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MIMO RADAR SYSTEMS DESIGN BASED ON MAXIMUM CHANNEL CAPACITY

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Abstract—In this paper, we consider the problem of bistatic multiple-input multiple-output (MIMO) radar systems design for parameters estimation. Maximum channel capacity is used as criterion for the problem of optimal systems design under transmitted power constraint and channel constraint. We obtain that the system design based on maximum channel capacity can be expressed as a joint optimization problem. Given the number of transmit antenna, the number of receive antenna and signal-noise ratio (SNR), the maximum channel capacity can be determined. This maximum channel capacity can be obtained from a unique appropriate power allocation and antenna placement strategy, which is very important for system design.

1. INTRODUCTION

MIMO radar has gotten considerable attention in a novel class of radar system, where the term MIMO refers to the use of multiple-transmit as well as multiple-receive antennas [1–4]. MIMO radar can transmit, via its antennas, multiple probing signals that may be correlated or uncorrelated with each other. There are two basic regimes of architecture considered in the current literature [5]. One is called statistical MIMO radar with widely separated antennas, which capture the spatial diversity of the target's RCS [2, 6]. This spatial diversity gain can improve the target detection and estimation of variations parameters. The other is called coherent MIMO radar with colocated antennas, which can obtain the waveform diversity and larger degrees of freedom to improve the target parameters estimation, parameter identifiability and much flexibility for transmit beampattern design [1].

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Furthermore, based on the location of transmit and receive antennas, the coherent MIMO radar can be distributed into two classes. One is bistatic MIMO radar; the other is monostatic MIMO radar. The former is with the quite different locations of transmit antennas from the receive antennas'. However, all transmit antennas are colocated, so are all receive antennas. Then the directions of the target to all transmit antennas are almost the same in the far-field scenario, so are all receive antennas'. While the latter is with all transmit and receive antennas colocated, then the directions of the target to all transmit and receive antennas are almost the same in the far-field scenario. For simplicity, we only discuss a bistatic MIMO radar architecture here. As monostatic MIMO radar can be taken as a special bistatic one.

MIMO radar system optimization has been extensively investigated. While the most authors concentrates on waveform optimization problem under certain given criterion. For example, in [7], waveform design methods for the optimization the Cramér-Rao Bound (CRB) matrix is discussed, under a total power constraint. In [8], waveform design methods for maximizing the conditional mutual information (MI) or minimizing the mean-square error (MMSE) are discussed. The two criteria lead to the same solution under the same total power constraint. In [9], the uncertainty of the target power spectrum is considered and the MI and MMSE criteria lead to different minimax robust waveforms. In [10], two information theoretic measures, maximizing the MI and maximizing the relative entropy for two hypotheses in detection, are used as criterions for optimal waveform design under transmitted power constraint. Both optimal solutions require that transmitted waveform should "match" with the target and noise. However, the optimal solutions of the two problems lead to different power allocation strategies. In [11], signal design for MIMO radar with colocated antennas based on transmit beam pattern is considered, focusing the power around the locations of the target, parameter estimation accuracy can be significantly improved. Among these criterions for waveform optimization, information theoretic criteria play an important role for MIMO radar waveform design, just like for conventional radar [12].

It is worth to note that antenna placement is an other key point for MIMO radar systems design. Some literature has studied up on antenna placement optimization problem. In [13], optimal antenna placement is discussed based on CRB for velocity estimation using separated MIMO radar. Assuming all antennas are located equidistant from the target, it is shown that symmetrically placing the transmit and receive antennas is the best choice, and the optimal achievable performance is not affected by the relative position of the transmit and receive antennas under an orthogonal received signal assumption. In [14], the target localization performance as a function of antennas placement is constructed. Then the optimal antennas placement have received from minimizing the variance of the localization error.

The predominant parameter estimation performance of bistatic MIMO radar results from taking advantages of the degrees of freedom greatly increased by virtual array. Considering a MIMO radar with M_t transmit antennas and M_r receive antennas, the number of the virtual array is M_tM_r. Since it can form M_tM_r channels of line-of-sight (LOS) (i.e., only one path between each transmit antenna and each receive antenna channel), like in communication theory [15]. We notice that the greater the MI between the target impulse response and the measurement, the better capability of parameter estimation [12]. Then channel capacity, defined as the maximum of the MI [17], is a good tool used to evaluate the system performance. In this paper, the MIMO radar system design for parameters estimation based on maximum channel capacity criterion will be focused on. This design strategy contains the factors of waveform and antenna geometry.

Notation: In this paper, we use boldface lowercase letters for vectors and boldface uppercase letters for matrices. We use $\{\cdot\}^*$, $\{\cdot\}^c$, $\{\cdot\}^T$ and $\{\cdot\}^{-1}$ for the complex conjugate transpose, complex conjugate, transpose and inverse of a matrix, respectively. We use $|\cdot|$ and $tr\{\cdot\}$ for the determinant and trace of a matrix, and $E\{\cdot\}$ for expectation with respect to all the random variables with the brackets. The symbol $\mathrm{diag}\{\mathbf{a}\}$ denotes a diagonal matrix with its diagonal given by the vector $\mathbf{a}$. We let I_N denote the identity matrix of size $N \times N$, and $\mathbf{0}$ denote a zero matrix with appropriate size. Finally, $(a)^+$ denotes the positive part of a, i.e., $(a)^+ = \max[0, a]$, and $\lceil a \rceil$ denotes the smallest integer greater than or equal to a, and $\lfloor a \rfloor$ denotes the largest integer smaller than or equal to a.

2. PROBLEM FORMULATION

Consider a bistatic MIMO radar system with M_t transmit antennas and M_r receive antennas, which all the antennas are identical omnidirectional, see Fig. 1. All transmit antennas are colocated, so are all receive antennas. The transmit and receive antennas are located in a three-dimensional (3-D) Cartesian coordinate system (x, y, z). Assume the transmit antennas located at $\mathbf{t}_k = [tx_k, ty_k, tz_k]^T$, $k = 1, \ldots, M_t$, and the receive antennas located at $\mathbf{r}_l = [rx_l, ry_l, rz_l]^T$, $l = 1, \ldots, M_r$. The gravity centers of transmit and receive antennas are at $\mathbf{t}_c = [x_{tc}, y_{tc}, z_{tc}]^T$ and $\mathbf{r}_c = [x_{rc}, y_{rc}, z_{rc}]^T$, respectively. Here the

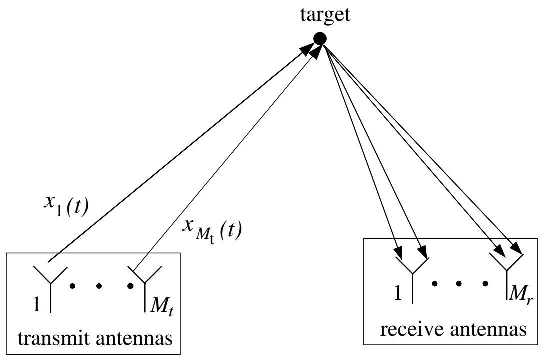

Figure 1. Illustration of bistatic MIMO radar.

centers of gravity are defined as in [18], given by

$$\begin{aligned} x_{tc} &= \frac{1}{M_t}\sum_{i=1}^{M_t} tx_i, \quad y_{tc} = \frac{1}{M_t}\sum_{i=1}^{M_t} ty_i, \quad z_{tc} = \frac{1}{M_t}\sum_{i=1}^{M_t} tz_i \\ x_{rc} &= \frac{1}{M_r}\sum_{k=1}^{M_r} rx_k, \quad y_{rc} = \frac{1}{M_r}\sum_{k=1}^{M_r} ry_k, \quad z_{rc} = \frac{1}{M_r}\sum_{k=1}^{M_r} rz_k \end{aligned} \tag{1}$$

Let $x_m(n)$, $m = 1, \ldots, M_t$; $n = 1, \ldots, N$ denote the discrete-time baseband signal transmitted by the mth transmit antenna, where n and N denote the sampled time and the number of snapshots of each transmitted signal pulse, respectively. Furthermore, the aggregate power transmitted by the transmitters is constant P, regardless of the number of transmit antennas. Let $\boldsymbol{\theta}_t = [\phi_t, \varphi_t]^T \in \boldsymbol{\Theta} \triangleq [0, 2\pi) \times [-\pi/2, \pi/2]$ and $\boldsymbol{\theta}_r = [\phi_r, \varphi_r]^T \in \boldsymbol{\Theta} \triangleq [0, 2\pi) \times [-\pi/2, \pi/2]$ denote the direction-of-departure (DOD) and direction-of-arrival (DOA), respectively. When all the antennas are colocated, i.e., $\boldsymbol{\theta}_t \approx \boldsymbol{\theta}_r$, the bistatic MIMO radar becomes a monostatic MIMO radar.

Under the assumption that transmitted probing signals are narrowband and that the propagation is nondispersive, the baseband signal at the target location can be described by the expression without considering the velocity of the target (see, [1], [16] and [20], chapter 6)

$$\mathbf{a}^*(\boldsymbol{\theta}_t)\,\mathbf{x}(n), \qquad n = 1, \ldots, N \tag{2}$$

where $\mathbf{x}(n) = [x_1(n), \ldots, x_{M_t}(n)]^T$, and $\mathbf{a}(\boldsymbol{\theta}_t)$ is the antenna's steering vector for transmit antennas. If the carrier frequency of every transmit antennas is the same, and also $\mathbf{t}_1, \ldots \mathbf{t}_{M_t}$ are in units of wavelengths corresponding to the carrier frequency, $\mathbf{a}(\boldsymbol{\theta}_t)$ is given by

$$\mathbf{a}\left(\boldsymbol{\theta}_t\right) = \left[e^{j2\pi((\mathbf{t'}_1)^T\mathbf{u}(\boldsymbol{\theta}_t)}, \ldots, e^{j2\pi((\mathbf{t'}_{M_t})^T\mathbf{u}(\boldsymbol{\theta}_t))}\right]^T \tag{3}$$

where $\mathbf{t}_i^{'} = \mathbf{t}_i - \mathbf{t}_c = [tx_i', ty_i', tz_i']^T$, and where $\mathbf{u}(\boldsymbol{\theta}_t) = [\cos\phi_t \cos\varphi_t, \sin\phi_t \cos\varphi_t, \sin\varphi_t]^T$ is the steering vector of the isotropic transmit antennas (i.e., the unit length vector pointing from the origin toward the target). Assume the transmit antennas is calibrated. Let $y_k(n)$, $k = 1, \ldots, M_r$, $n = 1, \ldots, N$ denote the signal received by the kth receive antenna; let

$$\mathbf{y}(n) = [y_1(n), \ldots, y_{M_r}(n)]^T, \; n = 1, \ldots, N \tag{4}$$

Under the simplified assumption of a point target, the received data vector can be described by the equation

$$\mathbf{y}\left(n\right) = \beta \mathbf{b}^c\left(\boldsymbol{\theta}_r\right)\mathbf{a}^*\left(\boldsymbol{\theta}_t\right)\mathbf{x}\left(n\right) + \boldsymbol{\epsilon}\left(n\right), \; n = 1, \ldots, N \tag{5}$$

where β, modeled an unknown deterministic constant during the CPI, is the complex amplitude proportional to the RCS of the point target. $\boldsymbol{\epsilon}(n) = [w_1(n), \ldots, w_{M_r}(n)]^T$ denotes the noise term. Without loss of generality, we assume $w_j(n)$, $j = 1, \ldots, M_r$; $n = 1, \ldots N$ is independent identically distributed (i.i.d.) zero-mean complex Gaussian process. The variance of this process is $E\{w_j(n)w_j^c(n)\} = \sigma_n^2$. $\mathbf{b}(\boldsymbol{\theta}_r)$ is the antenna's steering vector for receive antennas, given by

$$\mathbf{b}\left(\boldsymbol{\theta}_r\right) = \left[e^{j2\pi\left((\mathbf{r'}_1)^T\mathbf{u}(\boldsymbol{\theta}_r)\right)}, \ldots, e^{j2\pi\left((\mathbf{r'}_{M_r})^T\mathbf{u}(\boldsymbol{\theta}_r)\right)}\right]^T \tag{6}$$

where $\mathbf{r'}_k = \mathbf{r}_k - \mathbf{r}_c = [rx_k', ry_k', rz_k']^T$, and where $\mathbf{u}(\boldsymbol{\theta}_r) = [\cos\phi_r \cos\varphi_r, \sin\phi_r \cos\varphi_r, \sin\varphi_r]^T$ is the steering vector of the isotropic receive antennas.

Let $\mathbf{H} = \beta\mathbf{b}^c(\boldsymbol{\theta}_r)\mathbf{a}^*(\boldsymbol{\theta}_t)$ denote the $M_r \times M_t$ channel matrix, which is dependent of the antennas placement seriously. Then (5) can be reduced to

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \boldsymbol{\epsilon}(n), \; n = 1, \ldots, N \tag{7}$$

Let R_x denote the covariance matrix of the input signal vector $\mathbf{x}(n)$, and let R_y be the covariance matrix of the received signal vector $\mathbf{y}(n)$, and let R_ϵ be the covariance matrix of the noise vector. Assuming that $\mathbf{x}(n)$ and $\mathbf{y}(n)$ are uncorrelated with one another, we have that

$$\begin{aligned} R_y &\triangleq E\left[\mathbf{y}\left(n\right)\mathbf{y}\left(n\right)^*\right] = \mathbf{H}R_x\mathbf{H}^* + R_\epsilon \\ R_{xy} &\triangleq E\left[\mathbf{x}\left(n\right)\mathbf{y}\left(n\right)^*\right] = R_x\mathbf{H}^* \\ R_\epsilon &= \sigma_n^2 I_{M_r} \end{aligned} \tag{8}$$

Then the capacity of channel $\mathbf{H}$ is given by (see, e.g., [17, 19, 20])

$$C = \log_2 \frac{|R_\epsilon + \mathbf{H} R_x \mathbf{H}^*|}{|R_\epsilon|} = \log_2 |I_{M_r} + \mathbf{H} R_{x0} \mathbf{H}^*| \tag{9}$$

where $R_{x0} = E[\mathbf{x}(n)\mathbf{x}(n)^*]/\sigma_n^2$ is a noise-normalized transmit covariance matrix. We denote by μ_i, $i = 1, \ldots, n_{\min}$, $n_{\min} = \min(M_t, M_r)$, the ith eigenvalue of the Hermitian matrix $\mathbf{HH}^*$. Without loss of generality, we assume that $|\beta|^2 = 1$. Then we can easily get

$$tr\left[\mathbf{HH}^*\right] = \sum\nolimits_i \mu_i = tr\left[\mathbf{b}^c(\boldsymbol{\theta}_r)\,\mathbf{a}^*(\boldsymbol{\theta}_t)\,\mathbf{a}(\boldsymbol{\theta}_t)\,\mathbf{b}^T(\boldsymbol{\theta}_r)\right] = M_r M_t \tag{10}$$

From (10), small perturbations of the antenna locations do not change the trace of the Hermitian matrix $\mathbf{HH}^*$. Then we can get the channel constraint condition: $\sum_i \mu_i = M_r M_t$. Therefore, our goal is to find the transmitted waveforms (their covariance matrix must be satisfied) and the antennas placement that maximize the channel capacity C under the power constraint $tr\{\mathbf{x}(n)\mathbf{x}(n)^*\} \leqslant P$ and channel constraint $\sum_i \mu_i = M_r M_t$. Therefore we can express the problem of systems design based on maximum channel capacity as a joint optimization problem

$$\begin{aligned} \max_{\boldsymbol{\vartheta}, \mathbf{x}(n)} \quad & \log_2 |I_{M_r} + \mathbf{H} R_{x0} \mathbf{H}^*| \\ s.t. \quad & tr\{R_{x0}\} \leqslant P/\sigma_n^2 = P_0, \quad tr\left[\mathbf{HH}^*\right] = M_r M_t \end{aligned} \tag{11}$$

where $\boldsymbol{\vartheta} = [\mathbf{t}_1^T, \ldots, \mathbf{t}_{M_t}^T, \boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_{M_r}^T]^T$ is antennas placement vector, and P_0 is the signal-noise ratio (SNR).

3. SYSTEMS DESIGN CRITERION

The following lemma of Hadamard's inequality (see [21]) is useful in developing equation specifying optimal bistatic MIMO radar systems design.

LEMMA 1 Let A be an $N \times N$ positive semidefinite Hermitian matrix with (i, j)th entry a_{ij}. Then the following inequality,

$$|A| \leqslant \prod_{i=1}^{N} a_{ii} \tag{12}$$

holds, where equality is achieved if and only if A is diagonal.

Substituting USV^*, the magnitude-ordered singular value decomposition (SVD), for $\mathbf{H}$, (11) can be written as

$$\begin{aligned} \max_{Q} \quad & \log_2 |I_{n_{min}} + Q| \\ s.t. \quad & tr\left\{Q\left(S^*S\right)^{-1}\right\} \leqslant P_0, \quad Q \equiv SV^* R_{x0} V S^* \end{aligned} \tag{13}$$

where S is a diagonal $n_{\min} \times n_{\min}$ matrix, and U and V are $M_r \times n_{\min}$ and $M_t \times n_{\min}$ matrices containing the selected columns of unitary matrices, respectively. From LEMMA 1, we know Q is a diagonal $n_{\min} \times n_{\min}$ matrix. Then we can write $Q = \mathrm{diag}(q_{11}, \ldots, q_{n_{\min}n_{\min}})$, where $q_{ii} = \mu_i p_i$, and $p_1, \ldots, p_{n_{\min}}$ is some noise-normalized power allocation so that $\sum_i p_i \leqslant P_0$. Then (10) can also be written as

$$\begin{aligned} \max_{\boldsymbol{\mu},\mathbf{p}} \quad & \sum_{i=1}^{n_{\min}} \log_2\left(1+\mu_i p_i\right) \\ s.t. \quad & \sum_{i=1}^{n_{\min}} p_i \leqslant P_0, \quad \sum_{i=1}^{n_{\min}} \mu_i = M_r M_t \end{aligned} \tag{14}$$

where $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_{n_{\min}}]^T$ and $\mathbf{p} = [p_1, \ldots, p_{n_{\min}}]^T$. This is a constrained joint optimization problem which can be solved using the method of Lagrangian multiplier [22]. Towards this goal we form

$$\begin{aligned} J \quad & (p_1, \ldots, p_{n_{\min}}, \mu_1, \ldots, \mu_{n_{\min}}) = \sum_{i=1}^{n_{\min}} \log_2\left(1+\mu_i p_i\right) \\ & +\lambda_1 \left(\sum_{i=1}^{n_{\min}} p_i\right) + \lambda_2 \left(\sum_{i=1}^{n_{\min}} \mu_i\right) \end{aligned} \tag{15}$$

and differentiating J with respect to p_i and μ_i and setting equal to zero, respectively, we get

$$\frac{\mu_i}{1+\mu_i p_i} + \lambda_1 = 0, \quad \frac{p_i}{1+\mu_i p_i} + \lambda_2 = 0 \tag{16}$$

or

$$p_i = -\frac{1}{\lambda_1} - \frac{1}{\mu_i}, \quad \mu_i = -\frac{1}{\lambda_2} - \frac{1}{p_i} \tag{17}$$

where λ_1 and λ_2 have to be selected such that they satisfy

$$\sum_{i=1}^{n_{\min}} p_i = P_0, \quad \text{and} \quad \sum_{i=1}^{n_{\min}} \mu_i = M_r M_t \tag{18}$$

This discussion assumes that Q is full rank. Furthermore, with the positive constraint on μ_i and p_i, the optimal solutions are

$$p_i = \left(-\frac{1}{\lambda_1} - \frac{1}{\mu_i}\right)^+, \quad \mu_i = \left(-\frac{1}{\lambda_2} - \frac{1}{p_i}\right)^+ \tag{19}$$

These positive constraints are enforced by employing only the top n_+ (the number of positive eigenvalues of Q) modes of the $n_{\min}$ channel modes. Then the optimum Q_{opt} is given by [23]

$$Q_{opt} = \begin{bmatrix} \boldsymbol{\Lambda}_{n_+ n_+} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{n_{\min} n_{\min}} \tag{20}$$

Theorem III.1: Under the power constraint $tr\left\{\mathbf{x}\left(n\right)\mathbf{x}\left(n\right)^{*}\right\} \leqslant P$ and channel constraint $\sum_i \mu_i = M_r M_t$, the unique solution of the maximization channel capacity problem is:

$$\begin{aligned} p_1 &= p_2 = \ldots = p_{n_+} = P_0/n_+ \\ \mu_1 &= \mu_2 = \ldots = \mu_{n_+} = (M_r M_t)/n_+ \end{aligned} \tag{21}$$

and the maximization channel capacity is given to

$$C_{\max} = n_+ \log_2\left(1 + \frac{P M_r M_t}{\sigma_n^2 n_+^2}\right) \tag{22}$$

n_+ can be chosen by following the supplement illustration in the end of this section.

Proof: Firstly, observe that (21) satisfies (18)–(19) obviously, so (21) is a solution for our problem in (11). Now, we consider that this solution is unique for our maximization problem. Consider the set $\mu_1, \ldots, \mu_{n_+}$ and the corresponding set $p_1, \ldots, p_{n_+}$. The expression of the channel capacity becomes

$$\begin{aligned} C &= \sum_{i=1}^{n_+} \log_2\left(1 + \mu_i p_i\right) = n_+ \sum_{i=1}^{n_+} \frac{1}{n_+} \log_2\left(1 + \mu_i p_i\right) \\ &= n_+ E\left\{\log_2\left(1 + \mu_i p_i\right)\right\} \leqslant n_+ \log_2\left(1 + E\left\{\mu_i p_i\right\}\right) \end{aligned} \tag{23}$$

with equality iff $\mu_i p_i$ is constant for all $i \in [1, n_+]$ (see Jensen's inequality in [17]). So the maximum channel capacity is achieved for $\mu_1 p_1 = \mu_2 p_2 =, \ldots, \mu_{n_+} p_{n_+}$. Then we can get (21).

Remarks:

a): It is worth to note that Q should not be a full rank. It depends on the antennas placement vector and the transmitted signals. And only to choose $rank(Q) = n_+$ can be guaranteed the maximization channel capacity. From (22), we can easily know that n_+ depend on P, M_r, M_t and σ_n^2. It will be illustrated in detail in later part of this section. This maximum channel capacity can be attained from an appropriate power allocation and antenna placement strategy, by which $rank(Q) = n_+$ holds.

b): *Theorem III.1* is the result under the assumption of the simpler case of LOS channels with far field approximation. This assumption ensure that one can modify the number and the value of the positive eigenvalues of matrix $\mathbf{HH}^*$ by perturbing the locations of the antennas while keep $tr[\mathbf{HH}^*]$ constant. So *Theorem III.1* can not ensure its validity in a multipath environment.

c): It satisfies the maximum condition (21a) which almost all the published literature about MIMO radar uses the narrow-band signals with the same transmit power for all transmit antennas. Whereas, they do not consider the effect on the channel capacity by the antennas placement. If the rank of the matrix Q satisfies $n_+ = M_t$, the model of the transmit signals in the published literature is appropriate for maximizing the channel capacity. If the rank of the matrix Q satisfies $n_+ \neq M_t$ (it is easy to know that $n_+ < M_t$ holds in this time), the model of the transmit signals in the published literature is not appropriate for maximizing the channel capacity. We can compute the loss of channel capacity with respect to the maximum channel capacity. It is easy to know that $n_+ \leqslant M_t$ holds. Then the following definition of the channel capacity loss is positive semi-definite (only $n_+ = M_t$, then $C_{loss} = 0$ holds), given by

$$C_{loss} = C_{n_+} - C_{M_t} = n_+ \log_2\left(1 + \frac{PM_rM_t}{\sigma_n^2 n_+^2}\right) - n_+ \log_2\left(1 + \frac{PM_r}{\sigma_n^2 M_t}\right) \tag{24}$$

Now we are going to choose n_+ for the maximum channel capacity. Given P, M_r, M_t and σ_n^2, and let $k = (PM_rM_t)/\sigma_n^2$, we are look for n_0 (n_0 is an integer) which satisfies the following equation

$$C_{\max} = \max_{1 \leqslant n_0 \leqslant \min\{M_r, M_t\}} n_0 \log_2\left(1 + \frac{k}{n_0^2}\right) \tag{25}$$

First we consider the maximum of the function

$$C = n \log_2\left(1 + \frac{k}{n^2}\right) \tag{26}$$

and differentiating C with respect to n and setting equal to zero, we get

$$\log_2\left(1 + \frac{k}{(n')^2}\right) = \frac{2k}{(n')^2 + k} \tag{27}$$

Using the numerical analysis technique, we have the unique solution $n' = 0.505\sqrt{k}$. Then we consider the monotony characteristic of (26). Taking the first order derivative of (26) with respect to n, we get

$$\frac{\partial C}{\partial n} = \log_2\left(1 + \frac{k}{n^2}\right) - \frac{2k}{n^2 + k} \begin{cases} \geqslant 0, & n \leqslant n' \\ < 0, & n > n' \end{cases} \tag{28}$$

Then n' is the maximum point of (26) [22]. Whereas, n' may be not an integer, so we have the choose strategy of n_0 based on n' in the following table.

It is obvious that n_0 is not necessary for satisfying $n_+ = n_0 \cdot n_+$ is chosen by following the rule below:

Theorem III.1 supplement illustration:

- if $n_0 \geqslant n_{\min} = \min(M_r, M_t)$, then we have to take $n_+ = n_{\min}$,
- if $n_0 < n_{\min}$, we have to take $n_+ = n_0$.

Table 1. Choose strategy of n_0 based on n'.

n'	n_0
integer	n'
non-integer and $0 < n' < 1$	1
non-integer and $n' > 1$, $C\|_{\lceil n' \rceil} \geqslant C\|_{\lfloor n' \rfloor}$	$\lceil n' \rceil$
non-integer and $n' > 1$, $C\|_{\lceil n' \rceil} < C\|_{\lfloor n' \rfloor}$	$\lfloor n' \rfloor$

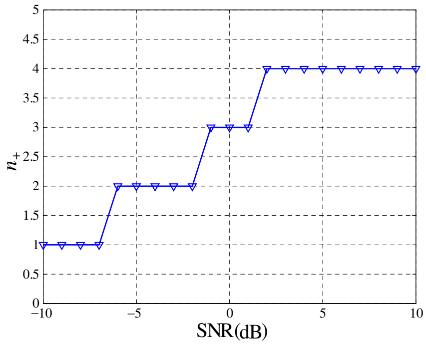

Figure 2. The value of n_+ as a function of the SNR, $M_r = 8$, $M_t = 4$.

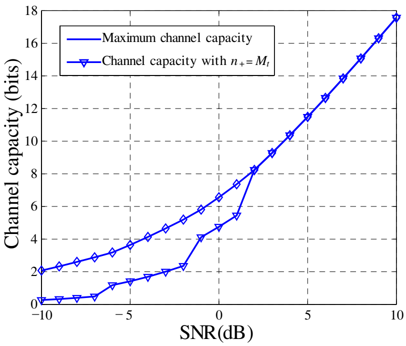

Figure 3. Comparison between the maximum channel capacity and the channel capacity using $n_+ = M_t$.

4. NUMERICAL RESULTS

In this section, we present numerical examples that illustrate the systems design solution derived in this paper.

In our first example we consider that the channel capacity varies with SNR. Assume a MIMO radar system having eight receive antennas and four transmit antennas, that is $M_r = 8$ and $M_t = 4$. Fig. 2 depicts the value of n_+ as a function of the SNR, based on *Theorem III.1 supplement illustration*. At lower SNR, n_+ increases with SNR's gain non-linearly, just like a ladder. While at enough large SNR (SNR $>$ 2 dB here), n_+ fixes with a constant at 4 (for $n_+ \leqslant \min(M_r, M_t)$), i.e., the number of the transmit antenna. Fig. 3 depicts the comparison between the maximum channel capacity and the channel capacity using $n = M_t$ usually used system strategy. It is clearly seen that both of channel capacities increase with SNR. However, at lower SNR (< 3 dB), the channel capacity of our proposed system strategy is superior. In

Fig. 4 we plot the C_{loss} as a function of the SNR corresponding to two system strategies in Fig. 3, where C_{loss} is defined as in the Section 3, i.e., we take the transmit model that all transmitting antennas transmit the same power regardless of whether $n_+ = M_t$ holds. It likes a sawtooth when $n_+ < 4$. And C_{loss} keeps zero when the SNR is large enough for keeping $n_+ = 4$.

In our second example we consider that the channel capacity varies with the number of receive antenna. Obviously, there are the similar results with respond to the number of transmit antenna. Assume a MIMO radar system having four transmit antennas and working in a SNR $= 2$ dB environment. Fig. 5 depicts the value of n_+ as a function of the number of receive antenna M_r, based on *Theorem III.1 supplement illustration*. n_+ increases with M_r non-linearly when $M_r < 8$, just like in Fig. 2. And n_+ keeps a constant 4, i.e., M_t, when $M_r \geqslant 8$. Fig. 6 depicts the the comparison between the maximum channel capacity and the channel capacity using $n = M_t$. It is clearly seen that both of channel capacities increase with M_r. However, at smaller M_r, the channel capacity of our proposed system strategy is superior. In Fig. 7 we plot the C_{loss} as a function of the number of receive antenna corresponding to the conditions in Fig. 6. It also likes a sawtooth when $M_r \leqslant 8$. And C_{loss} keeps zero when M_r is large enough for keeping $n_+ = 4$ ($M_r \geqslant 8$ here), which is the similar results in Fig. 4.

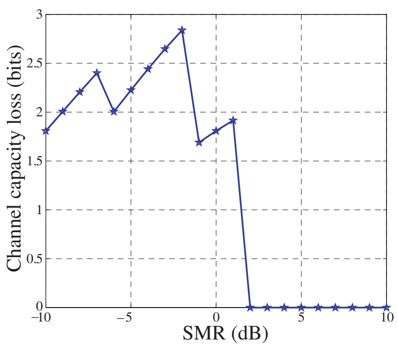

Figure 4. Channel loss corresponding to the conditions in Fig. 3 varying with SNR.

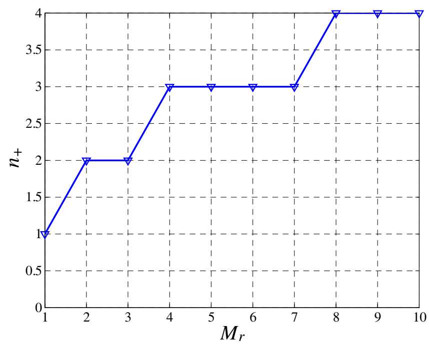

Figure 5. The value of n_+ as a function of the number of receive antenna M_r, SNR $= 2$ dB, $M_t = 4$.

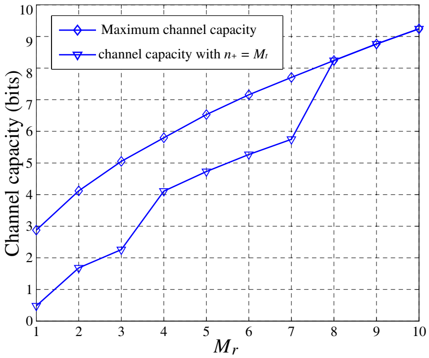

Figure 6. Comparison between the maximum channel capacity and the channel capacity using using $n_+ = M_t$.

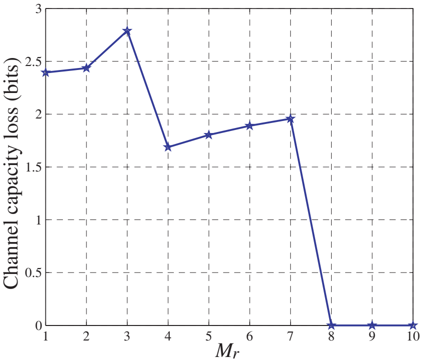

Figure 7. Channel loss corresponding to the conditions in Fig. 6 varying with M_r.

5. DISCUSSIONS AND CONCLUSION

In this paper, we have shown the solution of the optimum system design problem with bistatic MIMO radar for parameters estimation, which contains the factors of waveform and antenna geometry. We have used maximum channel capacity as criterion for this optimal problem under transmitted power constraint and channel constraint. And this problem of systems design based on maximum channel capacity can be expressed as a joint optimization problem. Given the number of transmit antenna, the number of receive antenna and SNR, then the maximum channel capacity can be determined, which can be obtained from a unique appropriate power allocation and antenna placement strategy, which is very important for systems design.

In practical application, there are many factors involved in the choice of a particular antenna geometry in MIMO radar system. It is obvious that the antenna placement strategy is greatly decided by the mission of radar system. We may get the completely different antenna configuration under the different design criterion.

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