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A MIXING VECTOR BASED AN AFFINE COMBINATION OF TWO ADAPTIVE FILTERS FOR SENSOR ARRAY BEAMFORMING

S. T. Lu[1, *], J. P. Sun[1], G. H. Wang[2], and Y. L. Lu[3]

[1]School of Electronic and Information Engineering, Beihang University, Beijing, China

[2]Sensor Array Research Programme, Temasek Laboratories at Nanyang Technological University, Singapore

[3]School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore

Abstract—In this paper, a novel beamformer for adaptive combination of two adaptive filters is proposed for interference mitigation of sensor array. The proposed approach adaptively combines two individual filters by coefficient weights vector instead of one scale parameter and takes the constraint of affine combination into consideration rather than previous studies. Due to the more degrees of freedom offered by the mixing vector, the proposed beamformer significantly improves the convergence and tracking performances of the combined filter under both stationary and non-stationary environments, respectively. Based on the generalized sidelobe canceller (GSC) structure, the optimal mixing vector is derived by Lagrange method, and then several new effective iterative algorithms are developed for its updating in practical implementation. Furthermore, theoretical discussions of the convergent performances and complexities of the proposed iterative algorithms are also investigated to verify the feasibility of the proposed beamformer. Moreover, the proposed methods in application of beamforming for interference mitigation of antenna array are simulated based space-time processing technique. When compared to existing methods, the proposed approach exhibits faster convergence rate and higher output signal to interference plus noise ratio (SINR). Its good behavior is illustrated through simulation results.

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* Corresponding author: Songtao Lu (lusongtao@ee.buaa.edu.cn).

1. INTRODUCTION

Adaptive beamforming techniques have been successfully developed to improve the reception of a desired signal while suppressing interferences at the output of a sensor array [1–4]. Adaptive or smart antennas have become a key component for array signal processing applications in radar, astronomy, sonar, wireless communication, and navigation systems [5–11]. Many of these techniques have employed adaptive algorithms to track the desired signals and suppress the interference signals [12–19].

The reduced-rank space-time adaptive processing (STAP) filters constitute a powerful and effective technique [20]. They exhibit faster convergence rate, better tracking capability, and lower computational complexity than full rank techniques. Several reduced-rank methods have been proposed in the last decade, such as auxiliary vector filter (AVF), conjugate gradient reduced-rank filter (CGRRF) [21], multistage nested Wiener filter (MNWF) [22] and its modified approaches applied in a wide area of adaptive array beamforming [23–25]. Many important results on how to improve the convergence rate and/or how to reduce the computational complexity of reduced-rank adaptive filters have been obtained in the literature (see, e.g., [26, 27]). However, there is a tradeoff between convergence rate and steady-state signal to interference plus noise ratio (SINR). Therefore, the design of an adaptive filter with a good tradeoff among these factors is a problem of wide interest in array processing.

Recently, combination schemes have been proposed to improve the fundamental tradeoff between convergence rate and steady-state excess mean-square error (EMSE) in adaptive filters. The basic idea is that two (or more) adaptive filters with complementary capabilities combine adaptively their outputs by means of a mixing parameter, in order to obtain an overall filter of improved performance. Among these schemes, convex [28–30], linear [31] and affine [32, 33] combinations have received attention due to their simplicity and universal behavior in steady-state, i.e., the combined estimate is at least as good as the best of the component filters. And the convex combination proposed in [34] for knowledge-aided STAP obtains a significant improvement to estimate the inverse interference covariance matrix by combining the inverse of a priori known covariance matrix and a sample covariance matrix with a scale weighting parameter. Although these schemes achieve a lower EMSE in theory, they cannot improve the performance over that of a single filter of the same type with optimal selection of the step-size. However, combining adaptive filters of different families, it is possible to achieve smaller errors than optimally adjusted individual filters [35].

In this paper, we propose a novel general affine combination structure of two adaptive filters for interference mitigation of antenna array. Instead of using a scalar as mixing parameter, a vector is used to combine each two component filters' weight for minimizing the mean-square error (MSE) of the combined filter, which exploits every entry of the two filters' weights to ameliorate quality of the estimates. In order to make the implementation of the adaptive array efficiently, a general sidelobe canceller (GSC) is adopted by the proposed beamformer. And then the optimal mixing parameter vector is derived based on GSC for beamforming. In addition, least mean-square (LMS) and recursive least square (RLS) based algorithms are proposed respectively for updating the mixing vector iteratively, where the theoretical investigations for these algorithms in terms of MSE and EMSE are also introduced. Furthermore, the output SINRs and complexities are discussed following to demonstrate the practical feasibilities of the proposed iterative algorithms. In particular, we apply the proposed beamformer into a combination of the Lanczos reduced-rank MNWF with the LMS filter to mitigate interference in space-time processing. The Lanczos reduced-rank MNWF presents a fast convergence rate to acquire the desired signals and suppress the interferences quickly. For another, the LMS filter, one of the most popular adaptive array beamforming techniques, is computationally efficient, exhibiting a low distortion when its step-size is appropriately adjusted. Thus, the combined filter should acquire the good properties of both filters, exhibiting better performance under both stationary and non-stationary environments. From the simulations, the results show that the proposed beamformer has a good robustness to mitigate the interferences with very fast convergent speed and high output SINR.

Throughout the paper all vectors are column vectors and represented by boldface lowercase letters. Matrices are represented by boldface capital letters. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation, and $(\cdot)^H$ stands for the Hermitian transpose while $(\cdot)^*$ for complex conjugation, respectively. The symbol $\odot$ indicates the Hadamard or Schur product and the symbol $\otimes$ is Kronecker product, and $|\cdot|$ denotes the absolute operation.

2. PROBLEM FORMULATION

Space-time processing techniques with multiple antennas can significantly increase the degrees of freedom (DOFs) in signal processing and adaptively offer the capability of nulling multiple narrowband and wideband jammers.

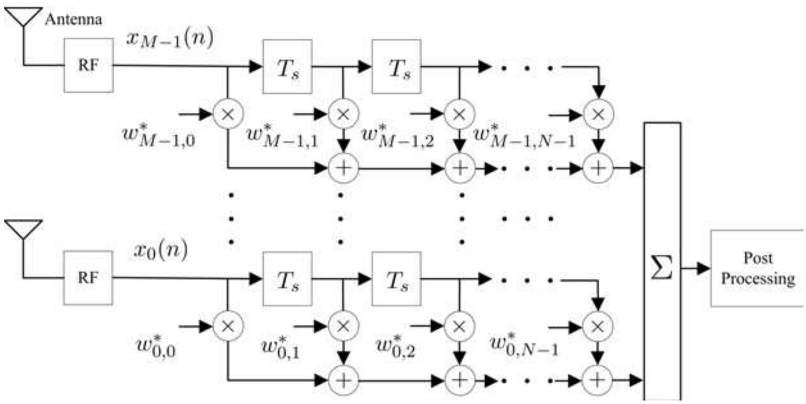


Figure 1. Block diagram of the joint space time adaptive processing beamformer for antenna array.

2.1. Configuration of the Space-time Processing

The STAP processor linearly combines the observational samples from antenna array, each of them are followed by a tapped delay line, forming a finite impulse response (FIR) filter in time domain as well as an adaptive filter in the space domain as shown in Figure 1. This scheme can reject whatever wideband and narrowband interference simultaneously at two dimensions, direction of arrival (DOA) and frequency (or time of arrival, TOA).

Therefore, with the prior known of the DOA of each desired signal $d(n)$, the optimal space time steering vector with dimension $MN \times 1$ can be written as

$$\mathbf{s}(\phi, \omega) = \mathbf{s}_s(\phi) \otimes \mathbf{s}_t(\omega) \tag{1}$$

where the space dimension steering vector $\mathbf{s}_s(\phi)$ and the time dimension steering vector $\mathbf{s}_t(\omega)$ are defined as followings

$$\begin{aligned} \mathbf{s}_s(\phi) &\triangleq \left[1, e^{-j\phi_1}, \ldots, e^{-j\phi_i}, \ldots, e^{-j\phi_{M-1}}\right]^H, \\ \mathbf{s}_t(\omega) &\triangleq \left[1, e^{-j\omega T_s}, \ldots, e^{-j\omega(N-1)T_s}\right]^H \end{aligned} \tag{2}$$

where ϕ_i represents the incidence angle of the desired signal at a certain type of arrays, M means the number of antenna array elements, ω denotes the response frequency, and N is the length of the delayed line with sampling interval T_s. The antenna locations and sampling frequency $f_s = 1/T_s$ are chosen to meet the Nyquist spatial sampling criterion to avoid aliasing effects in both spatial and frequency domains. In particular, spatial adaptive processing (SAP) is an example of STAP, i.e., $N = 1$ and $T_s = 0$.

2.2. Linear Multiple Constraint Generalized Sidelobe Canceller (LC-GSC)

The general purpose of problem in adaptive beamforming is to receive the desired signals $d(n)$ coming from specific direction or directions while minimizing the reception of unwanted signals emanating from other directions. The linearly constrained minimum variance (LCMV) beamformer minimizes the output power subject to linear constraints on the weight vector $\widetilde{\mathbf{w}}_{\mathrm{LCMV}}$. In general, the LCMV beamforming problem is formulated as

$$\min_{\widetilde{\mathbf{w}}_{\mathrm{LCMV}}} \widetilde{\mathbf{w}}_{\mathrm{LCMV}}^H \mathbf{R}_{\mathbf{x}} \widetilde{\mathbf{w}}_{\mathrm{LCMV}} \text{ subject to } \mathbf{C}^H \widetilde{\mathbf{w}}_{\mathrm{LCMV}} = \mathbf{f} \tag{3}$$

where $\mathbf{R}_{\mathbf{x}} \triangleq \mathbb{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\}$ is auto-correlation of input vector $\mathbf{x}(n)$, and $\mathbf{C} = [\mathbf{s}_1, \ldots, \mathbf{s}_j, \ldots, \mathbf{s}_{M_o}]$, $j = 1, \ldots, M_o$, is a $MN \times M_o$ constraints matrix, including the desired multiple directions or other constraints, whose columns are linear independent, so that its rank is M_o, the number of constraints. The vector $\mathbf{s}_j$ is given by (1) and $\mathbf{f}$ represents the constraint vector corresponding to $\mathbf{C}$, which can be expressed by many forms [12] with M_o entries. If $\mathbf{R}_{\mathbf{x}}$ is nonsingular and $\mathbf{C}$ has full-column rank, the solution to (3) is $\widetilde{\mathbf{w}}_{\mathrm{LCMV}} = \mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C})^{-1}\mathbf{f}$. Apparently, the optimal $\widetilde{\mathbf{w}}_{\mathrm{LCMV}}$ needs to find the inverse of the matrix, e.g., $\mathbf{R}_{\mathbf{x}}^{-1}$ and $(\mathbf{C}^H\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{C})^{-1}$, which inevitably consumes a lot of computations.

However, the GSC structure for beamforming makes the implementation much more efficient with adaptive algorithms, which assumes the DOAs of desired signals are prior known to receiver array according to signal estimation techniques [36] or other ways, such as inertial navigation system (INS) or ephemeris information of navigation receiver, etc.. And the LC-GSC weighting vector is expressed by

$$\widetilde{\mathbf{w}}_{\text{LC-GSC}}(n) = \mathbf{w}_q - \mathbf{B}^H\mathbf{w}(n) \tag{4}$$

where the quiescent weighting vector $\mathbf{w}_q$ is set as the matched filter, i.e., $\mathbf{w}_q = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f}$, which ensures that the desired signal passes through the wideband beamformer without distortion, and blocking desired signal matrix $\mathbf{B}$ satisfies $\mathbf{B}\mathbf{B}^H = \mathbf{I}$ and $\mathbf{B}\mathbf{C} = \mathbf{0}$, where $\mathbf{I}$ is an appropriate sized identity matrix. In this way, the linear constrained optimization problem converts to the unconstrained one such that the weighting vector $\mathbf{w}(n)$ can adjust $\widetilde{\mathbf{w}}_{\text{LC-GSC}}$ through unconstrained adaptive algorithms [40].

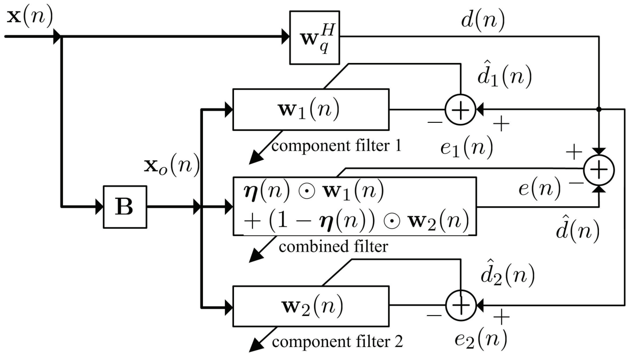


Figure 2. Proposed generalized sidelobe canceller for beamforming of sensor array.

3. PROPOSED COMBINATION APPROACH FOR BEAMFORMING USING A VECTOR $\boldsymbol{\eta}$

In this paper, a novel adaptive array beamformer using an affine combination of two filters based on GSC structure is proposed as shown in Figure 2, which directly combines the each component adaptive weights' elements by vector $\boldsymbol{\eta}(n) = [\eta_1(n), \ldots, \eta_k(n), \ldots, \eta_{MN}(n)]^H$, $k = 1, \ldots, MN$. Hence, the unconstrained adaptive weight vector is assumed by

$$\mathbf{w}(n) = \boldsymbol{\eta}(n) \odot \mathbf{w}_1(n) + (1 - \boldsymbol{\eta}(n)) \odot \mathbf{w}_2(n) \tag{5}$$

where the symbol 1 denotes a vector of all entries one, and $\mathbf{w}_i(n)$, $i \in 1, 2$ denote the unconstrained weight vectors for component filters. And the output of combined filter at time instant n is given by $e(n) \triangleq \widetilde{\mathbf{w}}^H_{\text{LC-GSC}}(n)\mathbf{x}(n)$. For simplicity of expression, we define the vector $\mathbf{w}_{12}(n)$ as the difference of two individual filters' weights, i.e., $\mathbf{w}_{12}(n) \triangleq \mathbf{w}_1(n) - \mathbf{w}_2(n) = [w_{12_1}(n), \ldots, w_{12_k}(n), \ldots, w_{12_{MN}}(n)]^H$.

Besides, each sub-filter's adaptive weights based on GSC structure for beamforming are written by $\widetilde{\mathbf{w}}_i(n) \triangleq \mathbf{w}_q - \mathbf{B}^H\mathbf{w}_i(n)$. And their outputs are defined by $e_i(n) \triangleq (\mathbf{w}_q - \mathbf{B}^H\mathbf{w}_i(n))^H\mathbf{x}(n) = \mathbf{w}_q^H\mathbf{x}(n) - \mathbf{w}_i^H(n)\mathbf{x}_o(n) = d(n) - \hat{d}_i(n)$, where $d(n) = \mathbf{w}_q^H\mathbf{x}(n)$, $\mathbf{x}_o(n) = \mathbf{B}\mathbf{x}(n)$ and $\hat{d}_i(n) = \mathbf{w}_i^H(n)\mathbf{x}_o(n)$ have been illustrated as in Figure 2.

4. OPTIMAL ADAPTIVE COMBINATION WEIGHTS FOR BEAMFORMING

Considering the interference power is higher than the desired signal, we need to restrict the desired DOAs, so the minimum variance (MV) criterion is adopted based on GSC for beamforming. Once the $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$ from the two component filters are acquired, then the goal of optimization turns to the coefficient weighing vector $\boldsymbol{\eta}(n)$ at nth snapshot. As the discussions above, the optimization problem of (3) under the proposed approach reduces to

$$\min_{\boldsymbol{\eta}} \widetilde{\mathbf{w}}_{\text{LC-GSC}}^H \mathbf{R}_{\mathbf{x}} \widetilde{\mathbf{w}}_{\text{LC-GSC}} \text{ subject to } \mathbf{C}^H \widetilde{\mathbf{w}}_{\text{LC-GSC}} = \mathbf{f}. \tag{6}$$

Using the method of Lagrange multiplier, the optimization equations can be written with the multiplier vector $\boldsymbol{\lambda}$. Replacing (3)–(5) in (6), and noting that $\widetilde{\mathbf{w}}_2 = \mathbf{w}_q - \mathbf{B}^H \mathbf{w}_2$, then we have the Lagrangian,

$$\begin{aligned} \mathcal{L}(\boldsymbol{\eta}, \boldsymbol{\lambda}) \;=\; & \frac{1}{2} \left(\widetilde{\mathbf{w}}_2 - \mathbf{B}^H \boldsymbol{\eta} \odot \mathbf{w}_{12}\right)^H \mathbf{R}_{\mathbf{x}} \left(\widetilde{\mathbf{w}}_2 - \mathbf{B}^H \boldsymbol{\eta} \odot \mathbf{w}_{12}\right) \\ & + \Re\left\{\boldsymbol{\lambda}^H \left[\mathbf{f} - \mathbf{C}^H \left(\widetilde{\mathbf{w}}_2 - \mathbf{B}^H \boldsymbol{\eta} \odot \mathbf{w}_{12}\right)\right]\right\} \end{aligned} \tag{7}$$

where $\Re\{\cdot\}$ denotes the real part of a complex value. The optimal solution can be obtained when

$$\nabla_{\boldsymbol{\eta}} \mathcal{L}(\boldsymbol{\eta}, \boldsymbol{\lambda}) = 0. \tag{8}$$

From (7) and (8), we get

$$\begin{aligned} \frac{\partial \mathcal{L}(\boldsymbol{\eta}, \boldsymbol{\lambda})}{\partial \boldsymbol{\eta}^*} \;=\; & \frac{\partial \left(-\frac{1}{2} (\boldsymbol{\eta} \odot \mathbf{w}_{12})^H \mathbf{B} \mathbf{R}_{\mathbf{x}} \widetilde{\mathbf{w}}_2\right)}{\partial \boldsymbol{\eta}^*} \\ & + \frac{\partial \left(\frac{1}{2} (\boldsymbol{\eta} \odot \mathbf{w}_{12})^H \mathbf{B} \mathbf{R}_{\mathbf{x}} \mathbf{B}^H \boldsymbol{\eta} \odot \mathbf{w}_{12}\right)}{\partial \boldsymbol{\eta}^*} \\ & + \frac{\partial \left(\Re\left(\boldsymbol{\lambda}^H \mathbf{C}^H \mathbf{B}^H (\boldsymbol{\eta} \odot \mathbf{w}_{12})^H\right)\right)}{\partial \boldsymbol{\eta}^*} = 0. \end{aligned} \tag{9}$$

After some algebraic manipulations, we arrive at

$$\mathbf{B} \mathbf{R}_{\mathbf{x}} \mathbf{B}^H \mathbf{w}_{12} \odot \boldsymbol{\eta}_{\text{opt}} + \mathbf{B} \mathbf{C} \boldsymbol{\lambda} = \mathbf{B} \mathbf{R}_{\mathbf{x}} \widetilde{\mathbf{w}}_2. \tag{10}$$

Note that, since $\mathbf{B}\mathbf{x}(n) = \mathbf{x}_o(n)$, so $\mathbf{B} \mathbf{R}_{\mathbf{x}} \mathbf{B}^H = \mathbf{R}_{\mathbf{x}_o}$. Thus, the optimal $\boldsymbol{\eta}(n)$ from the above equation can be expressed by

$$\boldsymbol{\eta}_{\text{opt}} = (\mathbf{R}_{\mathbf{x}_o})^{-1} \left(\mathbf{B} \mathbf{R}_{\mathbf{x}} \widetilde{\mathbf{w}}_2 - \mathbf{B} \mathbf{C} \boldsymbol{\lambda}\right) \odot \mathbf{w}_{12}^{-1} \tag{11}$$

where $\mathbf{w}_{12}^{-1}$ represents a vector containing all the inverse elements of $\mathbf{w}_{12}$, such that $\mathbf{w}_{12} \odot \mathbf{w}_{12}^{-1} = 1$. In addition, $\mathbf{w}_{12}^{-1}$ can also be expressed by $\mathbf{w}_{12}^* \odot \bar{\mathbf{w}}_{12}$, where $\bar{\mathbf{w}}_{12} = [1/\|w_{12_1}\|^2, \ldots, 1/\|w_{12_{MN}}\|^2]^H$ and $\|\cdot\|$ denotes the Euclidean norm. Taking the constraints of $\mathbf{C}^H \widetilde{\mathbf{w}}_{\text{LC-GSC}} = \mathbf{f}$ into account, we obtain

$$\boldsymbol{\lambda} = \left(\mathbf{C}^H (\mathbf{R}_{\mathbf{x}})^{-1} \mathbf{C}\right)^{-1} \mathbf{f}. \tag{12}$$

From (11) and (12), we can find that according to the prior DOA knowledge, i.e., $\mathbf{C}$ and $\mathbf{f}$, the coefficient weighting vector $\boldsymbol{\eta}_{\text{opt}}$ will adaptively minimize the total output power under constraints.

5. PROPOSED ITERATIVE ADAPTIVE COMBINATION APPROACHES USING η-LMS AND η-RLS

In practical implementation of beamforming, iterative adaptive approach is necessarily developed to update estimation of $\boldsymbol{\eta}_{\text{opt}}$ at every observation $\mathbf{x}(n)$. In this section, the iterative adaptive combination approaches based on LMS and RLS, i.e., $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS, are proposed respectively for achieving this goal.

5.1. The Proposed Iterative Combination Algorithm Using η-LMS

The LMS update is derived using the gradient of the instantaneous squared error $e_{\text{LMS}}^2(n)$ to update the combination vector $\hat{\boldsymbol{\eta}}_{\text{LMS}}(n)$, the estimation of $\boldsymbol{\eta}_{\text{opt}}(n)$, i.e.,

$$\begin{aligned}\hat{\boldsymbol{\eta}}_{\text{LMS}}(n+1) &= \hat{\boldsymbol{\eta}}_{\text{LMS}}(n) - \frac{1}{2}\alpha\nabla_{\boldsymbol{\eta}}^{H}\left\{e_{\text{LMS}}^2(n)\right\} \\ &= \hat{\boldsymbol{\eta}}_{\text{LMS}}(n) + \alpha e_{\text{LMS}}^*(n)\mathbf{x}_o(n)\odot\mathbf{w}_{12}^*(n) \end{aligned} \tag{13}$$

where *a priori* output estimation errors of $\boldsymbol{\eta}$-LMS is $e_{\text{LMS}}(n) \triangleq d(n) - \mathbf{w}_{\text{LMS}}^H(n)\mathbf{x}_o(n)$ and α is the step-size parameter. Additionally, we define the input data as $\widetilde{\mathbf{x}}_o(n) \triangleq \mathbf{w}_{12}^*(n)\odot\mathbf{x}_o(n)$ instead of $\mathbf{x}_o(n)$ for expression of convenience. Hence, (13) can be also expressed by

$$\hat{\boldsymbol{\eta}}_{\text{LMS}}(n+1) = \hat{\boldsymbol{\eta}}_{\text{LMS}}(n) + \alpha e_{\text{LMS}}^*(n)\widetilde{\mathbf{x}}_o(n). \tag{14}$$

Moreover, due to the combined adaptive weighting vector of $\boldsymbol{\eta}$-LMS is $\hat{\mathbf{w}}_{\text{LMS}}(n) = \hat{\boldsymbol{\eta}}_{\text{LMS}}(n)\odot\mathbf{w}_1(n) + (\mathbf{1} - \hat{\boldsymbol{\eta}}_{\text{LMS}}(n))\odot\mathbf{w}_2(n)$, so the $\boldsymbol{\eta}$-LMS error can be also rewritten as

$$\begin{aligned} e_{\text{LMS}}(n) &= d(n) - \left(\mathbf{w}_2(n) + \hat{\boldsymbol{\eta}}_{\text{LMS}}(n)\odot\mathbf{w}_{12}(n)\right)^H\mathbf{x}_o(n) \\ &= d(n) - \hat{d}_2(n) - \hat{\boldsymbol{\eta}}_{\text{LMS}}^H(n)\left(\mathbf{w}_{12}^*(n)\odot\mathbf{x}_o(n)\right) \\ &= d(n) - \hat{d}_2(n) - \hat{\boldsymbol{\eta}}_{\text{LMS}}^H(n)\widetilde{\mathbf{x}}_o(n). \end{aligned} \tag{15}$$

5.2. Convergence Analysis of the Iterative Combination Algorithm Using η-LMS

We investigate the first-order and second order convergence of the proposed algorithm by adopting the similar method as in [12, p. 814]. Firstly, the weight-error vector of $\boldsymbol{\eta}$-LMS algorithm is defined as

$$\widetilde{\boldsymbol{\eta}}_{e_{\text{LMS}}}(n) \triangleq \hat{\boldsymbol{\eta}}_{\text{LMS}}(n) - \boldsymbol{\eta}_{\text{opt}}(n). \tag{16}$$

Next, using (16) in (14) with optimal adaptive time-variant weights $\mathbf{w}_{\text{opt}}(n) = \mathbf{w}_2(n) + \boldsymbol{\eta}_{\text{opt}}(n)\odot\mathbf{w}_{12}(n)$, we obtain the iterative equation of the weight-error $\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)$

$$\begin{aligned}\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n+1) \;=\; & \widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)+\alpha\widetilde{\mathbf{x}}_o(n)\left(d^*(n)\right.\\ & \left.-\mathbf{x}_o^H(n)\left(\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)\odot\mathbf{w}_{12}(n)+\mathbf{w}_o(n)\right)\right). \quad (17)\end{aligned}$$

Taking the expectation on of both sides of (17), and using Wiener-Hopf equations $\mathbf{R}_{\mathbf{x}_o}\mathbf{w}_{\mathrm{opt}} = \mathbf{r}_{\mathbf{x}_o,d}$ under the independence assumption as in [12], where $\mathbf{r}_{\mathbf{x}_o,d} \triangleq \mathbb{E}\{\mathbf{x}_o(n)d^*(n)\}$ is cross-correlation of input vector $\mathbf{x}_o(n)$ and desired signal $d(n)$, we have

$$\mathbb{E}\{\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n+1)\} = (\mathbf{I}-\alpha\mathbf{R}_{\widetilde{\mathbf{x}}_o})\,\mathbb{E}\left\{\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)\right\} \quad (18)$$

where $\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n) = \mathbb{E}\{\widetilde{\mathbf{x}}_o(n)\widetilde{\mathbf{x}}_o^H(n)\}$. With $0 < \alpha < 2/\lambda_{\max}(\mathbf{R}_{\widetilde{\mathbf{x}}_o})$, it is ready to show $\lim_{n\to\infty}\mathbb{E}\{\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n+1)\} = 0$, where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix in the bracket and 0 is a zero vector.

Thus, $\lim_{n\to\infty}\mathbb{E}\{\widetilde{\boldsymbol{\eta}}_{\mathrm{LMS}}(n)\} = \boldsymbol{\eta}_{\mathrm{opt}}$. It is obviously

$$\lim_{n\to\infty}\mathbb{E}\{\hat{\mathbf{w}}_{\mathrm{LMS}}(n)\} = \mathbf{w}_{\mathrm{opt}}. \quad (19)$$

In order to obtain the MSE of the proposed algorithm, the auto-correlation matrix of $\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)$ is defined by

$$\mathbf{R}_{e_{\mathrm{LMS}}}(n) \triangleq \mathbb{E}\left\{\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)^H\right\}. \quad (20)$$

To analyze the above equation, we rewrite (17) as

$$\begin{aligned}\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n+1) \;=\; & \widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)+\alpha\widetilde{\mathbf{x}}_o(n)\left(e_o^*(n)-\widetilde{\mathbf{x}}_o^H(n)\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)\right)\\ =\; & \left(\mathbf{I}-\alpha\widetilde{\mathbf{x}}_o(n)\widetilde{\mathbf{x}}_o^H(n)\right)\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{LMS}}}(n)+\alpha\widetilde{\mathbf{x}}_o(n)e_o^*(n) \quad (21)\end{aligned}$$

where the orthogonal error $e_o(n) \triangleq d(n) - \mathbf{w}_{\mathrm{opt}}^H(n)\mathbf{x}_o(n)$, which is a zero-mean random process uncorrelated with $\mathbf{x}_o(n)$, whose variance is denoted by $\xi_o \triangleq \lim_{n\to\infty}\mathbb{E}\{|e_o(n)|^2\}$, i.e., the MSE of the combined filter by optimal mixing vector $\boldsymbol{\eta}_{\mathrm{opt}}$. In order to make performance analysis more tractable, the sequences $\{\mathbf{x}_o(n)\}$ and $\{e_o(n)\}$ are assumed stationary and we will use the common assumption that $e_o(n)$ is independent of $\mathbf{x}_o(n)$.

Substituting (21) into (20) gives

$$\begin{aligned}\mathbf{R}_{e_{\mathrm{LMS}}}(n+1) \;=\; & \alpha^2\mathbb{E}\left\{e_o^*(n)\widetilde{\mathbf{x}}_o(n)\widetilde{\mathbf{x}}_o(n)^He_o(n)\right\}\\ & +(\mathbf{I}-\alpha\mathbf{R}_{\widetilde{\mathbf{x}}_o})\,\mathbf{R}_{e_{\mathrm{LMS}}}(n)\,(\mathbf{I}-\alpha\mathbf{R}_{\widetilde{\mathbf{x}}_o}). \quad (22)\end{aligned}$$

Using the moment factorization of jointly Gaussian random variables and the independence assumption as in [12], we have

$$\mathbb{E}\left\{e_o^*(n)\widetilde{\mathbf{x}}_o(n)\widetilde{\mathbf{x}}_o(n)^He_o(n)\right\} = \xi_o\mathbf{R}_{\widetilde{\mathbf{x}}_o}. \quad (23)$$

Then, the $\mathbf{R}_{e_{\mathrm{LMS}}}(n)$ is obtained.

Substitution of (16) with $e_o(n)$ into (15) leads to $e_{\text{LMS}}(n) = e_o(n) - (\widetilde{\boldsymbol{\eta}}_{e_{\text{LMS}}}(n) \odot \mathbf{w}_{12}(n))^H \mathbf{x}_o = e_o(n) - \widetilde{\boldsymbol{\eta}}^H_{e_{\text{LMS}}}(n)\widetilde{\mathbf{x}}_o$. Accordingly, the MSE of $\boldsymbol{\eta}$-LMS, i.e., $\xi^{\boldsymbol{\eta}\text{-LMS}}(n) \triangleq \mathbb{E}\{|e_{\text{LMS}}(n)|^2\}$ at iteration n is

$$\begin{aligned}
\xi^{\boldsymbol{\eta}\text{-LMS}}(n) &= \mathbb{E}\left\{\left|e_o(n) - \widetilde{\boldsymbol{\eta}}^H_{e_{\text{LMS}}}(n)\widetilde{\mathbf{x}}_o(n)\right|^2\right\} \\
&= \xi_o + \mathbb{E}\left\{\widetilde{\boldsymbol{\eta}}^H_{e_{\text{LMS}}}(n)\widetilde{\mathbf{x}}_o(n)\widetilde{\mathbf{x}}^H_o(n)\widetilde{\boldsymbol{\eta}}_{e_{\text{LMS}}}(n)\right\} \\
&= \xi_o + \text{Tr}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)\mathbf{R}_{e_{\text{LMS}}}(n)) \qquad (24)
\end{aligned}$$

where $\text{Tr}(\cdot)$ stands for the trace of the matrix in the bracket. Thus, EMSE of proposed iterative algorithm based on $\boldsymbol{\eta}$-LMS is

$$\xi^{\boldsymbol{\eta}\text{-LMS}}_{ex}(n) = \xi^{\boldsymbol{\eta}\text{-LMS}}(n) - \xi_o = \text{Tr}\left(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)\mathbf{R}_{e_{\text{LMS}}}(n)\right). \qquad (25)$$

And the steady-state EMSE is

$$\xi^{\boldsymbol{\eta}\text{-LMS}}_{ex}(\infty) = \lim_{n\to\infty} \xi^{\boldsymbol{\eta}\text{-LMS}}_{ex}(n) = \xi_o \sum_{k=1}^{N} \frac{\alpha\lambda_k}{2 - \alpha\lambda_k} \qquad (26)$$

where λ_k is the kth eigenvalue of $\mathbf{R}_{\widetilde{\mathbf{x}}_o}$. If the step-size α is small enough in the sense that $\alpha\lambda_k \ll 1$, then the expression of EMSE simplifies to

$$\xi^{\boldsymbol{\eta}\text{-LMS}}_{ex}(\infty) = \xi_o \frac{\alpha\text{Tr}(\mathbf{R}_{\widetilde{\mathbf{x}}_o})}{2 - \alpha\text{Tr}(\mathbf{R}_{\widetilde{\mathbf{x}}_o})}. \qquad (27)$$

5.3. Proposed Adaptive Combination Approach Using $\boldsymbol{\eta}$-RLS

The proposed adaptive combination approach can also be applied for other supervised adaptive filters, such as RLS, whose iterative updating equation for RLS combination filter could be also derived by the definition of $\widetilde{\mathbf{x}}_o(n)$. Using RLS, we need to minimize the least square cost function

$$\mathcal{J} = ||\mathbf{e}||^2 = \mathbf{e}^H\mathbf{e} \qquad (28)$$

where $\mathbf{e} \triangleq \mathbf{d} - \hat{\mathbf{d}} = \mathbf{d} - \mathbf{w}(n)\mathbf{B}^H\mathbf{A}$, $\mathbf{e} \triangleq [e(1), \ldots, e(n)]^H$, $\mathbf{d} \triangleq [d(1), \ldots, d(n)]^H$, $\hat{\mathbf{d}} \triangleq [\hat{d}(1), \ldots, \hat{d}(n)]^H$ and $\mathbf{A}^H \triangleq [\mathbf{x}(1), \ldots, \mathbf{x}(n)]$. In order to find the optimal values of the mixing vector that minimize the power of the global errors, we take the derivation of (28), obtaining

$$\begin{aligned}
\nabla_{\boldsymbol{\eta}}\mathcal{J} &= 2\frac{\partial \mathcal{J}}{\partial \boldsymbol{\eta}^*} = 2\left(\mathbf{W}^*_{12} \odot \mathbf{B}^H\mathbf{A}\right)^H \mathbf{d} \\
&\quad -2\left(\mathbf{W}^*_{12} \odot \mathbf{B}^H\mathbf{A}\right)^H(\mathbf{W}^*_{12} \odot \mathbf{B}^H\mathbf{A})\,\hat{\boldsymbol{\eta}}(n) \qquad (29)
\end{aligned}$$

where $\mathbf{W}_{12} \triangleq [\mathbf{w}_{12}(1), \ldots, \mathbf{w}_{12}(n)]$. Setting (29) to zero, and after some manipulations, we get $\hat{\boldsymbol{\eta}}(n)$ to estimate proposed mixing vector $\boldsymbol{\eta}_{\text{opt}}$ by deterministic normal equations

$$\mathbf{A}^H_o\mathbf{A}_o\hat{\boldsymbol{\eta}}(n) = \mathbf{A}^H_o(\mathbf{d} - \mathbf{B}\mathbf{A}\mathbf{W}_2) \qquad (30)$$

where $\mathbf{A}_o \triangleq \mathbf{W}_{12}^* \odot \mathbf{B}^H\mathbf{A} = [\mathbf{w}_{12}^*(1) \odot \mathbf{x}_o(1), \ldots, \mathbf{w}_{12}^*(n) \odot \mathbf{x}_o(n)]$, and $\mathbf{W}_2 \triangleq [\mathbf{w}_2(1), \ldots, \mathbf{w}_2(n)]^H$. Considering non-stationary scenario, forgetting factor μ is adopted into auto-correlation matrix $\Phi(n)$ of the input vector and cross-correlation vector $\mathbf{z}(n)$ between $\widetilde{\mathbf{x}}_o(n)$ and $\mathbf{d}(n) - \mathbf{BAW}_2$ as followings

$$\Phi(n) = \sum_{i=1}^{n} \mu^{n-i}\widetilde{\mathbf{x}}_o(i)\widetilde{\mathbf{x}}_o^H(i) + \Phi(0), \tag{31}$$

$$\mathbf{z}(n) = \sum_{i=1}^{n} \mu^{n-i}\widetilde{\mathbf{x}}_o(i)(d^*(i) - \mathbf{w}_2^H(i)\mathbf{x}_o(i)) \tag{32}$$

where $0 < \mu \leqslant 1$ and $\Phi(0) = \epsilon\mathbf{I}$, ϵ is a small positive number. Hence, the solution of the problem is easily obtained from

$$\hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) = \Phi^{-1}(n)\mathbf{z}(n). \tag{33}$$

5.4. The Proposed Iterative Combination Algorithm Using $\boldsymbol{\eta}$-RLS

The iterative combination algorithm can be derived according to matrix inversion lemma. Define $\mathbf{P}(n) \triangleq \Phi^{-1}(n)$, we get the recursive equation of $\mathbf{P}(n)$

$$\mathbf{P}(n+1) = \mu^{-1}\mathbf{P}(n) - \mu^{-1}\mathbf{k}(n+1)\widetilde{\mathbf{x}}_o\mathbf{P}(n) \tag{34}$$

where $\mathbf{k}(n+1)$ is the gain vector which updates $\hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n)$ at its each element and expressed by

$$\mathbf{k}(n+1) = \frac{\mu^{-1}\mathbf{P}(n)\widetilde{\mathbf{x}}_o(n+1)}{1 + \mu^{-1}\widetilde{\mathbf{x}}_o^H(n+1)\mathbf{P}(n)\widetilde{\mathbf{x}}_o(n+1)}. \tag{35}$$

Hence, the final recursion of $\hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n)$ is given by

$$\begin{aligned} \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n+1) &= \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) + \Phi^{-1}(n+1)e_{\mathrm{RLS}}^*(n+1)\widetilde{\mathbf{x}}_o(n+1) \\ &= \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) + \mathbf{k}(n+1)e_{\mathrm{RLS}}^*(n+1) \end{aligned} \tag{36}$$

where *a priori* output estimation errors of $\boldsymbol{\eta}$-RLS is $e_{\mathrm{RLS}}(n+1) \triangleq d(n+1) - \mathbf{w}_{\mathrm{RLS}}^H(n)\mathbf{x}_o(n+1) = d(n+1) - (\mathbf{w}_2(n) + \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) \odot \mathbf{w}_{12}(n))^H\mathbf{x}_o(n+1)$, where $\hat{\mathbf{w}}_{\mathrm{RLS}}(n) = \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) \odot \mathbf{w}_1(n) + (1 - \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n)) \odot \mathbf{w}_2(n)$.

5.5. Convergence Analysis of the Iterative Combination Algorithm Using $\boldsymbol{\eta}$-RLS

Similarly, the weight-error vector of $\boldsymbol{\eta}$-RLS algorithm is defined as

$$\widetilde{\boldsymbol{\eta}}_{e_{\mathrm{RLS}}}(n) \triangleq \hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n) - \boldsymbol{\eta}_{\mathrm{opt}}(n). \tag{37}$$

For $\mu = 1$, substituting $d(n) = \mathbf{w}_{\text{opt}}^H(n)\mathbf{x}_o(n) + e_o(n)$ into (32) and using (31), we can rewrite the (33) as

$$\begin{aligned}\hat{\boldsymbol{\eta}}_{\text{RLS}}(n) &= \boldsymbol{\eta}_{\text{opt}}(n) - \Phi^{-1}(n)\Phi(0)\boldsymbol{\eta}_{\text{opt}}(n)\\ &\quad +\Phi^{-1}(n)\sum_{i=1}^{n}\tilde{\mathbf{x}}_o(i)e_o^*(i).\end{aligned} \tag{38}$$

Using the independence assumption as in [37], i.e., $\mathbb{E}\{\Phi^{-1}(n)\} = \frac{1}{n}\mathbf{R}_{\widetilde{\mathbf{x}}_o}^{-1}(n)$, $n > MN$, and taking the expectation on of both sides of (38), we have

$$\mathbb{E}\left\{\hat{\boldsymbol{\eta}}_{\text{RLS}}(n)\right\} \approx \boldsymbol{\eta}_{\text{opt}}(n) - \frac{\epsilon}{n}\mathbf{R}_{\widetilde{\mathbf{x}}_o}^{-1}(n)\boldsymbol{\eta}_{\text{opt}}(n). \tag{39}$$

As shown from (39), when $n \gg MN$, $\lim_{n\to\infty}\mathbb{E}\{\hat{\boldsymbol{\eta}}_{\text{RLS}}(n)\} = \boldsymbol{\eta}_{\text{opt}}$, so it is obviously

$$\lim_{n\to\infty}\mathbb{E}\left\{\hat{\mathbf{w}}_{\text{RLS}}(n)\right\} = \mathbf{w}_{\text{opt}}. \tag{40}$$

The error-correlation matrix of $\hat{\boldsymbol{\eta}}_{e_{\text{RLS}}}(n)$ is similarly defined by

$$\mathbf{R}_{e_{\text{RLS}}}(n) \triangleq \mathbb{E}\left\{\widetilde{\boldsymbol{\eta}}_{e_{\text{RLS}}}(n)\widetilde{\boldsymbol{\eta}}_{e_{\text{RLS}}}(n)^H\right\}. \tag{41}$$

Substituting (38) with (37) into (41) and ignoring the effect of initialization of $\Phi(0)$ yields

$$\begin{aligned}\mathbf{R}_{e_{\text{RLS}}}(n) &= \mathbb{E}\left\{\Phi^{-1}(n)\sum_{i=1}^{n}\sum_{j=1}^{n}\widetilde{\mathbf{x}}_o(i)\widetilde{\mathbf{x}}_o^H(j)(\Phi^{-1}(n))^H e_o^*(i)e_o(j)\right\}\\ &= \xi_o\mathbb{E}\left\{\Phi^{-1}(n)\sum_{i=1}^{n}\widetilde{\mathbf{x}}_o(i)\widetilde{\mathbf{x}}_o^H(i)\Phi^{-1}(n)\right\}\\ &= \xi_o\mathbb{E}\left\{\Phi^{-1}(n)\right\}.\end{aligned} \tag{42}$$

Therefore, as the derivation of [37], the MSE of the $\boldsymbol{\eta}$-RLS, i.e., $\xi^{\boldsymbol{\eta}\text{-RLS}}(n) \triangleq \mathbb{E}\{|e_{\text{RLS}}(n)|^2\}$ at iteration n, when $n \gg MN$, is given by

$$\begin{aligned}\xi^{\boldsymbol{\eta}\text{-RLS}}(n) &= \mathbb{E}\left\{\left|e_o(n) + (\mathbf{w}_{\text{opt}}(n-1) - \mathbf{w}_{\text{RLS}}(n-1))^H\mathbf{x}_o(n)\right|^2\right\}\\ &= \mathbb{E}\left\{\left|e_o(n) + \widetilde{\boldsymbol{\eta}}_{e_{\text{RLS}}}^H(n-1)\widetilde{\mathbf{x}}_o(n)\right|^2\right\}\\ &\approx \xi_o + \frac{1}{n}\xi_o\text{Tr}\left\{\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)\mathbf{R}_{\widetilde{\mathbf{x}}_o}^{-1}(n-1)\right\}\\ &\approx \xi_o + \frac{MN}{n}\xi_o.\end{aligned} \tag{43}$$

It is easy to find out that the EMSE of the $\boldsymbol{\eta}$-RLS is $\xi_{ex}^{\boldsymbol{\eta}\text{-RLS}}(n) \triangleq \xi^{\boldsymbol{\eta}\text{-RLS}}(n) - \xi_o = MN\xi_o/n$, i.e., the performance of $\boldsymbol{\eta}$-RLS is independent of the input covariance matrix $\mathbf{R}_{\mathbf{x}}$, which benefits the $\hat{\boldsymbol{\eta}}_{\text{RLS}}$ to trend $\boldsymbol{\eta}_{\text{opt}}$ quickly with few snapshots in stationary scenario.

In summary of the Section 5, due to the similarity of updated Equations (14) and (36), the updates of $\hat{\mathbf{w}}_{\text{LMS}}(n)$ and $\hat{\mathbf{w}}_{\text{RLS}}(n)$ are not only convergent but also have the similar expressions of MSE and EMSE under the plausible assumptions as in [12, 31].

6. TRACKING PERFORMANCES ANALYSIS OF THE PROPOSED ITERATIVE COMBINATION ALGORITHMS

In non-stationary environment, tracking performance of the proposed algorithms are needed to be investigated. After a general comment, we study a particular model for the statistics of the desired data, commonly used to model non-stationarity in tracking analysis [39], which assumes that the variation of in optimal solution $\mathbf{w}_{\text{opt}}$ follows a random-walk model, i.e.,

$$\mathbf{w}_{\text{opt}}(n) = \mathbf{w}_{\text{opt}}(n-1) + \mathbf{q}(n). \tag{44}$$

In this model, $\mathbf{q}(n)$ is an independent identically distributed (i.i.d.) vector with positive-definite auto-correlation matrix $\mathbf{Q} \triangleq \mathbb{E}\{\mathbf{q}(n)\mathbf{q}(n)^H\}$, independent of the initial conditions as in [5] and also assumed independent of $\{\mathbf{x}(m), d(m), e_o(m)\}$, for all $m \leq n$. From its definition, $\text{Tr}(\mathbf{Q})$ can be seen as a measure of the speed of changes in $\mathbf{w}_o$. For sufficiently small α and $(1-\mu)$, analytical EMSE expressions of the proposed $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS algorithms are given separately by [39], i.e.,

$$\xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty) = \frac{\alpha\xi_o\text{Tr}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}) + \alpha^{-1}\text{Tr}(\mathbf{Q})}{2} \tag{45}$$

and

$$\xi_{ex}^{\boldsymbol{\eta}\text{-RLS}}(\infty) = \frac{\xi_o(1-\mu)MN + (1-\mu)^{-1}\text{Tr}(\mathbf{Q}\mathbf{R}_{\widetilde{\mathbf{x}}_o})}{2}. \tag{46}$$

Theorem I: from the (45) and (46), the proposed general affine combination approach has faster convergence rate and less steady-state EMSE determined by the relationship between the two individual weights $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$.

Proof: Because the auto-correlation matrix of $\widetilde{\mathbf{x}}_o(n)$ satisfies $\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n) = \mathbb{E}\{\mathbf{x}_o(n) \odot \mathbf{w}_{12}(n)(\mathbf{x}_o(n) \odot \mathbf{w}_{12}(n))^H\} = \mathbf{R}_{\mathbf{x}_o}(n) \odot \mathbf{R}_{\mathbf{w}_{12}^*}(n)$, where the matrix $\mathbf{R}_{\mathbf{w}_{12}^*}(n) \triangleq \mathbb{E}\{\mathbf{w}_{12}^*(n)(\mathbf{w}_{12}^*(n))^H\} = [w_{ij}]_{MN \times MN}$, so the improvements of $\xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty)$ and $\xi_{ex}^{\boldsymbol{\eta}\text{-RLS}}(\infty)$ depend on the correlation among the individual adaptive filtering weights.

According to Schur's theorem, since the relationship between the eigenvalues of $\mathbf{R}_{\mathbf{x}_o}(n)$ and $\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)$ satisfies

$$\begin{aligned}&\min(w_{ii})\lambda_{\min}(\mathbf{R}_{\mathbf{x}_o}(n)) \leq \lambda_{\min}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n))\\ &\leq \lambda_{\max}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)) \leq \max(w_{ii})\lambda_{\max}(\mathbf{R}_{\mathbf{x}_o}(n))\end{aligned} \tag{47}$$

where $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of the matrix in the bracket. Therefore, the combined filter will accelerate the convergent rate by $w_{ii}(n)$ to adjust flexibly $\lambda_{\max}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n))/\lambda_{\min}(\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n))$ rather than the fixed one of $\lambda_{\min}(\mathbf{R}_{\mathbf{x}_o}(n))$ and $\lambda_{\max}(\mathbf{R}_{\mathbf{x}_o}(n))$, which can be also interpreted that the combined filter has different step-sizes which are assigned appropriately by each entry of the mixing vector.

Again, from the inequality equation of the Schur product, we know that $\|\mathbf{R}_{\widetilde{\mathbf{x}}_o}(n)\| \leq \|\mathbf{R}_{\mathbf{x}_o}(n)\|\|\mathbf{R}_{\mathbf{w}_{12}^*}(n)\|$. Regarding that two individual filters deal with the same input $\mathbf{x}(n)$, they are correlated so that $\|\mathbf{R}_{\mathbf{w}_{12}^*}(n)\|$ becomes small (generally, $|w_{ij}(n)| \ll 1$), which benefits the proposed overall filter with less $\xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty)$ or $\xi_{ex}^{\boldsymbol{\eta}\text{-RLS}}(\infty)$ in non-stationary environment.

7. SINRS OF THE PROPOSED ITERATIVE COMBINATION ALGORITHMS FOR BEAMFORMING

The primary measure of filters' performances is output signal power P_s to interference plus noise power P_{i+n} ratio (SINR) of an antenna array, which is computed as following

$$\text{SINR} = \frac{P_s}{P_{i+n}} = \frac{\widetilde{\mathbf{w}}_{\text{LC-GSC}}^H \mathbf{R}_s \widetilde{\mathbf{w}}_{\text{LC-GSC}}}{\widetilde{\mathbf{w}}_{\text{LC-GSC}}^H \mathbf{R}_{i+n} \widetilde{\mathbf{w}}_{\text{LC-GSC}}} \tag{48}$$

where $\mathbf{R}_s$ is the desired signal's auto-correlation matrix; $\mathbf{R}_{i+n}$ is the auto-correlation matrix of interference signal plus noise.

Theorem II: If the Theorem 1 stands, then the proposed general affine combination approach based on GSC also has higher output SINR relatively to other conventional algorithms, when $n \to \infty$.

Proof: The denominator of the output SINR in (48) can be also changed to

$$\begin{aligned}P_{i+n} &= \lim_{n\to\infty} \mathbb{E}\left\{\left|\widetilde{\mathbf{w}}_{\text{LC-GSC}}^H(n)\mathbf{x}(n) - \widetilde{\mathbf{w}}_{\text{LC-GSC}}^H(n)\mathbf{s}_d(n)\right|^2\right\}\\ &= \lim_{n\to\infty} \mathbf{w}_q^H \mathbf{R}_{\mathbf{x}}(n)\mathbf{w}_{\text{opt}} + \text{Tr}\left\{\mathbf{B}\mathbf{R}_{\mathbf{x}}(n)\mathbf{B}^H\mathbf{R}_e(n)\right\} - P_s\\ &= \xi_o + \xi_{ex}(\infty) - P_s\end{aligned} \tag{49}$$

where $\mathbf{s}_d = \mathbf{s}(\phi, \omega)d(n)$ is the desired optimal vector and $\xi_{ex}(\infty)$ is the EMSE of the iterative algorithm, e.g., $\xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty)$ and $\xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty)$, and $\mathbf{R}_e(n)$, e.g., $\mathbf{R}_{e_{\text{RLS}}}(n)$ and $\mathbf{R}_{e_{\text{LMS}}}(n)$, represents the auto-correlation matrix of the weight-error vector $\widetilde{\boldsymbol{\eta}}_e(n)$, e.g., $\widetilde{\boldsymbol{\eta}}_{e_{\text{LMS}}}(n)$ and $\widetilde{\boldsymbol{\eta}}_{e_{\text{RLS}}}(n)$. Equation (49) shows $\xi_{ex}(\infty)$ affects the output SINR of beamformer, i.e., the smaller the EMSE value is, the larger the steady-state SINR becomes. Therefore, as the EMSEs of the proposed $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS shown in (45) and (46), the output SINRs of $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS are given by $\text{SINR}_{\boldsymbol{\eta}\text{-LMS}} = P_s/(\xi_o + \xi_{ex}^{\boldsymbol{\eta}\text{-LMS}}(\infty) - P_s)$ and $\text{SINR}_{\boldsymbol{\eta}\text{-RLS}} = P_s/(\xi_o + \xi_{ex}^{\boldsymbol{\eta}\text{-RLS}}(\infty) - P_s)$, respectively, which are higher than conventional adaptive GSC because of the role of the general affine combination method as illustrated in Theorem 1.

8. COMPLEXITY ANALYSIS OF THE PROPOSED ITERATIVE COMBINATION ALGORITHMS FOR BEAMFORMING

Finally, another essential description of adaptive filters depends on their complexities, which plays a critical role of their practical implementations. The complexities of typical algorithms used in this paper have been shown in Table 1. There is no doubt that the adaptive combination of multiple individual filters will increase the complexity of the overall filter and computational cost for beamformer. But as the rapid development of the processing abilities of electronic chips and thanks to the high efficient algorithms, e.g., a family of reduced-rank algorithms, the proposed approach may be a promising way to enhance the beamforming performance for interference mitigation.

For example, the Lanczos reduced-rank algorithm represents an efficient way to compute the mutual orthogonal basis $\mathbf{t}_i$ in r-dimensional Krylov subspace $\kappa^r = \text{span}(\mathbf{r}_{\mathbf{x}_o,d}, \mathbf{R}_{\mathbf{x}_o}\mathbf{r}_{\mathbf{x}_o,d}, \ldots, \mathbf{R}_{\mathbf{x}_o}^{(r-1)}\mathbf{r}_{\mathbf{x}_o,d})$ as in literature [38]. Because the covariance matrix $\mathbf{R}_{\mathbf{x}_o}$ is Hermitian, so only upper (or lower) triangular portion of $\mathbf{R}_{\mathbf{x}_o}$ needs to be computed, resulting in the complexity $(1 + K)K/2$ and the backward iteration process of the Lanczos reduced-rank MNWF by constructing tridiagonal matrix can be processed in parallel with the complexity of $\mathcal{O}(r)$ as in [38], where K is the total number of sensors, i.e., $K = M \times N$ for space-time processing. Consequently, the complexity of the Lanczos reduced-rank MNWF is $\mathcal{O}(rK^2)$ per iteration. Furthermore, multiple constrained reduced-rank MNWF based on correlations subtractive structure (CSS) as proposed in [41] only involve complex vector-vector products in forward recursion, not complex matrix-vector products (for the single space-time weight constraint), thereby implying com-

Table 1. Computational complexity of the algorithms per iteration for complex-valued data in terms of the number of complex multiplications, complex additions and complex divisions.

Algorithm	**Additions**	**Multiplications**	**Divisions**
LMS	$2K$	$2K+1$	
NLMS	$3K$	$3K+1$	
RLS	K^2+3K	K^2+5K+1	1
Lanczos MNWF	$r(K^2+5K+3)$	$r(K^2+6K+7)$	$r(K+1)$
$\boldsymbol{\eta}$-LMS (combined filter)	$4K$	$4K+1$	
$\boldsymbol{\eta}$-RLS (combined filter)	K^2+5K	K^2+7K+1	1
$\boldsymbol{\eta}$-PLMS (combined filter)	$3K+N_p$	$3K+N_p+1$	
$\boldsymbol{\eta}$-PRLS (combined filter)	K^2+4K+N_p	K^2+6K+N_p+1	1

putational complexity $\mathcal{O}(rK)$ per snapshot. Therefore, the Lanczos reduced-rank MNWF based on CSS is suitable for being the component filter.

Meanwhile, the combined filter of the proposed adaptive approaches will increase the complexity slightly because of $2K$ additions for computing $\mathbf{w}_{12}(n)$ and $\mathbf{w}_2(n)+\boldsymbol{\eta}(n)\odot\mathbf{w}_{12}(n)$ and $2K$ multiplications for $\boldsymbol{\eta}(n)\odot\mathbf{w}_{12}(n)$ and $\mathbf{w}_{12}^*(n)\odot\mathbf{x}(n)$. Overall, the complexity of the $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS is proportional to LMS $\mathcal{O}(K)$ and RLS $\mathcal{O}(K^2)$. In order to reduce the complexity of the proposed algorithms further in hardware application, partially updated $\hat{\boldsymbol{\eta}}(n)$ algorithms ($\boldsymbol{\eta}$-PLMS and $\boldsymbol{\eta}$-PRLS) are proposed to reduce to the amount of float additions and multiplications. Hence, the proposed $\boldsymbol{\eta}$-PLMS and $\boldsymbol{\eta}$-PRLS only update the N_p entries of their updated $\hat{\boldsymbol{\eta}}_{\mathrm{LMS}}(n+1)$ and $\hat{\boldsymbol{\eta}}_{\mathrm{RLS}}(n+1)$ which change significantly than other ones, separately. This way will save $N-N_p$ multiplications to compute the incremental values of $\hat{\boldsymbol{\eta}}(n+1)$ and $N-N_p$ additions to update $\hat{\boldsymbol{\eta}}(n+1)$ respectively per iteration.

Otherwise, beamformer can also adopt inverse square-root RLS rather than RLS for reducing its complexity. The inverse square-root RLS algorithm can be implemented based on Systolic structure by parallel computing and holds better numerical stability, fast convergence speed and applicable for non-stationary environment. As the discussions above, the proposed approach imposes acceptable computational burden to antenna array processor.

9. SIMULATION RESULTS

In this section, we apply our proposed adaptive combination approach into wideband receiver antenna array for space-time processing. The Lanczos reduced-rank MNWF (rank = 4) and LMS are adopted as two component filters due to their low complexities and complementary characteristics. On the one hand, the Lanczos reduced-rank MNWF enjoys a fast convergent rate but suffers more errors in non-stationary scenario. Because the Lanczos reduced-rank MNWF minimizes an erroneous MSE function in this case in the Krylov subspace, so there is a mismatch in estimate of the auto-correlation matrix and the cross-correlation vector such that the $\mathbf{t}_i$ cannot hold orthogonality with each other, i.e., $\mathbf{t}_i^H \mathbf{t}_j \neq 0$, $i, j \in r$. Therefore, the solution obtained at each iteration is no longer optimal in the sense of minimizing the true MSE within the Krylov subspace [26]. On the other hand, the MSE of LMS filter is determined by the step-size, if it has a small MSE, then it converges very slowly. However, the LMS filter has good performance for anti-narrowband interference because of its nonlinear filtering rather than Lanczos reduced-rank MNWF. Therefore, adaptive combination of this two kind of filters may achieve a better performance both on convergence rate and output SINR in stationary and non-stationary environment.

Besides, the proposed $\boldsymbol{\eta}$-LMS, $\boldsymbol{\eta}$-PLMS, $\boldsymbol{\eta}$-RLS and $\boldsymbol{\eta}$-PRLS are exploited as the combined iterative combination algorithm respectively. The combined filter uses the combination vector $\boldsymbol{\eta}(n)$ to weight these two individual filters, which makes their adaptive convergent rates independently in order to enhance the convergent speed. Attributing to the more DOFs offered by the mixing vector $\boldsymbol{\eta}(n)$, the combination approach presents significant performance improvement over the current approaches. The simulation results demonstrate the proposed adaptive combination filters have faster convergent speed, higher output SINR and better robustness performances than other existing adaptive filters under various interference environments including stationary, non-stationary, mixed wideband and narrowband interferences scenarios via 100 Monte Carlo trials. Specific simulation parameters are as shown in Table 2.

Two-dimensional beam pattern is one of important measure to evaluate the results of beamforming from the view of space and frequency dimensions, which is defined as

$$G(\theta, f)(\mathrm{dB}) \triangleq 10 \log \frac{|F(\theta, f)|^2}{\max |F(\theta, f)|^2} \tag{50}$$

where θ represents the DOA and f denotes the normalized frequency, and the array factor $F(\theta, f)$ is calculated by multiplication of $\widetilde{\mathbf{w}}_{\text{LC-GSC}}$

with each elementary direction vector which covers the dimensions from both space and frequency.

Figure 3 shows clearly that the antenna gain pattern of proposed algorithm based on space-time processing can resist more than the number of $M-1$ interferences simultaneously because of which not only suppressing the wideband interferences at the spatial dimension

Table 2. Simulation parameters.

Parameters	**Specifications**
Antenna type	uniform linear array (ULA)
Antenna element space	half of a wavelength
Antenna element number (M)	5 elements
Number of delay line taps (N)	4 taps
Desired direction (θ_d)	0°
Sampling frequency (f_s)	5.5 MHz
Spread code rate	1 MHz
Wideband interference rate	10 MHz
Narrowband interference rate	1 MHz
Incidence angle (wideband interference)	40° -20° -60°
Incidence angle (narrowband interference)	60°
Signal to interference ratio (SIR)/per	-40 dB
Signal to noise ratio (SNR)	-10 dB

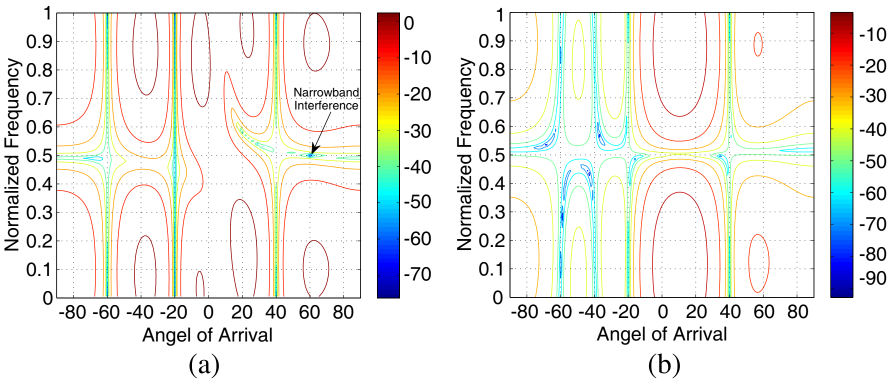


Figure 3. Two-dimensional beam pattern of the proposed beamformer. (a) There are 3 wideband interferences and 1 narrowband interference. (b) There are 4 wideband interferences, one of them comes from -40° with SIR $= -40$ dB and 1 narrowband interference which is the same as in Figure 3(a).

but also forming a notch at the frequency dimension to mitigate narrowband interference effectively, where the snapshot $n = 1000$ and the proposed $\boldsymbol{\eta}$-LMS ($\alpha = 1 \times 10^{-7}$) algorithm is adopted. The carrier frequency of narrowband interference is the same as the desired signal. More other rest of specific parameters are as shown in Table 2.

In Figure 4, there are 3 wideband interferences generated. We evaluate the proposed the beamformer by iterative algorithms in terms of output SINR, where the step-size of the component filter LMS is 1×10^{-7}, and the one of the traditional affine approach proposed in [33] is $\mu_\eta = 1 \times 10^{-4}$. In addition, $\alpha = 1 \times 10^{-7}$ and $\mu = 0.995$ are the step-sizes of the proposed $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS, respectively. More other rest of specific parameters are as shown in Table 2. In Figure 4(a) the square green curve plots the result of adaptive affine combination algorithm proposed in [32, 33] which is applied into the proposed beamformer based on GSC in this paper, whose output SINR has outperformed the any signal individual filter's SINR whatever in transient-state and steady-state. However, we can easily find out that the proposed adaptive combination method has even more fast convergent rate with high output SINR. Because the proposed iterative approach based on mixing vector $\boldsymbol{\eta}(n)$ has more DOFs to deal with the problem of convergence, so it can further suppress the residual errors and adaptively weight the two sub-filters' weights thoroughly, especially for combining two individual filters at both space and time dimensions. In particular, this result can be also verified in terms of MSE, mean value of weights and mean square derivation (MSD) of every filter as in Figures 4(b)–(d), where the MSD is defined by $\mathcal{D}(n) = \mathbb{E}\{\|\mathbf{w}_{\mathrm{opt}}(n) - \mathbf{w}(n)\|^2\}$. The results of partially updated $\hat{\boldsymbol{\eta}}(n)$ performances are depicted in Figures 4(e) and (f) which illustrate that the $\boldsymbol{\eta}$-PRLS has better performance than $\boldsymbol{\eta}$-PLMS regarding that the gain vector $\mathbf{k}(n)$ provides more accurate weights than α for updating the mixing vector $\hat{\boldsymbol{\eta}}(n)$.

Figure 5 plots that there is a strong narrowband existing in stationary environment. And there are still 2 wideband interferences and 1 narrowband interference. Rather than the values as in Table 2, their SIRs are -30 dB, -40 dB and -50 dB corresponding to their DOAs, respectively. And the step-size of the component filter LMS is 5×10^{-8}, and the step-size $\mu_\eta = 1 \times 10^{-5}$ is for the traditional affine approach proposed in [33]. Meanwhile, $\alpha = 1 \times 10^{-8}$ and $\mu = 0.995$ are the step-sizes of the proposed $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS, respectively. More other rest of specific parameters are still as shown in Table 2. From Figures 5(a) and (b), we can find out although the Lanczos reduced-rank MNWF doesn't work well, the proposed iterative algorithms still hold perfect tracking performance with faster convergent rate than

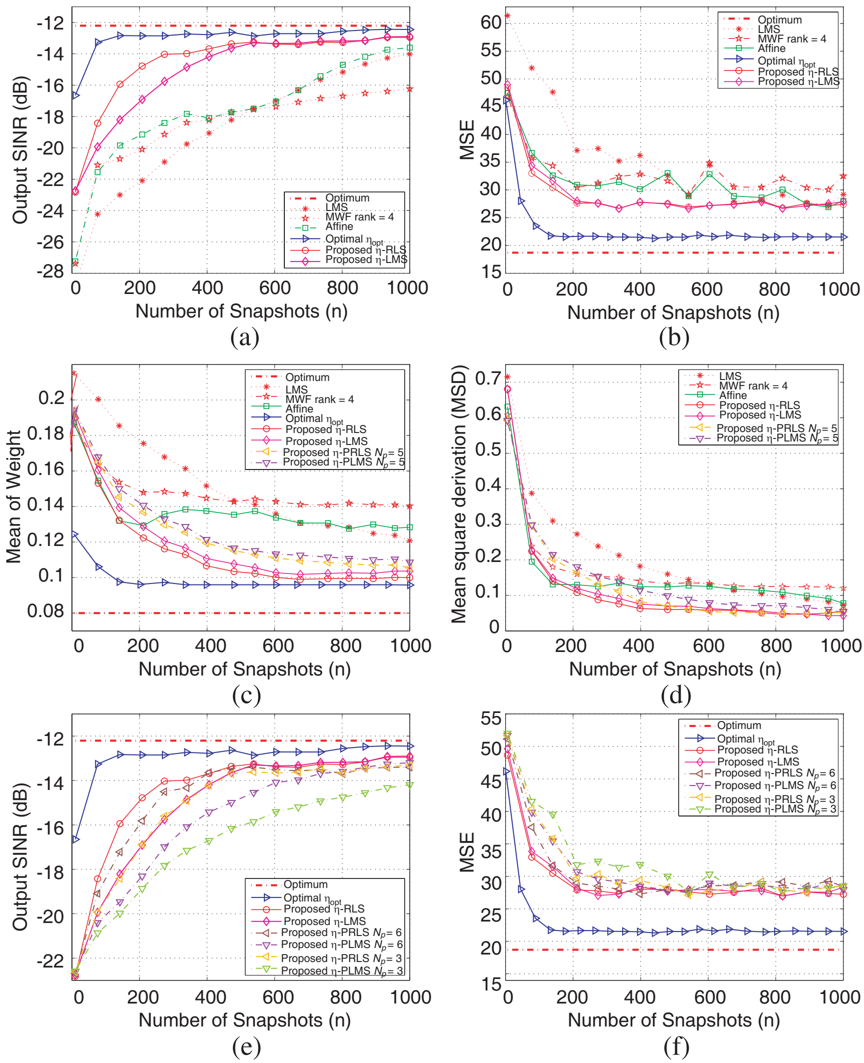


Figure 4. Output performances evaluation of the proposed adaptive combination approach with iterative algorithms in stationary environment. (a) Relationship of the combined filters and their component filters. (b) MSE of the filters in the left figure. (c) Mean value of the 1st entry of weight. (d) Mean square derivation of the filters. (e) Comparison of the proposed iterative algorithms with their partially updated $\boldsymbol{\eta}(n)$ algorithms. (f) MSE of the filters in the left figure.

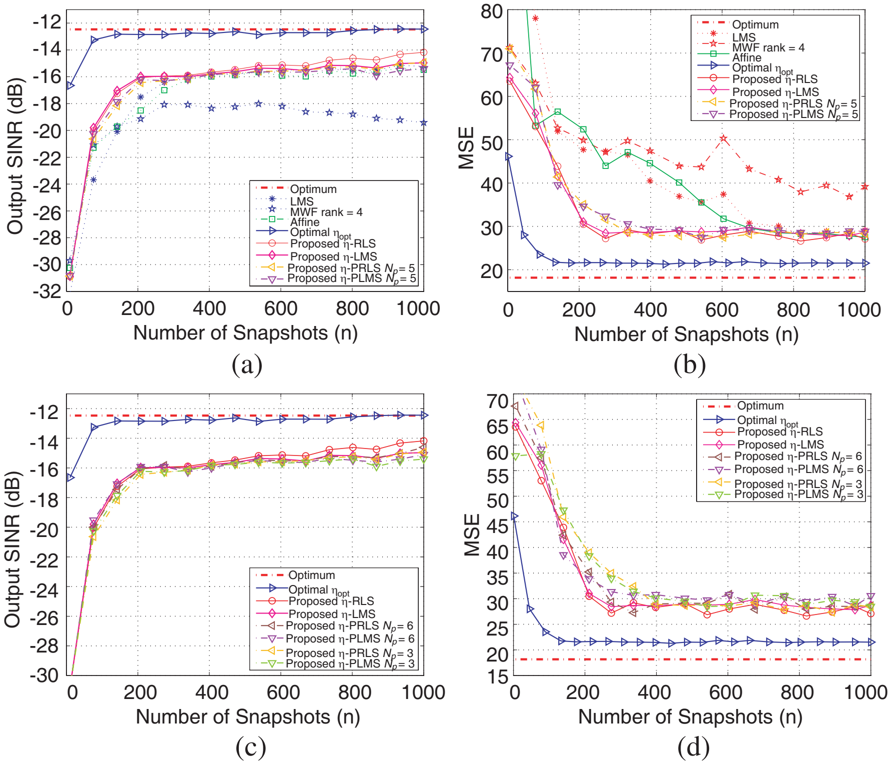


Figure 5. Output performances evaluation of the proposed adaptive combination approach with iterative algorithms in a mixed bandwidth interference environment. (a) Relationship of the combined filters and their component filters. (b) MSE of the filters in the left figure. (c) Comparison of the proposed iterative algorithms with their partially updated $\hat{\boldsymbol{\eta}}(n)$ algorithms. (d) MSE of the filters in the left figure.

the traditional affine combination approach only used a scale μ_η being the mixing parameter as in [33] and similar high output SINRs at steady-state. Figures 5(c) and (d) illustrate that partially updated $\hat{\boldsymbol{\eta}}(n)$ algorithms will not be affected by the N_p significantly, because of the sparsity of the narrowband interference in space-time processing as depicted in Figure 3(a). Hence, the partially updated $\hat{\boldsymbol{\eta}}(n)$ algorithms, i.e., $\boldsymbol{\eta}$-PLMS and $\boldsymbol{\eta}$-PRLS with $N_p = 3$, $N_p = 6$ and full updated $\hat{\boldsymbol{\eta}}(n)$ have the analogous curves of SINRs and the differences of their SINRs are only within 1 dB.

We also implement these new algorithms and other conventional ones, such as RLS, normalized LMS (NLMS) and MNWF (rank

$= 5$) in non-stationary scenario in Figure 6. There are 3 wideband interferences as in Table 2 and 1 wideband interference with the power of SIR $= -40$ dB comes from $-40°$ at 500th snapshot, where the step-size of the component filter LMS is 1×10^{-7}, and the ones of the traditional NLMS and RLS algorithms are 1×10^{-4} and 0.995. Again, $\alpha = 1 \times 10^{-7}$ and $\mu = 0.995$ are the step-sizes of the proposed $\boldsymbol{\eta}$-LMS and $\boldsymbol{\eta}$-RLS, respectively. Simultaneously, other rest of specific parameters are as shown in Table 2. We observe that when an unexpected wideband interference starts suddenly at 500th snapshot, the output SINRs of all filters slump down quickly because of the effect of outdated data. In particular, the output SINRs of the Lanczos reduced-rank MNWF (rank $= 4$ and rank $= 5$) fall down significantly caused by erroneous estimate of the auto-correlation matrix $\mathbf{R}_{\mathbf{x}_o}(n)$ and the cross-correlation vector $\mathbf{r}_{\mathbf{x}_o,d}(n)$. Comparably, the proposed approach recovers faster than any other algorithms, especially for $\boldsymbol{\eta}$-RLS and $\boldsymbol{\eta}$-PRLS. Their EMSEs are changed by $\mathrm{Tr}(\mathbf{QR}_{\tilde{\mathbf{x}}_o})$ as proved in Theorem 1, which enable filters to update their weights timely by acquired new data and enhance the proposed overall filter's ability to minimize the output signal variance under constraints.

We also investigate the effect of SINR loss at the output of beamformer, when a jammer arrives from different DOAs as [42] in Figure 7. There are 2 wideband interferences coming from $40°$ and $-40°$, and their SIRs are -30 dB and -50 dB, respectively. And one wideband interference with the power of SIR $= -40$ dB is arriving over a range of DOA from $90°$ to $-90°$, where the snapshot is $n = 400$.

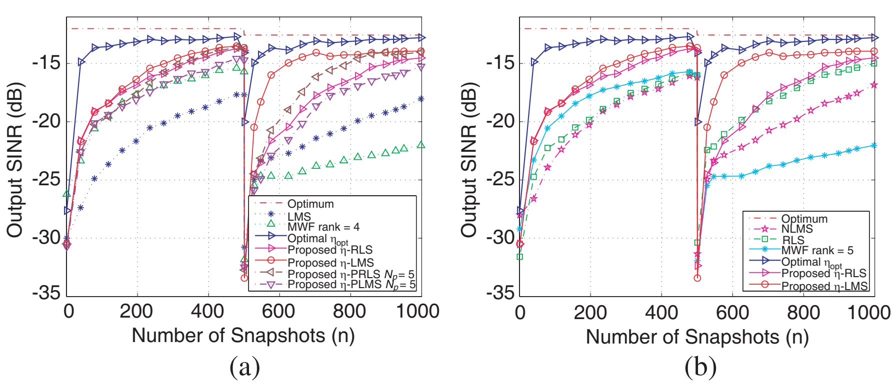
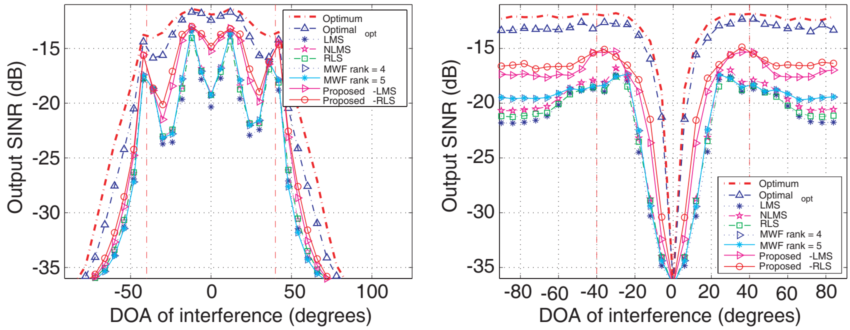


Figure 6. Output SINR performances evaluation of the proposed adaptive combination approach with iterative algorithms in a nonstationary scenario. (a) Relationship of the combined filters and their component filters. (b) Comparison of the proposed iterative algorithms with other adaptive algorithms.



(a) (b)

Figure 7. DOAs of the interference versus output SINRs, when the DOA of desired signal arrives from two directions (a) The DOA of desired signal is $\theta_d = 90°$. (b) The DOA of desired signal is $\theta_d = 0°$.

The step-sizes of all filters and rest of specific parameters are the same as Figure 6. From Figure 7, the proposed beamformer reveals less fluctuation than conventional approach in output SINR as expected, when the interference is arriving outside the main beam of the array. This is true for both $\theta_d = 0°$ and $\theta_d = 90°$.

10. CONCLUSION

To sum up, there are four primary contributions of this paper: i) A novel versatile array beamforming approach using an affine combination of two different adaptive filters by a mixing vector is proposed to mitigate interference of sensor array. ii) The optimal combination vector and its iterative algorithms are provided based on LC-GSC for implementation. iii) The theoretical MSEs, EMSEs and SINRs of the proposed algorithms are studied which show that the proposed approach has fast convergent rate and high output SINR by employing appropriately component filters. iv) Besides, the complexities of the proposed iterative algorithm are discussed as well in terms of additions and multiplications to verify the effectiveness of the algorithm in practical application. In conclusion, the proposed methods are developed to achieve a good performance among the convergence rate, tracking ability, robustness and complexity over comprehensive considerations. Simulation results for space-time processing of antenna array illustrate that the proposed adaptive filter greatly improves the two sub-filters' performances by utilizing the two individual filters from two families, LMS and Lanczos reduced-rank MNWF, respectively.

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