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SPECKLE SUPPRESSION BY INTEGRATED SUM OF FULLY DEVELOPED NEGATIVELY CORRELATED PATTERNS IN COHERENT IMAGING

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Abstract—A coherent imaging system images a frame or an object onto a changing diffuser and projects the resulting pattern which generally contains speckles. Using a spatial light modulator (SLM) as the changing diffuser, the speckles in the pattern are suppressed without the need for any other mechanisms. With M random phasor arrays being displayed in the SLM during the integration time of a detector, a suppression factor (C_f) of speckles, $1/\sqrt{M}$, is achievable in the projected pattern, which is the sum of the intensity of M uncorrelated patterns. This paper shows both theoretically and in simulations that the C_f of the sum pattern was considerably reduced when two elementary patterns with fully developed speckles were negatively correlated. With the correlation coefficients of the elementary patterns found at $[-0.3, -0.25]$, the C_f of the sum of 10 negatively-correlated speckle patterns was 48% lower than the C_f of the sum of 10 uncorrelated speckle patterns. The negatively correlated patterns can be implemented using spatial light modulators or diffractive optical elements, and are used to suppress speckle noise in digital holography, laser projection display, and holographic display projections with relatively high efficiency.

1. INTRODUCTION

Coherent sources, such as laser, are advantageous in projecting images due to their wider color gamut, higher resolution, and higher light efficiency as compared to incoherent sources. However, the presence of speckles, a random granular appearance in images resulting from the interference of the coherent waves scattered from a rough object,

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degrades the image quality and leaves fine details irresolvable [1–3]. Although the speckles in the patterns have been successfully applied to identify fine features of objects [3, 4], suppression of speckles has been a critical issue and has been widely studied in coherent imaging. Numerous methods have been proposed to reduce speckle noise: using a source of multiple wavelengths, inserting a moving (vibrating or rotating) diffuser or ground glass, coupling with a vibrating multimode fiber, moving an optical aperture in the Fourier plane, and spatial averaging in the detection plane (Readers may refer to references [3] and [5] for critical reviews and categorization of the speckle suppression methods).

Recently, a new technique of diffractive optical element (DOE) has been widely employed to reduce speckle noise. Using DOEs in laser scanning displays, the scanning spot was split into several randomly-spread small spots [6], or modulated with phase arrays [7–10]. In systems projecting full-frame images, the DOE functioned as a changing diffuser to generate partially coherent illumination [11–14]. In holographic projection displays, a phase-only DOE encoding a target image was displayed in an LCoS spatial light modulator (SLM), and the Fourier transform of the DOE, showing the target image, was projected onto the screen [15–18]. Although static DOEs, such as in [7] and [9], require spatial motion or rotation to reduce speckle noise, most DOEs can be implemented using an SLM and take advantage of the rapid display of a sequence of phase arrays of DOEs to suppress speckles. The phase content of the DOE can be carefully calculated and accurately realized by an SLM to provide effective de-correlation of the light, resulting in low speckle noise.

To evaluate the intensity fluctuation resulting from speckles in a speckle pattern, the speckle contrast C is commonly used, and is defined as the ratio of the standard deviation to the mean of the intensity of the pattern [1], as given by

$$C = \frac{\sigma}{\bar{I}}, \tag{1}$$

where $\bar{I}$ and σ are the mean and the standard deviation of the intensity, respectively. The speckle contrasts of the speckle patterns with the original phase uniformly distributed over $[0, 2\pi]$ are always unity (i.e., $\sigma = \bar{I}$). The patterns with suppressed speckle have a smaller C, and the patterns with constant intensity (no speckled fluctuations) have the minimum value of C, which is zero. When the intensities of M uncorrelated speckle patterns, generated from M phasor arrays with uniformly-distributed phase being sequentially displayed in an SLM, are summed, the speckle contrast of the integrated sum is reduced to $1/\sqrt{M}$ [1–3, 5, 7, 12–16].

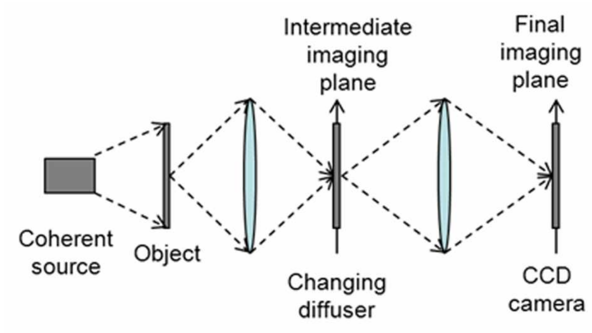

Figure 1. Simplified geometry for imaging an object or a frame onto a changing diffuser (intermediate plane) and projecting the resulting pattern onto the image plane.

In a coherent imaging system, which uses an SLM as a changing diffuser, the speckled image of an object is projected onto an intermediate plane where the changing diffuser is located (214, [3]). The intermediate image is modulated by the changing diffuser with a random phase array, and is projected by a secondary optical imaging lens onto the final imaging plane as shown in Figure 1. With a number of random phase arrays appearing rapidly in the SLM, the speckle noise in the final image, which is the sum of the corresponding images integrated over time using CCD cameras, is reduced. This paper shows that the speckles of the integrated image can be efficiently reduced by identifying the correlation coefficients of the corresponding images. In Section 2, the relationship between the speckle contrast of the integrated sum of two fully-developed speckle patterns and the correlation coefficients of the two individual patterns is derived. Section 3 shows the simulation results in which the speckle contrasts of the sums of two fully-developed speckle patterns are presented with the corresponding correlation coefficients. The proposed method is then applied to two-dimensional images with negative correlation coefficients and the summed images with low speckle contrasts are demonstrated. In addition, a scheme to produce a sequence of speckle patterns based on successively negative correlation coefficients is proposed to reduce the speckle contrast of their sum more efficiently.

2. THEORY OF SPECKLE CONTRAST VERSUS CORRELATION COEFFICIENT

Speckle noise can be suppressed more effectively by an integrated sum of multiple negatively correlated patterns, rather than uncorrelated speckle patterns. To obtain the relationship between the speckle contrast and the correlation of two speckle patterns, we assume that I_1 and I_2 are the intensity arrays of two arbitrary speckle patterns. The individual speckle contrasts of the patterns are given by $C_m = \sigma_m/\bar{I}_m$ for $m = 1$ and 2, where σ_m and $\bar{I}_m$ are the standard deviation and the mean of the intensity of pattern m, respectively. The correlation coefficient of the two elementary patterns is defined as [19, 20]

$$\rho_{1,2} = \frac{\overline{(I_1 - \bar{I}_1)(I_2 - \bar{I}_2)}}{\sigma_1\sigma_2} = \frac{\overline{I_1 I_2} - \bar{I}_1\bar{I}_2}{\sigma_1\sigma_2}. \tag{2}$$

Although different correlation functions are used to evaluate the speckle noise [21–23], Equation (2) can be adopted to attain the speckle contrast of the sum of two speckle patterns in terms of individual speckle contrasts. The correlation coefficient $\rho_{1,2}$ (or ρ for simplicity) is bounded in the range of $[-1,\ 1]$. Assuming that the intensity sum of the integration of two patterns is denoted by I_S, which is given by $I_1 + I_2$, the speckle contrast of the sum of the two patterns is modified from (1), and written as

$$C_S = \frac{\sigma_S}{\bar{I}_S}. \tag{3}$$

By substituting $\bar{I}_1$ and $\bar{I}_2$ and further simplifying, the intensity mean $\bar{I}_S$ and the variance σ_S^2 are given by

$$\bar{I}_S = \bar{I}_1 + \bar{I}_2 \tag{4}$$

and

$$\sigma_S^2 = \overline{I_S^2} - \bar{I}_S^2 = \left(\overline{I_1^2} - \bar{I}_1^2\right) + \left(\overline{I_2^2} - \bar{I}_2^2\right) + 2\rho_{1,2}\sigma_1\sigma_2, \tag{5}$$

respectively. Substituting (4) and (5) into (3) yields the speckle contrast of the integrated pattern

$$C_S = \sqrt{\frac{C_1^2\bar{I}_1^2 + 2\rho_{1,2}C_1C_2\bar{I}_1\bar{I}_2 + C_2^2\bar{I}_2^2}{\bar{I}_1^2 + 2\bar{I}_1\bar{I}_2 + \bar{I}_2^2}}. \tag{6}$$

Note that the speckle contrast in terms of an intensity ratio can be used to evaluate the effect of speckle suppression for the summed pattern of multiple partially developed speckle patterns that are uncorrelated [24]. Using Equation (6), however, will result in a direct comparison of the summed pattern and the individual patterns with speckles not only fully developed but also partially developed.

We now consider the case in which the elementary patterns have equal mean intensities; i.e., $\bar{I}_1 = \bar{I}_2$. The speckle contrast is simplified to

$$C_S = \sqrt{\frac{C_1^2 + 2\rho_{1,2}C_1C_2 + C_2^2}{4}}. \tag{7}$$

To characterize the effect of speckle suppression using the sum of two speckle patterns of equal mean intensity, we define the suppression factor C_f as

$$C_f \equiv \frac{C_S}{C_{avg}}, \tag{8}$$

where C_{avg} is the average of the speckle contrasts C_1 and C_2 of the two elementary speckle patterns; i.e., $C_{avg} = (C_1 + C_2)/2$. Using (2) and (7), C_f is written as

$$C_f = \sqrt{1 - \left(\frac{1-\rho_{1,2}}{2}\right)\frac{C_1 C_2}{C_{avg}^2}}. \tag{9}$$

Figure 2 shows the relationship between the suppression factor C_f of the summed pattern and the correlation coefficient ρ of the two corresponding elementary speckle patterns with equal mean intensities ($\bar{I}_1 = \bar{I}_2$) and fully developed speckle ($C_1 = C_2 = 1$). For two uncorrelated (or independent) patterns ($\rho = 0$) with fully developed speckle, the suppression factor of their intensity sum C_f is equal to $1/\sqrt{2}$, which is consistent with the result in [1]. An extreme case occurs when the two elementary patterns are identical; that is, when $\rho = 1$ and $C_f = 1$, no speckles are suppressed because the summed pattern is identical to the elementary patterns. For positively correlated patterns ($\rho > 0$), the suppression factor is larger than $1/\sqrt{2}$. However, the negatively correlated patterns ($\rho < 0$) provide higher

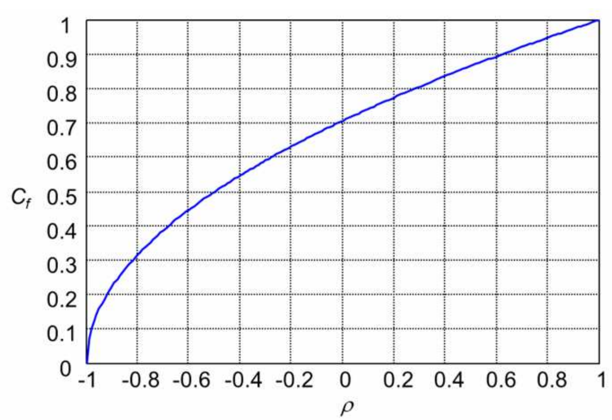

Figure 2. Suppression factor C_f of the sum of two fully-developed speckle patterns with equal mean intensity as a function of the correlation coefficient ρ.

efficiency in suppressing the speckle noise. In the extreme case of negatively correlated patterns ($\rho = -1$), the suppression factor C_f is zero, meaning that the integrated pattern contains no intensity fluctuations; i.e., $C_S = 0$, with the speckle noise being eliminated completely. Note that the symbol ρ without the subscript represents a general correlation coefficient for any two arrays, and the symbol $\rho_{m,n}$ represents a correlation coefficient for two specified patterns m and n.

3. SIMULATION RESULTS

To illustrate the general relationship between the suppression factor C_f of the integrated pattern of two speckle patterns (with equal $\bar{I}$, but arbitrary C values) and their correlation coefficient ρ, we conducted a simulation in which several million pairs of speckle patterns, resulting from a uniform illumination with a uniformly-distributed phase, were randomly generated using a method similar to that in Appendix G2 [3]. The suppression factors of their integrated sums were calculated and are discussed.

3.1. Suppression Factor of the Sum of Two Fully-developed Speckle Patterns

We first conducted a simulation in which several million pairs of one-dimensional (1D) speckle patterns with a uniformly-distributed phase were randomly generated. Using symbols similar to [3], the number of samples per 1D array, N, and the number of samples per speckle, k, were 256 and 4, respectively. Ten to twelve pairs of 1D speckle patterns were randomly selected on every 0.01 interval of ρ. A total of 905 pairs of 1D speckle patterns were obtained with ρ ranging between $(-0.431, 0.591)$, and the intensity of each selected pair of elementary speckle patterns was summed. The suppression factor C_f of the sum with respect to ρ of the elementary patterns is shown in Figure 3 in which a solid curve, a segment of the curve in Figure 2, is also plotted for comparison. The smallest ρ obtained was -0.431, and the corresponding C_f was 0.535. In Equation (9), when the factor $C_1C_2/C_{avg}^2 < 1$ for $C_1 \neq C_2$, the sum pattern has a larger C_f than the C_f at $C_1 = C_2$. This means that the lowest suppression result for a certain ρ occurs when the speckle contrasts of the two elementary patterns are equal. Figure 4 shows three pairs of speckle patterns with positive, zero, and negative $\rho's$ and their corresponding sum obtained from Figure 3.

We applied the speckle suppression method of negative correlation to two-dimensional (2D) coherent imaging. Figure 5 shows the

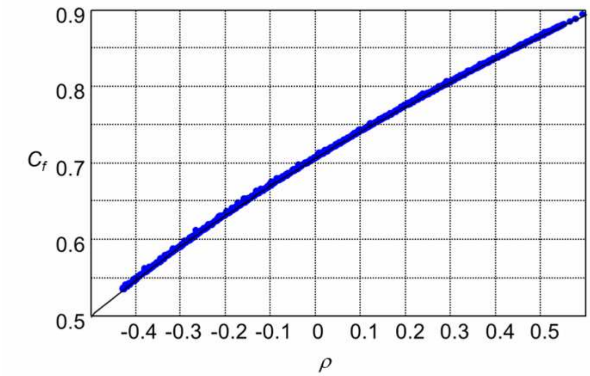

Figure 3. Suppression factor (C_f) vs. correlation coefficient (ρ) for 905 pairs of one-dimensional (1D) 256-sample speckle patterns.

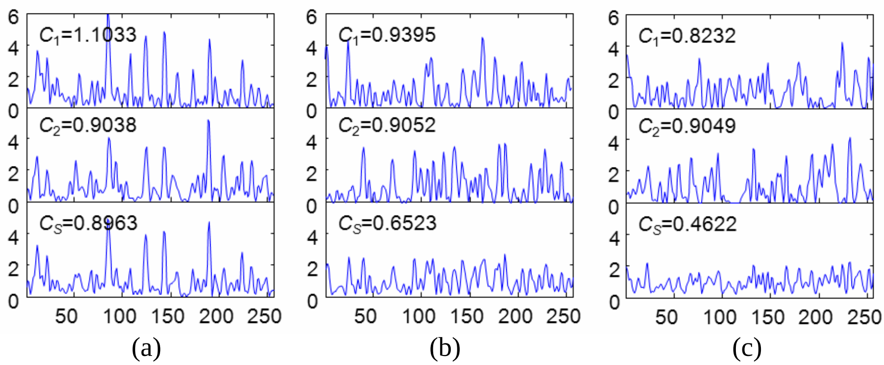

Figure 4. Speckle suppression of three pairs of 1D 256-sample speckle patterns with (a) positive correlation ($\rho = 0.591$, $C_f = 0.893$), (b) zero-correlation ($\rho = -0.001$, $C_f = 0.707$), and (c) negative correlation ($\rho = -0.431$, $C_f = 0.535$).

simulation result of a 128-by-128-pixel original object consisting of four square regions of different gray scales, the two elementary speckle patterns with $\rho = -0.113$, and the sum of the two speckle patterns.

3.2. Suppression Factor of the Sum of Multiple Fully-developed Speckle Patterns

Pairs of highly negatively correlated patterns are difficult to obtain because of the negative exponential density functions of the intensity distributions of fully-developed speckle patterns. However, a series of

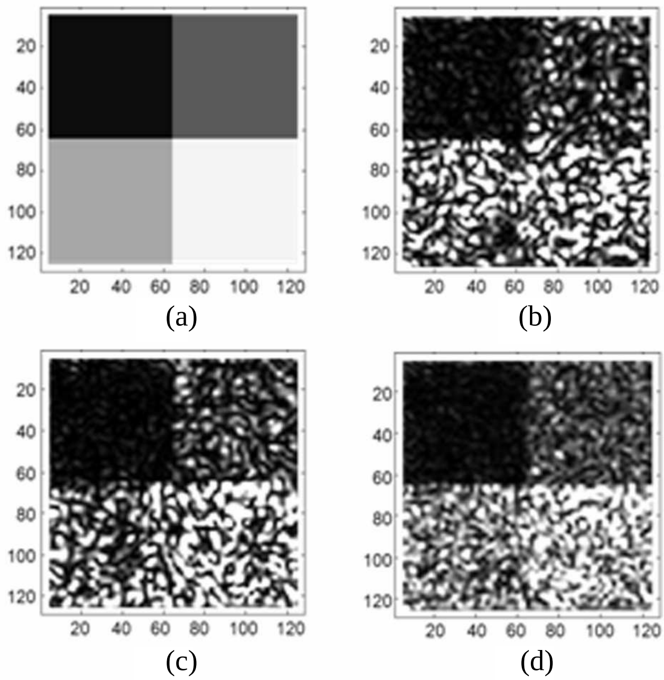

Figure 5. Speckle suppression of 2D negatively-correlated patterns ($N = 128^2$, $k = 4^2$). (a) Original object, (b) speckle pattern 1, (c) speckle pattern 2, and (d) the sum of patterns 1 and 2 with $\rho = -0.133$.

speckle patterns with small negative $\rho's$ correlated to the sum pattern, as found in the previous step, can be identified in a large number of fully-developed speckle patterns. The integrated sum of the intensity of these patterns suppresses speckle noise, as do two highly negatively correlated speckle patterns. A scheme is developed to produce a series of speckle patterns with successively negative $\rho's$; the procedures are listed below.

Step 1- Determine T, the number of searches for each elementary pattern. T depends on N/k.

Step 2- Generate a fully-developed N-sample speckle pattern. Set it as the initial sum pattern.

Step 3- Generate TN-sample speckle patterns with random phases, and choose the one with the lowest correlation coefficient ρ to the sum pattern obtained in the previous step.

Step 4- The new sum is generated by adding the intensities of the sum and the chosen pattern.

Step 5- Go to Step 3 for the next pattern until the set of speckle patterns is obtained.

In the simulations, ten sets of random-phase arrays for generating elementary speckle patterns with successively negative correlations were obtained and each set contained ten ($M = 10$) 1D 1024-sample phasor arrays. Each phasor array was identified in 1,000,000 (T) randomly-generated arrays based on the ρ of the corresponding speckle pattern and the sum pattern. In the negatively correlated patterns, the $\rho's$ were generally in the range of $-0.3 \sim -0.25$. Figure 6 shows the mean (solid curve), the highest, and the lowest $C_f's$ of 10 accumulated patterns of the 10 sets of negatively correlated patterns. Table 1 shows the averaged data of the ten sets of negatively correlated patterns. In the table, the letter m denotes the results at Step m, and the capital M denotes the accumulated result obtained by iteration from 1 to m. The mean C_f of ten sets of ten such patterns is considerably lower (48%) than that of the sum of uncorrelated patterns. For comparison, another two 10 sets of speckle patterns (each containing 10 patterns) were obtained in the simulation. In the first 10 sets (dashed curve) each new elementary pattern was zero correlated (uncorrelated) to the sum pattern by identifying the smallest absolute ρ identified in T randomly generated patterns (Step 3). Here, $|\rho| < 10^{-5}$ was obtained in all cases. The dashed curve of the simulated results coincides with the theoretical value of $1/\sqrt{M}$, and the data are shown in Table 2. The other 10 sets of speckle patterns were positively correlated by searching for the largest ρ between the sum pattern and the speckle pattern in T

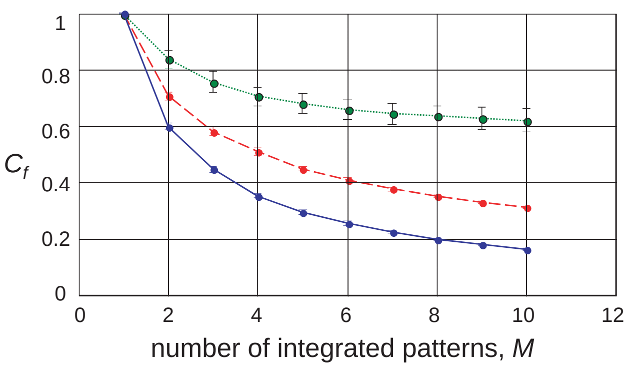

Figure 6. Suppression factor (C_f) of the sum of multiple speckle patterns with negative correlation $\rho = -0.3 \sim -0.25$ (solid), uncorrelation $\rho \sim 0$ (dashed), and positive correlation $\rho = 0.3 \sim 0.5$ (dotted).

Table 1. Averaged speckle suppression factors (C_f) and correlation coefficients (ρ) of ten sets of negatively correlated patterns. Each set contains ten speckle patterns scattered from the phasor arrays with the phase randomly distributed over $[0,\ 2\pi]$. Note that the symbol C_m denotes the speckle contrast of the m-th pattern, $\rho_{M-1,m}$ the correlation coefficient of the m-th pattern and the pattern summed from the 1st to $(m-1)$-th patterns, $C_{S(M)}$ the speckle contrast of the pattern summed from the first to m-th patterns, and $C_{f(M)}$ the suppression factor of the sum pattern of M speckle patterns.

m	1	2	3	4	5	6	7	8	9	10
C_m	0.951	0.927	0.967	0.929	0.923	0.987	0.950	0.957	0.917	0.946
$\rho_{M-1,m}$		−0.262	−0.275	−0.286	−0.295	−0.287	−0.284	−0.290	−0.279	−0.294
$C_{S(M)}$		0.571	0.426	0.334	0.275	0.237	0.208	0.185	0.169	0.153
$C_{f(M)}$	1.0	0.600	0.448	0.355	0.294	0.257	0.226	0.201	0.181	0.164

Table 2. Averaged speckle suppression factors (C_f) and correlation coefficients (ρ) of ten sets of uncorrelated (independent) patterns. Each set contains ten speckle patterns scattered from the phasor arrays with the phase randomly distributed over $[0,\ 2\pi]$.

m	1	2	3	4	5	6	7	8	9	10
C_m	0.951	0.948	0.978	1.002	0.913	0.933	0.929	0.929	0.924	0.955
$\rho_{M-1,m}$ ($\times 10^{-6}$)		−0.051	0.405	−0.050	0.375	−0.421	0.391	−0.040	0.518	−0.098
$C_{S(M)}$		0.671	0.554	0.485	0.429	0.390	0.360	0.336	0.316	0.300
$C_{f(M)}$	1.0	0.706	0.583	0.511	0.451	0.410	0.378	0.353	0.332	0.315

randomly generated patterns. The dotted curve in Figure 6 shows the C_f of the integrated pattern by adding speckle patterns with positive ρ in the range of $0.3 \sim 0.5$.

According to the simulation results shown in Tables 1 and 2, the C_f resulting from four negatively-correlated speckle patterns ($C_f \sim 0.334$) is 31% lower than the C_f (~ 0.485) of the sum of four uncorrelated patterns. It is also equivalent to the C_f of eight uncorrelated speckle patterns ($C_f \sim 0.336$). Therefore, when the phasor arrays, used to generate negatively correlated patterns, appear in the SLM with a limited frame rate, the speckle noise in the projected image can be reduced much more than when using the uncorrelated phasor arrays. In summary, negatively correlated patterns work more effectively with double the efficiency of uncorrelated patterns because a lower C_f can be achieved with only half the number of speckle patterns when they are negatively correlated.

4. CONCLUSION

In applications of coherent imaging such as digital holographic microscopy, laser projection displays, and holographic display projections, an SLM-type changing diffuser with elaborate phase characteristics can generate speckle patterns to suppress speckle noise and improve the image quality. The integration of the intensity of multiple fully-developed speckle patterns is a promising method for suppressing speckle noise. However, due to the limited frame rate of SLM, only a few phasor arrays can be present so that the corresponding speckle patterns are summed in a time integration to reduce the speckle noise of an image. In general, the use of M uncorrelated speckle patterns reduces the suppression factor (C_f) to $1/\sqrt{M}$. This paper shows a 31% decrease in the suppression factor when using four negatively correlated speckle patterns ($\rho \sim -0.274$) compared with four uncorrelated ($\rho \sim 0$) patterns, and a 48% decrease in the suppression factor when using ten negatively correlated speckle patterns ($\rho \sim -0.284$) compared with ten uncorrelated ($\rho \sim 0$) patterns.

The fully-developed speckle patterns with negative correlation and un-correlation were scattered from the phasor arrays with a randomly computer generated phase. The phasor arrays were displayed in the phase-modulated SLM which was used as the changing diffuser, as shown in Figure 1. In the application of holographic display [15–18], the phase-only DOEs, designed by using numerical methods, generated the diffractive images with negative correlation. Therefore, the merit function in the numerical methods has to contain the correlation coefficient in order to train the DOEs to generate the negatively correlated images.

The relationship between the speckle contrast of the integrated sum of two fully-developed speckle patterns and the correlation coefficients of the two individual patterns is derived. In theory, only two speckle patterns with high negative correlation are required to completely eliminate speckle noise. However, when speckle is fully developed, speckle patterns with high negative correlation are difficult to obtain. An iterative scheme for searching for a series of negatively correlated patterns is devised such that the speckle noise is reduced with considerable efficiency. In future research, more efficient methods will be developed to achieve phasor arrays that generate highly negatively correlated speckle patterns to reduce speckle noise with considerable efficiency.

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