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PHOTONIC BAND STRUCTURES AND ENHANCEMENT OF OMNIDIRECTIONAL REFLECTION BANDS BY USING A TERNARY 1D PHOTONIC CRYSTAL INCLUDING LEFT-HANDED MATERIALS

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Abstract—In this paper, we use the Bloch theorem and transfer matrix method to calculate the dispersion relation of a ternary 1D photonic crystal with left-handed materials. Then, we obtain the total omnidirectional reflection band gaps of this structure. We demonstrate that the omnidirectional reflected frequency bands are enlarged in comparison with ordinary materials with positive index of refraction.

1. INTRODUCTION

Photonic crystals (PCs) are novel class of optical media represented by natural or artificial structures with periodic modulation of refractive index [1–7]. They can prohibit the propagation of electromagnetic waves within a certain frequency range so the light can be totally reflected. Such forbidden bands are called Photonic Band Gaps (PBG) which is similar to the electronic band gaps for electrons in semiconductors [8, 9]. PCs has many interesting applications such as filters, optical switches, light-emitting diodes (LEDs) [10], fibers [11] wave guides [12, 13] etc. 1D photonic crystals are dielectric structures which optical properties changed in one direction is called the axis of periodicity, while in two other directions, the structure is uniform [2].

The periodic medium acts as a perfect mirror, totally reflecting wave incident from any direction and with any polarization under the certain conditions. An omnidirectional dielectric mirror (also known as a 1D photonic band gap crystal) exhibit 100% reflectivity at all angles of incidence and for all states of incident polarization. Unlike metallic mirrors which absorb a small fraction of incident optical power,

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dielectric reflectors are lossless. We can enhance the total reflection frequency range by choosing the proper refractive index, thickness of layers and by using the photonic heterostructures [14–18].

Left-handed materials are artificial composites with simultaneously negative permittivity ε and permeability μ [19–21]. In these materials the direction of Poynting vector $\vec{S} = \vec{E} \times \vec{H}$ is opposite to the wave vector $\vec{k}$, so the wave vector and refractive index should be negative and $\vec{k}$, $\vec{E}$ and $\vec{H}$ form a left-handed set of vectors [22–24]. The omnidirectional reflection bands of a binary PC with alternate right and left-handed materials were investigated by Srivastava and Ojha in 2007 [16].

Here we use a ternary 1D photonic crystal in which a left-handed layer PC (LHM) is sandwiched by two dielectric layers to calculate the dispersion relation. Using the dispersion relation, we plot the band structures of this material and compare with an ordinary structure with positive refractive index layer PC (RHM) in different angles for both polarizations. Then, we plot the reflectance spectra of two types of structures and demonstrate that the omnidirectional total reflected frequency bands of PC (LHM) are enhanced in compared with PC (RHM).

2. CALCULATION OF DISPERSION RELATION

Assuming a plane electromagnetic wave incident on 1D photonic crystal as we have shown in Figure 1, where n_1, n_2 and n_3 are the refractive indices, d_1, d_2 and d_3 are the thickness of layers and $\Lambda = d_1 + d_2 + d_3$ is the period of the structure. Suppose that the wave vector $\vec{k}$ lie in the x-z plane.

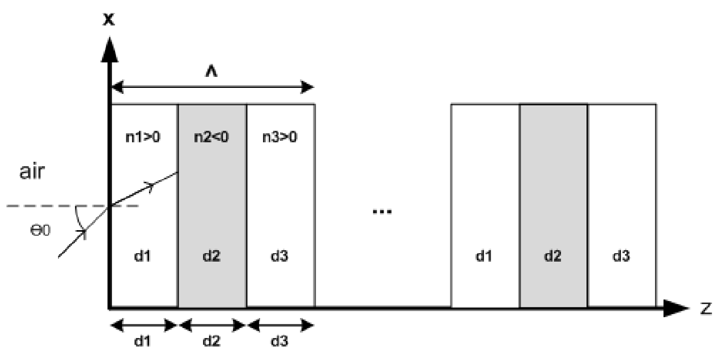

Figure 1. The structure of a ternary 1D photonic crystal with a left-handed layer that is sandwiched by two dielectric layers.

Using Maxwell's equations we obtain Helmholtz equation for the periodic structure as [14].

$$\nabla^2\vec{E}+\frac{\omega^2}{c^2}\varepsilon_r\mu_r\vec{E}=0, \tag{1}$$

where ε_r and μ_r are relative permittivity and relative permeability, respectively. The electric field of proposed wave is the function of x and z, [14],

$$E(x,z)=E(z)e^{i\beta x}. \tag{2}$$

Substituting (2) in (1) we obtain the electric field within each layer of unit cell for TE polarization as

$$\begin{aligned} E_{1y}&=A_1e^{ik_1z}+B_1e^{-ik_1z},\\ E_{2y}&=C_1e^{ik_2(z-d_1)}+D_1e^{-ik_2(z-d_1)},\\ E_{3y}&=E_1e^{ik_3(z-(d_1+d_2))}+F_1e^{-ik_3(z-(d_1+d_2))}, \end{aligned} \tag{3}$$

where $k_i=[(\frac{n_i\omega}{c})^2-\beta_i^2]^{\frac{1}{2}}$, $i=(1,\ 2,\ 3)$ is the wave number of each layer, $\beta_i=\frac{n_i\omega}{c}\sin\theta_i$ is the wave number along x axis, $n_i=-\sqrt{\varepsilon_i\mu_i}$ is refractive index for the left-handed material, ε_i and μ_i are the permittivity and permeability of each layer, respectively, θ_1, θ_2 and θ_3 are the incidence angles in the layers corresponding to refractive indices. These are related to each other by using Snell's law in air and the first layer of unit cell as

$$n_0\sin\theta_0=n_1\sin\theta_1. \tag{4}$$

The magnetic field vector $\vec{H}$ can be obtained along x-axis via Maxwell's equation as

$$\vec{H}=\frac{i}{\omega\mu}\vec{\nabla}\times\vec{E}. \tag{5}$$

The coefficients A_1, B_1, C_1, D_1, E_1 and F_1 are related through the continuity conditions at the interfaces $z=d_1$, $z=d_1+d_2$ and $z=\Lambda$. These continuity conditions lead to the matrix of each layer of unit cell as

$$M_j=\frac{1}{2}\begin{bmatrix}(1+\delta_j)e^{i\phi_j} & (1-\delta_j)e^{-i\phi_j}\\ (1+\delta_j)e^{i\phi_j} & (1-\delta_j)e^{-i\phi_j}\end{bmatrix},\quad (j=1,2,3). \tag{6}$$

In Equation (6) $\delta_1=\frac{k_1\mu_2}{k_2\mu_1}$, $\delta_2=\frac{k_2\mu_3}{k_3\mu_2}$ and $\delta_3=\frac{k_3\mu_1}{k_1\mu_3}$ are for TE polarization and $\delta_1=\frac{k_1\varepsilon_2}{k_2\varepsilon_1}$, $\delta_2=\frac{k_2\varepsilon_3}{k_3\varepsilon_2}$ and $\delta_3=\frac{k_3\varepsilon_1}{k_1\varepsilon_3}$ for TM polarization. $\phi_j=k_jd_j$ is the phase of each medium.

The transfer matrix of unit cell is given by

$$M_0=M_3\cdot M_2\cdot M_1. \tag{7}$$

If all unit cells were identical and by assuming the system is lossless, we can obtain the reflectance of the whole system as [2]

$$R_N = \frac{\psi_N^2 R}{1 - R + \psi_N^2 R}, \tag{8}$$

here N and R are number of layers and intensity reflectance of unit cell respectively. The reflectance R can be calculated by

$$R = \frac{M_0(2,1)}{M_0(1,1)} \left(\frac{M_0(2,1)}{M_0(1,1)}\right)^* . \tag{9}$$

According to the Bloch theorem [2,25], a wave propagating in a periodic medium is of the form

$$E_k(x,z) = E_k(z) e^{ikz} e^{i\beta x}, \tag{10}$$

where $E_k(z)$ is periodic with a period, Λ

$$E_k(z+\Lambda) = E_k(z), \tag{11}$$

and k is known as Bloch wave number. To determinate k, we can find relation between the electric field amplitudes of nth and $(n-1)$th layer as [25]

$$\begin{bmatrix} \mathrm{A}_n \\ \mathrm{B}_n \end{bmatrix} = e^{ik\Lambda} \begin{bmatrix} \mathrm{A}_{n-1} \\ \mathrm{B}_{n-1} \end{bmatrix}. \tag{12}$$

From the continuity conditions, we can also obtain [2]

$$\begin{bmatrix} \mathrm{A}_{n-1} \\ \mathrm{B}_{n-1} \end{bmatrix} = M_0 \begin{bmatrix} \mathrm{A}_n \\ \mathrm{B}_n \end{bmatrix}. \tag{13}$$

From Equations (12) and (13), we obtain

$$M_0 \begin{bmatrix} \mathrm{A}_n \\ \mathrm{B}_n \end{bmatrix} = e^{-ik\Lambda} \begin{bmatrix} \mathrm{A}_n \\ \mathrm{B}_n \end{bmatrix}. \tag{14}$$

Equation (14) shows an eigenvalue problem, with eigenvalue $e^{-ik\Lambda}$. By solving this equation, we can obtain the dispersion relation as

$$k(\omega) = \arccos\{1/2(M_0(1,\ 1) + M_0(2,\ 2))\}. \tag{15}$$

So, the dispersion relation of a ternary 1D PC with a negative index of refraction layer PC (LHM), is

$$k(\omega) = \frac{1}{\Lambda} \arccos\left\{\cos\phi_1 \cos\phi_2 \cos\phi_3 + 1/2\left(\delta_1 + \frac{1}{\delta_1}\right)\sin\phi_1 \sin\phi_2 \cos\phi_3 \right. \\ \left. + 1/2\left(\delta_2 + \frac{1}{\delta_2}\right)\cos\phi_1 \sin\phi_2 \sin\phi_3 - 1/2\left(\delta_3 + \frac{1}{\delta_3}\right)\sin\phi_1 \cos\phi_2 \sin\phi_3\right\}. \tag{16}$$

The refractive index of second layer is negative $n_2 < 0$ hence its wave number is also negative $k_2 < 0$.

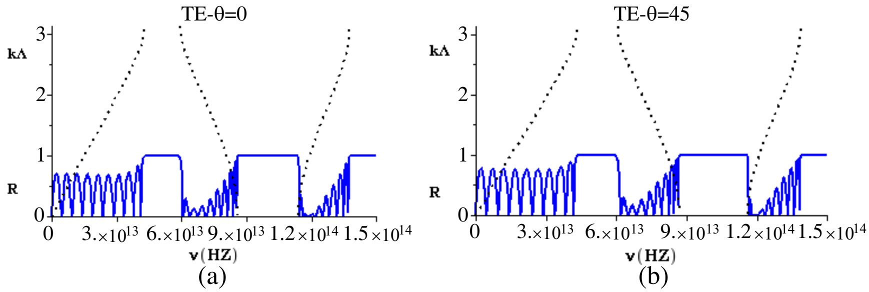

Figure 2. The frequency region of photonic band gap and propagating modes in dispersion relation (dotted line) and reflectivity (solid line) at (a) $\theta = 0°$, (b) $\theta = 45°$ for TE polarization of PC (LHM).

3. BAND STRUCTURES

In this section we plot the band structure of two types of photonic crystals PC (LHM) and PC (RHM) for both polarizations TE and TM under the different incident angles by using the dispersion relation. We consider a structure with the characteristics $n_1 = 1.35$, $n_2 = -3.6$ [16], $n_3 = 4.9$, $\mu_1 = \mu_3 = 1$, $\mu_2 = -1$, $d_1 = 0.3\,\mu\text{m}$, $d_2 = 0.4\,\mu\text{m}$, $d_3 = 0.8\,\mu\text{m}$ and $N = 10$ for PC (LHM) for TE polarization, and for PC (RHM) $n_2 = 3.6$ and the other parameters remain unchanged. The frequency range is considered from 0 to 1.5×10^{14} for all figures.

By using the profiles of dispersion relation, we can determine that how the electromagnetic wave can propagate and where the band gaps exist in PC. These comparisons between the reflectivity and dispersion relation are shown in Figure 2 for normal and oblique incident, ($\theta = 45°$) in the PC (LHM).

As we observe from dispersion relation profiles in Figure 2 (the dotted lines), the range of frequency where the dispersion relations are discontinuous correspond to photonic band gaps so that no propagating modes exist, and in the regions the dispersion relation are continuous, we have propagating modes (wave number) for each frequency.

As we observe from Figures 3 and 4, we conclude that for the structure PC (RHM) there are five band gaps in the considered range of frequency, while for PC (LHM) there are two band gaps. The band gaps of PC (LHM) are larger than the band gaps of PC (RHM).

4. OMNIDIRECTIONAL REFLECTION

To show the total omnidirectional reflection, we plot the reflectance spectra of two types of structures PC (LHM) and PC (RHM) for both polarizations TE and TM in Figures 5 to 8 under the different incident angles.

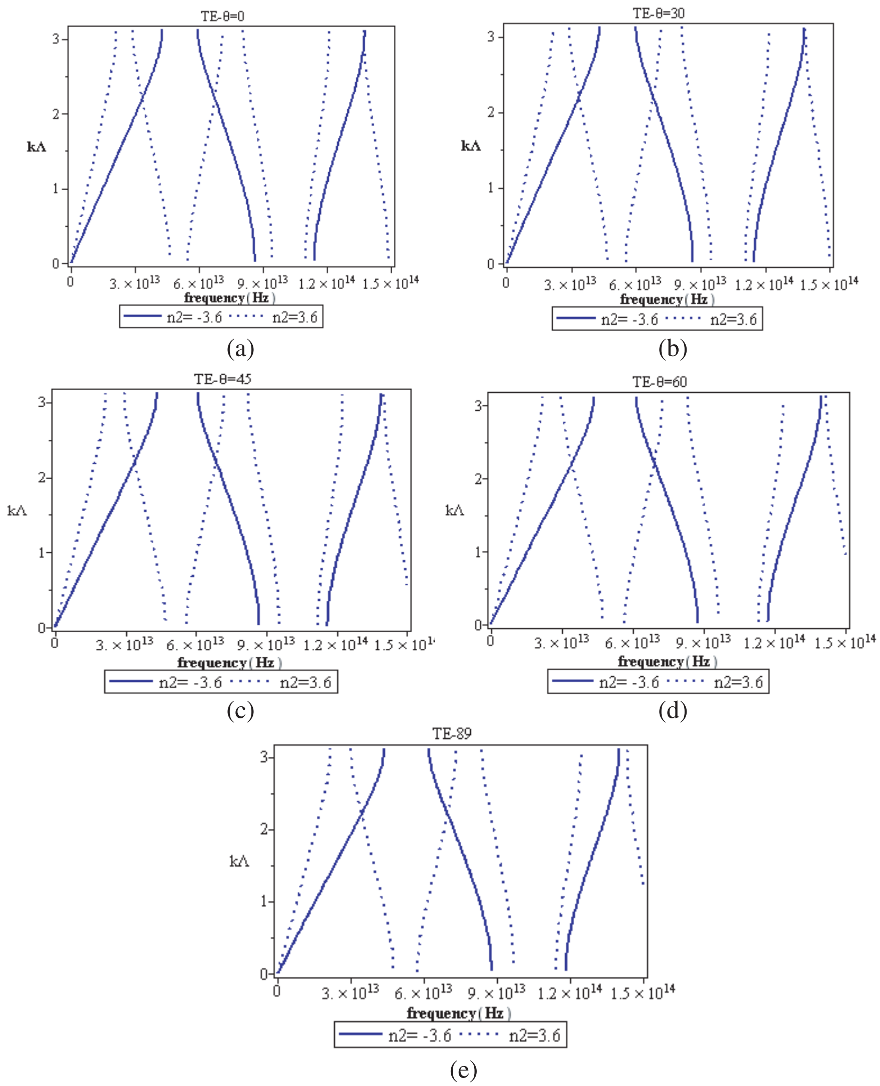

Figure 3. The plot of dispersion relation for both structures PC (LHM), (solid line) and PC (RHM), (dotted line) in TE polarization, (a) 0°, (b) 30°, (c) 45°, (d) 60° and (e) 89°.

We understand from figures that PC (LHM) has much wider band gaps than PC (RHM) for all incident angles, so using the left-handed layer can enhance the band gap widths. From Tables 1 and 2 it is clear that, the omnidirectional total reflection frequency for PC (RHM) in

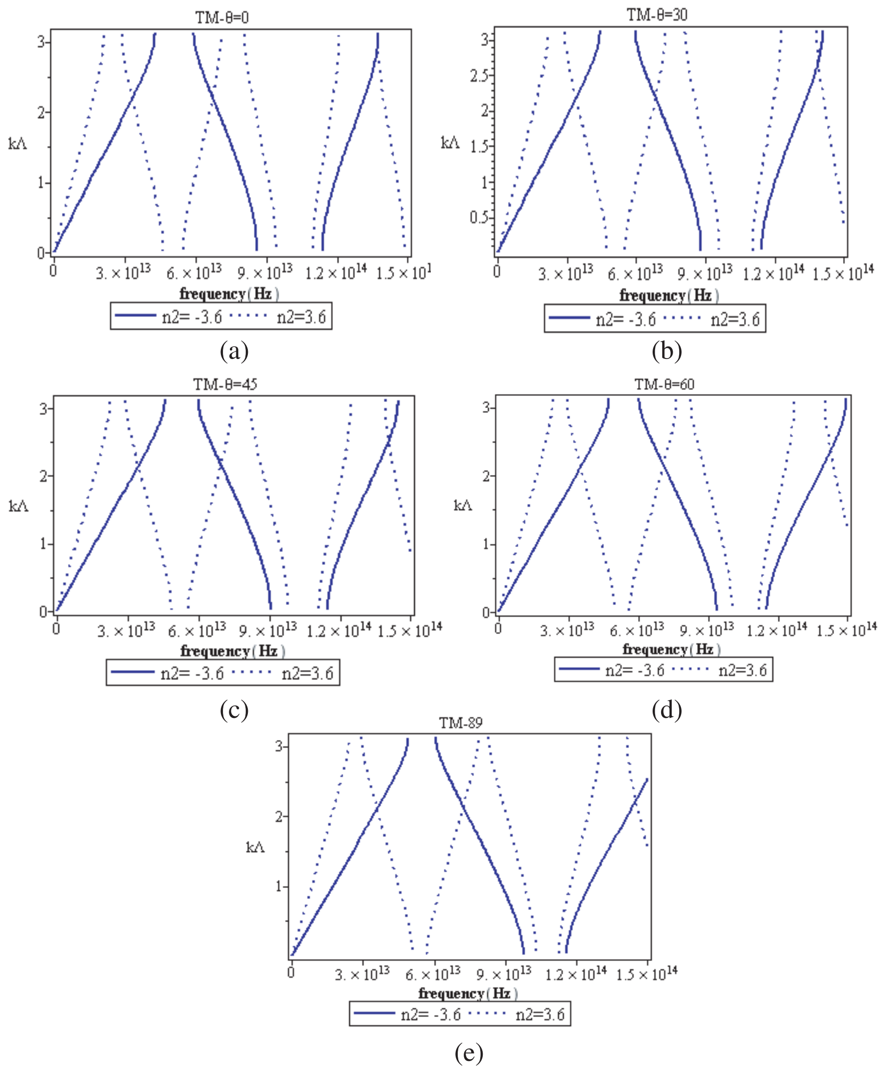

Figure 4. The plot of dispersion relation for both structures PC (LHM), (solid line) and PC (RHM), (dotted line) in TM polarization, (a) 0°, (b) 30°, (c) 45°, (d) 60° and (e) 89°.

TE and TM polarizations for the first band gap is about 2.478×10^{13} to 2.877×10^{13}, while for the PC (LHM) it lies in the range 4.924×10^{13} to 5.943×10^{13}. For the second band gap of PC (LHM) the omnidirectional total reflection frequency in both polarizations is about 9.776×10^{13} to

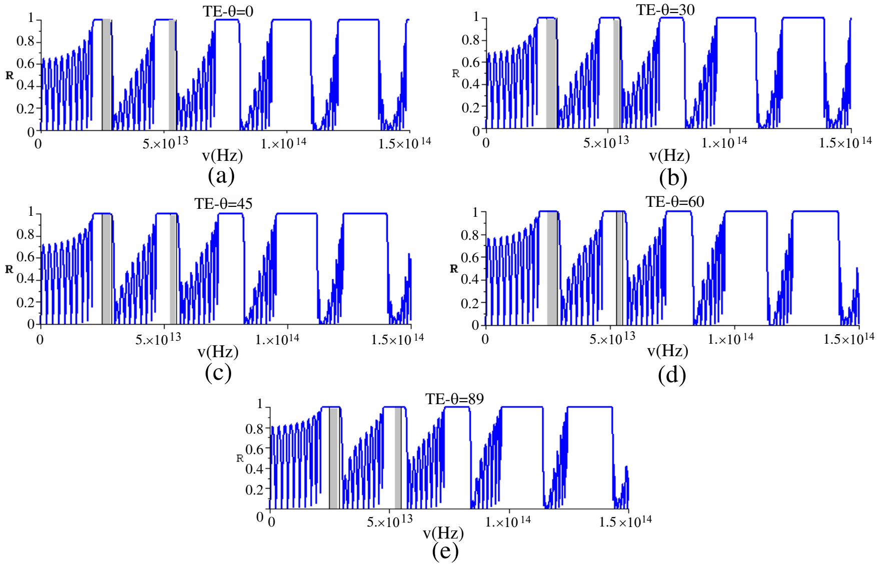

Figure 5. Reflectance spectra of PC (RHM) in TE polarization, (a) 0°, (b) 30°, (c) 45°, (d) 60°, (e) 89°.

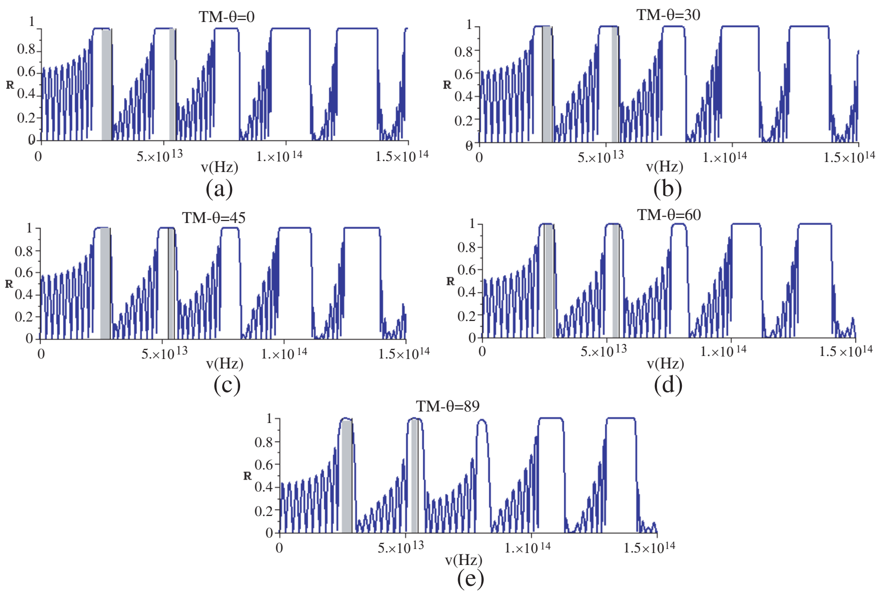

Figure 6. Reflectance spectra of PC (RHM) in TM polarization, (a) 0° (b) 30°, (c) 45°, (d) 60°, (e) 89°.

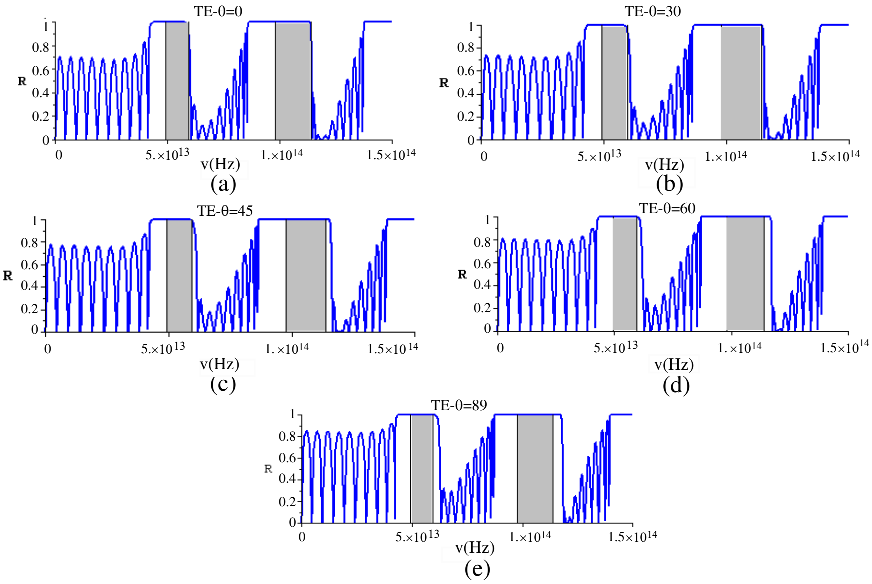

Figure 7. Reflectance spectra of PC (LHM) in TE polarization, (a) 0°, (b) 30°, (c) 45°, (d) 60°, (e) 89°.

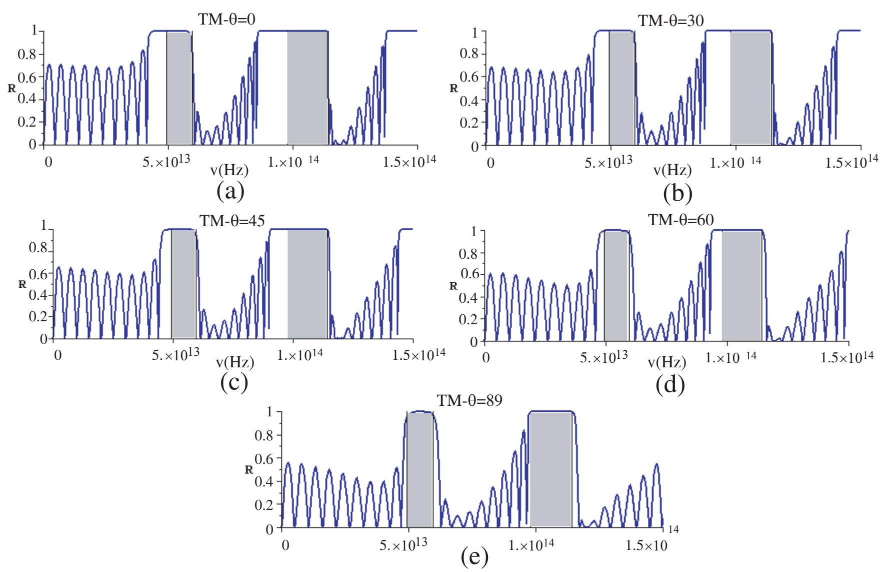

Figure 8. Reflectance spectra of PC (LHM) in TM polarization, (a) 0°, (b) 30°, (c) 45°, (d) 60°, (e) 89°.

11.395×10^{13} while for PC (RHM) there is an omnidirectional reflection frequency at the second band gap in 5.227×10^{13} to 5.482×10^{13}. We calculate the band gap widths of these structures in Tables 1 and 2.

Table 1. Total reflected frequency range for the structure PC (RHM) in both polarizations.

Incident angle	Band gaps for TE polarization ($\times 10^{13}$)	Band gaps for TM polarization ($\times 10^{13}$)
$\theta = 0°$	2.138-2.887 4.643-5.482 7.105-8.076 9.424-10.985 12.101-13.733	2.138-2.887 4.643-5.482 7.105-8.076 9.424-10.985 12.101-13.733
$\theta = 30°$	2.160-2.934 4.699-5.568 7.200-8.190 9.541-11.145 12.248-13.952	2.211-2.919 4.768-5.557 7.296-8.175 9.645-11.112 12.331-13.905
$\theta = 45°$	2.183-2.982 4.756-5.660 7.299-8.310 9.663-11.311 12.400-14.180	2.291-2.952 4.906-5.637 7.510-8.278 9.898-11.244 12.593-14.083
$\theta = 60°$	2.207-3.033 4.815-5.755 7.402-8.435 9.790-11.480 12.557-14.418	2.380-2.986 5.059-5.722 7.753-8.387 10.191-11.381 12.898-14.268
$\theta = 89°$	2.231-3.086 4.877-5.857 7.509-8.566 9.923-11.664 12.721-14.666	2.478-3.023 5.227-5.817 8.033-8.501 10.538-11.524 13.262-14.460
The omnidirectional reflected frequency range ($\times 10^{13}$): first band gap: 2.478-2.887, second band gap: 5.227-5.482		

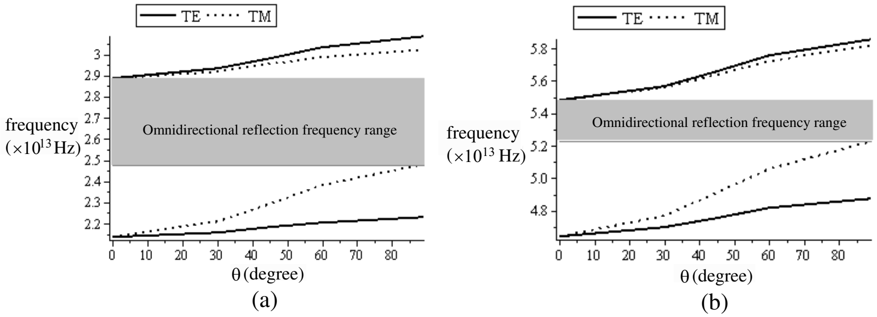

Figure 9. The calculated left and right band edges in both TE and TM of the (a) first (b) second band gap for PC (RHM).

Table 2. Total reflected frequency range for the structure PC (LHM) in both polarizations.

Incident angle	Band gaps for TE polarization ($\times 10^{13}$)	Band gaps for TM polarization ($\times 10^{13}$)
$\theta = 0°$	4.290-5.943 8.615-11.395	4.290-5.943 8.615-11.395
$\theta = 30°$	4.311-6.007 8.656-11.497	4.435-5.967 8.822-11.442
$\theta = 45°$	4.333-6.074 8.698-11.601	4.589-5.997 9.0719-11.491
$\theta = 60°$	4.356-6.143 8.740-11.708	4.753-6.035 9.380-11.539
$\theta = 89°$	4.378-6.214 8.783-11.815	4.924-6.085 9.776-11.587
The omnidirectional reflected frequency range ($\times 10^{13}$): first band gap: 4.924-5.943, second band gap: 9.776–11.395		

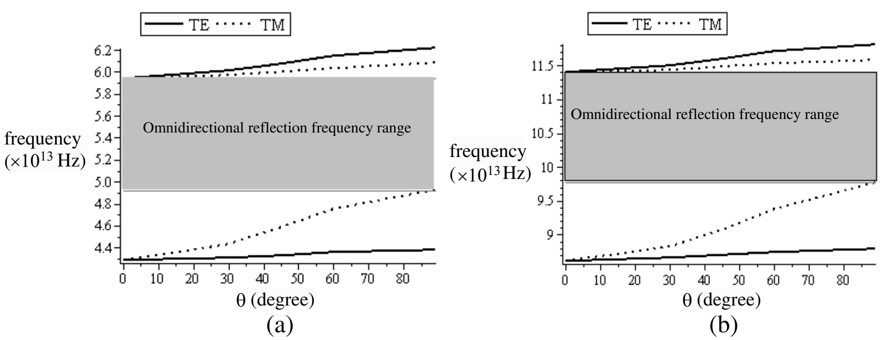

Figure 10. The calculated left and right band edges in both TE and TM of the (a) first (b) second band gap for PC (LHM).

We show the omnidirectional reflection bands for the first and second band gap of two structures and the variation of band gap width versus incident angles in Figures 9 to 10.

As we understand from these figures the calculated left and right band edges of the first and the second band gaps are increased by increasing the incident angle for both polarization TE and TM for two structures PC (LHM) and PC (RHM).

5. CONCLUSION

We calculated the dispersion relation for ternary PC (LHM). Using dispersion relation we plotted the band structures for both structures PC (LHM) and PC (RHM) under the different angles in TE and TM polarizations. We understood that the PC (LHM) has wider band gaps than PC (RHM). Furthermore, by increasing the incident angle, the bands are shifted toward the larger frequencies. We concluded that the omnidirectional reflection bands of PC (LHM) are larger than PC (RHM). Our results also showed that the calculated left and right band edges of the first and the second band gaps are increased, as the incident angle for both polarization TE and TM for two structures PC (LHM) and PC (RHM) is increased. Moreover this variation is deeper for TM polarization than TE polarization. In general, using left-handed material in photonic crystals can enlarge the omnidirectional band gaps.

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