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COMPARISON OF THE GUM AND MONTE CARLO METHODS FOR THE UNCERTAINTY ESTIMATION IN ELECTROMAGNETIC COMPATIBILITY TESTING

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Abstract—The rigorous uncertainty estimation in Electromagnetic Compatibility (EMC) testing is a complex task that has been addressed through a simplified approach that typically assumes that all the contributions are uncorrelated and symmetric, and combine them in a linear or linearized model using the error propagation law. These assumptions may affect the reliability of test results, and therefore, it is advisable to use alternative methods, such as Monte Carlo Method (MCM), for the calculation and validation of measurement uncertainty. This paper presents the results of the estimation of uncertainty for some of the most common EMC tests, such as: the measurement of radiated and conducted emissions according to CISPR 22 and radiated (IEC 61000-4-3) and conducted (IEC 61000-4-6) immunity, using both the conventional techniques of the Guide to the Expression of Uncertainty in Measurement (GUM) and the Monte Carlo Method. The results show no significant differences between the uncertainty estimated using the aforementioned methods, and it was observed that the GUM uncertainty framework slightly overestimates the overall uncertainty for the cases evaluated here. Although the GUM Uncertainty Framework proves to be adequate for the particular EMC tests that were considered, generally the Monte Carlo Method has features that avoid the assumptions and the limitations of the GUM Uncertainty Framework.

1. INTRODUCTION

Accredited test laboratories are required to estimate the uncertainty and to report it, according to the ISO/IEC 17025 — "General requirements for the competence of testing and calibration laboratories" [1].

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The aforementioned standard also recognizes that the complexity of the measurement systems may in some cases preclude a rigorous evaluation of uncertainty, as happens in Electromagnetic Compatibility (EMC) Testing.

The uncertainty estimation in EMC test is complex because commonly: the equipment that is being tested was not designed specifically for the test; the methods usually include setup factors that affect the measurement; the test system is itself complex and includes several separate but interconnected components (cables, antennas, spectrum analyzers, filters, amplifiers and preamplifiers, attenuators, RF switches, and others) and the quantities involved may be electromagnetic fields, varying in space, and may be transient or continuous [2]. Such complexity has been addressed through a conventional approach that attempts to simplify the calculation process while maintaining practical veracity in the uncertainties produced [3].

The conventional uncertainty analysis, stated in the Guide to the Expression of Uncertainty in Measurement (GUM) [4], is often difficult to implement in complex systems, such as EMC test systems, and requires several approximations and assumptions at each stage of processing which may cast doubt on the results of the tests. In that sense, the analysis of uncertainty in EMC testing using Monte Carlo Methods is an alternative to resolve many of the problems associated to the GUM uncertainty framework, including non-symmetrical measurement uncertainty distributions, non-linearity within the measurement system, input dependency and systematic bias [5]. Nowadays, the Monte Carlo Method (MCM) is recognized as a practical alternative by the Joint Committee for Guides in Metrology (JCGM) of the Bureau International des Poids et Mesureson (BIPM) and it has been included in the GUM as a supplement, since 2008 [6], and it has been widely used within many scientific disciplines, such as metrology, geodesy, optics, hydrology, electronics, structural mechanics, and various others [7–11].

The paper is organized as follows: on Section 2 it will be described the general methodology followed by the conventional uncertainty estimation and then, on Section 3, it will be explained the MCM approach used to describe measurement uncertainties in the context of GUM, next, on Sections 4 and 5 the conventional, and the MCM techniques will be applied in the estimation of the uncertainty of some of the most important EMC tests, respectively, and finally, a comparison between the results was performed to reach the conclusions of this study.

2. GUM UNCERTAINTY FRAMEWORK IN EMC TESTING

The functional relationship (i.e., measurement model or equation) between the measured (quantity intended for measurement) Y and the set of input quantities $\{X_1, X_2, \ldots, X_N\}$ in a EMC test measurement process is given by,

$$Y = f(X_1, X_2, \ldots, X_N). \tag{1}$$

The measurement model f includes both corrections for systematic effects and accounts for sources of variability, such as those due to different observers, instruments, EUT (Equipment Under Test), laboratories and times at which observations are made. Therefore, the general functional relationship expresses not only a physical law but also a measurement process. Some of these variables can be controlled directly or indirectly, others can be observed but not controlled and some others cannot even be observed.

An estimate of the measured Y, denoted by y, is obtained from (1) using input estimates $\{x_1, x_2, \ldots, x_N\}$ for the values of the N quantities $\{X_1, X_2, \ldots, X_N\}$. Thus the output estimate y, which is the result of the measurement, is given by,

$$y = f(x_1, x_2, \ldots, x_N). \tag{2}$$

The estimated standard deviation associated with the output estimate or measurement result y, is defined as the combined standard uncertainty and denoted by $u_c(y)$, and is determined from the estimated standard deviation associated with each input estimate x_i, termed the standard uncertainty and denoted by $u(x_i)$ [4].

Each input estimate x_i and its associated standard uncertainty $u(x_i)$ are obtained from a distribution of possible values of the input quantity X_i. This probability distribution may be experimentally determined, that is, based on a series of observations $X_{i,j}$ of X_i, or it may be an a-priori distribution. Type A evaluations of standard uncertainty components are founded on frequency distributions while Type B evaluations are founded on a-priori distributions. It must be recognized that in both cases, the distributions are models that are used to represent the state of our knowledge [4].

2.1. Contributions from Type A Evaluations

The estimation of the contribution from Type A evaluations of standard uncertainty of an input parameter is done by calculation from a series of repeated observations, using statistical methods, and resulting in an experimental probability distribution that is assumed to be normal. For any measurement method, this kind of contributions should be evaluated following the procedure and configuration that are typically involved in the test, using, if necessary, a standardized EUT. All the observations must be independent and obtained under the same conditions of measurement. The result will give a measure of the likely contribution due to random fluctuations, for instance, uncontrolled variations in the antenna position, the test environment, or losses through cable reconnection [2]. Generally, the diversity of EUT makes impractical to perform many repeated measurements on each type, make and model of EUT. A predetermination of the uncertainty due to random contributions is given by the experimental standard deviation $s(x_i)$ of a series of m such measurements $X_{i,j}$,

$$u(x_i) = s(X_{i,j}) = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}\left(X_{i,j} - \overline{X_i}\right)^2}, \tag{3}$$

where, $\overline{X_i}$ is the experimental average of the m measurements, as shown in (4).

$$\overline{X_i} = \frac{1}{m}\sum_{j=1}^{m} X_{i,j}. \tag{4}$$

Hence, $u(x_i)$ is used directly for the uncertainty due to random contributions, excluding the effects of the EUT, when only one measurement ($m = 1$) is made on the EUT and its result is the best estimate of the input parameter, that is, $x_i = X_{i,1}$. But if the result of the measurement is close to a non compliance respect to the standard that is being evaluated, it is advisable to perform several measurements on the EUT itself, at least at those frequencies that are critical. In this case ($x_i = \overline{X_i}$), and if the measurements are independent, the uncertainty is reduced to the standard deviation of the mean, as shown in (5).

$$u(x_i) = \frac{s(X_{i,j})}{\sqrt{m}}. \tag{5}$$

2.2. Contributions from Type B Evaluations

For an estimate x_i of an input quantity X_i that has not been obtained from repeated observations, the associated estimated standard uncertainty $u(x_i)$ is evaluated by scientific judgment based on all the available information on the possible variability of X_i [4]. The pool of information may include: previous measurements, experience, understanding of instrument behavior, manufacturer's specifications, data provided in calibration certificates (even if the reported uncertainty is calculated from results of replicate measurements) and uncertainties assigned to reference data taken from handbooks. This kind of contributions may be used to model systematic effects, that is, those that remain constant during the measurement but which may change if the measurement conditions, methods or equipments are altered [3]. If there is any doubt about whether a contribution is significant, or not, it should be included in the uncertainty budget in order to demonstrate that it has been considered. Some examples of uncertainty contributions from Type B evaluations in EMC test are: equipment calibration, mismatch errors, coupling effects, errors due to constant deviations in the physical setup (antenna height, separation form EUT and alignment),and instrument accuracy. If it is possible and practical, corrections for systematic effects should be applied.

In the EMC testing uncertainty estimations of contributions form Type B evaluations, it is required to choose a probability distribution that models each source of variability. The probability distributions that have been demonstrated to be more relevant, in EMC testing, are: normal for uncertainties derived from multiple contributions, such as calibration uncertainties with a statement of confidence; rectangular distribution for contributions taken from manufacturers' specifications

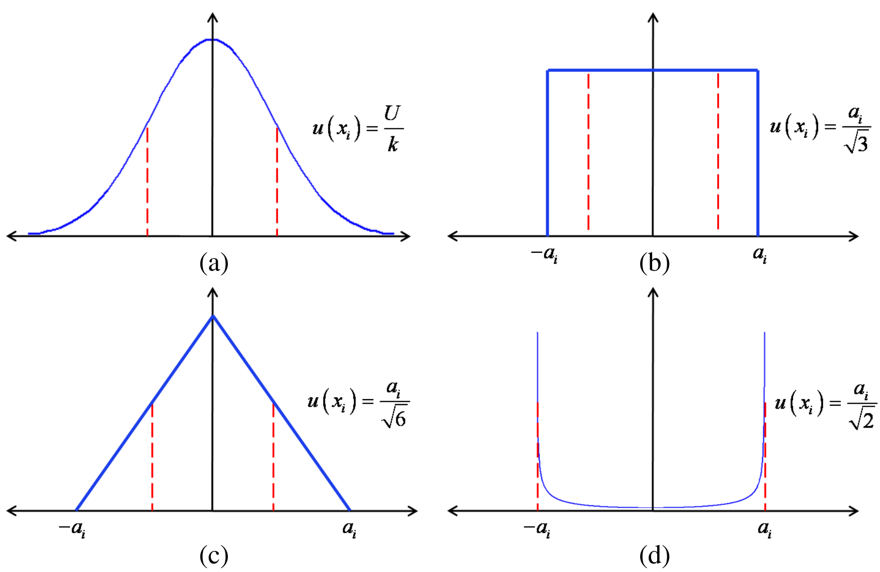

Figure 1. Probability density functions commonly used for Type B evaluations of uncertainty in EMC testing. (a) Normal. (b) Uniform. (c) Triangular and (d) U-shaped.

and reading from digital instruments, triangular distribution when the contribution has defined limits and where the majority of the values between the limits lie around the central point like the measurements taken with analog instruments and the U-shaped distribution is applicable to mismatch uncertainty, where the probability of the true value being close to the measured value is low [2, 3]. The standard uncertainty associated with the probability distributions quoted above, are shown in Figure 1.

It should be recognized that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation, especially in a measurement situation where a Type A evaluation is based on a comparatively small number of statistically independent observations [3].

It is important not to "double-count" uncertainty components. If a component of uncertainty arising from a particular effect is obtained from a Type B evaluation, it should be included as an independent component of uncertainty in the calculation of the combined standard uncertainty of the measurement result only to the extent that the effect does not contribute to the observed variability of the observations. This is because the uncertainty due to that portion of the effect that contributes to the observed variability is already included in the component of uncertainty obtained from the statistical analysis of the observations [4].

2.3. The Combined Standard Uncertainty

In order to calculate the combined standard uncertainty $u_c(y)$, the conventional approach uses the law of propagation of uncertainty based on a first-order Taylor series approximation of Y. This law, for the case of correlated input quantities, is given by [4],

$$u_c(y) = \sqrt{\sum_{i=1}^{N} {c_i}^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j)}, \tag{6}$$

where, c_i is the estimated sensitivity coefficients (7) and $r(x_i, x_j)$ is the estimated correlation coefficient (8) calculated using the covariance associated with x_i and x_j, $u(x_i, x_j)$.

$$c_i = \left.\frac{\partial f}{\partial X_i}\right|_{X_i = x_i}, \tag{7}$$

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i) u(x_j)}. \tag{8}$$

Usually in the EMC tests, the uncertainty budgets are calculated assuming that the measured variables are uncorrelated and that the value of the sensitivity coefficients is one (1) [3]. It is appreciated that this approach is mathematically imprecise. However, it is assumed that the approximations involved, do not cause an overall change in the expanded uncertainty of any significance and well within the five-percent criteria [2]. Consequently,the Equation (6) comes down to,

$$u_c(y) = \sqrt{\sum_{i=1}^{N} u^2(x_i)}. \tag{9}$$

2.4. The Expanded Uncertainty

The expanded uncertainty, U, defines an interval about the measurement result that will include the true value with a specified level of confidence. The expanded uncertainty is obtained by multiplying the combined standard uncertainty by a coverage factor, k, which is set to approximately 2 (1.96) for a level of confidence of 95% assuming that the results are normally distributed. The result is then expressed as follows,

$$Y = y \pm U = y \pm k u_c(y). \tag{10}$$

2.5. Disadvantages in Using the GUM Method of Estimating Uncertainties in EMC Testing

Summarizing, the greatest disadvantages presented in the conventional mathematical treatment of uncertainties in EMC testing are:

- Generally, the model of the measurement system is not linear. Hence, the expected value of the system output is not equal to the system output to the expected values of the inputs, that is, $E(y) \neq f(E(x_1), E(x_2), \ldots, E(x_N))$ [6]. However, some of the measurement models used in the EMC tests can be linearized by the decibel's logarithmic transformation of the measurements.
- Input variables may be correlated with each other, that is, $r(x_i, x_j) \neq 0$ [3].
- The sensitivity coefficients, generally, differs from the assumed value of 1, that is, $c_i \neq 1$ [6].
- The result is not necessarily normally distributed given the asymmetries of some contributions, such as, mismatch effects.
- The rigorous mathematical treatment of uncertainties using the conventional approach, in complex systems like those involved in EMC testing is undoubtedly impractical [2].

In this regard, an alternative is to use the Monte Carlo Method, which will be explained in general terms below.

3. UNCERTAINTY ESTIMATION USING MONTE CARLO METHOD

In Monte Carlo techniques, both, the random and the systematic components of the uncertainty are treated as having a random nature. It is important to notice that the systematic component is not modeled as random, it is the knowledge about it for which a probability distribution is introduced [6].

This method basically consists in randomly generate a number M of Monte Carlo trials (i.e., the number of model evaluations made) in where the distribution function of the value of the output quantity Y will be numerically approximated. It is further assumed that the probability densities of the considered input quantities are a priori known. Then, a sample vector of the input quantities can be drawn repeatedly using pseudo random number generators. For each input sample vector, the corresponding values of the output quantities are calculated by using the corresponding functional relation. The set of output sample vectors yields an empirical distribution which can be used to approximate the correct random distribution of the output quantities. All required measures (expectation value, variance and covariance) as well as higher-order central moments such as skewness and kurtosis can then be derived [7]. Before applying the MCM, the conditions for valid application should be verified [6]. It is recommended to use $M \geq 10^6$ to estimate a 95% coverage interval for the output quantity such that this length is correct to one or two significant decimal digits [6]. It is also recommended to validate the quality of the pseudo-random number generator to be used in the calculations.

The MCM is implemented using an algorithm that can be summarized as follows:

(i) There must be generated a set of N input parameters $\{x_1, x_2, \ldots, x_N\}$, which are random variables distributed according to a probability density function assigned to each input parameter. This process should be repeated M times for every input quantity.

(ii) The functional relationship that model the measurement system is then evaluated to obtain the output,

$$y_j = f(x_{1,j}, x_{2,j}, \ldots, x_{N,j}), \tag{11}$$

for $j = 1, 2, \ldots, M$. From this sample, it is possible to estimate the probability density function of y.

(iii) The relevant estimates of any statistical quantity can be calculated (average, variance, skewness and kurtosis of the output, among others).

(iv) The output vector $\{y_1, y_2, \ldots, y_N\}$ is sorted in ascending order to obtain a vector $\tilde{y} = \{\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_N\}$.

(v) The confidence interval $[\tilde{y}_r, \tilde{y}_s]$ is found approximately through (12) and (13) [12]:

$$r = \mathrm{round}((M+1)\gamma), \tag{12}$$

$$s = \mathrm{round}((M+1)(1-\gamma)), \tag{13}$$

where, γ is the significance level ($\gamma = 0.025$ for 95% of confidence) and the function round(x) is used to represent the nearest integer to x.

4. TYPICAL UNCERTAINTY BUDGETS IN EMC TESTING

This section presents four examples of typical uncertainty budgets relating to four different tests, two of which correspond to electromagnetic interference (EMI) and the other two referred to electromagnetic susceptibility (EMS). The examples were taken from the document "The Expression of Uncertainty in EMC Testing" of the United Kingdom Accreditation Service (UKAS) [3], with minor changes. The approach adopted for emission measurements, including sensitivity coefficients generally is to follow the methodology currently being implemented in the Comité International Spécial des Perturbations Radioélectriques, CISPR [13]. The contributions and values in the following examples are used for reference only for the later comparison with the results obtained applying the MCM. Laboratories shall determine the uncertainty contributions for the tests they are performing based on their own derived and available data.

4.1. Conducted Emissions According CISPR 22

The conducted disturbance level, C_{DL}, is modeled through the mathematical relationship shown in (14).

$$\begin{aligned} C_{DL} = & R_I + L_C + L_{AMN} + dV_{SW} + dV_{PA} + dV_{PR} \\ & + dV_{NF} + dZ + F_{ST} + M_M + R_S, \end{aligned} \tag{14}$$

where, R_I, L_C, L_{AMN}, dV_{SW}, dV_{PA}, dV_{PR}, dV_{NF}, dZ, F_{ST}, M_M and R_S represent, Receiver reading, Attenuation AMN (Artificial

Table 1. Uncertainty budget for conducted emissions according CISPR 22.

Source of Uncertainty	Type	Value B1	Value B2	Probability Distribution	c_i	u_i B1	u_i B2
R_I	B	0.05	0.05	Rectangular	1	0.03	0.03
L_C	B	0.40	0.40	Normal	1	0.20	0.20
L_{AMN}	B	0.20	0.20	Normal	1	0.10	0.10
dV_{SW}	B	1.00	1.00	Rectangular	1	0.58	0.58
dV_{PA}	B	1.50	1.50	Rectangular	1	0.87	0.87
dV_{PR}	B	1.50	1.50	Rectangular	1	0.87	0.87
dV_{NF}	B	0.00	0.00	Rectangular	1	0.00	0.00
dZ	B	3.60	2.70	Triangular	1	1.47	1.10
F_{ST}	B	0.00	0.00	Rectangular	1	0.00	0.00
M_M	A	0.89	0.89	U-Shaped	1	0.63	0.63
R_S	A	0.50	0.50	Normal	1	0.50	0.50
					$u_c =$	**2.17**	**1.94**
					$U =$	**4.3**	**3.9**

mains network) — Receiver, AMN voltage division factor, Receiver error due to sine wave voltage, Receiver error due to Pulse amplitude response, Receiver error due to Pulse repetition response, Error due to finite signal to noise ratio, AMN Impedance, Frequency step error, Mismatch and Measurement system repeatability, respectively [3].

The conventional uncertainty estimation is calculated using the budget shown in Table 1. The budget is divided into two bands: 9 kHz to 150 kHz (B1) and 150 kHZ to 30 MHz (B2), due to changes in the uncertainty contribution of the AMN impedance.

Therefore, the expanded uncertainty of the conducted disturbance level measurement estimated through the conventional approach, for a coverage factor, $k = 2$ (95% confidence level), is ± 4.3 dB between 9 kHz and 150 kHz and, ± 3.9 dB between 150 kHz and 30 MHz.

4.2. Radiated Field Strength Measurement According CISPR 22

The radiated filed strength, F_S, is modeled through the mathematical relationship shown in (15).

$$\begin{aligned} F_S = & \; R_I + dV_{SW} + dV_{PA} + dV_{PR} + A_F + C_L + A_D + A_H \\ & + A_P + A_I + S_I + D_V + D_B + D_C + F_{ST} + M_M + R_S, \end{aligned} \quad (15)$$

where, R_I, dV_{SW}, dV_{PA}, dV_{PR}, A_F, C_L, A_D, A_H, A_P, A_I, S_I, D_V, D_B, D_C, F_{ST}, M_M and R_S represent, Receiver indication, Receiver error due to sine-wave voltage, Receiver error due to Pulse amplitude response, Receiver error due to Pulse repetition response, Error due to finite signal to noise ratio, Antenna factor calibration uncertainty, Cable loss uncertainty, Antenna directivity uncertainty, Antenna factor height dependence, Mismatch and Measurement system repeatability, respectively [3].

The conventional uncertainty estimation is calculated using the budget shown in Table 2. The budget is divided into two bands: 30 MHz to 300 MHz (B1) and 300 MHz to 1 GHz (B2), due to changes between different antennas. Both bands were measured at vertical polarization and at 3 m measurement distance from EUT.

Hence, the expanded uncertainty of the Radiated Field Strength measurement estimated through the conventional approach, for a coverage factor, $k = 2$ (95% confidence level), is ±5.4 dB between

Table 2. Uncertainty budget for radiated field strength measurement according CISPR 22.

Source of Uncertainty	**Type**	**Value B1**	**Value B2**	**Probability Distribution**	c_i	u_i **B1**	u_i **B2**
R_I	B	0.05	0.05	Rectangular	1	0.03	0.03
dV_{SW}	B	1.00	1.00	Rectangular	1	0.50	0.50
dV_{PA}	B	1.50	1.50	Rectangular	1	0.87	0.87
dV_{PR}	B	1.50	1.50	Rectangular	1	0.87	0.87
dV_{NF}	B	0.50	0.50	Normal	1	0.25	0.25
A_F	B	1.00	1.00	Normal	1	0.50	0.50
C_L	B	0.50	0.50	Normal	1	0.25	0.25
A_D	B	0.00	3.00	Triangular	1	0.00	1.73
A_H	B	2.00	0.50	Rectangular	1	1.15	0.29
A_P	B	0.00	1.00	Rectangular	1	0.00	0.58
A_I	B	0.25	0.25	Rectangular	1	0.14	0.14
S_I	B	4.00	4.00	Triangular	1	1.63	1.63
D_V	B	0.60	0.60	Rectangular	1	0.35	0.35
D_B	B	0.30	0.30	Rectangular	1	0.17	0.17
D_C	B	0.00	0.90	Rectangular	1	0.00	0.52
F_{ST}	B	0.00	0.00	Rectangular	1	0.00	0.00
M_M	A	−1.25	−0.54	U-Shaped	1	−0.88	−0.38
R_S	A	0.50	0.50	Normal	1	0.50	0.50
					$u_c =$	**2.71**	**3.00**
					$U =$	**5.4**	**6.0**

30 MHz and 300 MHz and, ±6.0 dB between 300 MHz and 1 GHz.

4.3. Radiated Immunity According IEC 61000-4-3

The mathematical model for the measurement process of the Radiated Filed Strength, F_S, is shown in (16).

$$F_S = F_{SM} + F_{SAW} + P_D + P_{AH} + F_D + F_{EUT} + R_S, \qquad (16)$$

where, D_{SM}, F_{SAW}, P_D, P_{AH}, F_D and R_S represent, the uncertainty of the field probe used for monitoring the field strength, the field strength acceptability window, the drift in the forward power measurement, errors due to harmonics of the power amplifier, the effect of field disturbance and the measurement system repeatability, respectively [3]. The uncertainty contribution associated to the antenna-EUT coupling and reflections due to EUT presence, F_{EUT}, was added to the budget presented in [3] because it represents a significant source of uncertainty [14].

It is remarkable that in order to perform EMC immunity/susceptibility testing against a specified interference level it is recommended that, in the absence of other guidance, the test shall be performed at the specified immunity level increased by the standard uncertainty multiplied by a coverage factor k of 1.64 which under normal circumstances would give a confidence of approximately 90%, however, because the level is increased by this amount, there are only a 5% of probability that the immunity level remains below the required, hence it is achieved a confidence of 95% that the immunity level has been applied.

Table 3. Uncertainty budget for radiated immunity according IEC 61000-4-3.

Source of Uncertainty	**Type**	**Value**	**Probability Distribution**	c_i	u_i
F_{SM}	B	1.20	Normal	1	0.60
F_{SAW}	B	0.50	Rectangular	1	0.29
P_D	B	0.20	Rectangular	1	0.12
P_{AH}	B	0.35	Rectangular	1	0.20
F_D	B	0.35	Rectangular	1	0.29
F_{EUT}	B	0.32	U-Shaped	1	0.23
R_S	A	0.50	Normal	1	0.50
				$u_c =$	**0.94**
				$U =$	**1.5**

The conventional uncertainty estimation is calculated using the budget shown in Table 3. The budget is valid only for the "Re-establishment of precalibrated field level" method. The measurement uncertainty budget below is based on the assumption that it has been demonstrated during calibration that the 6 dB field uniformity has been achieved.

Hence, the expanded uncertainty of the Radiated Field Strength Immunity Level estimated through the conventional approach, for a coverage factor, $k = 1.64$ (90% confidence level), is ± 1.5 dB.

4.4. Conducted Immunity According IEC 61000-4-6

The mathematical model for the conducted induced voltage level, C_{VL}, is given by (17),

$$C_{VL} = V_{RMS} + V_{LAW} + P_D + P_{AH} + M_{VC} + M_{AC} + R_S, \tag{17}$$

where V_{RMS}, V_{LAW}, P_D, P_{AH}, M_{VC}, M_{AC} and R_S represent, the maximum induced current, the error specified for the RMS Voltmeter, the Voltage level acceptability window, the signal generator drift, the power amplifier harmonics, the contribution of the current coil, error due to spectrum analyzer, mismatch between voltmeter and CDN, mismatch between amplifiers and CDN and the measurement system repeatability, respectively [3]. This model only applies when the test is performed according to the CDN method described in IEC 61000-4-6. For this type of test, the conventional uncertainty estimation is calculated using the budget shown in Table 4.

Table 4. Uncertainty budget for conducted immunity according IEC 61000-4-6.

Source of Uncertainty	**Type**	**Value**	**Probability Distribution**	c_i	u_i
V_{RMS}	B	0.70	Rectangular	1	0.40
V_{LAW}	B	0.50	Rectangular	1	0.29
P_D	B	0.2	Rectangular	1	0.12
P_{AH}	B	0.70	Rectangular	1	0.40
M_{VC}	B	−0.54	U-Shaped	1	−0.38
M_{AC}	B	−0.16	U-Shaped	1	−0.82
R_S	A	0.50	Normal	1	0.50
				$u_c =$	**1.22**
				$U =$	**2.0**

Therefore, the expanded uncertainty of the Conducted Induced Voltage Level estimated through the conventional approach, for a coverage factor, $k = 1.64$ (90% confidence level), is ± 2.0 dB.

5. UNCERTAINTY ESTIMATION IN EMC TESTING USING MONTE CARLO SIMULATION METHOD

The expanded uncertainty of the aforementioned typical EMC test was estimated applying the MCM for $M = 10^6$ and taking the same probability density functions as proposed in [3] for the pseudo random number generators. The results are given in the following subsections.

5.1. Conducted Emissions According CISPR 22

The conducted disturbance level, C_{DL}, is modeled through the mathematical relationship shown in (14). The expanded uncertainty of C_{DL} estimated using the MCM is ± 4.2 dB for Band 1 and ± 3.8 dB for Band 2.

The histograms of relative frequency of the error in the conducted disturbance level measurement are shown in Figures 2 and 3. It is observed that the error in the conducted disturbance level measurement, follows an approximately normal distribution.

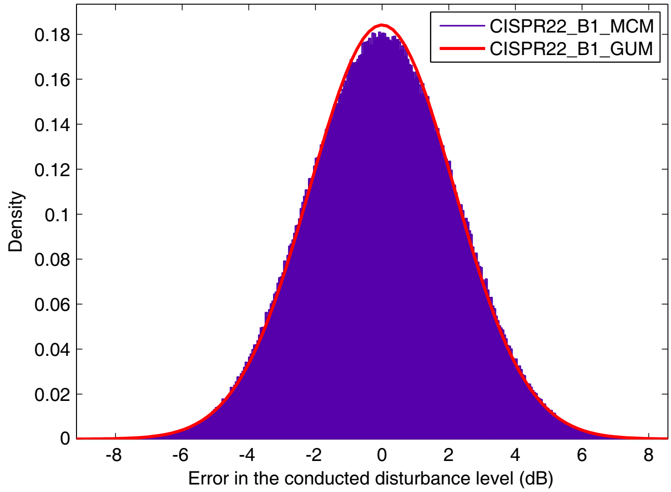

Figure 2. Histogram of relative frequency of the error in the conducted disturbance level measurements between 9 kHz and 150 kHz.

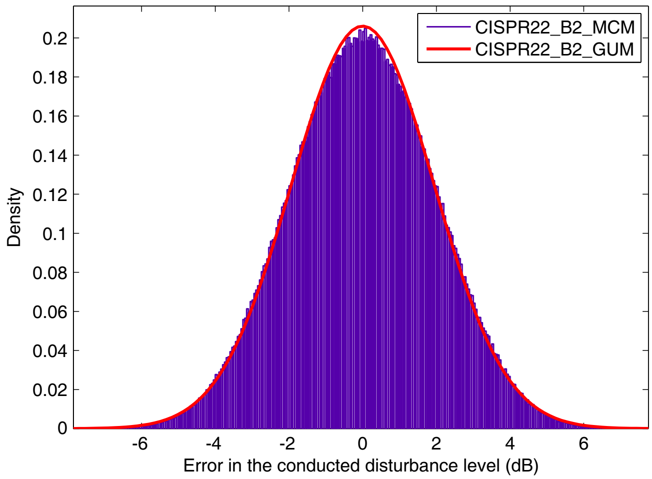

Figure 3. Histogram of relative frequency of the error in the conducted disturbance level measurements between 150 kHz and 30 MHz.

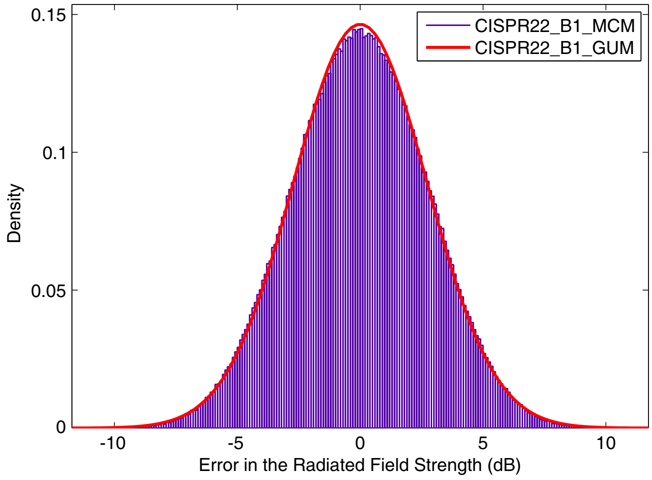

Figure 4. Histogram of relative frequency of the error in the radiated field strength measurements between 30 MHz and 300 MHz.

5.2. Radiated Field Strength Measurement According CISPR 22

The Radiated Field Strength, F_S, is modeled through the mathematical relationship shown in (15). The expanded uncertainty of F_S estimated using the MCM is ± 5.3 dB for Band 1 and ± 5.9 dB for Band 2.

The histograms of relative frequency of the error in the conducted disturbance level measurement are shown in Figures 4 and 5. It is observed that the error in the radiated field strength measurement, follows an approximately normal distribution.

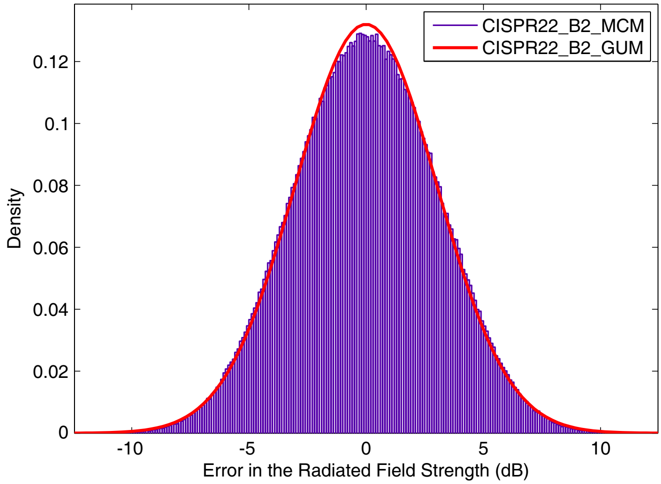

Figure 5. Histogram of relative frequency of the error in the conducted disturbance level measurements between 300 MHz and 1 GHz.

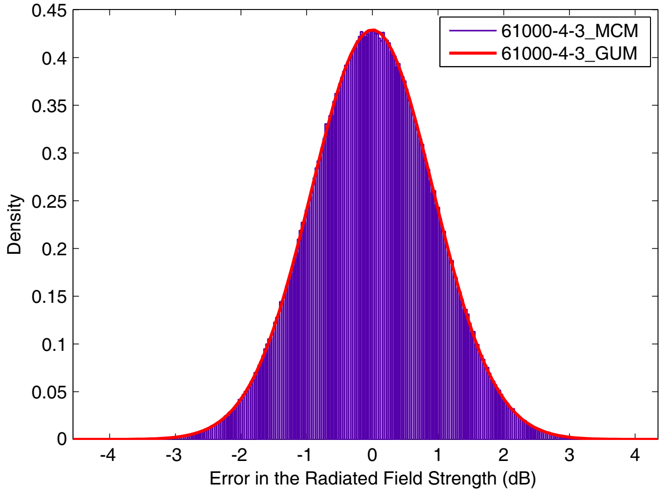

Figure 6. Histogram of relative frequency of the error in the radiated field strength applied.

5.3. Radiated Immunity According IEC 61000-4-3

The Radiated Field Strength, F_S, is modeled through the mathematical relationship shown in (16). The expanded uncertainty of F_S estimated using the MCM is ± 1.5 dB.

The histogram of relative frequency of the error in the radiated field strength applied is shown in Figure 6. It is observed that the error in the radiated field strength applied, follows an approximately normal distribution.

5.4. Conducted Immunity According IEC 61000-4-6

The Conducted Induced Voltage Level, C_{VL}, is modeled through the mathematical relationship shown in (17). The expanded uncertainty of C_{VL} estimated using the MCM is ± 2.0 dB. that even being more complex The histogram of relative frequency of the error in the Conducted Induced Voltage Level applied is shown in Figure 7. It is observed that the error in C_{VL}, follows an approximately normal distribution.

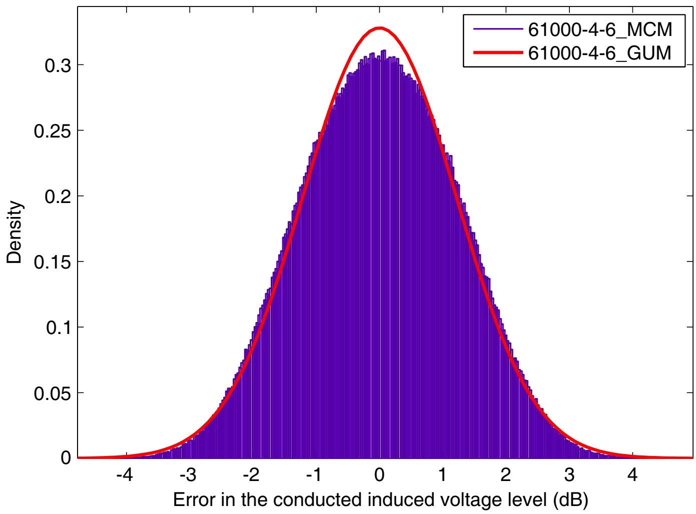

Figure 7. Histogram of relative frequency of the error in the conducted induced voltage Level applied.

6. CONCLUSIONS

The expanded uncertainty results estimated with the GUM Uncertainty Framework and the Monte Carlo Method showed no significant differences. This is because of the linearity of the measurement models used for each of the tests that were analyzed and the independence of effects of the uncertainty contributions considered. In all the cases evaluated, the uncertainty estimated using the GUM approach slightly overestimated the results obtained with the MCM. The largest difference found in the estimated uncertainties correspond to the Radiated Filed Strength measurement, and it was about 0.14 dB (3.3% of the reading). The combined error in the measurements of the particular EMC test that were considered are approximately normally distributed. The symmetry of the expanded uncertainty was to be expected because of the linearity of the models, and the symmetry of the probability density functions assigned to the contributions. The asymmetry of some contributions (such as, mismatch losses) was not considered, because it would have made it impossible to compare the results with the ones presented by the UKAS-LAB34. Although the GUM Uncertainty Framework proves to be adequate in these particular cases, generally the Monte Carlo Method has features that avoid the assumptions and the limitations of the GUM Uncertainty Framework, so we encourage EMC testing laboratories to model mathematically their particular measurement systems and validate the estimated uncertainty of the test using the MCM, incorporating different probability density functions that best fit to the random behavior of the sources of uncertainty.

APPENDIX A. TERMS AND DEFINITIONS

This paper uses terminology that has a specific definition in international standardized vocabularies. The terms related to metrology can be consulted in the "International vocabulary of metrology" (VIM) [15], which is available free of charge on the BIPMs web site. The terms associated with electromagnetic compatibility can be consulted in the International Electrotechnical Vocabulary [16].

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