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DESIGN OF UNEQUAL WILKINSON POWER DIVIDER FOR TRI-BAND OPERATION

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Abstract—This paper presents a novel tri-band unequal Wilkinson power divider. The proposed structure is derived from the conventional unequal Wilkinson power divider by replacing the quarter-wavelength branch lines and quarter-wavelength transformers with the extended T-shaped short stubs and three-section transformers respectively. The first and third operating frequencies of the proposed Wilkinson power divider can be flexibly controlled, while the second frequency is equal to the mean value of the other two frequencies. Both of the closed-form equations and design procedure are given. For verification, a tri-band unequal power divider with the power dividing ratio of 2 : 1 and operating frequencies of 1.3, 3.0 and 4.7 GHz is designed, fabricated and measured. The measurement results are in good agreement with the simulation ones. It is shown that the proposed power divider has simple topology and good performances in terms of insertion loss, port matching and isolation at all three operating frequency bands.

1. INTRODUCTION

The Wilkinson power divider is widely used in various microwave/RF circuits such as antenna arrays, power amplifiers, mixers and phase shifters [1]. Over past several years, the developments of modern wireless communication systems specify the Wilkinson power divider with high performances of flexible (equal or unequal) power division ratio, dual- or multi-band, reduced size, low cost, etc. Many efforts have been focused on obtaining dual-band operation with equal

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power division ratio by using two-section transformers together with lumped LC elements [2–4], transmission-line sections with stubs [5–8], port extensions [9, 10], artificial transmission lines [11], and coupled lines [12–14]. And for dual-band unequal division ratio case, some designs, corresponding to the dual-band equal topologies [3–10], have been presented in [15–24]. Recently, some new structures of tri-band equal Wilkinson power dividers have been reported [25, 26]. In [25], a tri-band Wilkinson power divider using lumped LC resonator was presented, however, it suffers from parasitic effects in high frequencies. In [26], a three-section transmission-line transformer was optimized for designing a tri-band Wilkinson power divider with three arbitrary operating frequency bands. Unfortunately, only approximated design equations were given, and the insertion loss was too large. To overcome these problems, a novel tri-band Wilkinson power divider using the extended T-shaped short stubs was proposed for equal power division ratio application in [27]. Additionally, the transmission-line transformer presented in [26] can also be optimized for designing tri-band unequal power divider [28].

In this paper, a new tri-band Wilkinson power divider is proposed for unequal power division ratio application, which is based on both the extended T-shaped short stub and three-section transformer. The proposed structure is more generalized and complicated than the equal one in [27]. Design equations are derived from the conventional Wilkinson power divider. As an example, a prototype of the proposed Wilkinson power divider operating at 1.3, 3.0 and 4.7 GHz is fabricated and measured. At the three design frequencies, the measured insertion loss between port 1 and 2 (3) is 4.93 (2.02), 4.99 (2.23) and 5.27 (2.61) dB respectively. The corresponding measured 20-dB isolation bandwidths are 230, 230 and 160 MHz. Over these three operating frequency bands, the measured return losses are better than 15 dB at all three ports.

2. STRUCTURE AND THEORY

2.1. Conventional Unequal Wilkinson Power Divider

Figure 1(a) shows a conventional unequal Wilkinson power divider. Z_{c1} and Z_{c2} denote the two quarter-wavelength branch lines. $Z_{2'}$ ($Z_{3'}$) represents the input impedance looking into port 2 (3) at node $2'$ ($3'$). Z_3 (Z_4) is the quarter-wavelength transformer for impedance matching between port 2 (3) and node $2'$ ($3'$). According to [1], the characteristic impedance of each transmission line can be calculated as follows,

$$Z_{c1} = Z_0\sqrt{k(1+k^2)} \tag{1}$$

$$Z_{c2} = Z_0\sqrt{\frac{1+k^2}{k^3}} \tag{2}$$

$$Z_{2'} = Z_0 k, \ \ Z_3 = Z_0\sqrt{k} \tag{3}$$

$$Z_{3'} = \frac{Z_0}{k}, \ \ Z_4 = \frac{Z_0}{\sqrt{k}} \tag{4}$$

$$R = Z_0\frac{k^2+1}{k} \tag{5}$$

where k^2 denotes the power dividing ratio of port 3 to 2, and Z_0 is the port impedance.

2.2. Tri-band Unequal Wilkinson Power Divider

Figure 1(b) demonstrates the proposed unequal Wilkinson power divider for tri-band operation. As shown in the dashed rectangular frames, the extended T-shaped short stubs are introduced to replace the quarter-wavelength branch lines Z_{c1} and Z_{c2} in Figure 1(a). Meanwhile, two three-section transformers (Z_{31}, Z_{32} and Z_{33}) and (Z_{41}, Z_{42} and Z_{43}) are utilized to realize tri-band port impedance matching. Therefore, the entire design procedure can be divided into two steps. The first step is to calculate the circuit parameters of the extended T-shaped short stubs, and the next is to design the three-section transformers.

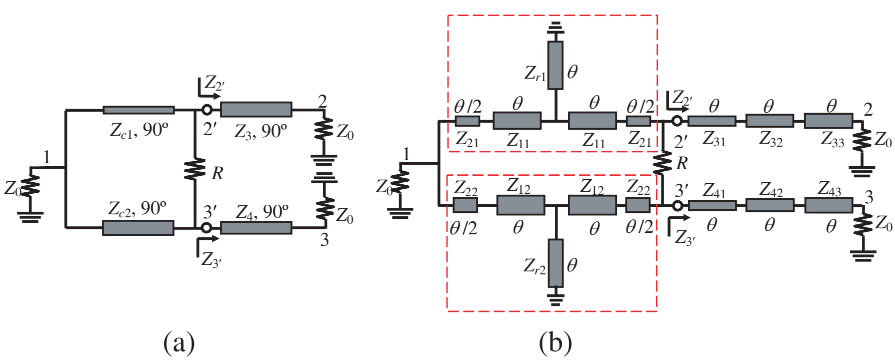


Figure 1. Schematic diagrams of (a) a conventional unequal Wilkinson power divider and (b) the proposed tri-band unequal Wilkinson power divider.

2.2.1. The Extended T-shaped Shot Stub Design

In [27], the extended T-shaped short stub has been derived to be equal to a quarter-wavelength transmission line by the authors, as demonstrated in Figure 2. The characteristic impedances Z_{1i} and Z_{ri} can be expressed as follows,

$$Z_{1i} = \frac{Z_{ci}Z_{2i} - Z_{2i}^2 \tan\frac{\theta}{2}}{Z_{2i}\tan\theta + Z_{ci}\tan\theta\tan\frac{\theta}{2}} \tag{6}$$

$$Z_{ri} = -\frac{Z_{1i}}{2\tan\theta} \cdot \frac{Z_{2i}\left(Z_{2i}\tan\frac{\theta}{2} + Z_{ci}\right) - Z_{1i}\left(Z_{ci}\tan\frac{\theta}{2} - Z_{2i}\right)\tan\theta}{Z_{2i}\left(Z_{1i} - Z_{ci}\tan\theta\right) - \left(Z_{2i}^2\tan\theta + Z_{ci}Z_{1i}\right)\tan\frac{\theta}{2}} \tag{7}$$

where i ($i = 1, 2$) denotes the two branch paths of the conventional or proposed Wilkinson power divider. For design simplicity, the characteristic impedance of the extended transmission line Z_{2i} is chosen to be equal to the required characteristic impedance of the quarter-wavelength transmission line Z_{ci}. In this way, (6) and (7) can be simplified as,

$$Z_{1i} = Z_{ci}\frac{1 - \sin\theta}{\sin\theta} \tag{8}$$

$$Z_{ri} = Z_{1i}\frac{1 - \sin\theta}{2\sin\theta - 1}. \tag{9}$$

From (8) and (9), two possible solutions of the electrical length θ (θ_L and $\theta_U = \pi - \theta_L$) between 0 and π can be found with the fixed values of Z_{1i} and Z_{ri}. The operating frequencies are defined as f_L and f_U, corresponding to the two electrical lengths θ_L and θ_U respectively. In particular, the electrical lengths θ_L and θ_U are symmetrical about $\theta = \pi/2$. In addition, in a special case of $\theta = \pi/2$ and $Z_{2i} = Z_{ci}$, this structure can also be used to replace the quarter-wavelength branch line (Z_{ci}, $\pi/2$) in the conventional Wilkinson power divider, as described in [27]. The operating frequencies corresponding to $\theta = \pi/2$ are defined as f_M. So far, all of the three operating frequencies (f_L, f_M

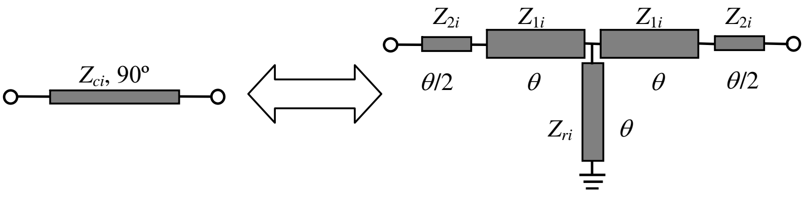


Figure 2. Equivalence between quarter-wavelength branch line and extended T-shaped stub.

and f_U) of the proposed Wilkinson power divider have been obtained. The general relationship between the electrical lengths and operating frequencies can be concluded as follows,

$$\theta_L = \pi/(1 + f_U/f_L), \theta_U = \pi/(1 + f_L/f_U) \quad (10)$$

$$\theta = \pi/2, f_M = (f_L + f_U)/2. \quad (11)$$

2.2.2. The Three-section Transformer Design

For unequal power division ratio, two three-section transformers (Z_{31}, Z_{32} and Z_{33}) and (Z_{41}, Z_{42} and Z_{43}) are utilized to obtain tri-band port matching. Based on the equations in [29], all the characteristic impedances of the two three-section transformers can be derived. Here, we take the three-section transformer (Z_{31}, Z_{32} and Z_{33}) for example, as displayed in Figure 3. Assuming that the electrical length of these three transmission lines at the first operating frequency f_L is defined as θ_L, Z_{33} and θ_L can be obtained by solving the following two equations for perfect matching at the three design frequencies (f_L, f_M and f_U),

$$u_1 = \frac{\arctan\left(\frac{(1-z_{2'})\cot u_1\theta_L - b\tan u_1\theta_L}{2a}\right) + q\pi}{\arctan\left(\frac{(1-z_{2'})\cot\theta_L - b\tan\theta_L}{2a}\right)} \quad (12)$$

$$u_2 = \frac{\arctan\left(\frac{(1-z_{2'})\cot u_2\theta_L - b\tan u_2\theta_L}{2a}\right) + r\pi}{\arctan\left(\frac{(1-z_{2'})\cot\theta_L - b\tan\theta_L}{2a}\right)} \quad (13)$$

where $u_1 = f_M/f_L$, $u_2 = f_U/f_L$, $z_{2'} = Z_{2'}/Z_0$, $a = \sqrt{z_{2'}}(\frac{Z_0}{Z_{33}} - \frac{Z_{33}}{Z_0})$, $b = \frac{Z_0^2}{Z_{33}^2}z_{2'} - \frac{Z_{33}^2}{Z_0^2}$; q and r are chosen as 0 or 1 to make sure u_1 and u_2 greater than one.

After we have calculated Z_{33} and θ_L, the other two characteristic impedances Z_{32} and Z_{31} can be simply obtained from

$$Z_{32} = \sqrt{z_{2'}}Z_0 \quad (14)$$

$$Z_{31} = Z_{32}^2/Z_{33}. \quad (15)$$

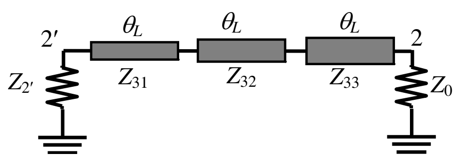


Figure 3. Three-section transmission-line transformer (Z_{31}, Z_{32} and Z_{33}).

In the same way, we can also get the characteristic impedances Z_{41}, Z_{42} and Z_{43}. All physical parameters of the transmission lines can be therefore synthesized from the obtained characteristic impedances and electrical lengths.

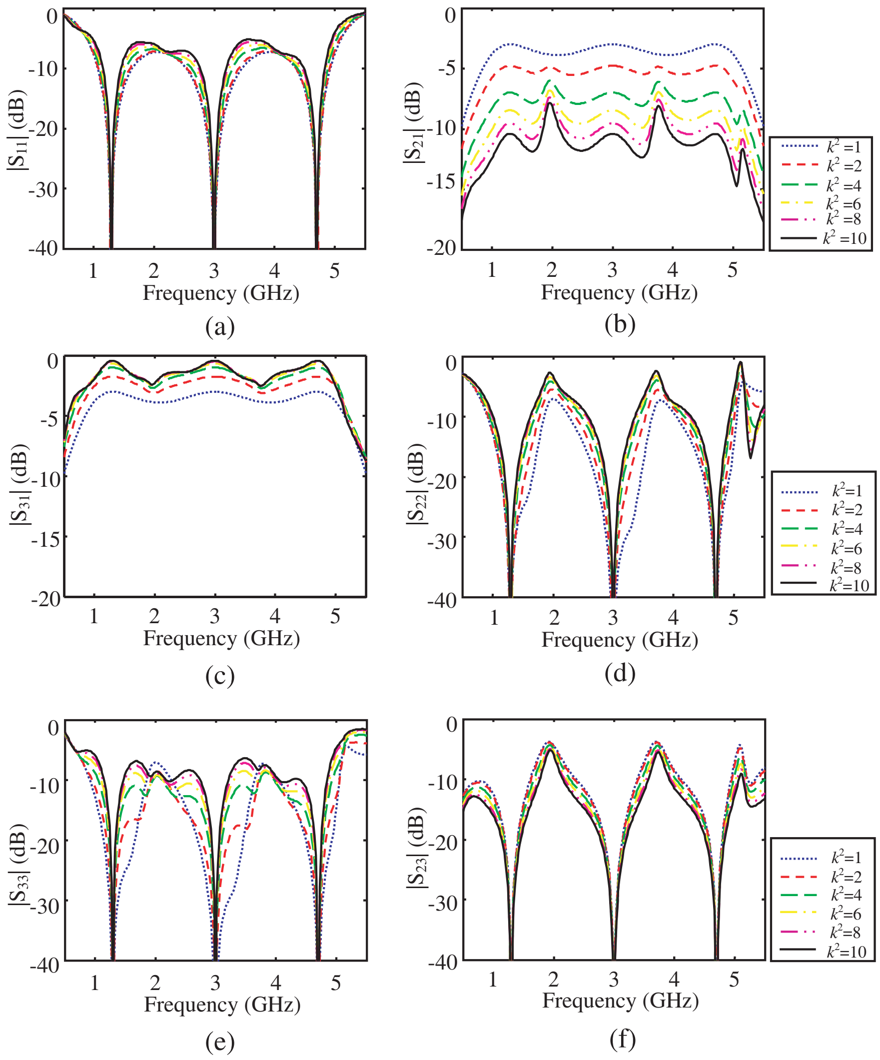


Figure 4. S parameters of the proposed Wilkinson power divider for different k^2.

3. DESIGN

Based on the above analyses, the design procedure can then be described as follows,

(a) According to the given power division ratio k^2, calculate the impedances of the conventional Wilkinson power divider by using (1)–(5).

(b) Choose three operating frequencies f_L, f_M and f_U, and then calculate the corresponding electrical lengths θ_L and θ_U using (10).

(c) The impedances of extended lines Z_{21} and Z_{22} are chosen to be equal to Z_{c1} and Z_{c2} respectively, and then calculate the impedances Z_{11} (Z_{12}) and Z_{r1} (Z_{r2}) with (8) ((9)).

(d) Derive the design parameters of the three-section transformer (Z_{31}, Z_{32} and Z_{33}) by using (12)–(15). Similarly, calculate the other three-section transformer (Z_{41}, Z_{42} and Z_{43}).

According to the design procedure, several design examples of the proposed tri-band Wilkinson power divider with different k^2 (f_U/f_L is chosen to be 3.62) are carried out and the design parameters are listed in Table 1. The corresponding frequency responses are displayed in

Table 1. Design parameters of the conventional and proposed Wilkinson power divider for different k^2 ($Z_0 = 50\,\Omega$).

	$Z(\Omega)$ \ k^2	$k^2 = 1$	$k^2 = 2$	$k^2 = 4$	$k^2 = 6$	$k^2 = 8$	$k^2 = 10$
Conventional power divider	Z_{c1}	70.71	102.99	158.11	207.04	252.27	294.89
	Z_{c2}		51.49	39.53	34.51	31.53	29.49
	Z_3	—	59.46	70.71	78.25	84.09	88.91
	Z_4		42.04	35.36	31.95	29.73	28.12
	R	100	106	125	143	159	174
Proposed power divider	Z_{11}	41.65	60.66	93.13	121.95	148.59	173.69
	Z_{21}	70.71	102.99	158.11	207.04	252.27	294.89
	Z_{r1}	59.69	86.94	133.47	174.78	212.96	248.94
	Z_{31}	—	63.36	80.22	91.97	101.28	109.09
	Z_{32}		59.46	70.71	78.25	84.09	88.91
	Z_{33}		55.79	62.33	66.58	69.82	72.47
	Z_{12}	41.65	30.33	23.28	20.33	18.57	17.37
	Z_{22}	70.71	51.49	39.53	34.51	31.53	29.49
	Z_{r2}	59.69	43.47	33.37	29.13	26.62	24.89
	Z_{41}	—	39.45	31.16	27.18	24.68	22.92
	Z_{42}		42.04	35.36	31.95	29.73	28.12
	Z_{43}		44.81	40.11	37.55	35.81	34.50

Figure 4. It is observed that good performances in terms of insertion loss, port matching and isolation at all three operating frequency bands can be achieved. Additionally, the operating bandwidth reduces with the growing k^2. In this design, the power division ratio k^2 is chosen to be 2. Figure 5 illustrates the design curves of characteristic impedances varying with frequency ratio f_U/f_L, which shows that the tendencies of Z_{11} (Z_{12}) and Z_{r1} (Z_{r2}). As plotted, the characteristic impedance Z_{11} (Z_{12}) exhibits a gradual increase with the frequency ratio, while the stub impedance Z_{r1} (Z_{r2}) grows rapidly. Assuming that the fabrication range of line impedance is $15\,\Omega < Z < 120\,\Omega$, the maximum limitation of power dividing ratio can be found as $3 < k^2 < 3.8$.

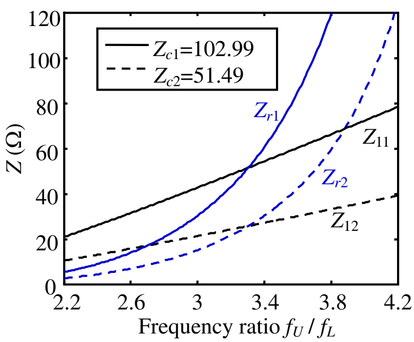
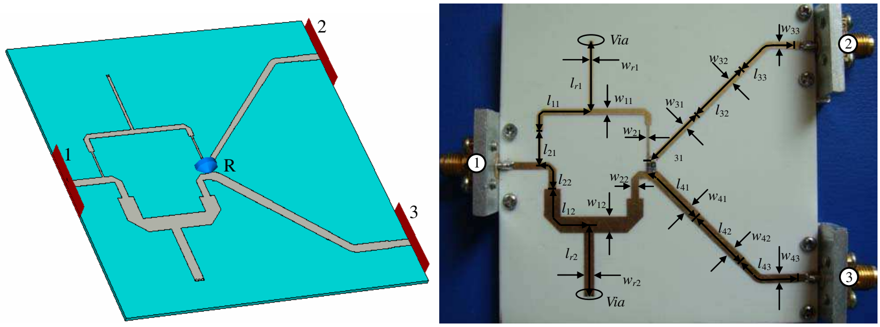


Figure 5. Characteristic impedances versus frequency ratio for tri-band unequal Wilkinson power divider design ($k^2 = 2$).



(a) (b)

Figure 6. (a) Topology of the proposed tri-band unequal Wilkinson power divider. (b) Photograph of the fabricated structure.

4. RESULTS AND DISCUSSION

A tri-band unequal ($k^2 = 2$) Wilkinson power divider is implemented in microstrip circuit. The three operating frequencies are chosen to be 1.3, 3.0 and 4.7 GHz. The detailed circuit layout on a substrate

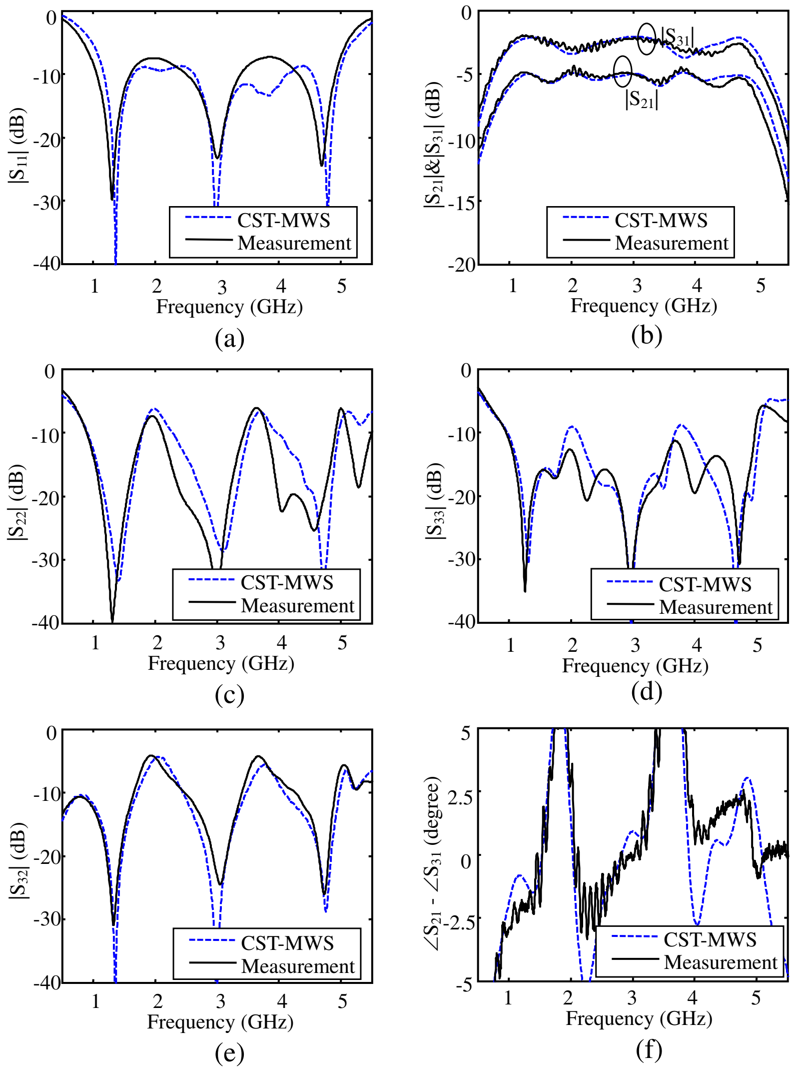


Figure 7. (a)–(e) S-parameters of the tri-band unequal Wilkinson power divider; (f) phase difference between output port 2 and 3.

Rogers4003C ($\varepsilon_r = 3.55$, $h = 0.813$ mm) is shown in Figure 6, and all the design parameters are listed in Table 2. The size of the whole circuit is 60 mm × 62 mm (not including 50 Ω feed lines). Figure 7 and Table 3 display the comparison between measurements and simulations using CST-MWS. As shown, the measurements agree well with the simulations except slight frequency shift and little extra insertion loss, which can be attributed to fabrication and measurement tolerance.

Table 2. Design parameters of the tri-band unequal Wilkinson power divider ($k^2 = 2$, $Z_0 = 50\,\Omega$).

Design frequency (GHz)	Structure	Impedance (Ω)	Electrical length (@f_L)	Physical dimension (mm)
$f_L = 1.3$ $f_M = 3.0$ $f_U = 4.7$	Extended T-shaped short stub	$Z_{11} = 60.65$	38.96°	$w_{11} = 1.29$, $l_{11} = 15.71$
		$Z_{r1} = 86.88$		$w_{r1} = 0.62$, $l_{r1} = 16.44$
		$Z_{21} = 102.99$	19.48°	$w_{21} = 0.40$, $l_{21} = 8.10$
		$Z_{12} = 30.32$	38.96°	$w_{12} = 3.74$, $l_{12} = 15.46$
		$Z_{r2} = 43.44$		$w_{r2} = 2.24$, $l_{r2} = 16.31$
		$Z_{22} = 51.49$	19.48°	$w_{22} = 1.71$, $l_{22} = 7.86$
	Three-section transformer	$Z_{31} = 63.36$	38.96°	$w_{31} = 1.19$, $l_{31} = 15.24$
		$Z_{32} = 59.46$		$w_{32} = 1.34$, $l_{32} = 15.17$
		$Z_{33} = 55.79$		$w_{33} = 1.5$, $l_{33} = 15.10$
		$Z_{41} = 39.45$		$w_{41} = 2.58$, $l_{41} = 14.75$
		$Z_{42} = 42.04$		$w_{42} = 2.35$, $l_{42} = 14.81$
		$Z_{43} = 44.81$		$w_{43} = 2.13$, $l_{43} = 14.87$
Calculated resistor $R = 106\,\Omega$			Practical resistor $R = (220//200) \pm 5\%\,\Omega$	

Table 3. Measured and simulated results at the designed frequencies.

		$f_M = 1.3\,\text{GHz}$	$f_M = 3.0\,\text{GHz}$	$f_U = 4.7\,\text{GHz}$
$\lvert S_{21}\rvert$ (dB)	*Measured*	−4.93	−4.99	−5.27
	Simulated	−4.99	−4.98	−5.13
$\lvert S_{31}\rvert$ (dB)	*Measured*	−2.02	−2.23	−2.61
	Simulated	−2.17	−2.06	−2.15
$\lvert S_{11}\rvert$ (dB)	*Measured*	−29.72	−23.27	−24.42
	Simulated	−31.87	−34.11	−26.75
$\lvert S_{22}\rvert$ (dB)	*Measured*	−38.99	−36.32	−21.17
	Simulated	−29.32	−28.64	−33.23
$\lvert S_{33}\rvert$ (dB)	*Measured*	−27.78	−31.87	−29.37
	Simulated	−29.53	−32.33	−32.01
$\lvert S_{32}\rvert$ (dB)	*Measured*	−28.78	−23.13	−24.49
	Simulated	−27.27	−38.73	−26.06

At the designed frequencies, the measured output magnitude balances ($|S_{31}| - |S_{21}|$) are 2.91, 2.76 and 2.66 dB. Meanwhile, the measured output phase differences ($\angle S_{21} - \angle S_{31}$) are $-2.14°$, $0.25°$ and $2.11°$. The measured 20-dB isolation bandwidths are 1.21–1.44 GHz, 2.93–3.16 GHz and 4.64–4.80 GHz, respectively. Over these three operating bands, the measured return losses are better than 15 dB at all three ports.

5. CONCLUSION

An unequal Wilkinson power divider has been proposed by using extended T-shaped short stubs and three-section transformers for tri-band operation. The detailed design equations and procedure are derived and given. A sample prototype with the power dividing ratio of 2 : 1 and operating frequencies of 1.3, 3.0 and 4.7 GHz has been designed, fabricated and measured. The results show that the proposed power divider has several advantages: 1) is convenient for design and fabrication owing to its simple structure and be free from complicated optimization procedures; 2) exhibits good performances in terms of insertion loss, port matching and isolation at all three operating frequency bands; 3) reduces parasitic effects especially at high frequencies because of only one resistor used in the whole structure.

REFERENCES

1. Pozar, D. M., *Microwave Engineering*, 3rd edition, Wiley, New York, 2005.
2. Wu, L., Z. Sun, H. Yilmaz, and M. Berroth, "A dual-frequency Wilkinson power divider," *IEEE Trans. Microw. Theory Tech.*, Vol. 54, No. 1, 278–285, Jan. 2006.
3. Kawai, T., Y. Nakashima, Y. Kokubo, and I. Ohta, "Dual-band Wilkinson power dividers using a series RLC circuit," *IEICE Trans. Electron.*, Vol. E91-C, No. 11, 1793–1797, 2008.
4. Wu, Y., Y. Liu, and S. Li, "A new dual-frequency Wilkinson power divider," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 4, 483–492, 2009.
5. Cheng, K.-K. M. and F.-L. Wong, "A new Wilkinson power divider design for dual band applications," *IEEE Microw. Wireless Compon. Lett.*, Vol. 17, No. 9, 664–666, Sep. 2007.
6. Park, M.-J. and B. Lee, "A dual-band Wilkinson power divider," *IEEE Microw. Wireless Compon. Lett.*, Vol. 18, No. 2, 85–87, Feb. 2008.
7. Cheng, K.-K. M. and C. Law, "A novel approach to the design and implementation of dual-band power divider," *IEEE Trans. Microw. Theory Tech.*, Vol. 56, No. 2, 487–492, Feb. 2008.
8. Wu, Y., Y. Liu, and X. Liu, "Dual-frequency power divider with isolation stubs," *Electron. Lett.*, Vol. 44, No. 24, 1407–1408, Nov. 2008.
9. Park, M. J. and B. Lee, "Wilkinson power divider with extended ports for dual-band operation," *Electron. Lett.*, Vol. 44, No. 15, 916–917, Jul. 2008.
10. Li, X., S.-X. Gong, L. Yang, and Y.-J. Yang, "A novel Wilkinson power divider for dual-band operation," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 2–3, 395–404, 2009.
11. Yang, T., C. Liu, L. Yan, and K. Huang, "A compact dual-band power divider using planar artificial transmission lines for GSM/DCS applications," *Progress In Electromagnetics Research Letters*, Vol. 10, 185–191, 2009.
12. Park, M.-J., "Two-section cascaded coupled line Wilkinson power divider for dual-band applications," *IEEE Microw. Wireless Compon. Lett.*, Vol. 19, No. 4, 188–190, Apr. 2009.
13. Park, M.-J., "Dual-band Wilkinson divider with coupled output port extensions," *IEEE Trans. Microw. Theory Tech.*, Vol. 57,

No. 9, 2232–2237, Sep. 2009.

14. Lin, Z. and Q.-X. Chu, "A novel approach to the design of dual-band power divider with variable power divider based on coupled-lines," *Progress In Electromagnetics Research*, Vol. 103, 271–284, 2010.
15. Wu, Y., H. Zhou, Y. Zhang, and Y. Liu, "An unequal Wilkinson power divider for a frequency and its first harmonic," *IEEE Microw. Wireless Compon. Lett.*, Vol. 18, No. 11, 737–739, Nov. 2008.
16. Ahn, S.-H., J. W. Lee, C. S. Cho, and T. K. Lee, "A dual-band unequal Wilkinson power divider with arbitrary frequency ratios," *IEEE Microw. Wireless Compon. Lett.*, Vol. 19, No. 12, 783–785, Dec. 2009.
17. Wu, Y., Y. Liu, and S. Li, "Unequal dual-frequency Wilkinson power divider including series resistor-inductor-capacitor isolation structure," *IET Microw. Antennas Propag.*, Vol. 3, No. 7, 1079–1085, Jul. 2009.
18. Wu, Y., Y. Liu, and S. Li, "An unequal dual-frequency Wilkinson power divider with optional isolation structure," *Progress In Electromagnetics Research*, Vol. 91, 393–411, 2009.
19. Li, X., Y.-J. Yang, L. Yang, S.-X. Gong, T. Hong, X. Chen, and Y.-J. Zhang, "Novel design of unequal Wilkinson power divider for dual-band operation," *Microwave and Optical Technology Letters*, Vol. 52, No. 8, 1736–1739, Aug. 2010.
20. Wu, Y., Y. Liu, Y. Zhang, J. Gao, and H. Zhou, "A dual band unequal Wilkinson power divider without reactive components," *IEEE Trans. Microw. Theory Tech.*, Vol. 57, No. 1, 216–222, Jan. 2009.
21. Li, X., Y.-J. Yang, L. Yang, S.-X. Gong, X. Tao, Y. Gao, K. Ma, and X.-L. Liu, "A novel design of dual-band unequal Wilkinson power divider," *Progress In Electromagnetics Research C*, Vol. 12, 93–100, 2010.
22. Wu, Y., Y. Liu, S. Li, and X. Liu, "A novel dual-frequency Wilkinson power divider with unequal power division," *Electromagnetics*, Vol. 29, No. 8, 627–640, Aug. 2009.
23. Li, X., Y. Yang, L. Yang, S. Gong, T. Hong, X. Chen, and Y. Zhang, "Design of unequal Wilkinson power divider for dual-band operation with isolation stubs," *Electron. Lett.*, Vol. 45, No. 24, 1245–1247, Nov. 2009.
24. Li, B., X. Wu, N. Yang, and W. Wu, "Dual-band equal/unequal Wilkinson power dividers based on coupled-line section with short-

circuited stub," *Progress In Electromagnetics Research*, Vol. 111, 163–178, 2011.

25. Wang, Z., J.-S. Jang, and C.-W. Park. "Tri-band Wilkinson power divider using resonators," *Proceedings of IEEE Radio and Wireless Symposium (RWS)*, 287–290, Phoenix, AZ, Jan. 2011.
26. Chongcheawchamnan, M., S. Patisang, M. Krairiksh, and I. D. Robertson, "Tri-band Wilkinson power divider using a three-section transmission-line transformer," *IEEE Microw. Wireless Compon. Lett.*, Vol. 16, No. 8, 452–454, Aug. 2006.
27. Li, B., X. Wu, C. Hua, N. Yang, D. Zhu, and W. Wu, "A novel tri-band Wilkinson power divider using extended T-shaped stubs," *China-Japan Joint Microwave Conference*, 322–324, Hangzhou, China, 2011.
28. Qaroot, A. M., N. I. Dib, and A. A. Gheethan, "Design methodology of multi-frequency unequal split Wilkinson power dividers using transmission line transformers," *Progress In Electromagnetics Research B*, Vol. 22, 1–21, 2010.
29. Chongcheawchamnan, M., S. Patisang, S. Srisathit, et al., "Analysis and design of a three-section transmission-line transformer," *IEEE Trans. Microw. Theory Tech.*, Vol. 53, No. 7, 2458–2462, Jul. 2005.