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ELECTROSTATIC ANALYSIS OF TRANSMISSION LINES TO SIMULATE CORONA DISCHARGE AT HIGH VOLTAGE

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Abstract—This paper presents the results of Boundary Element Method (BEM) numerical procedures of voltages distribution between transmission lines in order to investigate the theoretical corona discharges. The algorithm of the voltage distributions are coded in Mathematica studying size of the system under controlling Neumann and Dirichlet boundary conditions. Conducting experimental work at a high voltage (HV) is potentially very dangerous. Therefore, simulation is a vital research approach, and computer modeling offers significant advantages to estimate optimal calculation over established system to prevent dangerous voltage and not to exceed the corona voltage. In this paper, the BEM results are verified with Finite Element Methods (FEM) which is coded in Mathematica too.

1. INTRODUCTION

In recent years, the development of computational methods, as well as advances in computer technology and improved numerical research for solving boundary value problems have resulted in efficient computer analysis of various types of physical models. Specifically, in HV engineering the numerical methods play an important role in determining the voltage distribution around and between the overhead lines which is important to establish the properties and defects of discharge in conductor and to analyze the both, insulating capabilities and energy losses of overhead lines. It is known that more than 50% of the power system accidents are caused by high voltage (current) breakdown [1]. Therefore, studying the voltage distribution and additionally gas breakdown are of our utmost purpose in design

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and protection of high voltage systems. Whereas such electrostatic problems were previously approximated by rough models and verified by expensive tests, it is now solved by computer models. A large number of simulation programs have already been developed in various applications [1, 2] and are frequently presented to engineers to use, but their usage is limited. The goal of this work is therefore to develop a computer program that simplifies analysis of HV media to understand corona discharge and predict the overload effects of lines in order to analyze voltage distribution between lines. From previous simulation results, it was found that ∇V (nabla of voltage value) is the primary factor in the gas breakdown, whereas the ratio of $\frac{d\nabla V}{dt}$, atmospheric pressure and relative humidity are relatively insignificant, therefore, the Laplace equation with Neumann and Drichlet condition are considered to measure the corona discharge. Two dimensional boundary and finite element theory are demonstrated, providing a practical example of a medium (atmosphere), with a transmission line voltage of 220 kV, the line spacing of 6 m, and height of the lines from the ground is 24 m. Although the governing equations of BEM need strong fundamentals of electromagnetic basics, BEM was used in analyzing of the HV system since it requires a less computation time and provides an understandable concept of numerical calculation [3–7].

2. NUMERICAL DISCUSSION FOR A REGION OF HV

Electrostatic problems essentially consist of finding the unknown potential function Φ that satisfies Laplace's equation within a prescribed solution region D under certain boundary conditions. Usually these boundary conditions are the Dirichlet ($\Phi(\mathbf{x}) = f(\mathbf{x})$) and Neumann ($\frac{\partial \Phi}{\partial n} = g(\mathbf{x})$) types. We consider here finding the voltage (U) distribution of a two dimensional quadrilateral second order Laplace equation as the region shown in Figure 1:

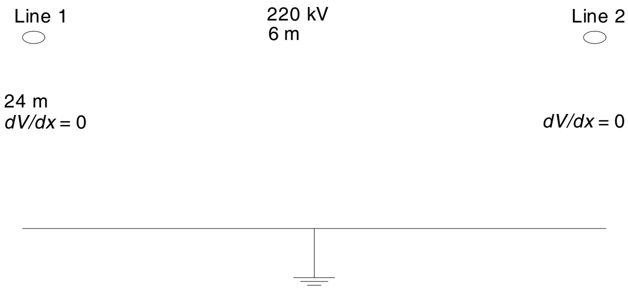

Figure 1. A plate with two dimensional transmission lines.

The line-line voltage and ground potential are taken as Dirichlet boundary conditions, and the rest of the boundary conditions are taken as Neumann conditions satisfying:

$$\frac{\partial U}{\partial x} = 0 \quad \text{and} \quad \frac{\partial U}{\partial y} = 0 \quad (x, y) \in \Gamma \tag{1}$$

where Γ symbolizes the boundary, and the Dirichlet conditions are considered at 20°C for natural convective air-cooling BEM formulation, Green's function as a solution of the Equation (2) through (7) as follows.

$$\nabla^2 G(\vec{r}, \vec{\rho}) = -\delta(\vec{r} - \vec{\rho}) \tag{2}$$

With free boundary conditions in two dimensions, Green's function is given by

$$G(\vec{r}, \vec{\rho}) = -\frac{1}{2\pi k} \ln |\vec{r} - \bar{\rho}| \, . \tag{3}$$

Here, $\vec{r}$ and $\vec{\rho}$ are field and source points, respectively. Together with equivalent boundary line charges on each subdivision or element for the solution of the above problem we get BEM equations

$$c(\vec{\rho})U(\vec{\rho}) = \int_S G(\vec{r}, \vec{\rho}) \frac{\partial U(\vec{r})}{\nabla n} - U(\vec{r}) \frac{\partial G(\vec{r}, \vec{\rho})}{\partial n} dS \tag{4}$$

where the constant element BEM formulation is then defined:

$$HU = G \frac{\partial U}{\partial n} \tag{5}$$

where

$$H = c(\vec{\rho}) - \sum_{e=1}^{N} \int_{\Gamma_e} \frac{(\vec{r} - \vec{\rho}) \cdot n}{|\vec{r} - \vec{\rho}|^2} d\Gamma_e \tag{6}$$

and

$$G = \sum_{e=1}^{N} \int_{\Gamma_e} \ln |\vec{r} - \vec{\rho}| \, d\Gamma_e. \tag{7}$$

where Γ_e is the boundary value for each element. The accuracy of BEM essentially depends on the accuracy of evaluation of integrals. The details of the integration schemes are covered in [1, 6–8, 11, 12]. The resulting Equation (8) can be separated into two parts defined as known and unknown terms, so that it can be reduced to a system of linear equations in the following form:

$$C\varphi = b \tag{8}$$

where C is a linear operator, and φ is the response of structure (voltage or temperature) while the second hand side vector b is the admissible (due to the boundary conditions imposed) excitation of the system. Of common knowledge, this unknown vector φ from Equation (8) is, in general, solved by iterative methods, due to the singularity of the matrix C [15–18].

The Corona Voltage (U_k) is calculated empirically from Peek's formula [12, 15]:

$$U_k = \sqrt{3} E_0 m r \delta \left(1 + \frac{0,3}{\sqrt{r\delta}}\right) \ln\left(\frac{a}{r}\right) \tag{9}$$

where $E_0 = 21.2\,\mathrm{kV/cm}$ is electric field of air; m represents line coefficient of aging which is 0.68; δ represents dependent air coefficient (760 mmHg and 20°C) which is 0.88; a represents the spacing between lines (m) which is 6 m; r represents line radius which is 14 mm. The result of the U_k using Equation (9) is 233 kV.

The corona thickness (coat) around the lines is determined by:

$$f = 0.301\sqrt{r} \tag{10}$$

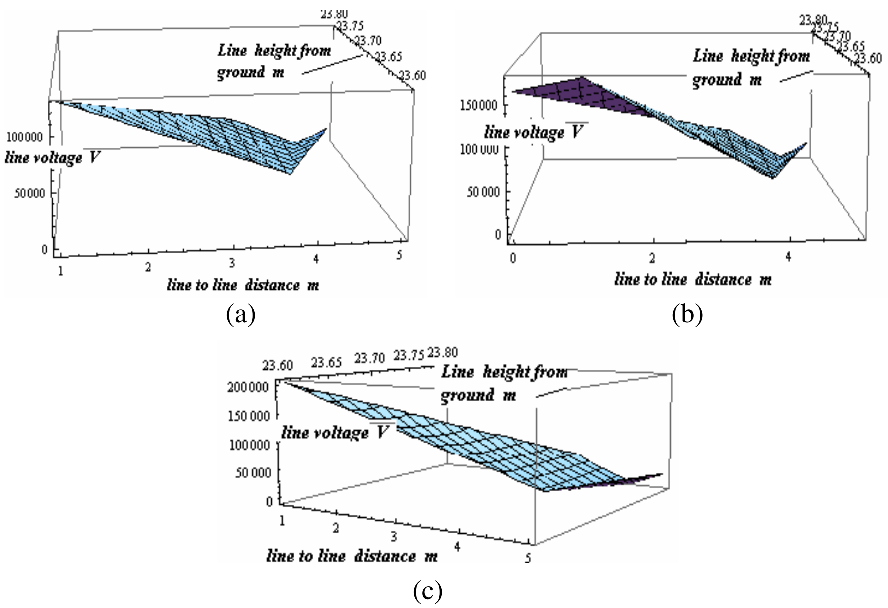

Figure 2. Voltage values versus interior nodes between lines for of (a) 220 kV, (b) 308 kV and (c) 352 kV.

3. NUMERICAL SOLUTIONS PROCEDURE

The voltage distribution was determined as if gas breakdown occurred around the lines at HV. This phenomenon causes big electrical losses, called corona losses. The corona discharge occurs around the naked lines so that the voltage distribution has values higher than U_k value, which is found from Peek's formula (Equation (9)). For the given example, shown in Figure 1, U_k is calculated as 233 kV.

As given in Section 2, electrostatic mathematical equations, identical for 2D Laplace equations, are solved numerically using indirect BEM and verified by FEM. The equations have been solved for nominal, 40% and 60% overloaded voltage values, and the results are shown as a graph in Figures 2(a), (b) and (c), respectively [13–16]. Those values are chosen to predict the changing rate for the transmission lines. The voltage distribution demonstrates that corona discharge is occurred on the line, surrounding the line as a coat, and the thickness (coat) of the corona depends on the radius of the transmission line (see Equation (10)).

The nominal voltage values of BEM are verified with FEM in Figure 3 which is almost closed. FEM result approached to that of BEM after fifty iterations. A certain number of nodes in the region of interest are taken intentionally, but it can be increased as required. For the point of eliminating the requirement for interior points, BEM model greatly reduces those (interior points) for a particular problem. This, in turn, reduces the time and system resources required for the solution of the problem. In a HV operation, it is necessary to know how voltages are distributed or what their maximum values are inside the insulated gases to avoid dielectric breakdown or flashover between two electrodes.

Robin's boundary condition may also be used instead of Neumann

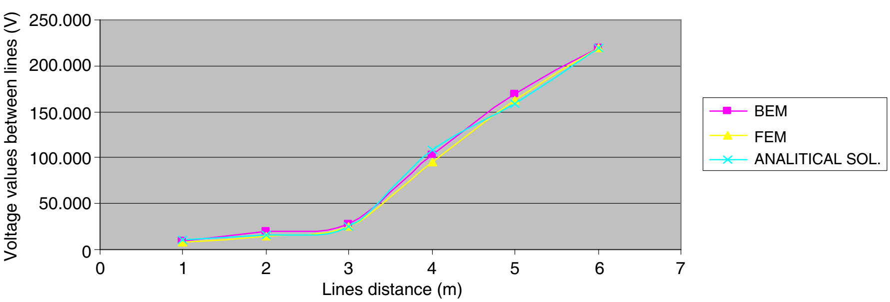

Figure 3. Voltage values BEM results for 220 kV verified with FEM.

boundary condition for the previously solved region just leaving all other conditions the same. Since medium is taken homogenous, the Robin condition does not make any change for the results of the problem. While studying with the numerical methods, putting the boundary condition with minimum error has vital importance. In this study, BEM results are compared to analytical and FEM results, and it shows that the region voltage distributions are solved with maximum approximation. Figure 3 indicates that the results are almost the same for BEM, FEM and Analytical Solution [9, 10, 12, 15].

4. CONCLUSION

Overall, the results presented here indicate that the numerical analysis of corona discharge at HV applications is suited to electrostatic computations. Corona discharge voltage and voltage distribution should be determined, either at the busbar or between the transmission lines. The study has shown that even at 40% and 60% overloads of nominal values, the voltage values did not change more than nominal distribution values, as shown in Figures 2(a), (b) and (c). This demonstrates that the corona thickness depends mostly on the radius of the transmission lines. Additionally, from the numerical studies, it is shown that corona discharge can be determined numerically. These are very important findings for the design of high voltage transmission lines. This study is also inspiring for calculating the thickness of the corona discharge. Furthermore, the study also has shown that BEM has some distinct advantages over FEM.

REFERENCES

1. Kuffel, E. and M. Abdullah, *High-voltage Engineering*, 32–43, Pergamon Press, 1970.
2. Cristina, S. and A. D. Napoli, "Combination of finite element and boundary element for magnetic field analysis," *IEEE Transaction on Magnetics*, Vol. 9, 2337–2339, IEEE Magnetic Society, 2003.
3. Wrobel, L., *The Boundary Element Method: Applications in Thermo-fluids & Acoustic*, John Wiley and Sons, 2002.
4. Gary, G. G. and A. G. Michael, "Boundary element methods for solving Poisson equations in computer vision problems," *IEEE Computer Society Conference on Computer Vision*, 3–6, IEEE Computer Society, 1991.

5. Trowbridge, C. W., "Electromagnetic computation: The way ahead," *IEEE Transaction on Magnetics*, Vol. 24, 13–18, IEEE Magnetic Society, 1998.
6. Sadiku, M. N. O., *Elements of Electromagnetic*, Oxford University Press, England, 2001.
7. Serteller, N. F. and A. Atalay, "Thermal analysis of ferromagnetic actuator by using finite element method," *Elsevier Pysica B Condensed Matter*, Vol. 372, 366–368, Elsevier, 2006.
8. Gipson, G. S., *Boundary Element Fundamentals — Topics in Engineering*, Computations Mechanics Publications, South Hampton, England, 1998.
9. Song, W. P., D. I. Yang, Y. H. Chung, and D. S. Kim, "A study of large current interrupting capability of SF/N2 mixtures," *IEEE International Symposium on Electrical Insulation*, 457–459, Boston, MA, USA, 2002.
10. Malik, N. H., A. A. Al-Aralny, and M. I. Qureshi, *Electrical Insulation in Power Systems*, Marcel Dekker Inc., 1998.
11. Buchau, A., W. Hafla, and W. W. Rucker, "Accuracy investigations of boundary element methods for solution of laplace equations," *IEEE Transaction on Magnetics*, Vol. 43, 1225–1228, 2007.
12. Tong, L. Z., F. X. Zgainski, J. C. Verite, and P. Thomas, "Investigation of gas phenomena in SF6 circuit breaker," *IEEE High Voltage Engineering Symposium*, 312–315, 1999.
13. Weillin, P., R. Gayllord, and S. Kamin, *An Introduction to Programming with Mathematica*, Cambridge University Press, England, 2005.
14. Smith, I. M. and D. V. Griffiths, *Programming with the Finite Element Method*, John Wiley and Son, England, 2004.
15. Kalenderli, Ö., C. Kocatepe, and O. Arıkan, *Çözümlü Problemlerle Yüksek Gerilim Tekniği Kitabı*, Birsen Yayınevi, 2005.
16. Abdelmageed, A. K., "Efficient evaluation of modal Green's functions arising in EM scattering by bodies of revolution," *Progress In Electromagnetics Research*, Vol. 27, 337–356, 2000.
17. Dean, T. R. and T. V. Hromadka, "A collocation CVBEM using program mathematica," *Engineering Analysis with Boundary Elements*, Vol. 34, 417–421, Elsevier, 2009.
18. Moroney, D. T. and P. J. Cullen, "The Green's function perturbation method for solution of electromagnetic scattering problems," *Progress In Electromagnetics Research*, Vol. 15, 221–252, 1997.