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DIFFRACTION OF PLANE WAVE BY STRIP WITH ARBITRARY ORIENTATION OF WAVE VECTOR

S. S. Sautbekov

Eurasian National University, 5 Munaitpassov St., Astana, Kazakhstan

Abstract—The classical problem for diffraction of a plane wave with an arbitrarily oriented wave vector at a strip is considered asymptotically by Wiener-Hopf method. The boundary-value problem has been broken down into distinct Dirichlet and Neumann problems. Each of these boundary-value problems is consecutively solved by a reduction to a system of singular boundary integral equations and then to a system of Fredholm integral ones of second kind. They are solved effectively by a reduction to a system of linear algebraic equations with the help of the etalon integral and the saddle point method.

1. INTRODUCTION

The problem of the electromagnetic wave diffraction on a conducting strip attracts attention since the publication of Sommerfeld's famous paper where an exact solution of the half-plane diffraction problem was presented [1]. Both the method developed and the results obtained are detailed in the numerous textbooks [2–5]. Following this technique, researchers have tried to construct the solution of the problem by combining two similar solutions for spaced edges of the conducting surface. The solution obtained with this approach depends critically on the width of the strip.

The exact solution for this problem, which removes the necessity for the Kirchhoft-Huygens (K-H) approximation, was first reported by Strutt [6] for the case of incidence in a plane normal to the strip axis. Generally, at arbitrary orientation of the incident propagation vector, the range of validity of the K-H approximation also depends on the polar angle between the incident wave propagation vector and strip axis. For small angle values, a K-H approximation gives

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* Corresponding author: Seil S. Sautbekov (sautbek@mail.ru).

poor results. Other methods as Kobayashi potential method [7], Maliuzhinet's techniques [8] could be used to solve the problem. The best result was achieved by Ufimtsev [9] who obtained the solution for a strip in the form of a series in sequential edge waves excited by different its edges. This series is formally convergent at any strip width, but its elements are integrals of rising multiplicity, and that is inconvenient from the computational viewpoint.

The classical diffraction problem of a plane wave, orthogonally impinged on a strip edge was considered in a closed-form [10]. In the present paper a generalization of the article [10] in case when the plane wave propagates in an arbitrary direction is carried out. For convenience the boundary-value problem is divided into two independent ones which are named after Dirichlet and Neumann and then are solved separately by the Wiener-Hopf method [5, 10]. It is important to observe that the obtained solution automatically satisfies the boundary Meixner's condition [11] on the strip edge which determines the uniqueness of the solution of the boundary-value problem.

2. STATEMENT OF THE PROBLEM

Let a plane electromagnetic wave (Fig. 1) impinges on an ideally conducting strip $|z| \leq a$, $y = 0$, $-\infty < x < \infty$ in arbitrary direction, given by the unit vector $\boldsymbol{n}$

$$\boldsymbol{E}^0 = \boldsymbol{e}E^0, \quad E^0 = Ae^{\boldsymbol{kr}}, \quad \boldsymbol{H}^0 = \boldsymbol{n}\times\boldsymbol{e}\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}E^0 \quad (\boldsymbol{e}\perp\boldsymbol{n}), \tag{1}$$

$$\boldsymbol{k} = \boldsymbol{n}k_0,\ k_0 = \omega\sqrt{\varepsilon_0\varepsilon\mu_0\mu},\ \boldsymbol{nr} = x\cos\beta + y\sin\beta\sin\psi_0 + z\sin\beta\cos\psi_0$$

where A is the electric field amplitude, β is the angle between the x-axis and the wave propagation direction $\boldsymbol{n}$, ψ_0 is the angle between the z-axis and the YOZ plane projection of $\boldsymbol{n}$. The electric field direction is given by any unit vector $\boldsymbol{e}$, perpendicular to $\boldsymbol{n}$. Further, the harmonic time factor $\exp(-i\omega t)$ is everywhere omitted.

As the incident wave has an x dependence in accordance with the harmonic law $\exp(ixk_0\cos\beta)$ in (1), the diffraction fields should have the same x-co-ordinate dependence. Hence, it is possible to present the electromagnetic field in the form

$$\begin{aligned} &\boldsymbol{E}(x,y,z) = \boldsymbol{E}(y,z)e^{ixk_0\cos\beta}, && \boldsymbol{H}(x,y,z) = \boldsymbol{H}(y,z)e^{ixk_0\cos\beta}, \\ &\boldsymbol{E} = \boldsymbol{E}^0 + \boldsymbol{E}^1, && \boldsymbol{H} = \boldsymbol{H}^0 + \boldsymbol{H}^1, \end{aligned} \tag{2}$$

where $\boldsymbol{E}^1$, $\boldsymbol{H}^1$ are the diffraction fields. Moreover, the harmonic x-dependence factor will be dropped in the expressions, too.

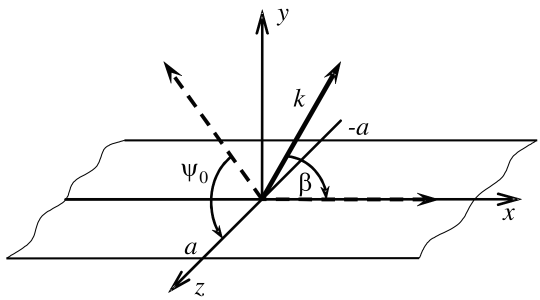

Figure 1. A strip.

Using the above mentioned dependence (2) in the Maxwell's equations, the electromagnetic field components can be written as [5]:

$$E_y = \frac{i}{k_0 \sin^2\beta}\left(\cos\beta\frac{\partial}{\partial y}E_x + \sqrt{\frac{\mu_0\mu}{\varepsilon_0\varepsilon}}\frac{\partial}{\partial z}H_x\right), \tag{3}$$

$$E_z = \frac{i}{k_0 \sin^2\beta}\left(\cos\beta\frac{\partial}{\partial z}E_x - \sqrt{\frac{\mu_0\mu}{\varepsilon_0\varepsilon}}\frac{\partial}{\partial y}H_x\right), \tag{4}$$

$$H_y = \frac{i}{k_0 \sin^2\beta}\left(\cos\beta\frac{\partial}{\partial y}H_x - \sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}\frac{\partial}{\partial z}E_x\right), \tag{5}$$

$$H_z = \frac{i}{k_0 \sin^2\beta}\left(\cos\beta\frac{\partial}{\partial z}H_x + \sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}\frac{\partial}{\partial y}E_x\right) \tag{6}$$

by the directed along the strip (x-axis) components E_x, H_x. As known these basic components of the electromagnetic field satisfy the two-dimensional Helmholtz equation for E_x and H_x

$$\frac{\partial^2}{\partial y^2}E_x + \frac{\partial^2}{\partial z^2}E_x + k_0^2\sin^2\beta E_x = 0, \tag{7}$$

$$\frac{\partial^2}{\partial y^2}H_x + \frac{\partial^2}{\partial z^2}H_x + k_0^2\sin^2\beta H_x = 0, \tag{8}$$

which follow from

$$E_x = \frac{i}{\omega\varepsilon_0\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right), \quad H_x = -\frac{i}{\omega\mu_0\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

and (3)–(6).

Thus, the boundary-value problem was transformed to the solution of Equations (7) and (8) for E_x, H_x with boundary conditions:

$$E_x = E_z = 0 \quad \text{at} \quad |z| \le a, \quad y = 0 \tag{9}$$

which corresponds to an absence of a tangential component of the electric field intensity on the strip. Since E_x and H_x are independent, like one can see, the boundary-value problem is divided in the next two boundary-value ones:

2.1. Dirichlet Problem

Equation (7) with a boundary condition

$$E_x = 0 \quad \text{at} \quad |z| \leq a, \quad y = 0.$$

Below it will be referred to as a magnetic problem.

2.2. Neumann Problem

Equation (8) with a boundary condition

$$\frac{\partial}{\partial y} H_x = 0 \quad \text{at} \quad |z| > a, \quad y = 0. \tag{10}$$

This problem is considered as an electric problem. It is easy to show that the condition (10) comes from (4) and (9). Further we will use the following notations

$$k \equiv k_0 \sin \beta, \quad h \equiv k \cos \psi_0. \tag{11}$$

3. SOLUTION OF THE ELECTRIC PROBLEM

We will present the general solution of Equation (8) in the form

$$H_x(y,z) = \text{sign} y \int_{-\infty}^{\infty} e^{i(wz+\sqrt{k^2-w^2}|y|)} \mathcal{F}(w) dw + H_x^0(y,z). \tag{12}$$

Here, the component of the magnetic field of the incident plane wave (1) along the strip is

$$H_x^0(y,z) = B_0 e^{i(yk \sin \psi_0 + zh)}, \; B_0 \equiv A \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}} \sin \beta (e_z \sin \psi_0 - e_y \cos \psi_0),$$

e_y and e_z are the y- and z-axes projections of the unit vector $\boldsymbol{e}$. The integral equation, expressing the absence of currents, is a consequence of the continuity condition of the magnetic field (12) on the continuation of the strip

$$\int_{-\infty}^{\infty} e^{iwz} \mathcal{F}(w) dw = 0 \quad \text{at} \quad |z| > a, \tag{13}$$

where $\mathcal{F}$ is the Fourier component of the current density for the electric problem. According to the boundary condition (9) we have the following integral equation

$$\int_{-\infty}^{\infty} e^{iwz}\sqrt{k^2-w^2}\mathcal{F}(w)dw + k\sin\psi_0 B_0 e^{ihz} = 0 \quad \text{at } |z| \leq a. \tag{14}$$

For concreteness the value h is fixed for example in the lower w half-plane (LHP).

We will construct the solution of the system of the singular Equations (13) and (14) by a technique, developed in Ref. [10] as analytical sources, localized on the edges of the strip. Thus, the Fourier component of the current density is written as a sum of two analytical sources:

$$\begin{aligned}\mathcal{F}(w) &= \mathcal{F}_1 + \mathcal{F}_2,\\ \mathcal{F}_2(w) &= \frac{1}{\sqrt{k-w}}\left(\mathcal{A}_2(w) + \mathcal{B}^+(w)\right)e^{-iwl_2},\\ \mathcal{F}_1(w) &= \frac{1}{\sqrt{k+w}}\left(\mathcal{A}_1(w) + \mathcal{B}^-(w)\right)e^{-iwl_1},\end{aligned} \tag{15}$$

where the co-ordinates of the edges 1 and 2 are l_1 and l_2, respectively. Let us choose the co-ordinates as $l_1 = a$ and $l_2 = -a$.

Note that $\mathcal{F}_1$ ($\mathcal{F}_2$) conforms to the current density on the semi-infinite plane in case $\mathcal{B}^- = 0$ ($\mathcal{B}^+ = 0$). The desired functions $\mathcal{A}_2$ and $\mathcal{A}_1$ correspond to plane wave amplitudes which provides a full cancelation of the field of the incident plane wave on the strip. Therefore they should be analytical functions on the entire complex w plane, except for a simple pole at $w = h$ on the LHP w. Note that the functions $\mathcal{B}^-$, $\mathcal{B}^+$ answer to the amplitudes of the reflected waves from the strip edges, as the singular points in the upper half plane (UHP) correspond to traveling waves from left to right. Therefore, the functions $\mathcal{B}^+$ and $\mathcal{B}^-$ should be analytical in the LHP, and $\mathcal{B}^+$ in the UHP.

It is convenient to present them in the form of contour integrals

$$\mathcal{B}^+(w) = \frac{1}{2\pi i}\int_{C^-}\frac{b_1(u)}{u-w}du, \quad \mathcal{B}^-(w) = -\frac{1}{2\pi i}\int_{C^+}\frac{b_2(u)}{u-w}du, \tag{16}$$

where b_1, b_2 are certain analytical functions in the band $|\mathrm{Im}u| < \mathrm{Im}k$, C^- and C^+ are integration contours (IC), laying parallel at a distance $\mp\delta$ $(0 < \delta < \mathrm{Im}k)$ from the real axis and consisting of an infinitely narrow loop, enveloping the point $w = \pm h$ from below or from above (Fig. 2).

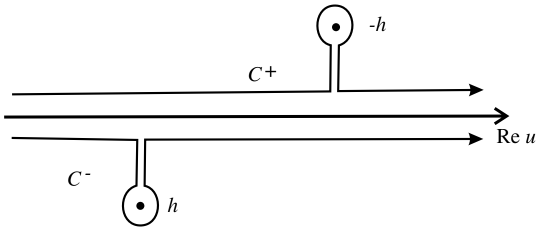

Figure 2. Integration contours.

Closing the IC in the LHP at $z \leq -a$, according to Jordan's lemma, in the integral Equation (14), we obtain

$$\mathcal{A}_1(w) = \frac{B_0\sqrt{k+h}}{2\pi i}\frac{e^{iha}}{w-h}. \tag{17}$$

We compensate the pole at this point in the LHP w at $z < -a$ and find

$$\mathcal{A}_2(w) = -\frac{B_0\sqrt{k-h}}{2\pi i}\frac{e^{-iha}}{w-h} \tag{18}$$

from (13). Note that (17) and (18) are valid when the plane wave is incident from the left. It is also important to observe that the $\mathcal{F}$ function satisfies the integral Equation (14) automatically. Really, each a summand of an integrand in (14) proves to be an analytical function in that half-plane where the integration contour is closed, according to the Jordan's lemma.

Representing the $\mathcal{B}^+(w)$ function in the form of a Cauchy type integral (16), taking a residue in the point $w = u$ and compensating the branch point in the LHP [10], we obtain the required function from (13):

$$\mathcal{B}^+(w) = -\frac{1}{2\pi i}\int\limits_{C^-}\frac{e^{-i2au}}{u-w}\sqrt{\frac{k-u}{k+u}}\left(\mathcal{A}_1(u)+\mathcal{B}^-(u)\right)du.$$

By means of the replacement $u \to -u$, the latter is represented as

$$\mathcal{B}^+(w) = \frac{1}{2\pi i}\int\limits_{C^+}\frac{e^{i2au}}{u+w}\sqrt{\frac{k+u}{k-u}}\left(\mathcal{A}_1(-u)+\mathcal{B}^-(-u)\right)du. \tag{19}$$

Analogously, eliminating the branch point in the w UHP at $z > a$ in (13), also as well as the poles, we get

$$\mathcal{B}^-(w) = \frac{1}{2\pi i}\int\limits_{C^+}\frac{e^{i2au}}{u-w}\sqrt{\frac{k+u}{k-u}}\left(\mathcal{A}_2(u)+\mathcal{B}^+(u)\right)du. \tag{20}$$

Hence, the boundary-value problem for the electric problem is reduced to the solution of a system of Fredholm integral Equations (19) and (20) of second kind.

That system may be integrated with a high precision by means of the saddle point method and the etalon integral [10]

$$\begin{aligned} J(w,l) &= \frac{1}{2\pi i}\int\limits_{C^+}\frac{e^{ilu}}{u-w}\sqrt{\frac{k+u}{k-u}}\,du \\ &= \frac{1}{2i}H_0^{(1)}(kl)-\sqrt{\frac{k+w}{k-w}}e^{ilw}\Upsilon(kl/2,w/k), \end{aligned}$$

where

$$\begin{aligned} \Upsilon(kl,\cos\beta) &= -\frac{\sqrt{k^2-w^2}}{2\pi i}e^{-i2lw}\int\limits_{C_1}\frac{e^{i2lu}}{(u-w)\sqrt{k^2-u^2}}du \\ &= \sin\beta\int\limits_{\infty}^{kl}H_0^{(1)}(2t)e^{-2it\cos\beta}dt, \end{aligned}$$

$H_0^{(1)}(x)$ is the Hankel function of the first kind and zero order, C_1 is the integration contour along the banks of the cut of the function $v=\sqrt{k^2-w^2}$ that is a parallel line to the imaginary axis upwards from the branch point.

Note that

$$J(-k,l)=\frac{i}{2}H_0^{(1)}(lk). \tag{21}$$

By deforming the integration contours in (19) and (20) up to C_1 which is the line of the steepest descent and applying the saddle point method and the etalon integral, the short-wave asymptotic behavior of the system of integral Equations (19), (20) is achieved:

$$\begin{aligned} \mathcal{B}^+(w) &\cong \mathcal{A}_1(w)\big(J(-w,2a)-J(-h,2a)\big)+\mathcal{B}^-(-k)J(-w,2a), \\ \mathcal{B}^-(w) &\cong \mathcal{A}_2(w)\big(J(w,2a)-J(h,2a)\big)+\mathcal{B}^+(k)J(w,2a). \end{aligned}$$

By solving the following system of linear algebraic equations:

$$\begin{aligned} \mathcal{B}^+(k) &= \mathcal{A}_1(k)\left(J(-k,2a)-J(-h,2a)\right)+\mathcal{B}^-(-k)J(-k,2a), \\ \mathcal{B}^-(-k) &= \mathcal{A}_2(-k)\left(J(-k,2a)-J(h,2a)\right)+\mathcal{B}^+(k)J(-k,2a), \end{aligned}$$

we find the function values in a branch point:

$$\begin{aligned} \mathcal{B}^+(k) &= \mathcal{A}_1(k)\frac{J(-k,2a)-J(-h,2a)}{1-J^2(-k,2a)}+\mathcal{A}_2(-k)J(-k,2a) \\ &\quad \frac{J(-k,2a)-J(h,2a)}{1-J^2(-k,2a)}, \end{aligned} \tag{22}$$

$$\mathcal{B}^-(-k) = \mathcal{A}_2(-k)\frac{J(-k,2a)-J(h,2a)}{1-J^2(-k,2a)} + \mathcal{A}_1(k)J(-k,2a)$$
$$\frac{J(-k,2a)-J(-h,2a)}{1-J^2(-k,2a)}. \tag{23}$$

It is important to observe that $1 - J^2(-k, 2a)$ is a resonance denominator in the foregoing expressions.

Thus, all required functions of a system of the integral equations were deduced. Now we will calculate the field.

3.1. Magnetic Field Calculation

By substituting (15) in (12) we will calculate the component of the magnetic field of the conical waves

$$H_x^1 = I_1 + I_2,$$

in the form of integrals sum

$$I_1 = \text{sign}y \int_{-\infty}^{\infty} e^{i(w(z-a)+v|y|)} \frac{\mathcal{A}_1(w) + \mathcal{B}^-(w)}{\sqrt{k+w}}\, dw,$$

$$I_2 = \text{sign}y \int_{-\infty}^{\infty} e^{i(w(z+a)+v|y|)} \frac{\mathcal{A}_2(w) + \mathcal{B}^+(w)}{\sqrt{k-w}}\, dw.$$

In a polar coordinate system:

$$z = r\cos\theta, \quad y = r\sin\theta; \quad w = k\sin\alpha, \quad v = k\cos\alpha, \tag{24}$$

the first integral is

$$I_1 = \frac{B_0}{\pi i}\int_S \frac{e^{ikr^-\sin(\alpha+\theta^-)}}{\sin\alpha - \cos\psi_0}\left|\sin\frac{\alpha-\frac{\pi}{2}}{2}\right| \{\cos\frac{\psi_0}{2}e^{ika\cos\psi_0} - \sin\frac{\psi_0}{2}$$
$$e^{-ika\cos\psi_0}[J(k\sin\alpha,2a) - J(k\cos\psi_0,2a)]\}d\alpha$$
$$+\sqrt{2k}\int_S e^{ikr^-\sin(\alpha+\theta^-)}\left|\sin\frac{\alpha-\frac{\pi}{2}}{2}\right| J(k\sin\alpha,2a)\mathcal{B}^+(k)d\alpha,$$

where S is the integration contour in the complex plane, passing from the top of the second quadrant down through the co-ordinate origin to the fourth one. Here, the following notation is introduced:

$$r^- \equiv \sqrt{r^2 - 2ar\cos\theta + a^2}, \quad \theta^- \equiv \theta + \arctan\left(\frac{a\sin\theta}{r - a\cos\theta}\right), \tag{25}$$

in accordance with a transfer of the coordinate origin to the strip edge by a polar coordinates transformation:

$$r\cos\theta - a = r^{-}\cos\theta^{-}, \quad r\sin\theta = r^{-}\sin\theta^{-}.$$

Their asymptotic behaviors are useful, too:

$$r^{-} \simeq r - a\cos\theta, \quad \theta^{-} \simeq \theta + \frac{a}{r}\sin\theta \quad \text{at} \quad r \gg a.$$

Putting a new integration variable $\tau = \alpha + \zeta - \pi/2$ and with a glance to the representation of the Hankel function

$$H_0^{(1)}(kr^{-}) = \frac{1}{\pi}\int\limits_S e^{ir^{-}k\cos\tau}d\tau,$$

by means of the saddle point method [12], we will deduce

$$\begin{aligned} I_1 \;=\; & -iB_0H_0^{(1)}(kr^{-})\frac{\sin\frac{\theta^{-}}{2}}{\cos\theta^{-} - \cos\psi_0}\left\{\cos\frac{\psi_0}{2}e^{ika\cos\psi_0} - \sin\frac{\psi_0}{2}\right. \\ & \left. e^{-ika\cos\psi_0}\left[J(k\cos\theta^{-}, 2a) - J(k\cos\psi_0, 2a)\right]\right\} + \sqrt{2k}\pi \\ & H_0^{(1)}(kr^{-})\sin\frac{\theta^{-}}{2}J(k\cos\theta^{-}, 2a)\mathcal{B}^{+}(k). \end{aligned} \tag{26}$$

The second integral is calculated similarly:

$$\begin{aligned} I_2 \;=\; & iB_0H_0^{(1)}(kr^{+})\frac{\cos\frac{\theta^{+}}{2}}{\cos\theta^{+} - \cos\psi_0}\left\{\sin\frac{\psi_0}{2}e^{-ika\cos\psi_0} - \cos\frac{\psi_0}{2}\right. \\ & \left. e^{ika\cos\psi_0}\left[J(-k\cos\theta^{+}, 2a) - J(-k\cos\psi_0, 2a)\right]\right\} + \sqrt{2k}\pi \\ & H_0^{(1)}(kr^{+})\cos\frac{\theta^{+}}{2}J(-k\cos\theta^{+}, 2a)B^{-}(-k). \end{aligned} \tag{27}$$

Here, the following notation

$$r^{+} \equiv \sqrt{r^2 + 2ra\cos\theta + a^2}, \quad \theta^{+} \equiv \theta - \arctan\left(\frac{a\sin\theta}{r + a\cos\theta}\right) \tag{28}$$

and its asymptotic forms

$$r^{+} \simeq r + a\cos\theta, \quad \theta^{+} \simeq \theta - \frac{a}{r}\sin\theta$$

are used. In a general way, the diffraction fields in Equations (26) and (27) are conical. They go over to cylindrical waves [10] at $\beta = \pi/2$.

4. SOLUTION OF THE MAGNETIC PROBLEM

The general solution of Equation (7) is presented in the form, also as well as in (12) [10]:

$$E_x(y,z) = \int\limits_{-\infty}^{\infty} e^{i(wz+v|y|)} \frac{F(w)}{v} dw + A_0 e^{i(hz+yk\sin\psi_0)}, \tag{29}$$

where $v = \sqrt{k^2 - w^2}$, $A_0 = e_x A$, e_x is the x-axis projection of the unit vector $\boldsymbol{e}$.

So, from (9) and (29), according to the boundary condition, we have the integral equation

$$\int\limits_{-\infty}^{\infty} e^{iwz} \frac{F(w)}{v} dw + A_0 e^{ihz} = 0 \quad \text{at} \quad |z| \le a. \tag{30}$$

For concreteness the value h is fixed for example in the lower w half-plane (LHP).

Due to the continuity condition of the magnetic field component H_z on the continuation of the strip with a glance to (29) and (6), a fulfillment of the following integral equation is necessary

$$\int\limits_{-\infty}^{\infty} e^{iwz} F(w) dw = 0 \quad \text{at } |z| > a. \tag{31}$$

Here, F is the Fourier component of the current density for the magnetic problem. The solution of a system of the singular integral Equations (30) and (31) will be constructed by the method of edge sources [10]

$$F(w) = F_1 + F_2, \tag{32}$$

where

$$F_2(w) = \sqrt{k-w}\left(A_2(w) + B^+(w)\right) e^{iwa},$$

$$F_1(w) = \sqrt{k+w}\left(A_1(w) + B^-(w)\right) e^{-iwa}.$$

Here, the required functions A_2 and A_1 correspond to the plane wave amplitude and should be analytical functions on the entire complex w-plane, except for the simple pole at $w = h$. The functions B^+ and B^- answer to the amplitudes of the reflected waves from the strip edges, B^- should be analytical in the LHP w, and B^+ in the UHP. Therefore, the required functions B^- and B^+ were represented as well as in (16).

Compensating the simple pole at the point $w = h$ in the w LHP, we define from (30)

$$A_1(w) = \frac{A_0\sqrt{k-h}}{2\pi i}\frac{e^{iha}}{w-h}. \tag{33}$$

Likewise, we obtain from (31) at $z < -a$

$$A_2(w) = -\frac{A_0\sqrt{k+h}}{2\pi i}\frac{e^{-iha}}{w-h}. \tag{34}$$

Compensating the branch point of the integrand in (31) in the w LHP at $z < -a$, similarly to (19), we get $B^+(w)$:

$$B^+(w) = \frac{1}{2\pi i}\int\limits_{-\infty+i\delta}^{\infty+i\delta} \frac{e^{i2au}}{u+w}\sqrt{\frac{k-u}{k+u}}\left(A_1(-u) + B^-(-u)\right)du. \tag{35}$$

Compensating the branching point in the w UHP at $z > a$ from (31), we also obtain:

$$B^-(w) = \frac{1}{2\pi i}\int\limits_{-\infty+i\delta}^{\infty+i\delta} \frac{e^{i2au}}{u-w}\sqrt{\frac{k-u}{k+u}}\left(A_2(u) + B^+(u)\right)du. \tag{36}$$

Thus, the system of singular boundary integral Equations (30) and (31) is reduced to a system of the functional Equations (35) and (36) which are Fredholm integral equations of second kind. The validity of the solution of (30) and (31) may be readily checked by substituting of (33)–(36) in the system and calculating the integrals, using the theory of residues.

Further, the system will be solved with the help of the saddle point method and the etalon integral:

$$I(w,l) = \frac{1}{2\pi i}\int\limits_{C^+} \frac{e^{ilu}}{u-w}\sqrt{\frac{k-u}{k+u}}du.$$

It can be expressed by virtue of the special functions of Hankel $H_0^{(1)}$ and Υ [10]:

$$I(w,l) = \frac{1}{2i}H_0^{(1)}(kl) - \sqrt{\frac{k-w}{k+w}}e^{ilw}\Upsilon(kl/2, w/k). \tag{37}$$

By deforming the integration contours in (35) and (36) up to the banks of the cut C_1 which is the line of the steepest descent and calculating the short-wave asymptotic behavior of the integral, employing the saddle point method and the etalon integral (37), the system of integral Equations (35), (36) is reduced to a system of algebraical linear equations:

$$\begin{aligned} B^{+}(k) &= A_1(k)\left(I(-k,2a)-I(-h,2a)\right)+B^{-}(-k)I(-k,2a),\\ B^{-}(-k) &= A_2(-k)\left(I(-k,2a)-I(h,2a)\right)+B^{+}(k)I(-k,2a). \end{aligned} \tag{38}$$

The functional values at the saddle point are found by solving the forgoing system of linear algebraic equations:

$$\begin{aligned} B^{+}(k) &= A_2(-k)I(-k,2a)\frac{I(-k,2a)-I(h,2a)}{1-I^2(-k,2a)}+A_1(k)\\ &\quad \frac{I(-k,2a)-I(-h,2a)}{1-I^2(-k,2a)}, \end{aligned} \tag{39}$$

$$\begin{aligned} B^{-}(-k) &= A_1(k)I(-k,2a)\frac{I(-k,2a)-I(-h,2a)}{1-I^2(-k,2a)}+A_2(-k)\\ &\quad \frac{I(-k,2a)-I(h,2a)}{1-I^2(-k,2a)}, \end{aligned} \tag{40}$$

where the required functions are defined in a short-wave approximation by these values as

$$\begin{aligned} B^{+}(w) &\cong A_1(w)\left(I(-w,2a)-I(-h,2a)\right)+B^{-}(-k)I(-w,2a),\\ B^{-}(w) &\cong A_2(w)\left(I(w,2a)-I(h,2a)\right)+B^{+}(k)I(w,2a). \end{aligned} \tag{41}$$

4.1. Electric Field Calculation

The diffraction field is calculated by substituting of (32) in (29)

$$E_x^1 = I_3 + I_4 \tag{42}$$

in the form of the following integrals:

$$I_3 = \int_{-\infty}^{\infty} e^{i(w(z-a)+v|y|)}\frac{A_1(w)+B^{-}(w)}{\sqrt{k-w}}dw, \tag{43}$$

$$I_4 = \int_{-\infty}^{\infty} e^{i(w(z+a)+v|y|)}\frac{A_2(w)+B^{+}(w)}{\sqrt{k+w}}dw. \tag{44}$$

The integral (43) in the notation of (25) in a polar coordinate system (24) becomes

$$I_3 = \frac{A_0}{\pi i}\int\limits_S \frac{e^{ikr^- \sin(\alpha+\theta^-)}}{\sin\alpha - \cos\psi_0}\left|\cos\frac{\alpha-\frac{\pi}{2}}{2}\right|\Big(\sin\frac{\psi_0}{2}e^{ika\cos\psi_0} - \cos\frac{\psi_0}{2} e^{-ika\cos\psi_0}[I(k\cos\alpha,2a) - I(k\cos\psi_0,2a)]\Big)d\alpha + \sqrt{2k}\int\limits_S e^{ikr^- \sin(\alpha+\theta^-)}\left|\cos\frac{\alpha-\frac{\pi}{2}}{2}\right| I(k\sin\alpha,2a)B^+(k)d\alpha.$$

Next, we obtain the short-wave asymptotic formula by the saddle point method

$$I_3 \cong -iA_0H_0^{(1)}(kr^-)\frac{\cos\frac{\theta^-}{2}}{\cos\theta^- - \cos\psi_0}\Big\{\sin\frac{\psi_0}{2}e^{ika\cos\psi_0} - \cos\frac{\psi_0}{2} e^{-ika\cos\psi_0}\left[I(k\cos\theta^-,2a) - I(k\cos\psi_0,2a)\right]\Big\} + \sqrt{2k}\pi H_0^{(1)}(kr^-)\cos\frac{\theta^-}{2}I(k\cos\theta^-,2a)B^+(k). \quad (45)$$

Analogously, we have for the integral (44):

$$I_4 \cong -iA_0H_0^{(1)}(kr^+)\frac{\sin\frac{\theta^+}{2}}{\cos\theta^+ - \cos\psi_0}\Big\{-\cos\frac{\psi_0}{2}e^{-ika\cos\psi_0} + \sin\frac{\psi_0}{2} e^{ika\cos\psi_0}\left[I(-k\cos\theta^+,2a) - I(-k\cos\psi_0,2a)\right]\Big\} + \sqrt{2k}\pi H_0^{(1)}(kr^+)\sin\frac{\theta^+}{2}I(-k\cos\theta^+,2a)B^-(-k). \quad (46)$$

It should be noted that the obtained diffraction electromagnetic waves are conical, they go over to cylindrical at $\beta = \pi/2$ and their asymptotic expressions provide a high precision [10].

5. CONCLUSION

Let us now consider a transition to a limit of the strip to a half plane when the first edge goes to infinity. If the second analytical source in Equation (15) is located in the origin of coordinates ($l_2 = 0$) and the first analytical source is at infinity ($l_1 \to \infty$) from Equation (27) we obtain the asymptotic magnetic field formula

$$H_x^1 = I_2 \simeq B_0\frac{\cos\frac{\theta}{2}\sin\frac{\psi_0}{2}}{\cos\theta - \cos\psi_0}\sqrt{\frac{2}{\pi k_0 r\sin\beta}}\,e^{i(k_0x\cos\beta + k_0r\sin\beta + \frac{\pi}{4})}. \quad (47)$$

Similarly from Equation (46) we have the next asymptotic expression for the electric field

$$E_x^1 = I_4 \simeq A_0 \frac{\sin\frac{\theta}{2}\cos\frac{\psi_0}{2}}{\cos\theta - \cos\psi_0}\sqrt{\frac{2}{\pi k_0 r \sin\beta}}\, e^{i(k_0 x \cos\beta + k_0 r \sin\beta + \frac{\pi}{4})}. \quad (48)$$

Thus, the formulae (47) and (48) coincide with the expressions for the diffraction of the plane wave by a half plane [5].

Notice that primary edge waves in Equations (26) and (27) are coincided with the results [9]

$$H_x^1 = I_1 + I_2 \sim \frac{2}{\cos\vartheta + \cos\vartheta_0}\left[\sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}e^{-ikl(\cos\vartheta+\cos\vartheta_0)} - \cos\frac{\vartheta}{2}\right.$$
$$\left.\cos\frac{\vartheta_0}{2}e^{ikl(\cos\vartheta+\cos\vartheta_0)}\right]\frac{e^{i(kr+\frac{\pi}{4})}}{\sqrt{2\pi kr}}, \quad (49)$$

in the far zone found from the solution of the half-plane diffraction problem, where the asymptotic form of Hankel function and the following notation $\theta_0 = \pi - \psi_0$, $a = l$, $\beta = \frac{\pi}{2}$, $k_0 = k$ are used.

So, the first terms of I_1, I_2, I_3, I_4 in Equations (26), (27), (45), (46) for H_x^1 and E_x^1 correspond to a diffraction of the plane wave, incident to the half plane. The second terms in equations listed above correspond to the diffraction of the primary conical wave on the edges of the strip.

It is interesting to observe that the precision of the dominating contribution turns out to be no less than that of the solution of the tertiary diffraction, achieved by the method of successive approximations.

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