EFFECT OF INHOMOGENEOUS PLASMA DENSITY ON THE REFLECTIVITY IN ONE DIMENSIONAL PLASMA PHOTONIC CRYSTAL

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Abstract—The dependence of reflectivity on inhomogeneous plasma density for one dimensional plasma photonic crystal is presented. The exponential varying and linear varying plasma density profiles have been chosen in such a way that the volume average permittivity remains constant. The transfer matrix method is used to derive the dispersion relation and reflectivity of the proposed structures by employing the continuity conditions of electric fields and its derivatives on the interface. The exponential varying plasma density profile gives high reflectivity than the linear varying plasma density profile in all considered cases. Also the exponential varying plasma density profile shows perfect reflection in considered volume average permittivity. This profile may be used in sensor applications or in plasma functional devices.

1. INTRODUCTION

Interaction of electromagnetic waves with plasma is very important in the realm of plasma physics and it has been explored for years in numerous fields, such as fusion plasma, ionospheres, and plasma materials processing reactors. When electromagnetic wave is launched near the plasma slab, depending upon its frequency, it gets attenuated or is passed through plasma slab. Electromagnetic waves with angular frequency smaller than plasma frequency are attenuated while those having higher frequency than plasma frequency get transmitted. These behaviors get modified when we have periodic arrangement of plasma and dielectric. These periodic structures are called plasma photonic crystals exhibit novel physical aspects such as band gaps.

Electromagnetic waves whose frequency is outside the band gap, even smaller than the plasma frequency can propagate through plasma. The electromagnetic waves with frequency inside the band gap, even higher than plasma frequency is get attenuated. Firstly, the propagation of electromagnetic wave in rapidly created spatially periodic plasma is studied by Kuo and Faith [1, 2]. The concept of plasma photonic crystals (PPCs) was given by Hojo et al. [3] and they have studied the dispersion relation of electromagnetic wave propagation in 1-D binary PPCs. Thereafter, lots of theoretical [4–7], simulation [8–11] and experimental [12–16] studies are being performed to observe the effect of plasma parameters and dielectric constants on the band gaps, reflectivity and transmissivity of the one, two and three dimensional PPCs.

In most of the abovementioned studies, generally it is found that the discussions regarding dispersion characteristics or reflection or transmission coefficients in which a unit cell is consist of homogeneous unmagnetized plasma layer and homogeneous dielectric layer (or vacuum). These properties are being controlled by using plasma parameters like plasma density, plasma frequency, relative plasma width and external magnetic field [17–20].

Moreover, the optical properties and dispersion characteristics of PPC can also be controlled by considering inhomogeneous plasma in the unit cell. It is more practical also because homogeneous plasma having uniform density is rarely realized in the laboratory plasma [21]. Therefore in the present study we have taken inhomogeneous plasma in the unit cell of binary 1D-PPC which is yet not discussed by any researcher. One of the reasons for inhomogeneities in the plasma can be spatial dependence of plasma density. Here the transfer matrix method is employed to study the propagation of electromagnetic waves in 1D-PPCs having inhomogeneous plasma in the unit cell which is suitable for the analysis of 1D-PPCs [22, 23]. The inhomogeneous plasma with linear density profile [24] and exponentially density profiles [25] have chosen because close form solutions can be obtained in terms of special mathematical functions. The paper is organized as follow: in Section 2 formulas for the dispersion relation of the proposed structure is given. The other necessary formulas used in this paper are also presented. Section 3 is devoted to result and discussion. A conclusion is drawn in Section 4.

2. THEORETICAL MODELING

To study the electromagnetic wave propagation in such inhomogeneous plasma photonic crystal, a plane electromagnetic wave with frequency

ω is obliquely incident onto present structure having inhomogeneous plasma layer and homogeneous dielectric materials in one unit cell is considered and shown in Fig. 1. It is assumed that inhomogeneous plasma is collision-less, isotropic and non-magnetic. The plasma density varies either linearly or exponentially in space and is given by $n(x)$. The plasma density for linearly varying profile in the space is given by

$$n(x) = n_{cr}px/b$$

Similarly, plasma density for an exponentially varying profile in the space is given by

$$n(x) = n_{cr}e^{-\frac{px}{b}}$$

where b is the width of plasma layer, p is gradation parameter for controlling variation of density in the plasma layer and n_{cr} is the critical density [26] $n_{cr} = m\varepsilon_0\omega^2/e^2$. The permittivity profile for inhomogeneous plasma and dielectric media is given by:

$$\varepsilon(x) = \begin{cases} 1 - \frac{\omega_p^2(x)}{\omega^2}; & (n-1)\Lambda < x < (n-1)\Lambda + b \\ \varepsilon_1; & (n-1)\Lambda + b < x < n\Lambda \end{cases} \quad (1a)$$

For linear varying plasma density profile, the permittivity is written as

$$\varepsilon(x) = \begin{cases} 1 - \frac{p\,x}{b}; & (n-1)\Lambda < x < (n-1)\Lambda + b \\ \varepsilon_1; & (n-1)\Lambda + b\ < x < n\Lambda \end{cases} \quad (1b)$$

with condition that $\varepsilon(x) = \varepsilon(x + \Lambda)$. Here ε_1 is the dielectric constant of dielectric layer, and $\Lambda = a + b$, a and b are width of dielectric and
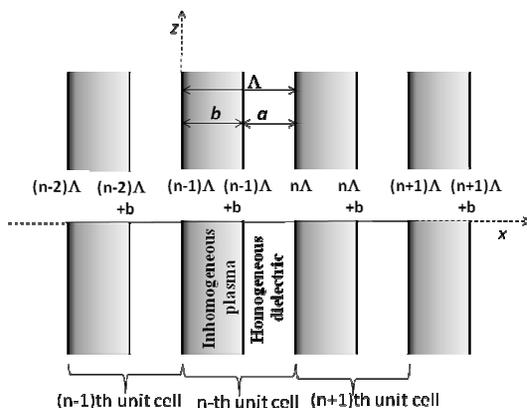

Figure 1. Schematic representation of the unit cell of binary 1D-PPCs having inhomogeneous plasma density layers with homogeneous dielectric layers.

plasma layer respectively. The permittivity in inhomogeneous plasma layer has linear spatial dependence and the variation of permittivity is along x-direction. Maxwell equations can be solved for both cases: TE and TM modes but in the present study only TE mode is considered. The electric field in the case of TE mode is in y-z plane. Along the z-direction there is no change in permittivity, so z-component of wave vector is conserved. The one dimensional wave equation for $E(x, z)$ is

$$\frac{\partial^2 E(x,z)}{\partial x^2} + \frac{\partial^2 E(x,z)}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon(x)E(x,z) = 0 \tag{2}$$

Typical field solution can be expressed as $E(x, z) = E(x)e^{i\beta z}$ and using this in Equation (2), we can write the above equation both regions: inhomogeneous plasma layer and dielectric medium in the nth unit cell as:

$$\frac{d^2 E(x)}{dx^2} + \left(\frac{\omega^2}{c^2}\left(1 - \frac{px}{b}\right) - \beta^2\right)E(x) = 0; \quad (n-1)\Lambda < x < (n-1)\Lambda + b \tag{3}$$

$$\frac{d^2 E(x)}{dx^2} + \left(\frac{\omega^2}{c^2}\varepsilon_1 - \beta^2\right)E(x) = 0; \quad (n-1)\Lambda + b < x < n\Lambda \tag{4}$$

where β is the z-component of wave-vector is $\beta = \frac{\omega}{c}\sqrt{\varepsilon_1}\sin(\theta_1)$ and angle θ_1 can be calculated using Snell's law. If θ is the angle of incidence then $\cos(\theta_1) = \sqrt{1 - \frac{\sin^2(\theta)}{n_1^2}}$ where $n_1 = \sqrt{\varepsilon_1}$.

The solutions of above equations for electric fields in the nth unit cell are given as:

$$E(x) = \begin{cases} c_n Ai(\zeta) + d_n Bi(\zeta); & (n-1)\Lambda < x < (n-1)\Lambda + b \\ a_n e^{-ik_x(x-n\Lambda)} + b_n e^{ik_x(x-n\Lambda)}; & (n-1)\Lambda + b < x < n\Lambda \end{cases} \tag{5}$$

where $k_{1x} = \frac{\omega}{c}\sqrt{\varepsilon_1}\cos(\theta_1)$ and $\zeta = (\frac{\omega^2 p}{c^2 b})^{\frac{1}{3}}(x - n\Lambda + a - \frac{b}{p}(1 - \varepsilon_1\sin^2(\theta_1)))$, A_i and B_i are Airy functions.

Now, using the continuity condition of electric field $E(x)$ and its derivatives at interfaces $x = (n-1)\Lambda$ and $x = (n-1)\Lambda + b$ and using transfer matrix method [27], we obtain following matrix relation:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \tag{6}$$

where A, B, C and D are elements of unit translation matrix [27], which relates complex amplitudes of incident and reflected wave in $(n-1)$th cell to corresponding amplitudes in nth cell. According to Floquet Theorem, wave propagating in a periodic medium is of form $E_K(x, z) = E_K(x)e^{iKx}e^{i\beta z}$, where $E_K(x)$ is periodic with period Λ,

that is, $E_K(x + \Lambda) = E_K(x)$. The constant K is known as the Bloch wave number. The dispersion relation [25] for proposed structure can be written as:

$$K = \left(\frac{1}{\Lambda}\right) \cos^{-1}\left[\frac{1}{2}(A + D)\right] \tag{7}$$

Here the constant K is Bloch wave number.

Considering N unit cells in the proposed structure, then Equation (6) can be written as:

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N \begin{pmatrix} a_N \\ b_N \end{pmatrix} \tag{8}$$

Now, the coefficient of reflection of the structure can be derived by using the relation:

$$r_N = \left(b_0/a_0\right)_{b_N = 0} \tag{9}$$

Here a_0 and b_0 represents the complex amplitudes of incident and reflected waves and the condition $b_N = 0$ implies the boundary condition that to the right of the periodic structure, there is no wave incident on the structure. Similar calculations can also be done for exponential varying plasma density profile.

After simplification, the coefficient of reflection can expressed as:

$$r_N = \left(\frac{C \, U_{N-1}}{AU_{N-1} - U_{N-2}}\right) \tag{10}$$

where $U_N = \sin(N + 1)K\Lambda/\sin K\Lambda$.

The reflectivity of the structure is given by:

$$Rf = |r_N|^2 = \frac{|C|^2}{|C|^2 + (\sin K\Lambda/\sin(N+1)K\Lambda)^2} \tag{11}$$

3. RESULTS AND DISCUSSION

Some sample numerical computations based on Equation (11) are obtained. For the comparison point of view, the density profile of linear inhomogeneous plasma layer and exponential inhomogeneous plasma layer are chosen in such a way that the average volume permittivity remain fixed at 0.5. The effects of incident angle, width of plasma layer, gradation parameter (p), dielectric constant of dielectric materials and number of unit cells (N) on the reflectivity of 1D-PPCs having inhomogeneous plasma layer for different sets of selection parameters have been analyzed. There are six selection parameters incident angles (θ), p, a, d, ε_m and N involved in the numerical calculation. In the

proposed 1D-PPCs, a unit cell is consist of a homogeneous dielectric layer of width a and an inhomogeneous plasma layer of width b with permittivity profile $\varepsilon = 1 - \frac{\omega_p^2(x)}{\omega^2}$. Here $b = d \times a$ and d is a constant related to the width of plasma layer (for $d = 1, a = b$).

Figure 2 show the variation of reflectivity with normalized frequency $(\frac{\omega\, a}{c})$ at various angle of incidence. These graphs are plotted to see the effect of incident angle on the reflectivity of 1D-PPCs for both inhomogeneous plasma density profiles. It is clear from figures that the inhomogeneities in the plasma layer highly affect the reflectivity. The
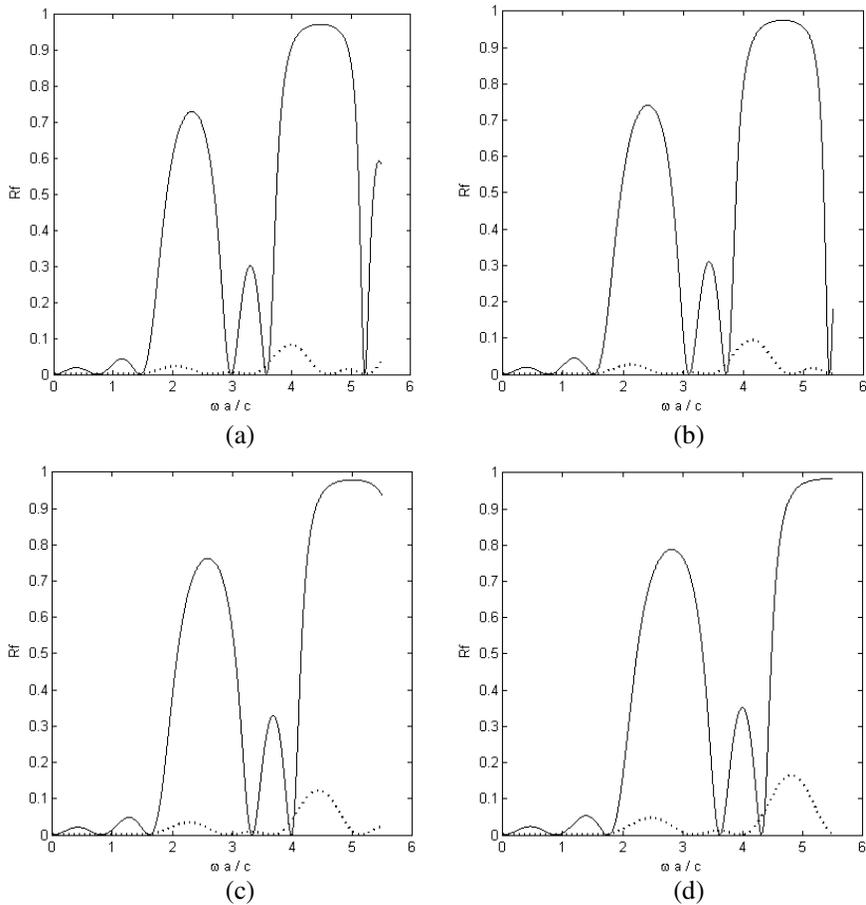

Figure 2. The variation of reflectivity with normalized frequency $(\frac{\omega a}{c})$ for $L = 5000\,\mu\text{m}$, $d = 0.1$, $N = 3$, $\varepsilon_1 = 2.25$ at (a) $\theta = \pi/10$, (b) $\theta = \pi/6$, (c) $\theta = \pi/4$ and (d) $\theta = \pi/3$.

reflectivity in exponential plasma density profile (solid line) always larger than the reflectivity obtained in linear plasma density profile (dotted line). This is due to facts that in the case of exponential density profile the contrast of permittivity changes rapidly than the linear density profile and hence produces large Bragg's reflections. If the angle of incidence changes from $\pi/10$ to $\pi/3$ then these reflectance bands are shifted toward higher frequency with a small increment in the corresponding reflectance amplitudes and reflectance bands for both plasma density profiles. These figures do not show any perfect reflection band due to small number of unit cell ($N = 3$). To see the effect of the number of unit cell on the reflectivity, graphs between the reflectivity and normalized frequency is plotted for $N = 15$ and $N = 45$ as shown in Fig. 3. It is clear that as the number of unit cell increases ($N = 15$) the perfect reflection bands are obtained in exponential density profile (solid line) while liner density profile (dotted line) approaches higher reflectivity. A further increase of number of unit cell ($N = 45$) provide the perfect reflection band at frequency ranges 1.8–2.5, 3.8–5.0 for exponential density profile (solid line) and the perfect reflections at frequencies 2.0, 4.0 for linear density profile (dotted line) under given frequency range.

The effect of the width of inhomogeneous layer on the reflectivity of the plasma photonic crystal is shown in Fig. 4. Fig. 4(a) shows that the perfect reflection bands are obtained only for exponential
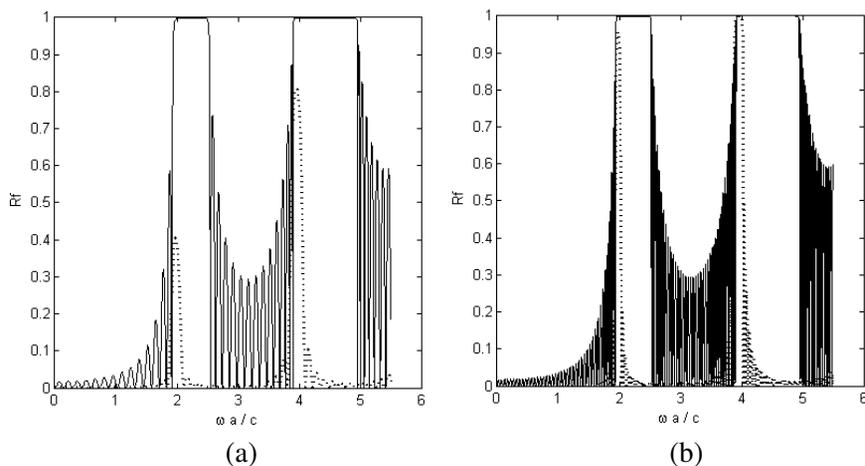

(a) (b)

Figure 3. The variation of reflectivity with normalized frequency $\left(\frac{\omega a}{c}\right)$ for $L = 5000\,\mu\text{m}$, $d = 0.1$, $\theta = \pi/10$, $\varepsilon_1 = 2.25$ at (a) $N = 15$, (b) $N = 45$.
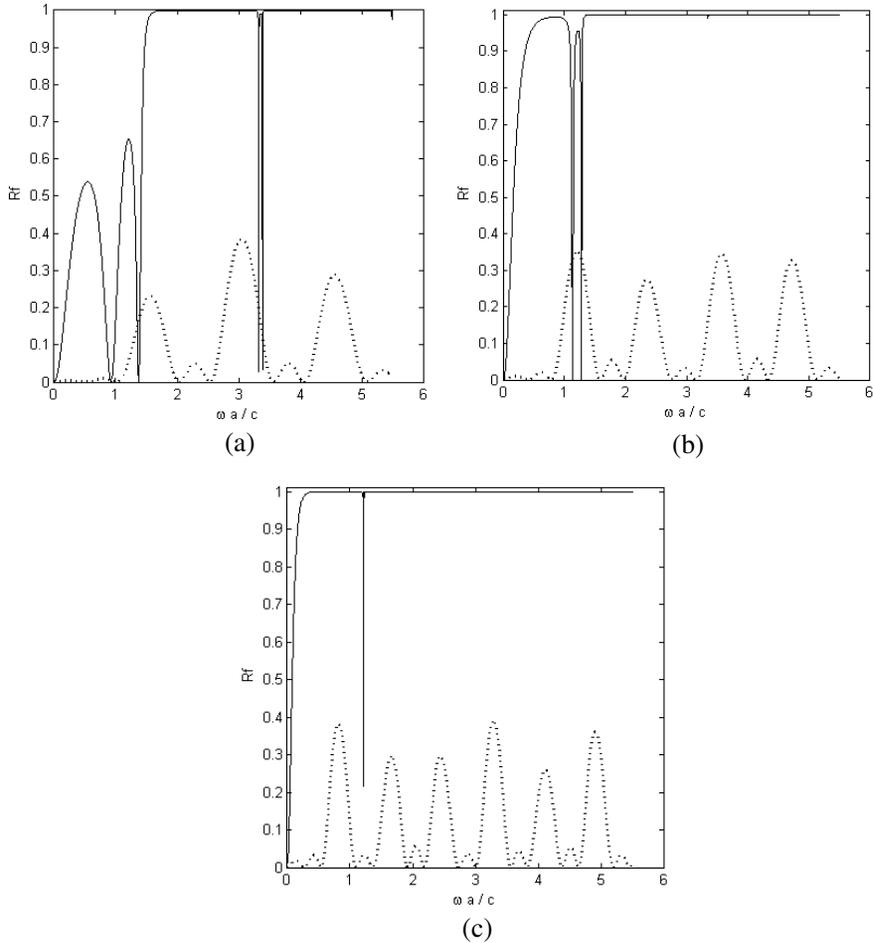
Figure 4. The variation of reflectivity with normalized frequency $(\frac{\omega a}{c})$ for $L = 5000\,\mu m$, $N = 3$, $\theta = \pi/10$, $\varepsilon_1 = 2.25$ at (a) $d = 0.5$, (b) $d = 1.0$ and (c) $d = 2.0$.

density profile (solid line) at frequency ranges 1.7–3.3 and 3.5–5.5. These perfect reflection bands are obtained in frequency ranges 1.5–5.5 with increase of inhomogeneous plasma layer width from $d = 0.5$ to $d = 1.0$, Fig. 4(b). With the increase of plasma layer width from $d = 1.0$ to $d = 2.0$, the exponential density profile (solid line) shows perfect reflectivity in almost all considered frequency range except the frequency region 1.20–1.23, Fig. 4(c). This frequency range may also be used to examine the sensitivity or plasma functional devices. Here
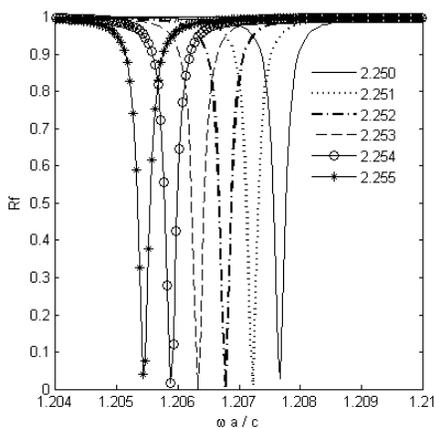
Figure 5. The variation of reflectivity for exponential plasma density profile with normalized frequency $\left(\frac{\omega a}{c}\right)$ for $L = 5000\,\mu\text{m}$, $N = 3$, $\theta = \pi/10$, $d = 2.0$ at $\varepsilon_1 = 2.250 - 2.255$.

we find that a small change (0.001) in the permittivity of dielectric material (glass, $\varepsilon_1 = 2.25$) is able to shift the position of dip of reflection. This shift of the position of reflectance dip is the sensor signal. Here the slope of the curve will give the sensitivity while the shift of dip gives the concentration/amount which cause to change in dielectric permittivity, Fig. 5. On the other hand in the case of linear density profile (dotted line) no such perfect reflectivity is observed Fig. 4(c).

4. CONCLUSION

The reflectivity of a plasma photonic crystal having inhomogeneous variations of plasma density has been studied at the first time in our knowledge. This analysis is more practical because homogeneous plasma having uniform density is rarely realized in the laboratory. The two inhomogeneous variations of plasma density profile, linear and exponential are considered. The reflectivity of exponential varying plasma density profile is much larger, due to its higher permittivity contrast at interface, than the reflectivity obtained in linear varying plasma density profile. Also, only exponential varying plasma density profile is able to give prefect reflection band for considered volume average permittivity and may be used for sensing purpose or other applications.

ACKNOWLEDGMENT

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