DIRECT COUPLING MATRIX SYNTHESIS OF BAND-STOP FILTERS

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Abstract—This work presents a straightforward way to obtain the coupling matrix of a bandstop filtering response from the original bandpass coupling matrix. The generated bandstop coupling matrix implements a response where the bandwidth is defined at the equiripple return loss of the lower and the upper passbands.

1. INTRODUCTION

The operation of some communication technologies such as WiFi, Bluetooth and WiMAX, which operate in adjacent frequency bands, may cause inter-system radio inferferences due to their frequency proximity. Interferences lead to a deterioration in terms of service provision for the users. Bandstop filters have been widely used in RF devices in several applications and technologies to prevent interferences with other applications and users since decades ago to nowadays [1–4]. Bandstop filters can meet the increasingly drastic required specifications in terms of rejection.

The coupling matrix representation of microwave filter circuits is particularly useful because matrix operations can be applied simplifying the synthesis, reconfiguration of the topology and performance simulation of filtering networks [5]. A suitable application of the coupling matrix technique to bandstop filters expands the use of matrix operations beyond passband responses. Starting from a Chebyshev type bandpass response which exhibits equi-ripple return loss inside the passband, a bandstop response can be generated. In [6], the bandstop response is generated in such a way that the bandwidth is defined at a level of equi-ripple rejection inside the stopband. However,

in some cases, it may be useful to define the bandwidth of the bandstop response at a level of equi-ripple return loss of the upper and the lower passbands.

This letter presents an approach to design bandstop filters by means of the coupling matrix. The particularity of the proposed approach is that the resulting coupling matrix uses any original bandpass matrix and produces a frequency inversion on the original response, converting the bandpass response into a bandstop response. Therefore, the bandwidth of the bandstop filter is defined at the return-loss level. The resulting configuration is a bandpass-like topology with bandstop filter characteristics.

2. COUPLING MATRIX SYNTHESIS OF BANDSTOP RESPONSES BY POLYNOMIAL EXCHANGING ON THE BANDPASS RESPONSE

The transfer and the reflection parameters for a two-port network and for a general bandpass Chebyshev filtering function, S_{21} and S_{11}, may be expressed as the ratio of two finite-degree polynomials [5]:

$$S_{21}(s) = \frac{P(s)}{\varepsilon E(s)} \qquad S_{11}(s) = \frac{F(s)}{\varepsilon_R E(s)} \tag{1}$$

where s is the complex frequency variable and it is related to the real frequency variable ω by $s = j\omega$. $E(s)$ and $F(s)$ are polynomials of degree N which coincides with the degree of the filtering functions. $P(s)$ is a polynomial of degree n_{fz} which contains the n_{fz} finite-position prescribed transmission zeros (TZs). The polynomials are normalized to their respective highest degree coefficients and constants ε and ε_R depend on the return loss (RL) and the polynomial coefficients.

In [6], it is proposed a technique to obtain a bandstop response from (1) exchanging the reflection and transfer functions as follows:

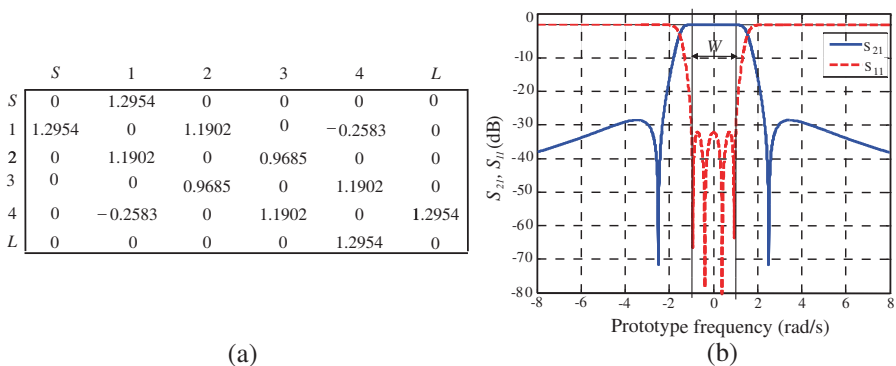$$S_{21}(s) = \frac{F(s)}{\varepsilon_R E(s)} \qquad S_{11}(s) = \frac{P(s)}{\varepsilon E(s)} \tag{2}$$

Note that for a Chebyshev response, the original prescribed equiripple return loss characteristic becomes the transfer response, with a minimum reject level equal to the original prescribed return loss level. Therefore, the bandwidth that is originally defined at the return loss of the passband becomes the bandwidth of the stopband defined at a level of rejection and not at the return loss of the upper and the lower band.

The generation of the coupling matrix for this bandstop response is similar to the generation of a bandpass response matrix as described
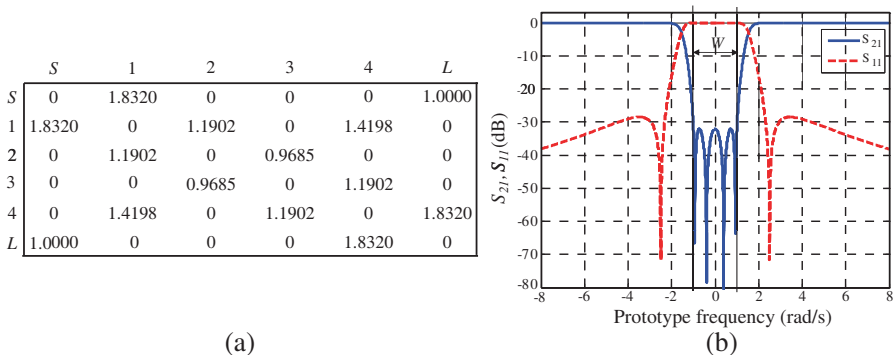
in [7] but it will need to incorporate a direct source-load coupling M_{SL} [6].

Figure 1(a) shows a bandpass coupling matrix for a response with $RL = 32\,\text{dB}$, $N = 4$ and transmission zeros at $\pm 2.5\,\text{rad/s}$. Fig. 1(b) shows the corresponding filtering response.

Figure 2(a) shows the coupling matrix in folded canonical form for a bandstop filter with equiripple rejection inside the stopband of $32\,\text{dB}$, $N = 4$ and reflection zeros at $\pm 2.5\,\text{rad/s}$. Fig. 2(b) shows the

	S	1	2	3	4	L
S	0	1.2954	0	0	0	0
1	1.2954	0	1.1902	0	−0.2583	0
2	0	1.1902	0	0.9685	0	0
3	0	0	0.9685	0	1.1902	0
4	0	−0.2583	0	1.1902	0	1.2954
L	0	0	0	0	1.2954	0

(a)

(b)

Figure 1. (a) Example of coupling matrix in folded canonical form for a bandpass filter generated using the technique proposed in [7] and (b) the corresponding bandpass filtering response. The bandwidth is defined at the RL level.

	S	1	2	3	4	L
S	0	1.8320	0	0	0	1.0000
1	1.8320	0	1.1902	0	1.4198	0
2	0	1.1902	0	0.9685	0	0
3	0	0	0.9685	0	1.1902	0
4	0	1.4198	0	1.1902	0	1.8320
L	1.0000	0	0	0	1.8320	0

(a)

(b)

Figure 2. (a) Example of coupling matrix in folded canonical form for a bandstop filter generated using the technique proposed in [6] and (b) the corresponding bandstop filtering response. The bandwidth is defined at the rejection level.

corresponding filtering response. The frequency range inside the cut-off frequencies of $\pm 1\,\mathrm{rad/s}$ contains the equiripple rejection and not an equiripple return loss in the lower and the upper passbands.

The bandpass and the bandstop responses are complementary due to the polynomial exchanging of the transfer and the reflection parameters.

3. COUPLING MATRIX SYNTHESIS OF BANDSTOP RESPONSES BY FREQUENCY INVERSION ON THE BANDPASS RESPONSE

The purpose of this section is to obtain a coupling matrix that implements a stopband response with equiripple RL in the lower and the upper passbands, below $-1\,\mathrm{rad/s}$ and above $1\,\mathrm{rad/s}$ respectively.

This work proposes a way to obtain this kind of stopband response by means of a frequency inversion of the response given by the transfer and the reflection parameters in (1). A quick and straightforward way to generate the bandstop coupling matrix from an original bandpass one producing the frequency inversion is by means of the inversion of the bandpass coupling matrix as follows:

$$\mathbf{M}_{\mathrm{bandstop}} = \mathbf{M}_{\mathrm{bandpass}}^{-1} \tag{3}$$

where $\mathbf{M}_{\mathrm{bandpass}}$ is the bandpass coupling matrix and $\mathbf{M}_{\mathrm{bandstop}}$ is the corresponding bandstop coupling matrix that produces the frequency inversion.

The transmission $S_{21}(\omega)$ and reflection $S_{11}(\omega)$ parameters depend on the admittance elements $[y]_{N+2,1}(\omega)$ and $[y]_{1,1}(\omega)$ as follows:

$$S_{21}(\omega) = 2\sqrt{R_S R_L}[y]_{N+2,1}(\omega) \tag{4}$$

$$S_{11}(\omega) = 1 - 2R_S[y]_{1,1}(\omega) \tag{5}$$

$[y]_{N+2,1}(\omega)$ and $[y]_{1,1}(\omega)$ are the elements of the admittance matrix $\mathbf{y}(\omega)$ in positions $(N+2, 1)$ and $(1, 1)$. R_S and R_L are the resistive terminations. The admittance matrix $\mathbf{y}(\omega)$ can be calculated from the inversion of the impedance matrix $\mathbf{z}(\omega)$:

$$\mathbf{y}(\omega) = \mathbf{z}^{-1}(\omega) \tag{6}$$

And the impedance matrix is calculated from:

$$\mathbf{z}(\omega) = j(\mathbf{M} + \omega \mathbf{U} + \mathbf{R}) \tag{7}$$

where $\mathbf{M}$ is the coupling matrix, $\mathbf{U}$ a identity matrix with $[U]_{1,1} = 0$ and $[U]_{N+2,N+2} = 0$ and $\mathbf{R}$ is zero except $[R]_{11} = R_S$ and $[R]_{N+2,N+2} = R_L$.
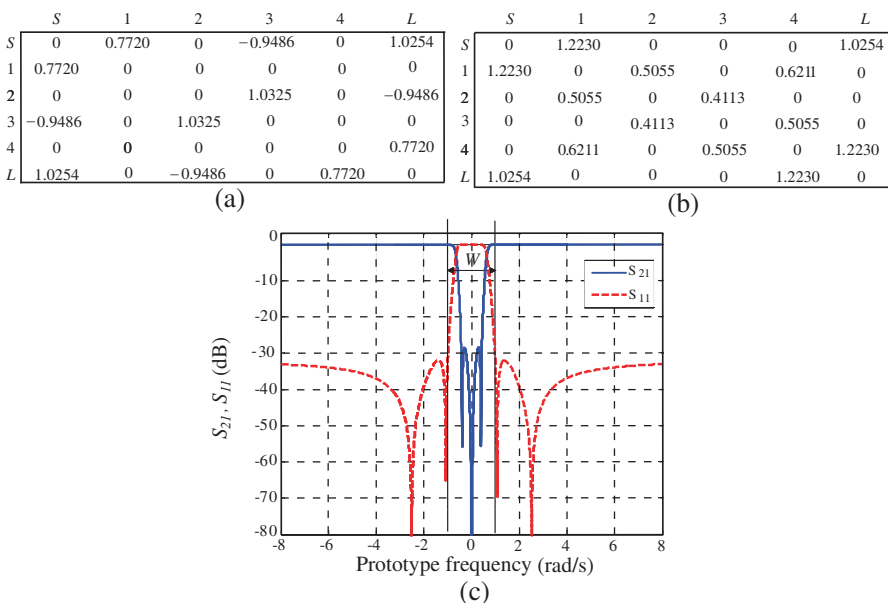
The absolute value of the parameters S_{21} and S_{11} calculated from the matrix $\mathbf{y}\left(1/\omega\right)$, where a frequency inversion has been produced, have the same absolute values that those ones calculated from the matrix $\mathbf{y}'\left(\omega\right)$ that is obtained from the inverted coupling matrix:

$$\mathbf{y}'(\omega) = \left(j\left(\mathbf{M}^{-1} + \omega\mathbf{U} + \mathbf{R}\right)\right)^{-1} \qquad (8)$$

So that, an inversion on the original coupling matrix provides a frequency inversion straightforwardly.

Figure 3(a) shows the result of the inversion of the bandpass coupling matrix of Fig. 1(a). The inverted coupling matrix offers a bandstop response. Fig. 3(a) shows the bandstop coupling matrix reconfigured to the folded canonical form. The filtering response of the bandstop matrixes is shown in Fig. 3(c).

The generated bandstop matrixes offer a frequency inversion of the response regarding the original bandpass matrix. In terms of the reflection coefficient, the reflection zeros in the passband of the bandpass response become reflection zeros split in the lower

S	1	2	3	4	L
S 0	0.7720	0	-0.9486	0	1.0254
1 0.7720	0	0	0	0	0
2 0	0	0	1.0325	0	-0.9486
3 -0.9486	0	1.0325	0	0	0
4 0	0	0	0	0	0.7720
L 1.0254	0	-0.9486	0	0.7720	0

(a)

S	1	2	3	4	L
S 0	1.2230	0	0	0	1.0254
1 1.2230	0	0.5055	0	0.6211	0
2 0	0.5055	0	0.4113	0	0
3 0	0	0.4113	0	0.5055	0
4 0	0.6211	0	0.5055	0	1.2230
L 1.0254	0	0	0	1.2230	0

(b)

(c)

Figure 3. (a) Bandstop coupling matrix obtained by inversion of the bandpass matrix, (b) bandstop coupling matrix after the reconfiguration to the folded canonical form and (c) bandstop filtering response of the coupling matrix generated using the proposed technique. The bandwidth is defined at the return loss.

and the upper passbands of the bandstop response following the frequency inversion of the response at $\omega = \pm 1/0.928$ rad/s and $\omega = \pm 1/0.397$ rad/s. A Chebyshev bandpass filter of order N exhibits N reflection zeros, so that after the frequency inversion, the number of reflection zeros is N as well. The original RL equiripple level keeps equiripple RL in the new side passbands.

In terms of the transmission coefficient, the original transmission zeros will lie inside the stopband of the new bandstop response at the positions $\omega = \pm 1/2.5$ rad/s and $\omega = 0$ rad/s. If a bandpass response exhibits a number of finite transmission zeros n_{fz}, after the frequency inversion, the bandstop response will have n_{fz} transmission zeros, and additional transmission zeros at frequency 0 rad/s that come from the transmission zeros at infinity in the original bandpass response. The position of the transmission zeros and the level of rejection inside the stopband is controlled by means of the frequency position of the transmission zeros in the stopbands of the original bandpass response. In the example, the maximum level of the rejection lobes is 28.4 dB which coincides with the level of the side rejection lobes in Fig. 1(b).

Note that the proposed technique to obtain the bandstop coupling matrix defines the bandwidth at the return loss and not at the rejection level. The return losses at the passbands are perfectly defined while the level of rejection and the bandstop bandwidth must be adjusted by means of the transmission zeros. Meanwhile, in the technique explained in the previous section, the equi-ripple level of rejection is defined inside a given bandwidth but the return losses are controlled adjusting the reflection zeros. Therefore, the proposed technique is specially useful to control the return loss in the passbands that sandwich a rejection frequency range.

4. CONCLUSION

The coupling matrix associated to a bandstop filtering response can be obtained straightforwardly from an original bandpass coupling matrix. An inversion of the bandpass coupling matrix leads to a bandstop coupling matrix that implements a filtering response where a frequency inversion is produced regarding the original bandpass response. Therefore, an equiripple return loss is obtained along the passbands that sandwich the stopband and the bandwidth is defined at the return loss and not at the level of rejection.

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REFERENCES

1. Young, L. and G. L. Matthaei, "Microwave bandstop filters with narrow stop bands," *PGMTT National Symposium Digest*, Vol. 62, No. 1, 46–51, 1962.

2. He, Z. N., X. L. Wang, S. H. Han, T. Lin, and Z. Liu, "The synthesis and design for new classic dual-band waveguide band-stop filters," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 1, 119–130, 2008.

3. Xiao, J.-K. and X.-P. Zu, "Multi-mode bandstop filter using defected equilateral triangular patch resonator," *Journal of Electromagnetic Waves and Applications*, Vol. 25, No. 4, 507–518, 2011.

4. Chen, H., Y. H. Wu, Y. M. Yang, and Y. X. Zhang, "A novel and compact bandstop filter with folded microstrip/CPW hybrid structure," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 1, 103–112, 2010.

5. Cameron, R. J., "General coupling matrix synthesis methods for chebyshev filtering functions," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 47, No. 4, 433–442, Apr. 1999.

6. Cameron, R. J., Y. Ming, and W. Ying, "Direct-coupled microwave filters with single and dual stopbands," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 53, No. 11, 3288–3297, Nov. 2005.

7. Cameron, R. J., "Advanced coupling matrix synthesis techniques for microwave filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 51, No. 1, 1–10, Jan. 2003.