

## K-BAND WILKINSON POWER DIVIDER BASED ON A TAPER EQUATION

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**Abstract**—In this paper, a microstrip circuit structure for a K-Band Wilkinson power divider is presented. The designed power divider is composed of two-step taper stubs based on empirical equations. The symmetry of this circuit allows a half circuit analysis through looking at the odd- and even-model excitations. To demonstrate its performance, the proposed Wilkinson power divider has been fabricated and tested. Results show that the measured insertion loss is less than 0.3 dB and that the output reflection, input reflection and isolation are better than 16 dB, 22 dB, 16 dB, respectively, in the frequency range from 18 GHz to 27 GHz.

### 1. INTRODUCTION

The power divider and power combiner are very popular components for a microwave power combining system or an arrayed antenna system [1]. With the development of broadband requirement in wireless communication systems, wideband components are required increasingly, and many ultra-wide band Wilkinson power dividers have been developed. A N-way dual-band Wilkinson hybrid power divider based on the closed-form design method or an enhanced particle swarm optimization is proposed for the design and optimization of equal split broadband microstrip Wilkinson power dividers [2, 3]. An unequal Wilkinson power divider is based on asymmetrical coupled-line sections to obtain reduced-size circuit layout [4]. The dual-band equal or unequal Wilkinson power dividers are based on a coupled-line section with short-circuited stub [5]. A wideband Wilkinson power divider based on exponentially taper line with dissatisfied insert loss

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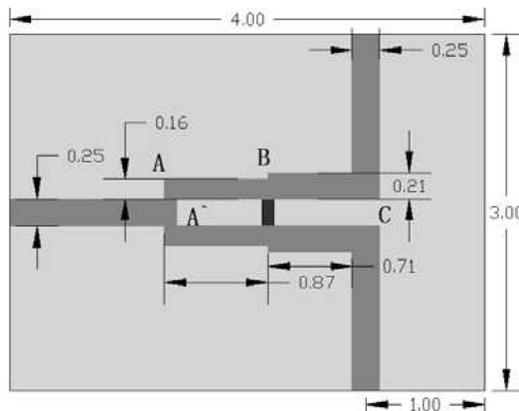
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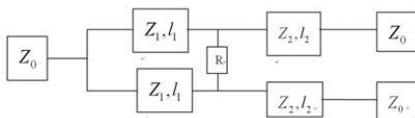
is shown in [6]. Although many advances have been made to the design of wideband power dividers, they do not describe a simplified structure which covers the K-Band in its entirety. Here, we describe a new wideband design which consists of two cascaded sections with isolation resistors in between. In addition, a taper empirical equation is adopted for two branches design in order to achieve a wideband impedance match. The symmetry of this circuit allows for the analysis through the even- and odd-mode half circuits and the taper of two branches using provides wideband for a characteristic impedance of  $50\ \Omega$  at each port. The analysis of taper lines and comparison between general lines and taper lines are also presented. To demonstrate that this type of Wilkinson circuit has a good performance in the whole K-Band, the proposed Wilkinson power divider has been fabricated and tested. The measured results indicate reasonable agreement with the designed results. The simulation results of the power divider show a reasonable agreement with those obtained with the tested fabricated power divider, designed according to the dimensions found by simulation.

## 2. CIRCUIT STRUCTURE AND THE THEORY

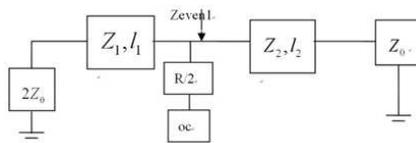
Figure 1 depicts the common two-steps Wilkinson power divider structure, which is capable of an equal power division over a wide band. According to Figure 1, we can consider its circuit diagram which is shown in Figure 2. This is a symmetrical power divider using two-section microstrip lines and one isolation resistor  $R$ . Each branch



**Figure 1.** Proposed the initial model of Wilkinson power divider. (unit in mm).



**Figure 2.** Proposed schematic diagram of Wilkinson power divider.



**Figure 3.** Circuit of the power divider for the even-mode analysis.

consists of two sections of transmission line with different characteristic impedances  $Z_1$  and  $Z_2$  and the physical lengths  $l_1$  and  $l_2$ , respectively. Since this divider is symmetric, the even- and odd-mode analyses will be adopted to solve the divider parameters for wideband operation.

### 2.1. Even-mode Analysis

In the case of even-mode, since no current flows in the plane of symmetry, the circuit element  $R$  can be omitted, and the impedance  $Z_0$  at port 1 is doubled [7]. The equivalent circuit of the divider in the even mode is shown in Figure 3.

According to the theory of transmission line, for a power divider with matched ports, the ideal input matching is necessary. Therefore, the relationship of the parameters in Figure 3 must be satisfied. We have

$$2Z_0 = Z_1 \frac{Z_{even1} + jZ_1 \tan(\beta_1 l_1)}{Z_1 + jZ_{even1} \tan(\beta_1 l_1)} \tag{1}$$

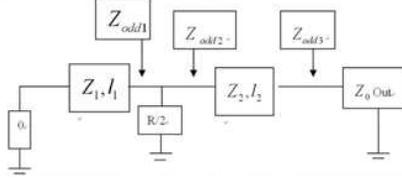
$$Z_{even1} = Z_2 \frac{Z_0 + jZ_2 \tan(\beta_2 l_2)}{Z_2 + jZ_0 \tan(\beta_2 l_2)} \tag{2}$$

Substituting (2) into (1), the real and imaginary parts of the combined equation can be written separately as

$$\begin{cases} Z_0 Z_1 Z_2 + (Z_0 Z_1^2 - 2Z_0 Z_2^2) \tan(\beta_1 l_1) \tan(\beta_2 l_2) = 0 \\ 2Z_0^2 [Z_1 \tan(\beta_2 l_2) + Z_2 \tan(\beta_1 l_1)] = Z_1 Z_2 [Z_1 \tan(\beta_1 l_1) + Z_2 \tan(\beta_2 l_2)] \end{cases} \tag{3}$$

Then,  $Z_1$ ,  $Z_2$  can be found as [6],

$$\begin{cases} Z_1 = \frac{(A_2 - Z_0)Z_2}{2Z_0 \tan(\beta_1 l_1) \tan(\beta_2 l_2)} \\ Z_2 = \sqrt{\frac{(Z_0 + A_2)(-Z_0^2) [1 + \tan^2(\beta_2 l_2)] + 4Z_0^3 \tan^2(\beta_2 l_2) [\tan^2(\beta_2 l_2) - \tan^2(\beta_1 l_1)]}{2A_1}} \end{cases} \tag{4}$$



**Figure 4.** Circuit of the power divider for the Odd-mode analysis.

where

$$A_1 = Z_0 \tan^2(\beta_2 l_2) [1 + \tan^2(\beta_2 l_2)] - 2Z_0 \tan^2(\beta_2 l_2) [1 + \tan^2(\beta_1 l_1)]$$

$$A_2 = \sqrt{Z_0^2 + 8Z_0^2 \tan^2(\beta_1 l_1) \tan^2(\beta_2 l_2)}$$

## 2.2. Odd-mode Analysis

In case of an odd mode, the two signals applied to output ports have the same magnitude. There is a voltage null along the middle of the power divider [7]. Thus, we can obtain the equivalent circuit, which is shown in Figure 4.

Similarly, to satisfy the ideal matching and isolation performance at the output ports, we have

$$Z_{odd1} = Z_1 \cdot \frac{0 + j \cdot Z_1 \cdot \tan(\beta_1 \cdot l_1)}{Z_1 + j \cdot 0 \cdot \tan(\beta_1 \cdot l_1)} = j \cdot Z_1 \cdot \tan(\beta_1 \cdot l_1) \quad (5)$$

$$Z_{odd2} = \frac{Z_{odd1} \cdot R/2}{Z_{odd1} + R/2} = \frac{R \cdot j \cdot Z_1 \cdot \tan(\beta_1 \cdot l_1)}{2Z_1 \cdot j \tan(\beta_1 \cdot l_1) + R} \quad (6)$$

$$Z_{odd3} = Z_2 \cdot \frac{Z_{odd2} + j \cdot Z_2 \cdot \tan(\beta_2 \cdot l_2)}{Z_2 + Z_{odd2} \cdot j \tan(\beta_2 \cdot l_2)} \quad (7)$$

Then, for a power divider with matched outputs, the ideal output matching is necessary, so  $Z_{odd3} = Z_0$ . By substituting (7) into (6) and separating the real and imaginary parts, we can obtain

$$R = \frac{2Z_0 Z_1 Z_2 \tan(\beta_1 l_1)}{Z_1 Z_2 \tan(\beta_1 l_1) + Z_2^2 \tan(\beta_2 l_2)} \quad (8)$$

$$Z_1 Z_2^2 \tan(\beta_1 l_1) + Z_2^3 \tan(\beta_2 l_2) = Z_0^2 Z_1 \tan(\beta_1 l_1) \tan(\beta_2 l_2) - Z_0^2 Z_2 \quad (9)$$

According to (3), (4), (8) and (9), all the electrical parameters of the proposed Wilkinson circuit are then solved. We can obtain the initial dimensions by MATLAB programming or evaluation, which are shown in Figure 1. The isolation resistor  $R$  value is about  $90 \Omega$ .

### 2.3. Taper Equation

Moreover, the continuous tapered line is synthesized from small reflections theory [8]. In paper [9], the method of coupled modes accompanied by the spectral domain solutions of uniform line is adopted for analyzing a broad class of nonuniform transmission lines, and the reflection coefficients are derived:

$$R = - \int_0^l C(y) \exp \left( -j2 \int_0^y \beta(\xi) d\xi \right) dz \quad (10)$$

$\beta(\xi)$  is phase constant.  $C(y)$  is the function of electric field  $E$  and magnetic field  $H$ .

$$C(y) = \frac{1}{2} \sum \left( k - \bar{k} \right)$$

where

$$k = b \sum \int \left( \bar{E}_x \frac{\partial}{\partial z} \bar{H}_y^* - \bar{E}_y \frac{\partial}{\partial z} \bar{H}_x^* \right) dy$$

$$\bar{k} = b \sum \int \left( \bar{H}_x \frac{\partial}{\partial z} \bar{E}_y^* - \bar{H}_y \frac{\partial}{\partial z} \bar{E}_x^* \right) dy$$

A transition connecting between input port and output ports of the Wilkinson circuit would be of importance for achieving wideband operation. Furthermore, the normal transverse field obeys the orthogonal relation [10]:

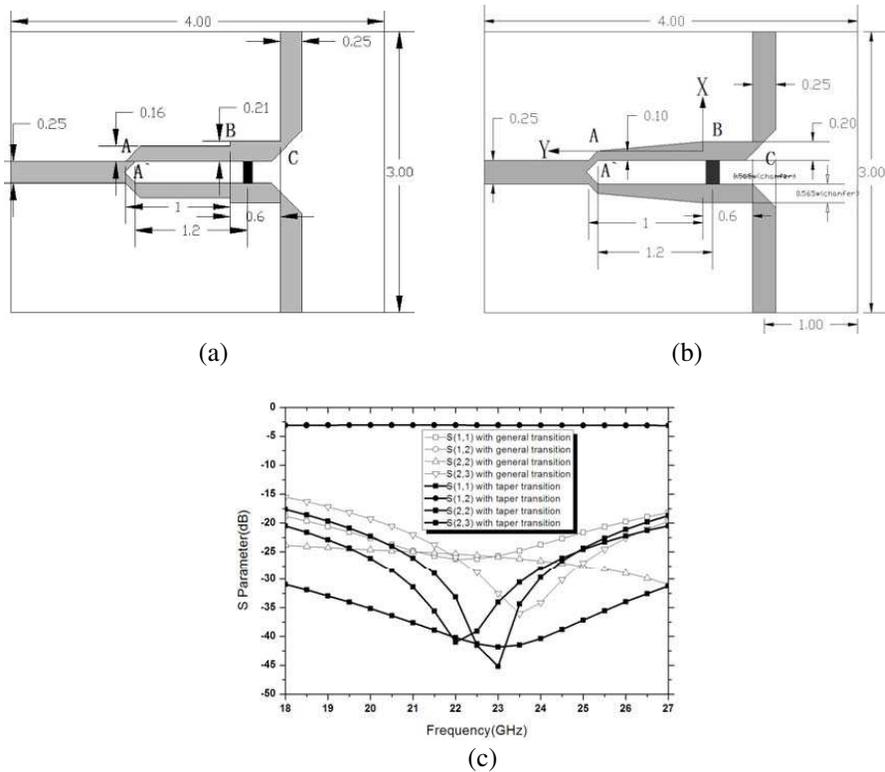
$$\int e_t(x, y, \beta_\gamma) \times h_t^*((x, y, \beta_\mu) \cdot a_z ds = \delta_{\gamma\mu} \quad (11)$$

where  $\delta_{\gamma\mu}$  is the Kronecker delta.

The transition microstrip lines profiles are considered to be as follows:

$$w(y) = (b - a) - (b - a) \sin^2(\pi y/2l) \quad (12)$$

According to (10) and (11), the profile (12) satisfies the requirements of both vanishing  $R$  at either end of the taper and smooth variation of geometry along the transition region [9]. The improved Wilkinson power divider and its assumed initial dimensions, according to the initial dimensions of the model in Figure 1, are shown in Figure 5(b).  $w(y)$  is the width of microstrip line along  $y$  axis,  $a$  the width of A place,  $b$  the width of B place, and  $l$  the length of AB section.



**Figure 5.** Comparison between general transition and taper transition: (a) general transition (unit in mm), (b) taper transition (unit in mm), (c)  $S$  parameter.

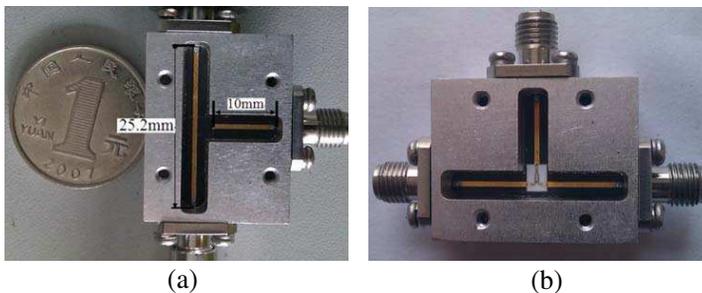
#### 2.4. Comparison between General Transition and Taper Transition

To further demonstrate the above analysis suitable for the Wilkinson power divider that we presented, the comparison between general transition and taper transition is indispensable. The two structures are designed under the same condition, which are shown in Figures 5(a) and (b), respectively. Both of the model dimensions are  $3\text{ mm} \times 4\text{ mm}$ , and the divider substrate is  $\text{Al}_2\text{O}_3$  ceramic, whose dielectric constant is 9.8 and thickness 0.254 mm. In addition, the Chamfer design is applied in the branch and output ports for achieving a good impedance matching performance. We defined that the chamfer in A' place was the half width of input ports and simulated the chamfer in A place by HFSS. In C place, we adopted 0.565 times of width of BC section and output ports based on engineering experiences. Obviously,

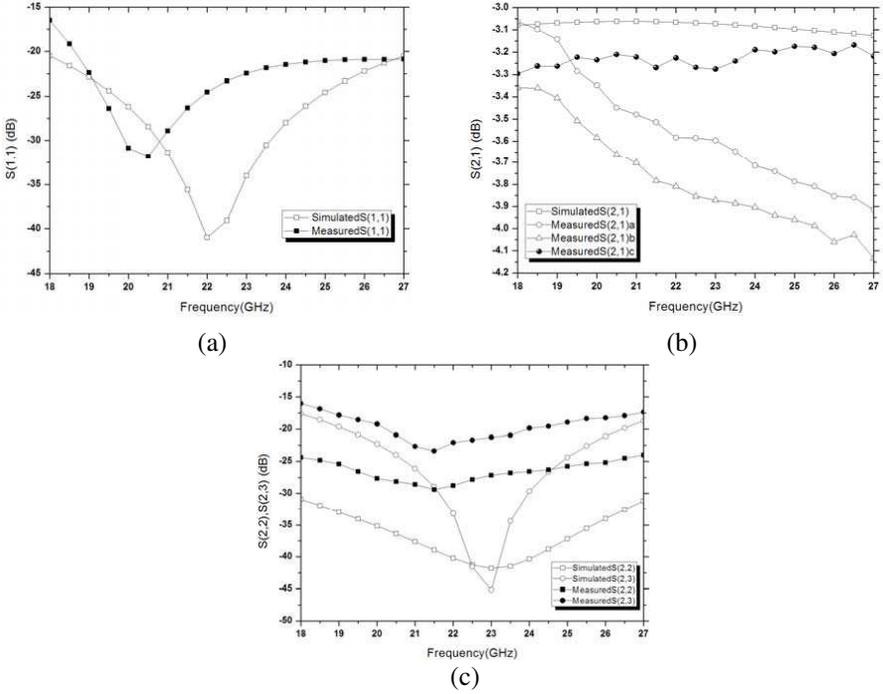
the performance of taper transition is better than general transition. The simulation results by Sequential Nonlinear Programming in HFSS between general transition and taper transition are shown in Figure 5(c).

### 3. MEASUREMENT RESULTS

Different dimensions of the power divider have been simulated by HFSS. The photograph of the fabricated divider is shown in Figure 6(b). The isolation resistor is about  $90\ \Omega$  and fabricated by thin film technology of  $50\ \Omega/\text{square}$ . To demonstrate that the designed circuit has a good performance, the fabricated Wilkinson power divider is tested, and its measured results are in good agreement with the simulated performance in the band of 18 GHz to 27 GHz. On the other hand, the fabricated dividers dimensions are  $3\ \text{mm} \times 4\ \text{mm}$ . Because the dimension of the Wilkinson divider is so small that it is not convenient for testing, the model of Figure 6(a) is fabricated. The loss of the microstrip line is shown in Figure 7(b)  $S(2,1)a$ . And then, we can obtain the insert loss in every millimeter of the extra microstrip line. We adopted gold bonding technology to connect the Wilkinson circuit and extra microstrip line. The fabricate model is shown in Figure 6(b), and the insert loss of this model is shown in Figure 7(b)  $S(2,1)b$ . The extra microstrip line is fabricated on the Rogers RT/duroid 5880 dielectric substrate with a relative dielectric constant of 2.2 and thickness of 0.254 mm. Then, we can obtain the single Wilkinson divider insertion loss by subtracting the extra insertion loss. The results are shown in Figure 7(b)  $S(2,1)c$ . Consequently, according to the measured results in Figure 7, the insertion loss is less than 0.3 dB, input reflection better than 16 dB, output reflection better than 22 dB,



**Figure 6.** The improved Wilkinson power divider circuit: (a) test circuit without Wilkinson divider, (b) test circuit with a Wilkinson divider.



**Figure 7.** Simulation and measurement results of  $S$ -parameters for the Wilkinson power divider operating at 18 GHz to 27 GHz: (a)  $S(1,1)$ , (b)  $S(1,2)$  or  $S(2,1)$ , (c)  $S(2,2)$  or  $S(3,3)$  and  $S(2,3)$  or  $S(3,2)$ .

and isolation better than 16 dB as well in the frequency range from 18 GHz to 27 GHz. The results show that the proposed power divider has good performance in the K-Band. The difference observed between simulation and experimental results is mainly due to the experimental model with an extra microstrip line, whereas the simulation model without the extra microstrip line, although the conduction band is connected by gold bonding wire in between Rogers RT/duroid 5880 and  $\text{Al}_2\text{O}_3$  ceramic.

#### 4. CONCLUSION

A novel K-Band operation scheme has been proposed for the Wilkinson power divider in the frequency range from 18 GHz to 27 GHz. The structure of this power divider and the design equations to determine its design parameters have been given, and a good agreement between the simulation and measurement has been achieved. The design equations are presented, and the corresponding solution results are

obtained. This design is wideband requiring only a simple taper structure. It is believed that the proposed Wilkinson divider can be a good candidate for microwave and millimeter wave system application.

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