partial s_1^2}+k_1^2\right]\vec{e}_{s_1}\left(\hat{G}_{0s_1}^{V_1}(s_1,s_1')\vec{e}_{s_1'}\right)\right\}ds_1'\\
&= -i\omega\varepsilon_1 E_{0s_1}(s_1)+i\omega\varepsilon_1 z_i(s_1)J_v(s_1), \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&\int\limits_{L_{sl}}\left\{\frac{1}{\mu_1}\left[\frac{\partial}{\partial s_2}\frac{\partial J_{sl}(s_2')}{\partial s_2'}+k_1^2(\vec{e}_{s_2}\vec{e}_{s_2'})J_{sl}(s_2')\right]G_{sl}^{V_1}(s_2,s_2')\right.\\
&\quad+\frac{1}{\mu_2}\left[\frac{\partial}{\partial s_2}\frac{\partial J_{sl}(s_2')}{\partial s_2'}+k_2^2(\vec{e}_{s_2}\vec{e}_{s_2'})J_{sl}(s_2')\right]G_{sl}^{V_2}(s_2,s_2')\\
&\quad+\frac{1}{\mu_1}J_{sl}(s_2')\left[\frac{\partial^2}{\partial s_2^2}+k_1^2\right]\vec{e}_{s_2}\left(\hat{G}_{0s_2}^{V_1}(s_2,s_2')\vec{e}_{s_2'}\right)\\
&\quad\left.+\frac{1}{\mu_2}J_{sl}(s_2')\left[\frac{\partial^2}{\partial s_2^2}+k_2^2\right]\vec{e}_{s_2}\left(\hat{G}_{0s_2}^{V_2}(s_2,s_2')\vec{e}_{s_2'}\right)\right\}ds_2'\\
&= -i\omega H_{0s_2}(s_2). \qquad (13)
\end{aligned}$$

Here $\vec{e}_{s_1'}$ and $\vec{e}_{s_2'}$ are unit vectors of vibrator and slot axes at the sources, and

$$G_v(s_1,s_1') = \int\limits_{-\pi}^{\pi}\frac{e^{-ik_1\sqrt{(s_1-s_1')^2+[2r\sin(\varphi/2)]^2}}}{\sqrt{(s_1-s_1')^2+[2r\sin(\varphi/2)]^2}}\psi(r,\varphi)r\,d\varphi, \quad (14)$$

$$G_{sl}^{V_{1,2}}(s_2,s_2') = \int\limits_{-d/2}^{d/2}\frac{e^{-ik_{1,2}\sqrt{(s_2-s_2')^2+(\xi)^2}}}{\sqrt{(s_2-s_2')^2+(\xi)^2}}\chi(\xi)\,d\xi, \quad (15)$$

$\hat{G}_{0s_1}$ and $\hat{G}_{0s_2}$ are regular components of tensor Green's functions which take into account a geometry of volumes.

Solution of the integral equation with the exact kernel expressions (14) and (15) may be very difficult. Therefore, we will use approximate expressions, the so called "quasi-one-dimensional" kernels [5, 15]

$$G_v(s_1, s_1') = \frac{e^{-ik_1\sqrt{(s_1-s_1')^2+r^2}}}{\sqrt{(s_1-s_1')^2+r^2}}, \tag{16}$$

$$G_{sl}^{V_{1,2}}(s_2, s_2') = \frac{e^{-ik_{1,2}\sqrt{(s_2-s_2')^2+(d/4)^2}}}{\sqrt{(s_2-s_2')^2+(d/4)^2}}, \tag{17}$$

derived with the assumption that source points belong to the geometric axes of the vibrator and slot while observation points belong to vibrator surface and to slot axis, having coordinates $\{s_2, \xi/2\}$. In that case, the functions $G_v(s_1, s_1')$ and $G_{sl}^{V_{1,2}}(s_2, s_2')$ are everywhere continuous, and equations for the currents are simplified significantly.

Thus, the problem of electromagnetic waves excitation by thin impedance vibrators and narrow slots, connecting two electrodynamic volumes, is reduced to integro-differential equations relative electrical current in a vibrator and equivalent magnetic current in a slot. The solution of these equations is the completion phase of the problem, since full electromagnetic fields in the volumes can be easily found by expressions (1) and (4).

It should be emphasized that the form of Green's functions in the above equations was not defined. Therefore, the equations are valid for any electrodynamic volume, provided that corresponding Green's functions are known or can be constructed.

4. VIBRATOR-SLOT STRUCTURE IN RECTANGULAR WAVEGUIDE

Now let us consider a problem of electromagnetic wave scattering by a narrow straight transverse slot in the broad wall of rectangular

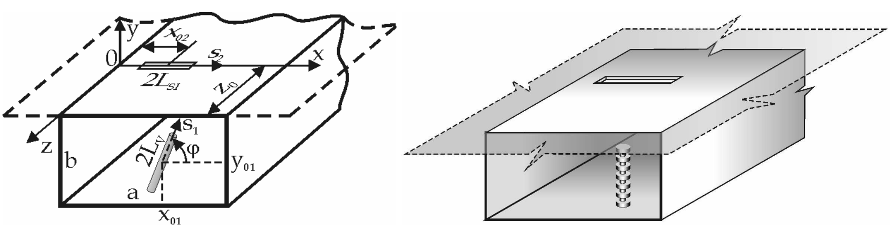

Figure 2. Problem geometry and notations.

waveguide with a passive impedance vibrator in it (Figure 2).

Let the fundamental wave H_{10} propagate in a hollow rectangular waveguide $\{a \times b\}$ (index Wg). A thin wire (radius r and length $2L_v$) with a variable surface impedance is placed in the waveguide cross section plane so that the angle between its axis, and the axis $\{0x\}$ in the Cartesian coordinate system is φ. A narrow transverse slot (width d and length $2L_{sl}$), radiating in a free half-space above an infinite perfectly conducting plane (index Hs), is cut in the broad waveguide wall, and its thickness is h. z_0 is the distance between the axes of the vibrator and the slot. In this case, system (9) can be transformed to

$$\begin{aligned} &\left(\frac{d^2}{ds_1^2}+k^2\right)\int_{-L_v}^{L_v} J_v(s_1')G_{s_1}^{Wg}(s_1,s_1')ds_1' - ik\int_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')\tilde{G}_{s_2}^{Wg}(s_1,s_2')ds_2' \\ &= -i\omega E_{0s_1}(s_1) + i\omega z_i(s_1)J_v(s_1), \end{aligned} \tag{18a}$$

$$\begin{aligned} &\left(\frac{d^2}{ds_2^2}+k^2\right)\int_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')[G_{s_2}^{Wg}(s_2,s_2') + G_{s_2}^{Hs}(s_2,s_2')]ds_2' \\ &-ik\int_{-L_v}^{L_v} J_v(s_1')\tilde{G}_{s_1}^{Wg}(s_2,s_1')ds_1' = -i\omega H_{0s_2}(s_2), \end{aligned} \tag{18b}$$

where

$$\begin{aligned} \tilde{G}_{s_1}^{Wg}(s_2,s_1') &= \frac{\partial}{\partial z}G_{s_1}^{Wg}[x(s_2),0,z;x'(s_1'),y'(s_1'),z_0], \\ \tilde{G}_{s_2}^{Wg}(s_1,s_2') &= \frac{\partial}{\partial z}G_{s_2}^{Wg}[x(s_1),y(s_1),z;x'(s_2'),0,0] \end{aligned}$$

after substituting $z = 0$ into $\tilde{G}_{s_1}^{Wg}$ and $z = z_0$ into $\tilde{G}_{s_2}^{Wg}$ after the first derivation.

If $z_0 = 0$, $\tilde{G}_{s_1}^{Wg} = \tilde{G}_{s_2}^{Wg} = 0$, the system of coupled Equations (18) is transformed into two independent equations:

$$\left(\frac{d^2}{ds_1^2}+k^2\right)\int_{-L_v}^{L_v} J_v(s_1')G_{s_1}^{Wg}(s_1,s_1')ds_1' = -i\omega E_{0s_1}(s_1) + i\omega z_i(s_1)J_v(s_1), \tag{19a}$$

$$\left(\frac{d^2}{ds_2^2}+k^2\right)\int_{-L_{sl}}^{L_{sl}} J_{sl}(s_2')[G_{s_2}^{Wg}(s_2,s_2') + G_{s_2}^{Hs}(s_2,s_2')]ds_2' = -i\omega H_{0s_2}(s_2). \tag{19b}$$

The solution of the coupled Equations (18) by the averaging method is impractical since it results in rather cumbersome expressions for the currents, unsuitable for practical use [3, 4]. Therefore, to find the solution of the coupled Equations (18), we will use the generalized method of induced EMMF, with approximate expressions for the currents $J_v(s_1) = J_{0v} f_v(s_1)$ and $J_{sl}(s_2) = J_{0sl} f_{sl}(s_2)$ (here J_{0v} and J_{0sl} are treated as unknown amplitudes), previously obtained by solving Equations (19) by the averaging method. So, we multiply Equation (18a) by the function $f_v(s_1)$ and Equation (18b) by the function $f_{sl}(s_2)$ and integrate Equation (18a) over the vibrator length and Equation (18b) over the slot length. The amplitudes J_{0v} and J_{0sl} are found as solution of the resulting system of linear algebraic equations. For the arbitrary vibrator-slot structures and coupled electrodynamic volumes expressions for $f_v^{s,a}(s_1)$ and $f_{sl}^{s,a}(s_2)$ (the subscripts s, a denote the symmetric and antisymmetric components of the currents with respect to the vibrator ($s_1 = 0$) and slot ($s_2 = 0$) centers, respectively), can be obtained from the following relations [3, 4]:

$$f_v^{s,a}(s_1) \sim \left\{ \sin\tilde{k}(L_v - s_1) \int\limits_{-L_v}^{s_1} E_{0s_1}^{s,a}(s_1') \sin\tilde{k}(L_v + s_1') ds_1' \right.$$
$$\left. + \sin\tilde{k}(L_v + s_1) \int\limits_{s_1}^{L_v} E_{0s_1}^{s,a}(s_1') \sin\tilde{k}(L_v - s_1') ds_1' \right\}, \qquad (20a)$$

$$f_{sl}^{s,a}(s_2) \sim \left\{ \sin k(L_{sl} - s_2) \int\limits_{-L_{sl}}^{s_2} H_{0s_2}^{s,a}(s_2') \sin k(L_{sl} + s_2') ds_2' \right.$$
$$\left. + \sin k(L_{sl} + s_2) \int\limits_{s_2}^{L_{sl}} H_{0s_2}^{s,a}(s_2') \sin k(L_{sl} - s_2') ds_2' \right\}, \qquad (20b)$$

where $E_{0s_1}^{s,a}(s_1)$ and $H_{0s_2}^{s,a}(s_2)$ are projections of symmetric and antisymmetric components of impressed sources on the vibrator and the slot axes. Here the sign $\sim$ means that after integration in expressions (20), only multipliers, depending upon coordinates s_1 and s_2, are left. Thus, for transverse slot with $x_{02} = a/2$ and for coordinate vibrator, i.e., $\varphi = 90°$, exited by fundamental wave, in accordance with (20), we have

$$f_v(s_1) = \cos\tilde{k}s_1 - \cos\tilde{k}L_v, \qquad (21a)$$
$$f_{sl}(s_2) = \cos ks_2 \cos\frac{\pi}{a}L_{sl} - \cos kL_{sl} \cos\frac{\pi}{a}s_2, \qquad (21b)$$

where $\tilde{k} = k - \frac{i2\pi z_i^{av}}{Z_0\Omega}$, $z_i^{av} = \frac{1}{2L_v}\int\limits_{-L_v}^{L_v} z_i(s_1)ds_1$ is the mean value of the internal impedance along the vibrator [4], $\Omega = 2\ln(2L_v/r)$.

If $z_0 = 0$, i.e., the interaction between the vibrator and slot is absent, $y_{01} = 0$, i.e., the vibrator is monopole, and the normalized surface impedance, distributed along vibrator as $\bar{Z}_S(s_1) = \bar{Z}_S\phi(s_1)$, where $\phi(s_1)$ is the given function, the solution of Equations (19) by generalized method of induced EMMF is as follows:

$$J_v(s_1) = -\frac{i\omega}{2k\tilde{k}}E_0\sin\frac{\pi x_{01}}{a}\frac{(\sin\tilde{k}L_v - \tilde{k}L_v\cos\tilde{k}L_v)(\cos\tilde{k}s_1 - \cos\tilde{k}L_v)}{Z^{Wg}(kr,\tilde{k}L_v) + F_z^{Wg}(\tilde{k}r,\tilde{k}L_v)}, \quad (22)$$

$$Z^{Wg}(kr,\tilde{k}L_v) = \frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_y^2)\tilde{k}^2}{kk_z(\tilde{k}^2-k_y^2)^2}e^{-k_z r}\sin^2 k_x x_{01}$$
$$[\sin\tilde{k}L_v\cos k_y L_v - (\tilde{k}/k_y)\cos\tilde{k}L_v\sin k_y L_v]^2,$$

$$F_z^{Wg}(\tilde{k}r,\tilde{k}L_v) = -\frac{i}{r}\int\limits_{-L_v}^{L_v} f_v^2(s_1)\bar{Z}_S(s_1)ds_1.$$

$$J_{sl}(s_2) = -\frac{i\omega}{2k}H_0\frac{f(kL_{sl})\left(\cos ks_2\cos\frac{\pi}{a}L_{sl} - \cos kL_{sl}\cos\frac{\pi}{a}s_2\right)}{[Y^{Wg}(kd,kL_{sl}) + Y^{Hs}(kd,kL_{sl})]}, \quad (23)$$

$$Y^{Wg}(kd_e,kL_{sl}) = \frac{2\pi}{ab}\sum_{m=1,3,\dots}^{\infty}\sum_{n=0,1,\dots}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{kk_z}e^{-k_z\frac{d_e}{4}}I^2(kL_{sl}),$$

$$I(kL_{sl}) = 2\left\{\frac{k\sin kL_{sl}\cos k_xL_{sl} - k_x\cos kL_{sl}\sin k_xL_{sl}}{k^2-k_x^2}\cos\frac{\pi L_{sl}}{a}\right.$$
$$\left.-\frac{\left(\frac{\pi}{a}\right)\sin\frac{\pi L_{sl}}{a}\cos k_xL_{sl} - k_x\cos\frac{\pi L_{sl}}{a}\sin k_xL_{sl}}{(\pi/a)^2-k_x^2}\cos kL_{sl}\right\},$$

$$Y^{Hs}(kd_e,kL_{sl}) = \frac{1}{2k}\left\{\left(k\cos\frac{\pi L_{sl}}{a}\sin kL_{sl} - \frac{\pi}{a}\cos kL_{sl}\sin\frac{\pi L_{sl}}{a}\right)\right.$$
$$-\int\limits_{-L_{sl}}^{L_{sl}} f_{sl}(s_2')\left[G_{s_2}^{Hs}(L_{sl},s_2') + G_{s_2}^{Hs}(-L_{sl},s_2')\right]ds_2'$$
$$-k_g\cos kL_{sl}\int\limits_{-L_{sl}}^{L_{sl}}\cos\frac{\pi s_2}{a}\left[\int\limits_{-L_{sl}}^{L_{sl}} f_{sl}(s_2')\right.$$
$$\left.\left.G_{s_2}^{Hs}(s_2,s_2')ds_2'\right]ds_2\right\},$$

$$f(kL_{sl}) = 2\cos\frac{\pi L_{sl}}{a}\frac{\sin kL_{sl}\cos\frac{\pi L_{sl}}{a} - \left(\frac{\pi}{ka}\right)\cos kL_{sl}\sin\frac{\pi L_{sl}}{a}}{1-(\pi/ka)^2}$$
$$-\cos kL_{sl}\frac{\sin\frac{2\pi L_{sl}}{a} + \frac{2\pi L_{sl}}{a}}{(2\pi/ka)}.$$

In the formulas (22), (23), $\varepsilon_n = \begin{cases} 1, & n=0 \\ 2, & n\neq 0 \end{cases}$, $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$, $k_z = \sqrt{k_x^2 + k_y^2 - k^2}$, m, n are integers, $d_e = de^{-\frac{\pi h}{2d}}$ is "equivalent" slot width [3] which takes into account a wall thickness h of the waveguide, and $G_{s_2}^{Hs}(s_2, s_2')$ is defined by the formula (17).

Let us consider the several simple functions of impedance distribution along the vibrator: the constant distribution $\phi_0(s_1) = 1$, the distribution, decreasing to the vibrator end linearly $\phi_1(s_1) = 2[1-(s_1/L_v)]$ and the linearly increasing distribution $\phi_2(s_1) = 2(s_1/L_v)$ with equal mean value $\overline{\phi_n(s_1)} = 1$ $(n = 0, 1, 2)$ over the vibrator length. The expression for $F_{z0}^{Wg}(\tilde{k}r, \tilde{k}L_v)$ has the form

$$F_{z0}^{Wg}(\tilde{k}r,\tilde{k}L_v) = -\frac{2i\bar{Z}_S}{\tilde{k}^2 L_v r}\left[\left(\frac{\tilde{k}L_v}{2}\right)^2(2+\cos 2\tilde{k}L_v) - \frac{3}{8}\tilde{k}L_v\sin 2\tilde{k}L_v\right] \quad (24)$$

for the constant distribution, and for the variable distributions

$$F_{z1}^{Wg}(\tilde{k}r,\tilde{k}L_v) = -\frac{2i\bar{Z}_S}{\tilde{k}^2 L_v r}\left[\left(\frac{\tilde{k}L_v}{2}\right)^2(2+\cos 2\tilde{k}L_v)\right.$$
$$\left. -\frac{7}{4}\sin^2\tilde{k}L_v - 2(\cos\tilde{k}L_v - 1)\right], \quad (25)$$

$$F_{z2}^{Wg}(\tilde{k}r,\tilde{k}L_v) = -\frac{2i\bar{Z}_S}{\tilde{k}^2 L_v r}\left[\left(\frac{\tilde{k}L_v}{2}\right)^2(2+\cos 2\tilde{k}L_v)\right.$$
$$\left. +\frac{7}{4}\sin^2\tilde{k}L_v - \frac{3}{4}\tilde{k}L_v\sin 2\tilde{k}L_v + 2(\cos\tilde{k}L_v - 1)\right]. \quad (26)$$

As seen, formulas (24)–(26) for the impedance distribution functions $\phi_n(s_1)$ differ though they have equal mean values along the vibrator. Thus, though the functional dependency $f_v(s_1) = \cos\tilde{k}s_1 - \cos\tilde{k}L_v$ in the formulas for the current are used for all three impedance distributions, the current amplitudes and the energy characteristics for the vibrator in the waveguide are different.

Let us consider the corrugated metallic conductor, located in the rectangular waveguide, as shown in Figure 3(b), as an example of the

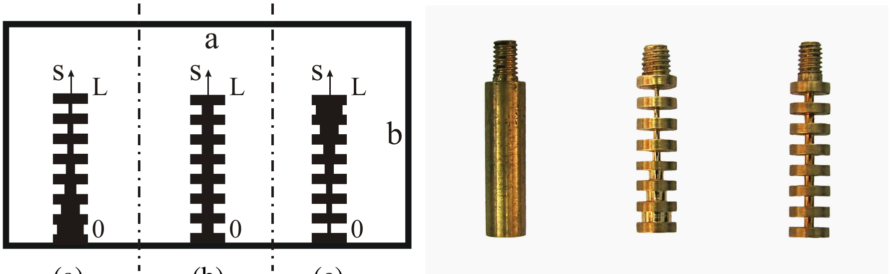

Figure 3. The schematic examples and experimental models of the impedance vibrators.

vibrator with the constant surface impedance. We should remind that the surface impedance for such vibrator is purely inductive (if the conductance of the metal is infinite) and is defined by the formula [4]

$$\bar{Z}_S = i\bar{X}_S = ikr\ln(r/r_i), \tag{27}$$

where r and r_i are the outer and inner radiuses of the corrugation, respectively, and the size of an elementary cell along the axis $\{0s_1\}$ is considerable less than the operating wavelength. Let the impedance vary along the vibrator as $\bar{Z}_S(s_1) = ikr\ln(r/r_i)\phi_n(s_1)$, where r_i corresponds to the case $\phi_0(s_1) = 1$. The impedance variation along the vibrator length can be realized by changing of the inner radius of corrugation as $r_i(s_1) = re^{-\ln(r/r_i)\phi_n(s_1)}$ (Figures 3(a), (c)).

Energy characteristics of vibrator-slot structure in rectangular waveguide (S_{11} and S_{12} are the field reflection and transmission coefficients, respectively, and $|S_\Sigma|^2$ is power radiation coefficient) are defined by the expressions:

$$S_{11} = \frac{4\pi i}{abkk_g}\left\{\frac{2k_g^2}{k^2}\frac{f^2(kL_{sl})}{Y^{Wg}(kd_e,kL_{sl})+Y^{Hs}(kd_e,kL_{sl})} - \frac{k^2}{\tilde{k}^2}\frac{\sin^2(\pi x_{01}/a)f^2(\tilde{k}L_v)}{Z^{Wg}(kr,\tilde{k}L_v)+F_{zn}^{Wg}(kr,\tilde{k}L_v)}\right\}e^{2ik_g z}, \tag{28}$$

$$S_{12} = 1+\frac{4\pi i}{abkk_g}\left\{\frac{2k_g^2}{k^2}\frac{f^2(kL_{sl})}{Y^{Wg}(kd_e,kL_{sl})+Y^{Hs}(kd_e,kL_{sl})} + \frac{k^2}{\tilde{k}^2}\frac{\sin^2(\pi x_{01}/a)f^2(\tilde{k}L_v)}{Z^{Wg}(kr,\tilde{k}L_v)+F_{zn}^{Wg}(kr,\tilde{k}L_v)}\right\}, \tag{29}$$

$$|S_\Sigma|^2 = 1-|S_{11}|^2-|S_{12}|^2. \tag{30}$$

In the formulas (28)–(30) $f(\tilde{k}L_v) = (\sin\tilde{k}L_v - \tilde{k}L_v\cos\tilde{k}L_v)$, $k_g = \sqrt{k^2 - (\pi/a)^2}$.

To illustrate the influence of passive vibrator with constant and variable surface impedance upon the electrodynamic characteristics of the slot, Figures 4–6 show the dependence of radiation coefficient $|S_\Sigma|^2(\lambda)$, reflection coefficient $|S_{11}|(\lambda)$ and transmission coefficient $|S_{12}|(\lambda)$, upon wavelength. As can be seen from the plots, the curves $|S_\Sigma|^2(\lambda)$ for all examined variations of impedance $\bar{Z}_S(s_1)$ and slots lengths are practically identical, both among themselves and for the slots without the vibrator $|S_\Sigma|^2(\lambda)$, i.e., radiation coefficient of the vibrator-slot system, in the absence of interaction between them, is determined primarily by the geometric slot dimensions and its position relative to the waveguide walls. At the same time, if passive vibrators of fixed length with different dependencies of surface impedance on longitudinal coordinate are placed in waveguide, the

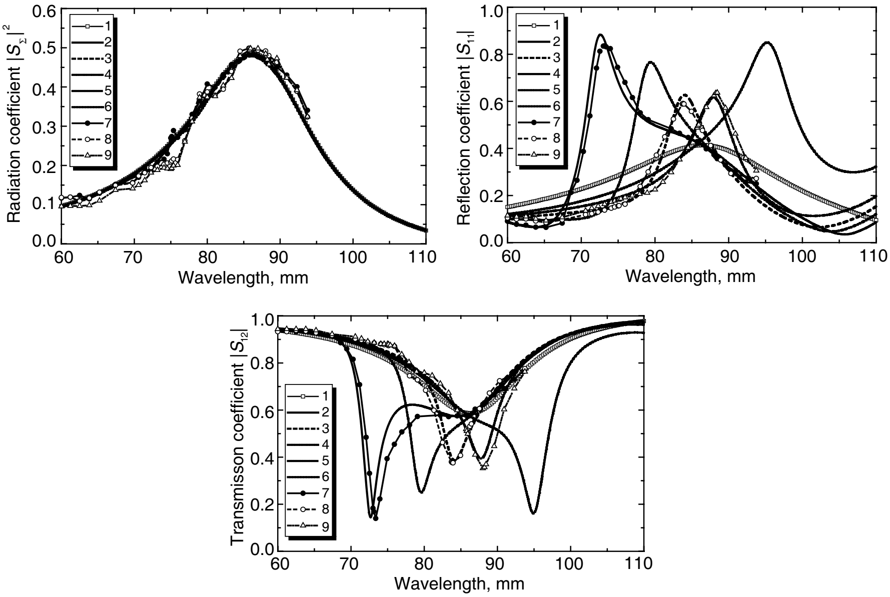

Figure 4. Energy characteristics of vibrator-slot system versus wavelength at $a = 58.0\,\text{mm}$, $b = 25.0\,\text{mm}$, $h = 0.5\,\text{mm}$, $r = 2.0\,\text{mm}$, $L_v = 15.0\,\text{mm}$, $\varphi = 90°$, $x_{01} = a/8$, $y_{01} = 0$, $d = 4.0\,\text{mm}$, $2L_{sl} = 40.0\,\text{mm}$, $x_{02} = a/2$, $z_0 = 0$: 1 — single slot; 2 — $\bar{Z}_S = 0$; 3 — $\bar{Z}_S = ikr\ln(4.0)$; 4 — $\bar{Z}_S(s_1) = ikr\ln(4.0)\phi_1(s_1)$; 5 — $\bar{Z}_S(s_1) = ikr\ln(4.0)\phi_2(s_1)$; 6 — $\bar{Z}_S(s_1) = ikr\ln(8.0)\phi_1(s_1)$; 7, 8, 9 — experimental data.

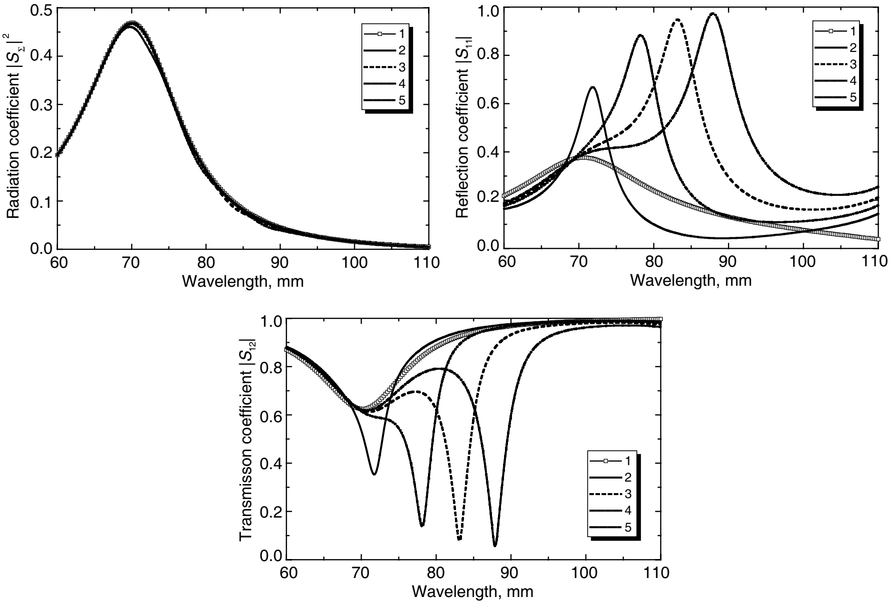

Figure 5. Energy characteristics of vibrator-slot system versus wavelength at $a = 58.0$ mm, $b = 25.0$ mm, $h = 0.5$ mm, $r = 2.0$ mm, $L_v = 15.0$ mm, $\varphi = 90°$, $x_{01} = a/8$, $y_{01} = 0$, $d = 4.0$ mm, $2L_{sl} = 32.0$ mm, $x_{02} = a/2$, $z_0 = 0$: 1 — single slot; 2 — $\bar{Z}_S = 0$; 3 — $\bar{Z}_S = ikr \ln(4.0)$; 4 — $\bar{Z}_S(s_1) = ikr \ln(4.0)\phi_1(s_1)$; 5 — $\bar{Z}_S(s_1) = ikr \ln(4.0)\phi_2(s_1)$.

substantial variation of $|S_{11}|(\lambda)$ and $|S_{12}|(\lambda)$ as compared with those for the single slot may be achieved.

The comparison of theoretical and experimental curves (Figure 4) indicates that the solution of integral equations for combined vibrator-slot structures by the generalized method of induced EMMF with approximating functions for the currents in the impedance vibrator and the slot, obtained by averaging method, is quite legitimate.

Note once more that for arbitrary orientations of the vibrator, or the slot relative to the waveguide walls, or for another impressed field sources, expressions (20) should be used to determine the distribution functions of electric and magnetic currents in the vibrator and slot. For example, for the longitudinal slot in the broad wall of waveguide, i.e., if axes $\{0s_2\}$ and $\{0z\}$ coincide, we obtain

$$\begin{aligned} f_{sl}^s(s_2) &= \cos ks_2 \cos k_g L_{sl} - \cos kL_{sl} \cos k_g s_2, \\ f_{sl}^a(s_2) &= \sin ks_2 \sin k_g L_{sl} - \sin kL_{sl} \sin k_g s_2. \end{aligned} \tag{31}$$

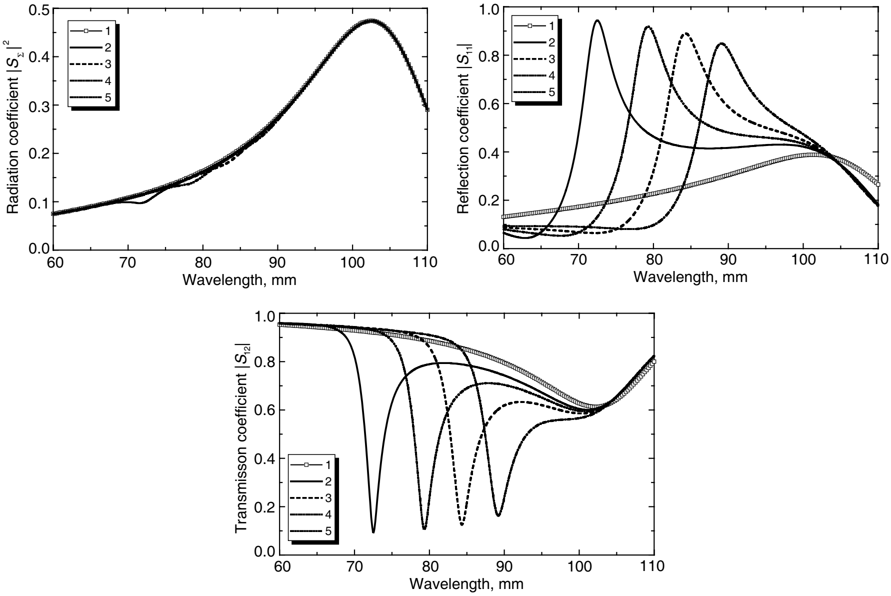

Figure 6. Energy characteristics of vibrator-slot system versus wavelength at $a = 58.0\,\text{mm}$, $b = 25.0\,\text{mm}$, $h = 0.5\,\text{mm}$, $r = 2.0\,\text{mm}$, $L_v = 15.0\,\text{mm}$, $\varphi = 90°$, $x_{01} = a/8$, $y_{01} = 0$, $d = 4.0\,\text{mm}$, $2L_{sl} = 48.0\,\text{mm}$, $x_{02} = a/2$, $z_0 = 0$: 1 — single slot; 2 — $\bar{Z}_S = 0$; 3 — $\bar{Z}_S = ikr\ln(4.0)$; 4 — $\bar{Z}_S(s_1) = ikr\ln(4.0)\phi_1(s_1)$; 5 — $\bar{Z}_S(s_1) = ikr\ln(4.0)\phi_2(s_1)$.

If vibrator is excited at its base by voltage δ-generator as in a waveguide-to-coaxial adapter we have

$$f_v(s_1) = \sin\tilde{k}(L_v - s_1). \tag{32}$$

5. CONCLUSION

This paper presents the methodological basis for application of the generalized method of induced EMMF for the analysis of electrodynamic characteristics of the combined vibrator-slot structures. Characteristic feature of the generalization to a new class of approximating functions consists in using them as a function of the current distributions along the impedance vibrator and slot elements; these distributions are derived as the asymptotic solution of integral equations for the current (key problems) by the method of averaging. It should be noted that for simple structures similar to that considered in the model problem, the proposed approach yields an analytic solution of the electrodynamic problem. For more complex structures, the method may be used to design effective numerical-analytical algorithms for their analyses. The applicability of the proposed method of the generalized method of induced EMMF for analyzing the vibrator-slot systems with an arbitrary structure under the adopted assumptions is proved by comparative analysis of theoretical and experimental results in the range of operating wavelengths. This method retains all benefits of analytical methods as compared with direct numerical methods and allows significantly expanding the boundaries of numerical and analytical studies of practically important problems, concerning the application of single impedance vibrator, including irregular vibrator, the systems of such vibrators and narrow slots. And this is a natural step in the further development of general fundamental theory of linear radiators of electric and magnetic types.

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