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# DISPERSION AND PEAK REFLECTIVITY ANALYSIS IN A NON-UNIFORM FBG BASED SENSORS DUE TO AN ARBITRARY REFRACTIVE INDEX PROFILE

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**Abstract**—This paper deals with a group velocity dispersion issue and a peak reflectivity issue in a non-uniform fiber Bragg gratings (FBG) due to an arbitrary refractive index profile along the length of grating. The paper shows that by using more complicated refractive index profile one can significantly reduce the group velocity dispersion and side lobes intensity and that in main lobe the bandwidth of reflectivity would also increase substantially due to a complicated refractive index profile. To the authors' knowledge, there has not been any work reported in this direction. Generally, coupled mode theory is used to analyze the uniform fiber Bragg grating (UFBG). The analysis results in two coupled first order ordinary differential equations with constant coefficients for which closed form solutions can be found for appropriate boundary conditions. Most fiber gratings designed for practical applications however are non uniform. The main reason for using non uniform grating is that it reduces the side lobes in the reflectivity spectrum. Due to the complexity of analysis, no particular method for an analysis of the non-uniform fiber Bragg grating would be found. The two standard approaches for calculating the reflection and transmission spectra of a non uniform FBG are direct numerical integration of coupled mode equations and piecewise uniform approximation approach. The former is more accurate but computationally intensive. In this paper, piecewise uniform approximation approach is used to study a dispersion characteristic due to an arbitrary refractive index profile. The usefulness in FBG based sensors has been demonstrated.

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## 1. INTRODUCTION

Grating based devices have received a wide popularity over the competing technologies due to their advantages such as all fiber geometry, versatility, low insertion loss, high return loss and low cost.

In high bit rate telecommunication applications, ultra short pulse propagation through fiber Bragg grating is of primary importance, where the grating can act as a narrow band reflector or pulse compressor [1]. There are two possibilities. One is the spectral bandwidth of the pulse narrower than the grating response bandwidth, and the other is the spectral bandwidth of the incident pulse larger than that of the grating response. In the former case, the FBGs may not introduce data distortion. However, in the later case data distortion takes place. This case is of our interest from both fundamental and application points of view. In this case, the quality of the data after passing through the grating is distorted due to delay in the signal which is also called the group velocity dispersion broadening and deformations of the data pulse and Inter Symbol Interference (ISI) in the neighboring and distance time slot. Here the reflectivity function of a uniform fiber Bragg grating has, in general, large out of band side lobes. Moreover, the peak reflectivity bandwidth has also narrowed. In dense wavelength division multiplexing (DWDM) systems where the channels are to be placed as closely as permitted by data bandwidth, the side-lobes of the grating response introduce crosstalk between the WDM channels. Many of the practical applications for FBG require non uniform grating. One advantage of taking this approach is the reduction of undesirable side lobes that are prevalent in the spectra of uniform gratings. Apodization and chirping of grating can help side lobe reduction and pulse shaping, respectively. Power reflectivity of the non uniform fiber Bragg gratings (NUFBGs) Gaussian apodized and zero mean Gaussian apodized are discussed elsewhere [2]. However, dispersion characteristics due to an arbitrary refractive index profile for the case of NUFBGs have not been discussed in literature to the authors' knowledge. This paper deals with the group velocity dispersion issue and peak reflectivity analysis by controlling the refractive index profile parameters in case of non-uniform fiber Bragg grating. The reflection characteristics of an arbitrary non uniform grating can be obtained by standard two approaches. In the first approach, the coupled mode equations are numerically integrated. In the second approach, the grating is divided into discrete uniform sections, and the closed form solutions to each of these uniform sections are combined by multiplying an array of matrices, which are associated with uniform sections. This has proven to be a popular approach. In the next section, we will discuss these techniques in details.

## 2. COMPUTATIONAL TECHNIQUES: PIECEWISE UNIFORM APPROXIMATION APPROACH

The direct numerical integration approach to solve the coupled mode equations is straightforward. Let the forward modal field be denoted by $R$ and the reflected field be denoted by $S$. The boundary conditions for a Bragg grating of length $L$ are $R(\frac{L}{2}) = 1$ and $S(\frac{L}{2}) = 0$, and then the mode fields for $R$ and $S$ integrate backward from $z = L/2$ to $-L/2$, thus obtaining $R\,(-\frac{L}{2})$ and $(-\frac{L}{2})$. Typically, adaptive step size Runge Kutta numerical integration works well. If at the input end $R$ and $S$ are denoted by $R_i$ and $S_i$ and at the output end by $R_0$ and $S_0$ respectively, the solution to the coupled wave equation can be written in a matrix form as [3]

$$\begin{bmatrix} R_0 \\ S_0 \end{bmatrix} = \begin{bmatrix} A & jB \\ jB & A^* \end{bmatrix} \begin{bmatrix} R_i \\ S_i \end{bmatrix} \tag{1}$$

where $A = \cosh\gamma L + j\frac{\alpha}{\gamma}\sinh\gamma L$, $B = -\frac{\kappa}{\gamma}\sinh\gamma L$ and $\gamma = \sqrt{\kappa^2 - \alpha^2}$. Here the meaning of the various symbols is as that in literature [3]. Considering the grating of length $L$ varying from $z = 0$ and $L$, the boundary condition leads the amplitude reflection coefficient $\rho = \frac{S_i}{R_i}$ expressed as [3],

$$\rho = -\frac{\kappa\sinh\gamma L}{\alpha\sinh\gamma L + j\gamma\cosh\gamma L} \tag{2}$$

A typical reflectivity response as a function of wavelength has a flat main lobe around center wavelength and decaying side lobes. The analysis of reflectivity response of a non uniform fiber Bragg gratings can be obtained by using the piecewise uniform approximation approach. In this approach, the grating is divided into a number of uniform sections, and the closed form solutions for each uniform section are combined using matrix approach. This method is simple to implement, sufficiently accurate and computationally less intensive. Let the grating of length $L$ be divided into $N$ sections. Each of these sections is assumed to be uniform FBG. If at the input end $R$ and $S$ are denoted by $R_0$ and $S_0$ and at the output end by $R_N$ and $S_N$, respectively, the solution to the coupled-wave equation can be written in a matrix form. For the $n$th section (approximated by a uniform grating) the parameters are taken as the mean parameter of that section. Let the parameters of the $n$th section be denoted by $\alpha_n$ and $\kappa_n$, the solution to the coupled wave equation can be written in a matrix form as,

$$\begin{bmatrix} R_n \\ S_n \end{bmatrix} = \begin{bmatrix} A_n & jB_n \\ jB_n & A_n^* \end{bmatrix} \begin{bmatrix} R_{n-1} \\ S_{n-1} \end{bmatrix} = [T_n] \begin{bmatrix} R_{n-1} \\ S_{n-1} \end{bmatrix} \tag{3}$$

where * indicates complex conjugate. $R_{n-1}$ and $S_{n-1}$ are the input to the $n$th section, and $R_n$ and $S_n$ are the output of the $n$th section. The coefficients $A_n$ and $B_n$ of the transfer matrix $T_n$ for the $n$th section are identical to $A$ and $B$ in Eq. (1) with proper modification such as the parameter $L$ replaced by $\frac{L}{N}$. The parameters $\kappa$, $\alpha$ and $\gamma$ in these equations are replaced by $\kappa_n$, $\alpha_n$ and $\gamma_n$, respectively, where $\kappa_n$, $\alpha_n$ and $\gamma_n$ represent the corresponding values for $\kappa$, $\alpha$ and $\gamma$ for $n$th section. The total transfer characteristics of the grating can then be described by

$$\begin{bmatrix} R_n \\ S_n \end{bmatrix} = T_n T_{n-1} \ldots T_1 \begin{bmatrix} R_0 \\ S_0 \end{bmatrix} \tag{4}$$

Applying the boundary conditions of $S_N$ and $R_0 = 1$, the reflectivity $\rho$ from Eq. (2) of the grating can be obtained. It should be noted that for reasonably accurate result, the grating has to be divided into fine sections. Computations show that $N$ greater than few hundred is required to get the desired accuracy. The grating period must be smaller to be accurate enough. There is an excellent accuracy obtained for a few hundred divisions of $N$ by piecewise method with the analytical method discussed elsewhere [4].

## 3. GROUP VELOCITY DISPERSION ISSUE

Recently, there has been growing interest in the dispersive properties of fiber Bragg gratings for applications such as dispersion compensation, pulse shaping in nonlinear fiber optics and semiconductor components. Many of them rely on the ability to tailor the dispersion in non-uniform gratings. Here we introduce the basis for determining delay and dispersion from the known (complex) reflectivity of a Bragg grating. The group delay and dispersion of the reflected light can be determined from the phase of the amplitude reflection coefficient $\rho$ in Eq. (2). If we denote $\theta_\rho = \text{phase}(\rho)$, then at a local frequency $\omega_0$ we may expand $\theta_\rho$ in a Taylor series about $\omega_0$. Since the first derivative $\frac{d\theta_\rho}{d\omega}$ is directly proportional to the frequency $\omega$, this quantity can be identified as a time delay. Thus, the delay time $\tau_\rho$ for light reflected off of a grating is

$$\tau_\rho = \frac{d\theta_\rho}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d\theta_\rho}{d\lambda} \tag{5}$$

$\tau_\rho$ is usually given in units of picoseconds. We know that for un-chirped uniform gratings both the reflectivity and the delay are symmetric about the center wavelength. Since the dispersion $d_\rho$ (in ps/km) is the rate of change of delay with wavelength, we find [5],

$$d_\rho = \frac{d\tau_\rho}{d\lambda} = \frac{2\tau_\rho}{\lambda} - \frac{\lambda^2}{2\pi c}\frac{d^2\theta_\rho}{d\lambda^2} = -\frac{2\pi c}{\lambda^2}\frac{d^2\theta_\rho}{d\omega^2} \tag{6}$$

In a uniform grating, the dispersion is zero near center wavelength and only becomes appreciable near the band edges and side lobes of the reflection spectrum, where it tends to vary rapidly with wavelength. Figure 1 shows the reflectivity, phase spectra and group velocity dispersion (GVD) of a uniform grating while refractive index profile variation is step index. Here the grating parameters, which correspond to filter bandwidth (BW) of 0.4 nm, are $\kappa L = 20$ and $N = 500000$, where $N$ is the total number of grating periods $N = \frac{L}{\Lambda}$. We consider the length of the grating 4 times larger than UFBG, and the required grating period $\Lambda$ should be too small for enough accuracy. If $N$ were larger or smaller, the reflection bandwidth would be narrower or broader, respectively, for a given value of $\kappa L$.

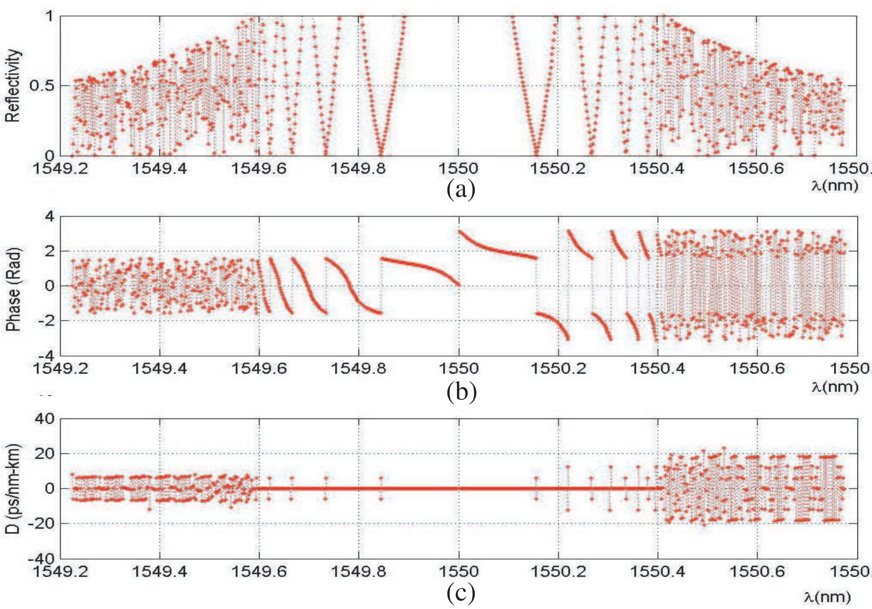


**Figure 1.** (a) Power reflectivity. (b) Phase spectra and (c) group velocity dispersion (GVD) of a uniform FBG having step index variation of refractive index.

## 4. ARBITRARY REFRACTIVE INDEX PROFILE

Non-uniform optical waveguide gratings manufactured by thermo migration or ion exchange are called planar graded optical waveguide grating because the refractive index changes gradually. This fabrication technique may be applied to make the change of refractive index profile along the length of fiber Bragg gratings. The refractive index distribution $n(y)$ (assume that $y$-axis is along the length of grating) can be obtained as follows

$$n(y) = n_s + \Delta n g(y) \tag{7}$$

Here $n_s$ is the refractive index of the substrate, and $\Delta n$ is the maximum amount of change in the refractive index. $g(y)$ describes the distribution function of the refractive index. The value of the distribution function is usually between 1 and 0. Exponential functions, Gaussian functions, error-compensating functions, linear functions, second-order functions, etc. are used as the distribution

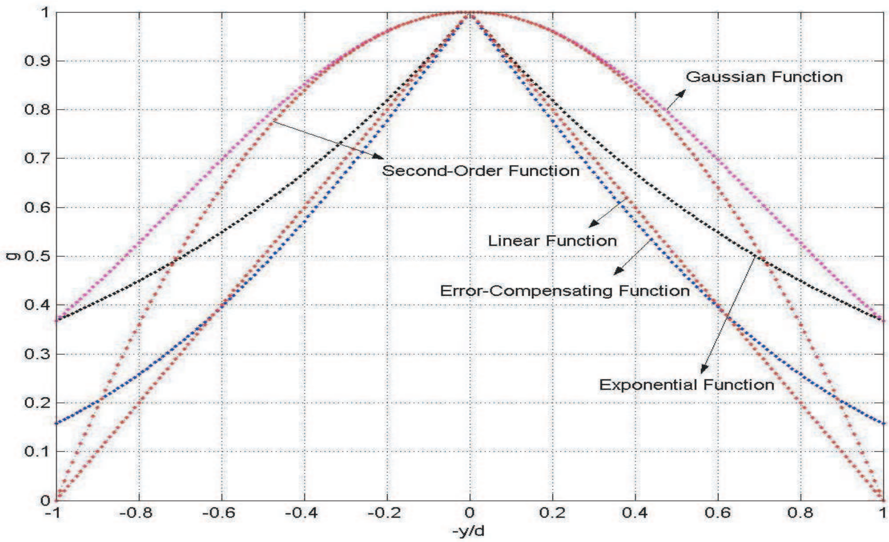


**Figure 2.** Refractive index distribution function, $g(y)$.

function and expressed, respectively, as follows

$$\left.\begin{array}{l} g(y) = \exp\left(y/d_y\right) \quad y \leq 0 \\ g(y) = \exp\left(-y^2/d_y^2\right) \quad y \leq 0 \\ g(y) = \exp\left(-y/d_y\right) \quad y \leq 0 \\ g(y) = \begin{cases} 1 + y/d_{y'} & -d_y \leq y \leq 0 \\ 0 & y \leq -d_y \end{cases} \\ g(y) = \begin{cases} 1 - y^2/d_{y'}^2 & -d_y \leq y \leq 0 \\ 0 & y \leq -d_y \end{cases} \end{array}\right\} \tag{8}$$

Here $d_y$ is the diffusion depth in the $y$-direction. Figure 2 shows the changes in the refractive index distribution function $g(y)$, in the direction of the depth of substrate.

We have already defined the arbitrary kinds of refractive index profile for the case of planar Bragg grating. Now we will define the more complicated refractive index profile, which may exist in Bragg fiber. In the design of the fiber profile, the first step is to define the profile in parametric form. The parametric form must be versatile enough to create a wide variety of profiles with few control parameters. A linear

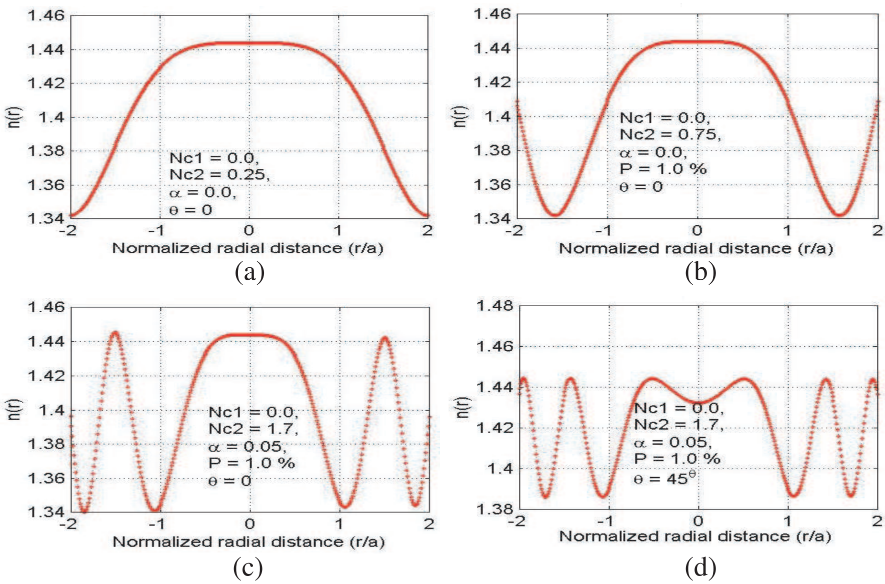


**Figure 3.** (a) A graded index profile. (b) A W-shaped profile. (c) A multiple cladding type index profile. (d) A multiple cladding type index profile with a dip at the fiber axis.

chirp type refractive index profile is a profile which has these features. The Bragg fiber investigated in this paper is assumed to have a circular symmetric refractive index profile. The generalized linear chirp type refractive index profile is defined as [6]

$$n(r) = \begin{cases} n_1\left\{1 - \left(\dfrac{\Delta + P(1-\Delta)}{2}\right)\Big[1 - \exp(-\alpha r) \right. & \\ \left. \cos\left\{2\pi\left(\dfrac{(N_{c2} - N_{c1})}{2}\dfrac{r^2}{a} + N_{c1} r\right) + \theta\right\}\Big]\right\} & r \leq a \\ n_1(1-\Delta) & r > a \end{cases} \tag{9}$$

where $\Delta = \frac{(n_1 - n_2)}{n_1}$, $n_1$ is the refractive index at the axis of the Bragg fiber, and $\alpha$ controls the decay or growth of the profile envelope. $N_{c1}$ or $N_{c2}$ is the number of cycles in a core radius, $\alpha$ the core radius, $\theta$ the arbitrary phase difference and $P$ the maximum permissible percentage change in refractive index below the cladding refractive

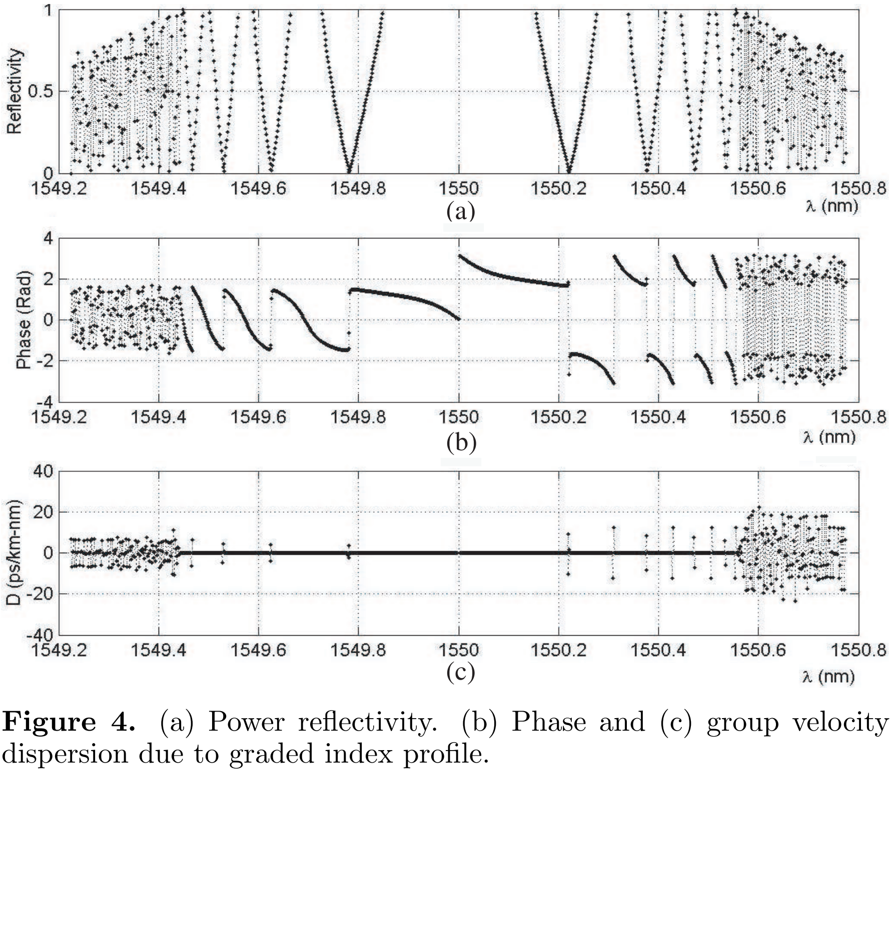


**Figure 4.** (a) Power reflectivity. (b) Phase and (c) group velocity dispersion due to graded index profile.

index $n_2$. It can be noted that the parameters, which define the index profile, can be divided in two parts. One, the fiber parameters as $a$, $n_1$, $\Delta$, $P$ and the other, the profile parameters like $N_{c1}$, $N_{c2}$ $\alpha$ and $\theta$. By varying eight parameters ($a$, $\Delta$, $n_1$, $N_{c1}$, $N_{c2}$, $\alpha$, $P$, $\theta$), one can generate profiles from simple step index type to complex multiple cladding type as shown in Figure 3. For example, the profile parameters are $N_{c1} = 0.0$, $\alpha = 0.0$ and $\theta = 0$, respectively for step index profile. The group velocity dispersion and reflectivity are examined in the next section for these cases.

## 5. CONTROLLING THE DISPERSION AND PEAK REFLECTIVITY DUE TO AN ARBITRARY REFRACTIVE INDEX PROFILES

Power reflectivity, phase and group velocity dispersion due to graded index profile, W-shaped and multiple cladding type profile with a dip at the fiber axis are shown in Figures 4, 5, and 6, respectively. Comparisons of Figures 1(a) and 4(a) lead to the bandwidth of main

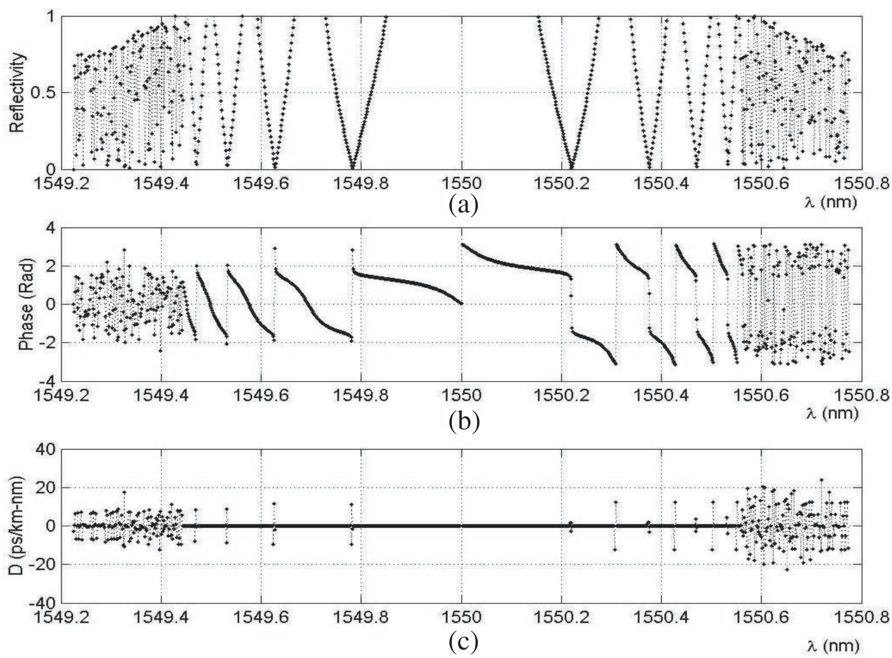


**Figure 5.** (a) Power reflectivity. (b) Phase and (c) group velocity dispersion due to a W-shaped profile.

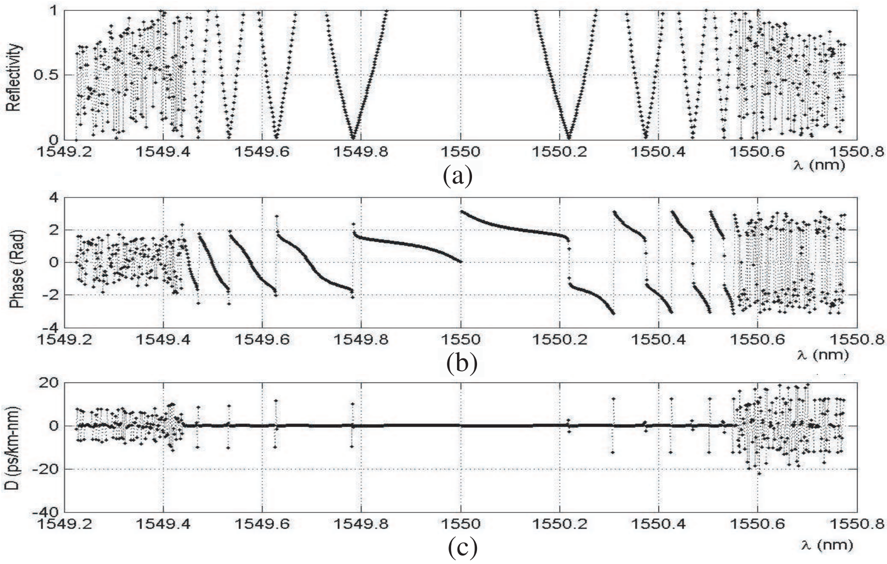


**Figure 6.** (a) Power reflectivity. (b) Phase and (c) group velocity dispersion due to a multiple cladding type index profile with a dip at the fiber axis.

lobe increased due to graded index profile with respect to a uniform grating. Moreover, the side lobes intensity has also been reduced in this case [7, 8].

It is apparent from Figures 1, 4–6 that as the bandwidth of FBG is increased, the number of side lobes is decreased, and it tends to decay slower. However, the phase spectrum shows a less deviation from linearity at the edges in the stop band, and consequently the group velocity dispersion tends to be reduced. Hence we can substantially increase the reflectivity by using more complex refractive index profile, which could be possible only by using the strong grating having high index contrast, as in the case of UFBG. Comparisons also reveal that the group velocity dispersion is reduced due to complicated refractive index profile up to some extent. It is also apparent that zero dispersion bandwidth is increased from $\sim 0.3$ nm to $\sim 0.42$ nm due to an arbitrary refractive index profile. The zero dispersion response of the grating would be helpful to the fabrication of DWDM optical system reliable even at high power application. At high optical power level ($\geq 100$ mw), the nonlinear effect may degrade the performance of DWDM system, but by using the proposed larger bandwidth zero dispersion grating may compensate it [9–11].

Figure 7 demonstrates how the side lobes have been eliminated

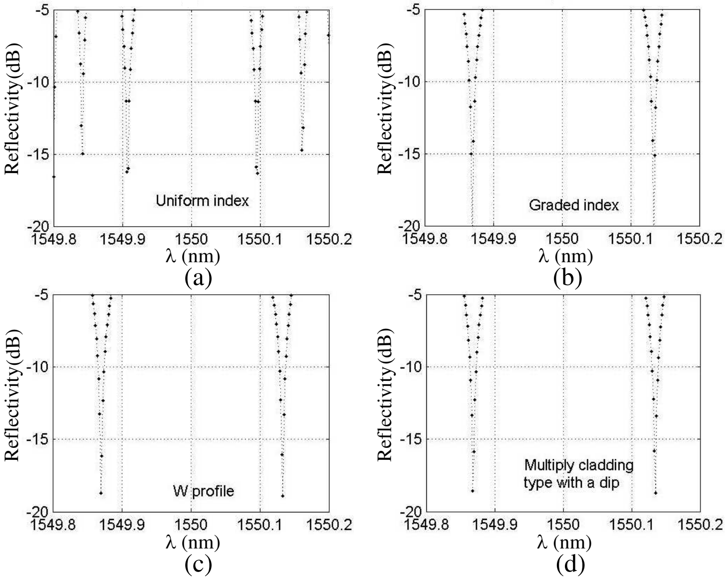


**Figure 7.** Reduction the number of reflectivity side lobes for various kinds of refractive index profile. It is obvious that the bandwidth of the reflectivity of main lobes has been increased.

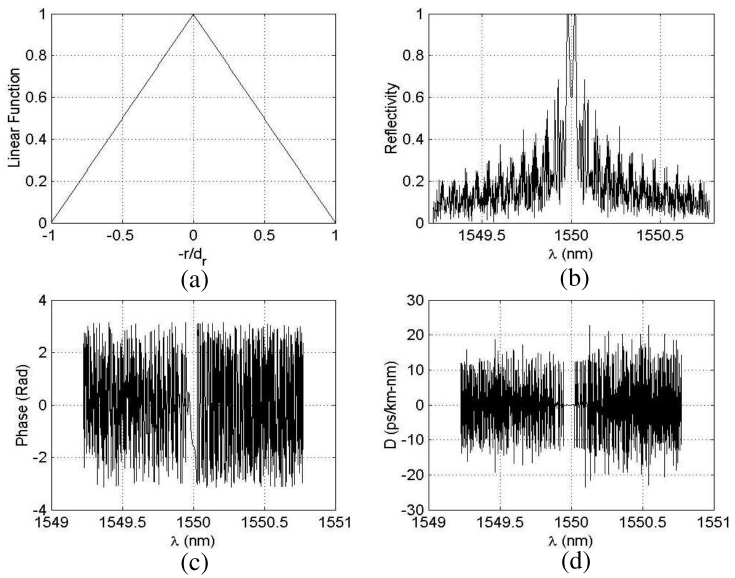


**Figure 8.** Power reflectivity, phase and group velocity dispersion due to linear function type index profile.

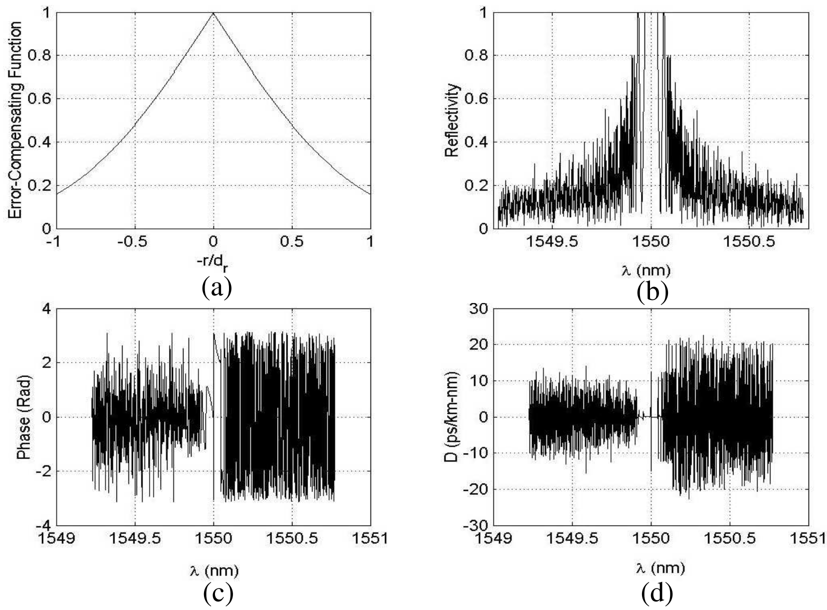


**Figure 9.** Power reflectivity, phase and group velocity dispersion due to error-compensating function type index profile.

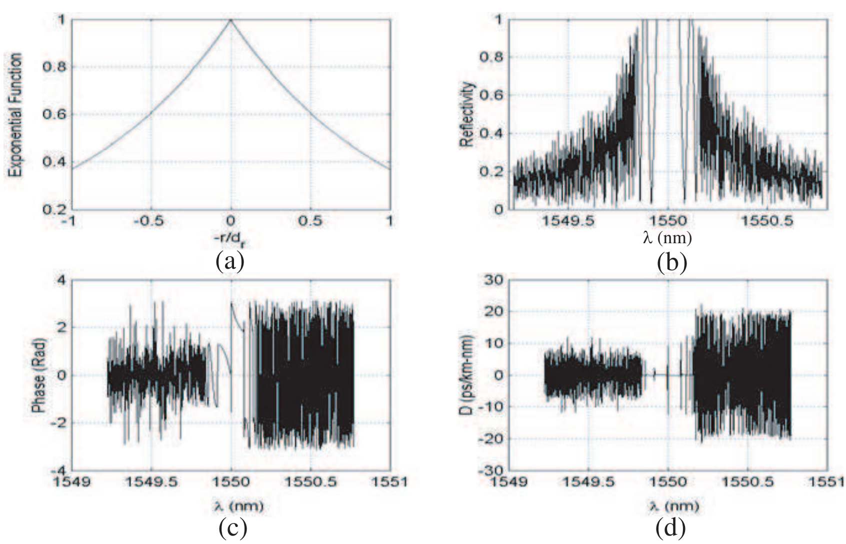


**Figure 10.** Power reflectivity, phase and group velocity dispersion due to exponential function type index profile.

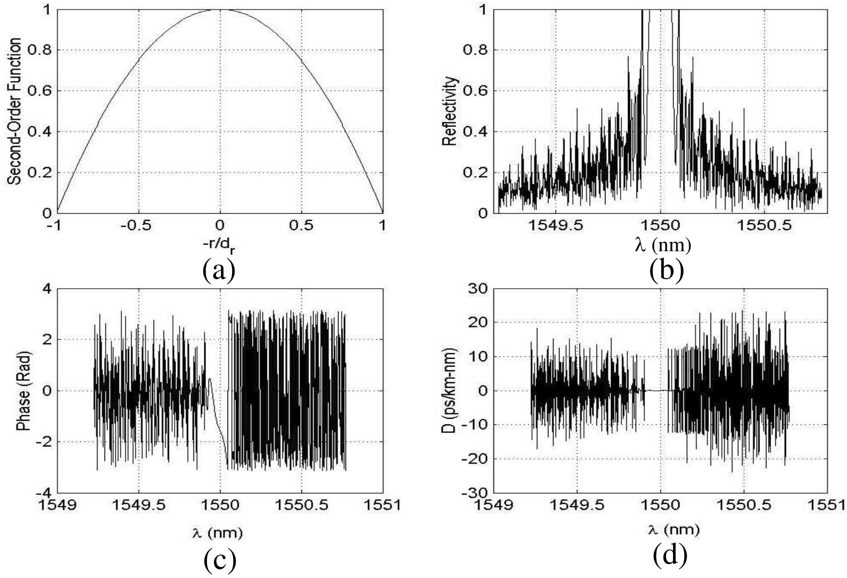


**Figure 11.** Power reflectivity, phase and group velocity dispersion due to second order function type index profile.

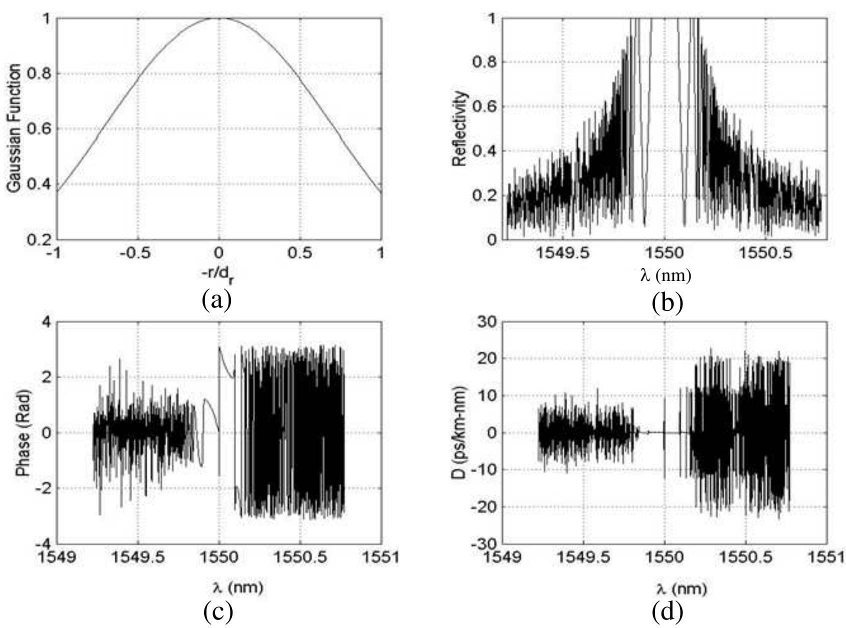


**Figure 12.** Power reflectivity, phase and group velocity dispersion due to Gaussian function type index profile.

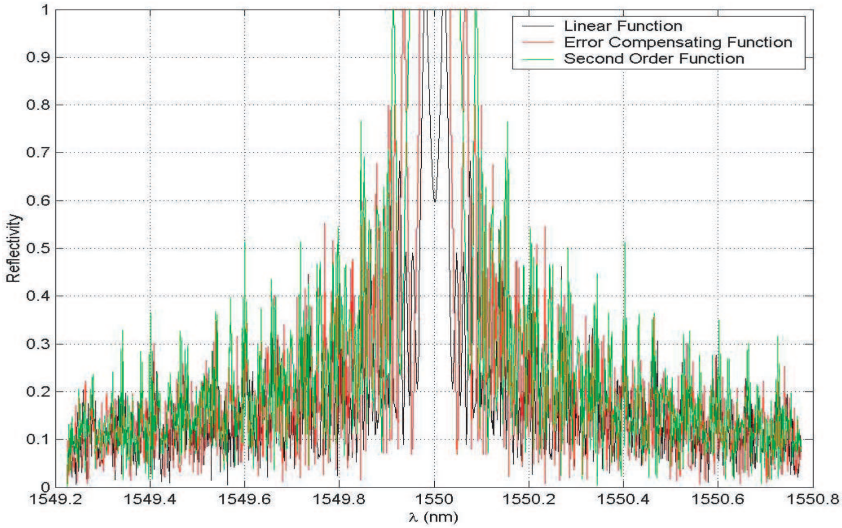


**Figure 13.** Reflectivity due to various kind of refractive index profile.

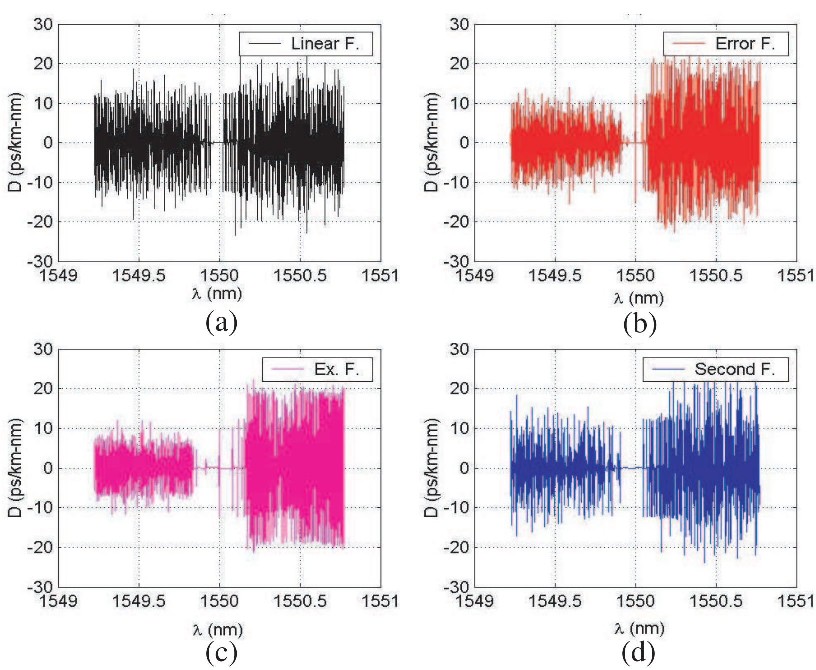


**Figure 14.** Group velocity dispersion due to various kind of refractive index profile.

near the main center lobe due to complicated refractive index profile, and the bandwidth of the reflectivity is increased. The spectrum is discrete due to finite number of grid size. The accuracy of our computation is also limited due to finite grid points, thus the reflectivity spectrum does not show much variation over the minor change of refractive index profile parameters. These properties can be used to design DWDM network to accommodate the large number of optical channels. The main advantage found due to complicated refractive index profile in the fiber Bragg grating is to reduce the nonlinear phase shift during pulse propagation; hence the optical power level can be significantly increased without much worrying about the nonlinear optical effects. From [5], it is apparent that in case of a UFBG when the non uniform input pulse would be applied to strong grating, the bandwidth of reflectivity would be increased, but the GVD has shown larger deviation with linearity in the central region of reflectivity spectrum. Using the complicated refractive index profile can compensate this nonlinearity [12, 13].

We can further conclude that this feature can be used to maintain an optical power level, in the same time maintaining a low dispersion in DWDM optical link. Both peak reflectivity and zero GVD bandwidth are increased due to an arbitrary refractive index profile. Comparisons

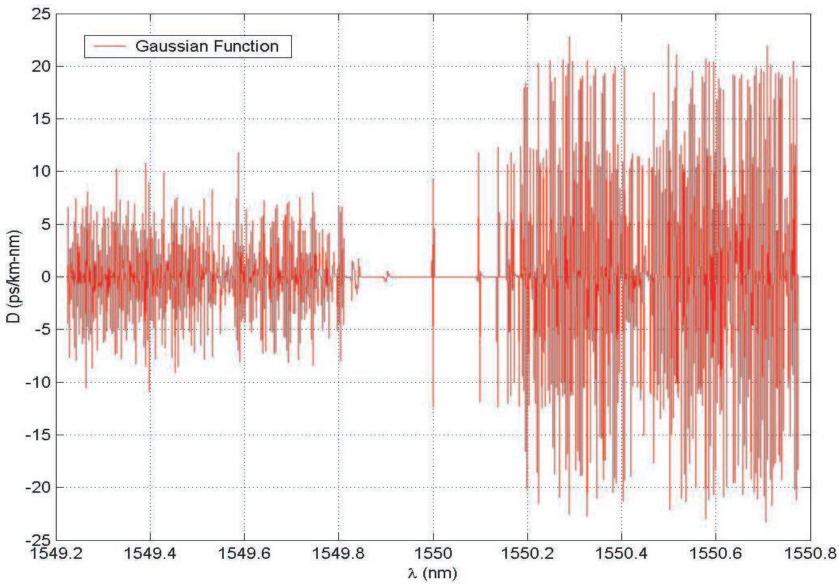


**Figure 15.** Group velocity dispersion due to Gaussian type refractive index profile.

are also given in Figures 8–12 due to various kinds of refractive index profile discussed in the beginning of Section 4. Figures 13–15 show the comparison between peaky reflectivity and group velocity dispersion $D$ due to various kinds of refractive index profile [14, 15]. It is apparent from Figure 13 that the bandwidth of reflectivity has been enhanced for the second order function which is a more complicated profile than other two cases. We conclude that complex refractive index profile can be used to achieve large zero GVD bandwidth near center wavelength of enhanced peak reflectivity spectrum, even it is impossible by using uniform grating with very high index contrast (strong grating).

## 6. CONCLUSION

This paper demonstrates how substantially we can reduce the nonlinear effects which are more dominated at high optical power level by using more complicated refractive index profile. Piecewise uniform approximation approach has been used here to the analysis of NUFBG. A dispersion characteristic due to an arbitrary refractive index profile has been discussed with great details. The paper provides the scheme to reduce the harmful nonlinear optical effects for high power optical sensor applications. Not only can we reduce the dispersion by the provided scheme but also can increase the peak reflectivity to main enhanced lobe, which is only possible by using strong grating. We can achieve the desired bandwidth requirement by using even weak grating but allowing the fiber Bragg grating with complicated profile. This study will be useful for high power optical sensor applications.

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