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A HYBRID METHOD BASED ON DIFFERENTIAL EVOLUTION AND CONTINUOUS ANT COLONY OPTIMIZATION AND ITS APPLICATION ON WIDEBAND ANTENNA DESIGN

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Abstract—An evolutionary learning algorithm based on differential evolution strategy (DES) and continuous ant colony optimization (CACO) for wideband antenna design is proposed. The advantages of this hybrid method are demonstrated with several mathematical functions and a linear array pattern synthesis. This method is applied to design an E-shaped wideband patch antenna, which achieves the impedance bandwidth $4.8 \sim 6.53$ GHz. We compare the hybrid method with the traditional DES and CACO optimization algorithms, and the advantage of this hybrid method over the DES and the CACO is also demonstrated.

1. INTRODUCTION

In recent years, many evolutionary algorithms have been proposed for solving electromagnetics problems such as genetic algorithm (GA) [1–4], particle swarm optimization (PSO) [5–8] and bees algorithm [9, 10]. In [11], an UWB microstrip antenna is designed based on differential evolution strategy (DES) [12].

Among existing global optimization algorithms, the DES pioneered by Storn and Price [12] is a stochastic search procedure and is able to memorize optimal individual. It optimizes problems with real valued variables and has been applied to electromagnetic inverse scattering problems [13] and design of microwave filter [14, 15]

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and antenna [15–19] as well as pattern synthesis [15, 20–23]. It has been proven to have strong global convergency and robustness. The ant colony optimization (ACO) algorithm proposed by Dorigo and Stützle [24] is a population-based heuristic bionics evolutionary algorithm. It has been extensively applied in solving combinatorial optimization problems, such as the traveling salesman problem (TSP) [25], and the job shop scheduling problem (JSP) [26]. This method has also been used to synthesize thinned arrays with low sidelobe level (SLL) [27]. However, the original ACO is invented for solving discrete optimization problems. Recently, the continuous ACO (CACO) [28, 29] has been proposed to solve some engineering optimization problems. Hosseini and Atlasbaf [30] employed the CACO to optimize the sum and difference patterns of linear monopulse antennas.

In order to improve the optimization performance of ACO and DES, some research on combining the discrete ACO and DES has been presented in [31, 32]. These hybrid methods set the pheromone on the path left by ants in ACO as the object of mutation, crossover and selection in DES. In this paper, we propose a new hybrid method based on DES and CACO, in which the possibility of premature can be reduced and the convergence rate is improved.

This paper is organized as follows. Section 2 introduces the new hybrid algorithm, as well as some modification in the DES and CACO. In Section 3 some examples including trial functions and an antenna array pattern synthesis are provide to demonstrate the advantages of our algorithm. We employ this new algorithm to design a wideband antenna. Conclusions are given in Section 4.

2. THE HYBRID ALGORITHM BASED ON DES AND CACO

2.1. Differential Evolution Strategy

The DES has four operations: mutation, crossover, selection and re-initialization [23], which are described as follows:

2.1.1. Mutation

A mutant vector is computed for each individual as:

$$\mathbf{v}_{i,G+1} = \mathbf{x}_{b,G} + F(\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}), \quad 1 \leq r_1 \neq r_2 \neq i \leq N_p \tag{1}$$

where N_p is the population size. $\mathbf{x}_{b,G}$ is the optimal vector of the parent population in the Gth generation. F is a real number that controls the amplification of differential variation.

2.1.2. Crossover

Crossover operator is then applied to produce a child vector $x_{i,G+1}^{j,c}$ in the $(G+1)$th generation, using the following scheme:

$$x_{i,G+1}^{j,c} = \begin{cases} v_{i,G+1}^{j}, & r_{i,G+1}^{j} \leq C_r \\ x_{i,G}^{j}, & \text{otherwise} \end{cases} \tag{2}$$

where $r_{i,G+1}^{j} \in [0,1]$ is a real random number, and C_r is the crossover probability.

2.1.3. Selection

The selection scheme for minimization problems is defined by

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{x}_{i,G+1}^{c}, & f\left(\mathbf{x}_{i,G+1}^{c}\right) < f\left(\mathbf{x}_{i,G}\right) \\ \mathbf{x}_{i,G}, & \text{otherwise} \end{cases} \tag{3}$$

2.1.4. Re-initialization

In [23], the refreshing distribution operation is used to maintain the population diversity with a period of N_{refresh}.

2.2. Continuous Ant Colony Optimization

The continuous ACO iterates over an ant population of k individuals and k is the number of ants [29, 30].

We define a model for the global optimization problem:

$$\begin{aligned} \text{minimize} \quad & F = f(\mathbf{X}) \\ \text{subjectto} \quad & \mathbf{l} \leq \mathbf{X} \leq \mathbf{u} \end{aligned}$$

where $\mathbf{X} = (X_1, X_2 \ldots, X_n)$ is a variable vector for a $n$dimensional problem, $f(\mathbf{X})$ is the objective function, and $\mathbf{l} = (l_1, l_2, \ldots, l_n)$ and $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ define the feasible solution space. The set of all pheromone trail parameters is denoted by $\mathbf{T} = (T_1, T_2, \ldots, T_n)$. The length of each dimension is

$$\text{Len}(i) = \frac{u_i - l_i}{k}, \quad (i = 1, 2, \ldots, n) \tag{4}$$

The procedure of CACO can be described as follows [29]:

```
Initialize Ant_1, Ant_2, ..., Ant_k
for each ant
calculate fitness and pheromone function
end for
Ant_best ← best ant
bestindex ← index number for the best ant
while termination conditions not met do
for i = 1 to k do
if i ≠ bestindex then
Global search
end if
end for
Local search for Ant_best,
if the new ant is betterthen replace the Ant_best
for i = 1 to k do
if i ≠ bestindex then
calculate fitness and pheromone function
end if
end for
update pheromone T
end while
Best ant found Ant_best
```

In this procedure, the global search employs the transition probability:

$$P(i) = \exp(d)/\exp(T(\text{bestindex})),\ i \neq \text{bestindex},\ (i = 1, 2, \ldots, k) \quad (5)$$

with $d = T(\text{bestindex}) - T(i)$. The new positions of ants are determined as:

$$X_i = \begin{cases} X_i + \lambda(X_{\text{bestindex}} - X_i), & P(i) < P_0 \\ X_i + \text{rand}(-1, 1) \times \text{Len}, & \text{otherwise} \end{cases} \quad (6)$$

where $0 < \lambda < 1$, $0 < P_0 < 1$.

In the local search the position of the best ant is updated as follows:

$$X_{\text{temp}} = \begin{cases} X_{\text{bestindex}} + w \times \text{step}, & \text{rand}\,(0, 1) < 0.5 \\ X_{\text{bestindex}} - w \times \text{step}, & \text{otherwise} \end{cases} \quad (7)$$

where step is the initial step for the local search. $w = w_{\max} + (w_{\max} - w_{\min}) \times current/total$, $1 < w_{\max} < 1.4$, $0.2 < w_{\min} < 0.8$. *current* denotes the current iteration number and *total* is the total iteration number. Then we calculate the fitness value of X_{temp}. If the fitness value is better, we replace $X_{\text{bestindex}}$ with X_{temp}.

2.3. The New Hybrid Method

The core of the hybrid algorithm is to integrate DES and CACO in parallel. Here the dynamic DES (DDES) [13] is employed for the DES method. The procedure of the hybrid algorithm can be described as follows:

1. Initialize N_p individuals and calculate the fitness values. Sort the individuals according to their fitness values.

2. If $N_G = m \cdot N_{\text{refresh}}$ ($m = 1, 2, \ldots$), goto step 4, else goto step 3. N_G is the number of evolution generations carried out.

3. The excellent group which contains $N_p/2$ individuals with the fitness values lower than that of the other individuals is applied with the global search of CACO. In order to increase the randomness and diversity, Eq. (6) is modified as

$$X_i = X_i + \text{rand}(0, 1) \times (X_{\text{bestindex}} - X_i). \tag{8}$$

The fitness values are calculated.

The rest individuals not included in the excellent group are used to produce child vectors by DDES (See Eq. (1) to Eq. (3)). In Eq. (1), the two individuals $\mathbf{x}_{r_1,G}$ and $\mathbf{x}_{r_2,G}$ are selected from the whole population.

All individuals are then sorted according to their fitness values, and the best individual I_{best} is obtained.

4. The individuals in the excellent group are applied with the mutation operator with

$$\mathbf{v}_{i,G+1} = \mathbf{x}_{i,G} + F(\mathbf{x}_{b,G} - \mathbf{x}_{i,G}) + F(\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}), \ 1 \le r_1 \ne r_2 \ne i \le N_p, \tag{9}$$

where the two individuals $\mathbf{x}_{r_1,G}$ and $\mathbf{x}_{r_2,G}$ are also selected from the whole population. The crossover and selection operations are applied to the excellent group.

The rest individuals are initialized and the fitness values are calculated. All individuals are sorted according to their fitness values and the best individual I_{best} is obtained.

5. The Local search (see Eq. (7)) is applied for the best individual I_{best}.

6. If the termination conditions are met, stop, otherwise, goto step 2.

The global search capability can be enhanced by the combination of two strategies Eqs. (1) and (9).

3. NUMERICAL RESULTS

3.1. Verifications of the Hybrid Algorithm

3.1.1. Test Functions

To demonstrate the performance of the hybrid algorithm, we first consider the N-dimensional Ackley function as one benchmark function

$$f(\bar{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i^2}\right) - \exp\left(\frac{1}{N}\sum_{i=1}^{N}\cos 2\pi x_i\right) + 20 + \exp(1.0). \quad (10)$$

where $\bar{x} = (0, \ldots, 0)_N$ with $N = 20$, and this function has a global minimum at 0.

We apply the hybrid algorithm to find the global minimum in the range $[-5.0, 5.0]$. The population size is set as 100. The average number of the evaluations for this function is 4747 in 20 test runs for the same termination condition (the value of the function is less than 1.0×10^{-8}) as in [23] are satisfied. In contrast, the modified differential evolution strategy (MDES) needs 6160 evaluations [23]. The convergence curve is shown in Fig. 1.

The second example we consider is the Rosenbrock function

$$f(\bar{x}) = \sum_{i=1}^{N-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + (1 - x_i)^2\right] \quad (11)$$

which has a minimum 0 for $x_i = 1$ with $i = 1, 2, \ldots, N$. We apply the hybrid algorithm to find this minimum. The optimization (finding minimum) is performed for the case of $N = 10$ and the range of $[-2.048, 2.047]$. The termination condition is set as when the value of $f(\bar{x})$ is less than 1.0×10^{-3}. The convergence curve is shown in

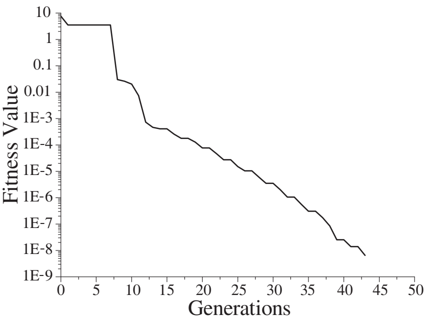

Figure 1. Convergence rate for 20-D Ackley function.

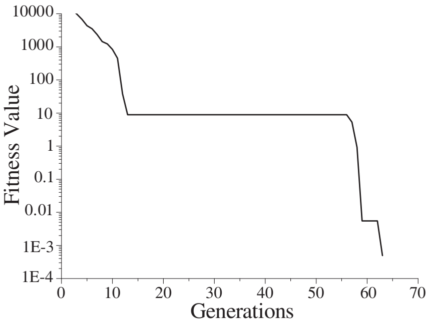

Figure 2. Convergence rate for 10-D Rosenbrock function.

Table 1. Best fitness values.

	The hybrid algorithm	IDE [21]	Range
Griewangk	0	0.0238	[−200, 200]
Rastrigin	0	1.9324	[−20, 20]

Fig. 2. The hybrid algorithm requires the average number of 6540 evaluations in 20 test runs while the MDES need 10,000 evaluations to obtain the minimum of 8.0×10^{-3} [23].

To further demonstrate the advantages of the proposed algorithm, we apply the hybrid algorithm to optimize another two 20-D test functions: Griewangk function and Rastrigin function [12]. A population size of 40 vectors is selected and the total number of iterations is set to 400. Table 1 presents the best fitness values obtained by the hybrid algorithm, as well as those got by the improved differential evolution (IDE) strategy. It can be seen from the Table that the average optimal values found by the proposed method in 20 test runs are smaller than those obtained by the IDE [21].

3.1.2. Linear-array Pattern Synthesis

We consider $2N$ elements symmetrically placed on the x-axis, and the array factor for uniform amplitude excitation can be written as [15]

$$AF\left(\theta,\bar{x},\bar{\varphi}\right)=2\sum_{n=1}^{N}\cos\left[\frac{2\pi}{\lambda}x_n\sin\theta+\varphi_n\right], \tag{12}$$

where λ is the wavelength, φ_n and x_n denote the phase and location of the array elements.

The design of a 32-element symmetric array for position-only synthesis is carried out. The angle resolution of θ is 0.1°. Our purpose is to find the optimal positions of array elements that would afford a pattern with a minimum side-lobe level (SLL). The population size is set as 100 and the total number of iterations is set to 2000. We assume that

$$d_{\min}=0.5\lambda\leq x_i-x_{i-1}\leq d_{\max}=0.6\lambda,\quad i=2,\ldots,16 \tag{13}$$

To investigate reliability, 10 independent runs of the hybrid algorithm are repeated. In average, the maximum SLL of −15.82 dB is obtained in 1608 iterations, in contrast to 2000 iterations by the self-adaptive differential evolution (SADE) algorithm [15].

Figure 3 shows the optimal pattern. The element positions for the pattern in Fig. 3 are shown in Table 2 for verification purpose. The average convergence rate of the hybrid algorithm is shown in Fig. 4.

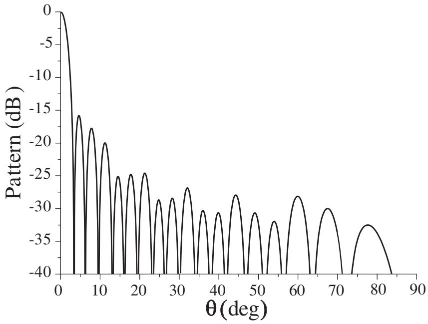

Figure 3. Radiation pattern for 32-element symmetric array.

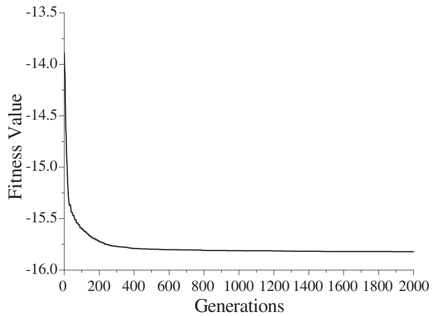

Figure 4. Average convergence rate for the 32 elements array.

Table 2. Element locations for a 32-element array.

i	1	2	3	4	5	6	7	8
x_i/λ	0.25	0.75	1.25	1.75	2.25	2.75	3.25	3.8499
i	9	10	11	12	13	14	15	16
x_i/λ	4.44989	5.04989	5.64989	6.24989	6.84988	7.44988	8.04988	8.64988

Table 3. Design parameters for the antenna (unit: mm).

	W	L	W_s	L_s	P	L_m
Lower bounds	10	10	0	0.5	-15	0.5
Upper bounds	30	50	30	20	15	40

3.2. Antenna design

The DES has been applied in designing the E-shaped patch antenna [15, 19]. The geometry of the E-shaped patch antenna is shown in Fig. 5. The radiuses of the coaxial inner conductor and outer conductor are 0.5 mm and 1.15 mm. The design parameters of the antenna are patch width W, patch length L, slot length W_s, slot width L_s, feed position P and the separation of the two slots width L_m. Their ranges are listed in Table 3, which are the same as in [19]. The relative dielectric constant of the substrate (air) is $\varepsilon_r = 1.0$. The design problem is defined as the minimization of the objective function:

$$F(\bar{x}) = 20 \log \{\max |S_{11}(\bar{x}, f)|, f \in [4.9\text{GHz}, 6.2\text{GHz}]\}, \qquad (14)$$

where $\bar{x} = (W, L, W_s, L_s, P, L_m)$. The E-shaped patch antenna is simulated in CST MWS. A VBA macro language is used to combine the hybrid algorithm and CST MWS. The population size is set to 20 and the total number of iterations is set to 30. It is worth noting that the calculation time of the optimization is dominated by the

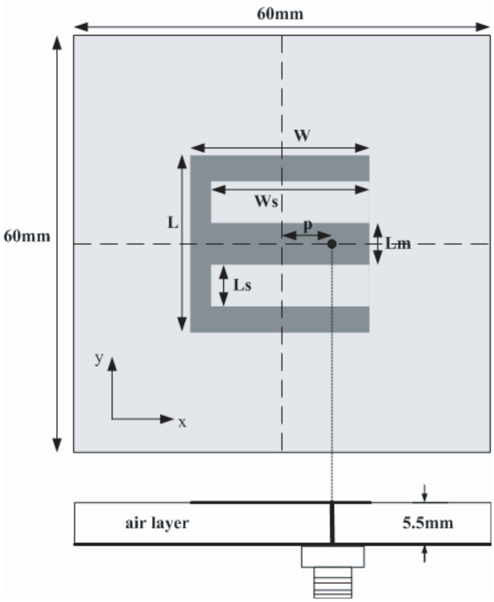

Figure 5. Geometry of the E-shaped patch antenna.

numerical simulation of S_{11} by the electromagnetic solver, therefore the performance of algorithm is indicated by iteration number.

The following additional restrictions apply to the design parameters for maintaining the E-shape [8, 15],

$$W_s < W, \tag{15}$$

$$L_m/2 + L_s < L/2, \tag{16}$$

$$|P| < W/2. \tag{17}$$

Table 4 compares the optimal parameters obtained by the hybrid algorithm, the DES and the CACO in 10 independent runs. The average convergence rates for these three optimization algorithm are shown in Fig. 6. We can see that the hybrid algorithm converge faster than the DES and the CACO, as well as the final objective-function value is smaller than those obtained by the DES and the CACO. The S_{11} based on the optimal parameters are given in Fig. 7. The antenna found by the hybrid algorithm has an impedance bandwidth between $4.8 \sim 6.53\,\text{GHz}$ with $S_{11} < -10\,\text{dB}$.

The radiation patterns for the hybrid algorithm, the CACO and the DES designs are presented in Fig. 8. It can be seen that the radiation patterns have same polarization at 5.1 and 5.9 GHz. The gain of the three designs is compared in Fig. 9. We can see that the gains among the three designs are comparable. In order to give readers some ideas about the computation cost for this antenna design, in our simulations, the electromagnetic solver CST MWS is employed and the average calculation time for our optimization is 37152 seconds (on ASUS server with 4 cores, CPU: Xeon X3330 @ 2.66 GHz).

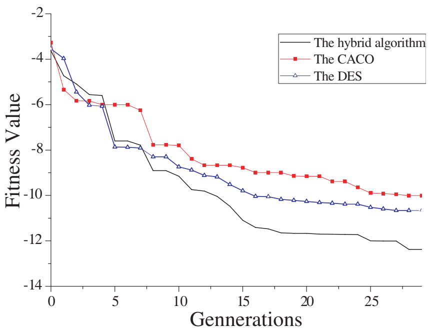

Figure 6. Comparisons of the average convergence rates for DES, CACO and the hybrid algorithm.

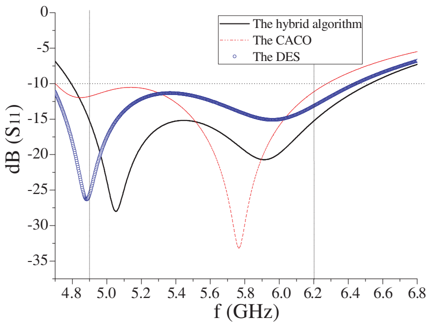

Figure 7. Simulated S_{11} curves obtained via DES, CACO and the hybrid algorithm.

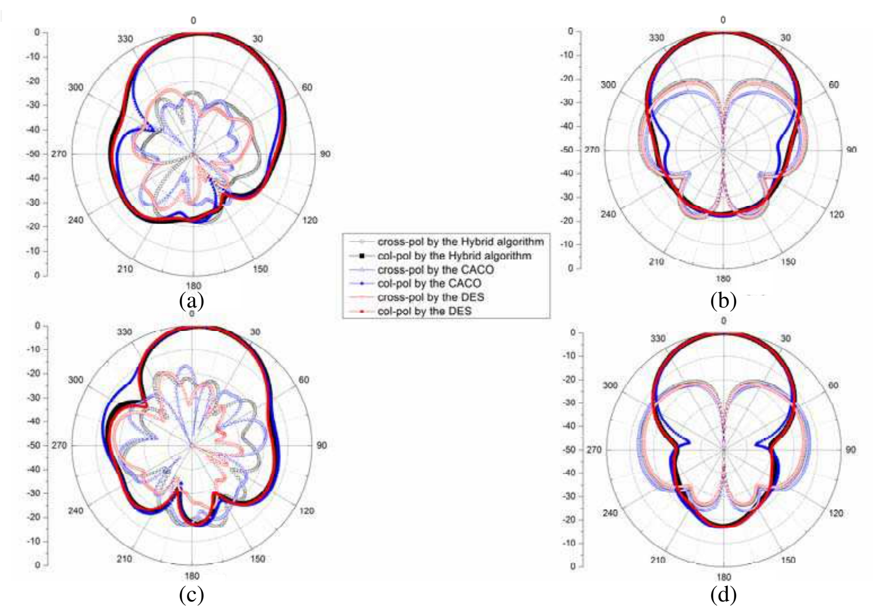

Figure 8. Comparisons of the simulated radiation patterns of the three optimized antennas. (a) E-plane at 5.1 GHz. (b) H-plane at 5.1 GHz. (c) E-plane at 5.9 GHz. (d) H-plane at 5.9 GHz.

Table 4. Optimal Parameters for the E-shaped patch antenna (unit: mm).

	W	L	W_s	L_s	P	L_m
The hybrid algorithm	19.9134	37.7541	16.8262	5.92375	4.95039	5.15766
The CACO	21.592	42.3633	15.0841	8.57912	4.59907	4.56407
The DES	19.8588	37.5544	15.4478	5.74849	5.82192	7.51514

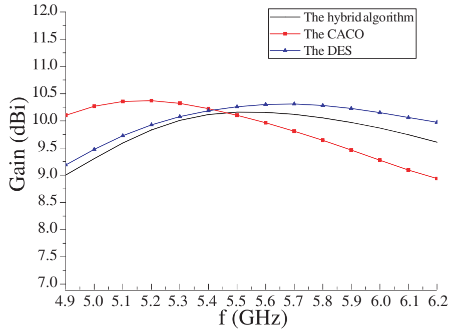

Figure 9. Comparisons of the simulated gains by DES, CACO, and the hybrid algorithm.

4. CONCLUSION

We propose a new hybrid algorithm based on the DES and the CACO. Optimization results of several benchmark trail functions and a linear-array pattern synthesis example show that the hybrid algorithm performs better than the DES. The hybrid algorithm is also applied to design a wideband E-shaped patch antenna combined with a numerical solver CST MWS. Compared with the traditional DES and CACO, the hybrid algorithm obtains better results of the convergence rate and the final objective-function value. It is shown that the design methodology is an effective way for antenna design.

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REFERENCES

1. Siakavara, K., "Novel fractal antenna arrays for satellite networks: Circular ring sierpinski carpet arrays optimized by genetic algorithms," *Progress In Electromagnetics Research*, Vol. 103, 115–138, 2010.
2. Reza, A. W., M. S. Sarker, and K. Dimyati, "A novel integrated mathematical approach of ray-tracing and genetic algorithm for optimizing indoor wireless coverage," *Progress In Electromagnetics Research*, Vol. 110, 147–162, 2010.
3. Dadgarnia, A. and A. A. Heidari, "A fast systematic approach for microstrip antenna design and optimization using ANFIS and GA," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 16, 2207–2221, 2010.
4. Xu, O., "Collimation lens design using AI-GA technique for gaussian radiators with arbitrary aperture field distribution," *Journal of Electromagnetic Waves and Applications*, Vol. 25, Nos. 5–6, 743–754, 2011.
5. Islam, M. T., M. Moniruzzaman, N. Misran, and M. N. Shakib, "Curve fitting based particle swarm optimization for UWB patch antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 17–18, 2421–2432, 2009.
6. Naghavi, A. H., M. Tondro-Aghmiyouni, M. Jahanbakht, and A. A. Lotfi Neyestanak, "Hybrid wideband microstrip Wilkinson power divider based on lowpass filter optimized using particle swarm method," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 14–15, 1877–1886, 2010.
7. Wang, W.-B., Q. Feng, and D. Liu, "Application of chaotic particle swarm optimization algorithm to pattern synthesis of antenna arrays," *Progress In Electromagnetics Research*, Vol. 115, 173–189, 2011.
8. Jin, N. B. and Y. Rahmat-Samii, "Parallel particle swarm optimization and finite-difference time-domain (PSO/FDTD) algorithm for multiband and wide-band patch antenna designs," *IEEE Trans. on Antennas and Propagat.*, Vol. 53, No. 11, 3459–3468, 2005.
9. Guney, K. and M. Onay, "Amplitude-only pattern nulling of linear antenna arrays with the use of bees algorithm," *Progress In Electromagnetics Research*, Vol. 70, 21–36, 2007.
10. Guney, K. and M. Onay, "Synthesis of thinned linear antenna arrays using bees algorithm," *Microwave and Optical Technology Letters*, Vol. 53, No. 4, 795–799, 2011.

11. Zheng, X., C. Liao, L. Chang, J. Fang, and H. J. Fu, "Automated design of an ultra-wideband microstrip patch antenna," *ICMTCE, The International Conference on Microwave Technology & Computational Electromagnetics*, 80–83, Beijing, China, 2009.
12. Storn, R. and K. Price, "Differential evolution — A simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Vol. 11, No. 4, 341–359, 1997.
13. Qing, A. Y., "Dynamic differential evolution strategy and applications in electromagnetic inverse scattering problems," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 44, No. 1, 116–125, 2006.
14. Goudos, S. K., Z. D. Zaharis, and T. V. Yioultsis, "Application of a differential evolution algorithm with strategy adaptation to the design of muti-band microwave filters for wireless communications," *Progress In Electromagnetics Research*, Vol. 109, 123–137, 2010.
15. Goudos, S. K., K. Siakavara, T. Samaras, E. E. Vafiadis, and J. N. Sahalos, "Self-adaptive differential evolution applied to real-valued antenna and microwave design problems," *IEEE Trans. on Antennas and Propagat.*, Vol. 59, No. 4, 1286–1298, 2011.
16. Goudos, S. K., K. Siakavara, E. Vafiadis, and J. N. Sahalos, "Pareto optimal Yagi-Uda antenna design using multi-objective differential evolution," *Progress In Electromagnetics Research*, Vol. 105, 231–251, 2010.
17. Li, J.-Y., "A bi-swarm optimizing strategy and its application of antenna design," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 14–15, 1877–1886, 2009.
18. Xie, L., Y.-C. Jiao, Y.-Q. Wei, and G. Zhao, "A compact band-notched UWB antenna optimized by a novel self-adaptive differential evolution algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 17–18, 2353–2361, 2010.
19. Zhang, L., Z. Cui, Y. C. Jiao, and F. S. Zhang, "Broadband patch antenna design using differential evolution algorithm," *Microwave and Optical Technology Letters*, Vol. 51, No. 7, 1692–1695, 2009.
20. Dib, N. I., S. K. Goudos, and H. Muhsen, "Application of taguchi's optimization method and self-adaptive differential evolution to the synthesis of linear antenna arrays," *Progress In Electromagnetics Research*, Vol. 102, 159–180, 2010.
21. Li, R., L. Xu, X.-W. Shi, N. Zhang, and Z.-Q. Lv, "Improved differential evolution strategy for antenna array pattern synthesis problems," *Progress In Electromagnetics Research*, Vol. 113, 429–441, 2011.

22. Li, F., Y.-C. Jiao, L.-S. Ren, Y.-Y. Chen, and L. Zhang, "Pattern synthesis of concentric ring array antennas by differential evolution algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 25, Nos. 2–3, 421–430, 2011.
23. Chen, Y. K., S. W. Yang, and Z. P. Nie, "The application of a modified differential evolution strategy to some array pattern synthesis problems," *IEEE Trans. on Antennas and Propagat.*, Vol. 56, No. 7, 1919–1927, 2008.
24. Dorigo, M. and T. Stützle, *Ant Colony Optimization*, The MIT Press, 2003.
25. Luo, D. L., H. P. Chen, S. X. Wu, and Y. X. Shi, "Hybrid ant colony multi-objective optimization for flexible job shop scheduling problems,"*Journal of Internet Technology*, Vol. 11, No. 3, 361–369, 2010.
26. Xing, L. N., Y. W. Chen, P. Wang, Q. S. Zhao, and J. Xiong, "Knowledge-based ant colony optimization for flexible job shop scheduling problems," *Applied Soft Computing*, Vol. 10, No. 3, 888–896, 2010.
27. Quevedo-Teruel, Q. and E. Rajo-Iglesias, "Ant colony optimization in thinned array synthesis with minimum sidelobe level," *IEEE Antennas and Wireless Propag. Lett.*, Vol. 5, 349–352, 2006.
28. Bilchev, B. and I. C. Parmee, "The ant colony metaphor for searching continuous design spaces," *Proceedings of the AISB Workshop on Evolutionary Computation, Lecture Notes in Computer Science*, Vol. 993, 25–39, 1995.
29. Zhang, H. M., M. Z. Gao, and X. F. Zhang, "Continuous ant colony optimization algorithm based on crossover and mutation," *Journal of China University of Metrology*, Vol. 20, No. 3, 259–262, 2009.
30. Hosseini, S. A. and Z. Atlasbaf, "Optimization of side lobe level and fixing quasi-nulls in both of the sum and difference patterns by using continuous ant colony optimization (ACO) method," *Progress In Electromagnetics Research*, Vol. 79, 321–337, 2008.
31. Coelho, L. D. and D. L. D. Bernert, "A modified ant colony optimization algorithm based on differential evolution for chaotic synchronization," *Expert Systems with Applications*, Vol. 37, No. 6, 4198–4203, 2010.
32. Khushaba, R. N., A. Al-Ani, A. AlSukker, and A. Al-Jumaily, "A combined ant colony and differential evolution feature selection algorithm," *ANTS, Proceedings of the 6th International Conference on Ant Colony Optimization and Swarm Intelligence*, 1–12, 2008.