

CYLINDRICAL INVISIBILITY CLOAK INCORPORATING PEMC AT PERTURBED VOID REGION

A. Shahzad, F. Qasim, S. Ahmed, and Q. A. Naqvi*

Department of Electronics, Quaid-i-Azam University, Islamabad 45320, Pakistan

Abstract—A cylindrical wave expansion method is used to obtain the scattering field for a two dimensional cylindrical invisibility cloak incorporating perfect electromagnetic conductor (PEMC) at perturbed void region. A near-ideal model of the invisibility cloak is set up to solve the boundary-value problem at the inner boundary of the cloak shell. It is confirmed that a cloak with the ideal material is a perfect cloak by observing the change of the scattering coefficients from the near-ideal case to the ideal one. However, because of the slow convergence of the zeroth-order scattering coefficients, a tiny perturbation on the cloak would induce a noticeable field scattering. A better convergence rate of the scattering coefficients has been observed for decreasing δ in PEMC case.

1. INTRODUCTION

To become invisible or make other things invisible has always been a great fantasy for the human mind. Many science-fiction novels or movies revolve around people with powers to become invisible. Modern science is converting this fantasy into a reality. Scientists have already developed certain devices which can render things invisible to certain specific electromagnetic waves. Such devices are known as electromagnetic invisibility cloaks or simply cloaks. A cloaking device can virtually conceal what is placed under it. This device is made up of special metamaterials (MTMs) with variable refractive index which does not allow waves to enter it. The waves falling on the cloaking material are allowed to flow around the cloak undistortedly and then emerge on the other side, while remaining in phase with other waves

Received 13 June 2011, Accepted 19 September 2011, Scheduled 27 September 2011

* Corresponding author: Qaisar Abbas Naqvi (nqaisar@yahoo.com).

which do not fall on the cloak, i.e., exactly in the same direction as they began. In this way the cloaks allow no reflection of waves as well as casts no shadow. This makes the object inside a cloak to virtually become invisible, that is, one can theoretically see behind the cloaked object.

Metamaterials used for making cloaks are required to have negative refraction index and known as left-handed (LH) metamaterials. Metamaterials are a class of artificially engineered composite materials having extraordinary electromagnetic properties (not found in natural materials). Negative refraction index material has been discussed by many scientists [1–4]. Smith et al. represent the metamaterials and negative refraction index in [1]. Shaleev then gave idea of optical negative-index metamaterials in [2]. Shelby et al. [3] experimentally showed the existence of (refractive index, permittivity and permeability) at microwave frequencies in a metamaterial and which do not violate any of Maxwell equations. Pendry showed that negative refraction makes a perfect lens in [4]. Pendry is the pioneer of the cloaking concept. Pendry and Schuring [5] first illustrated through theoretical simulations, that object can be cloaked from electromagnetic fields by exploiting coordinate transformation in inhomogeneous and anisotropic metamaterials. Pendry et al. constructed a cloak using metamaterial design and practically achieved electromagnetic cloaking at microwave frequencies. Cummer et al. analyzed the full-wave simulations of electromagnetic cloaking structures [6]. They worked on the electromagnetic simulations of the cylindrical version of this cloaking structure using ideal and non-ideal electromagnetic parameters that showed that the low reflections and power flow banding properties of electromagnetic cloaking structures are not especially sensitive to modest permittivity and permeability variation. The cloaking performance degrades smoothly with increasing loss, and effective low reflection shielding can be achieved with the cylindrical shell composed of an eight-(homogeneous) layered approximation of the ideal continuous medium. Schuring et al. described metamaterial electromagnetic cloak at microwave frequencies [7]. It is the first practical realization of such a cloak. In their work, a copper cylinder is hidden inside a cloak. They presented a design of cloak operating over a band of microwave frequencies. This cloak decreases scattering from the hidden object while at the same time reduces its shadow so that the cloak and object combined began to resemble empty space. Cai et al. discussed optical cloaking with metamaterials [8]. Ruan et al. [9] confirmed that a cloak with ideal material parameters in a perfect invisibility cloak by symmetrically studying the scattering coefficient from the near ideal case to the ideal one. They introduced a tiny perturbation because of

the slow convergence of the zeroth order scattering coefficient. Yan et al. discussed that cylindrical invisibility cloak with simplified material parameters is inherently visible [10]. They described that cylindrical cloak having simplified material parameter inherently allow the zeroth order cylindrical wave to pass through the cloak as if the cloak is made of a homogenous isotropic medium and thus visible to all higher order cylindrical waves. Their numerical simulation suggest that the simplified cloak inherits some properties of the ideal cloak, but some scattering exist. Greenleaf et al. worked on improvement of cylindrical cloak with the soft and hard surfaces (SHS) lining [11]. They showed that the cloak is significantly improved by the use of SHS lining with both the far field of the scattering wave significantly reduced and the blow up of electric field density and magnetic field density prevented. Yan et al. suggested that scattering characteristics of simplified cylindrical invisibility cloak [12]. They compared the scattering characteristics of three types of simplified cylindrical cloak, simplified linear cloak, improved linear cloak, and the simplified quadratic cloak. Among the these three cloaks both the improved linear cloak and quadratic cloak show better invisibility performance as compared to the simplified linear cloak. Zhang et al. show the response of cylindrical invisibility cloak to electromagnetic waves [13]. They discussed that both magnetic and electric surface current are induced at the inner boundary of the cylindrical cloak by incoming wave and the surface currents have no counterparts in the coordinate transform theory. Cai et al. introduced nanomagnetic cloak with minimized scattering [14]. They proposed an electromagnetic cloak using higher order transformation to create smooth rather than discontinuous module at the outer interface. Yan et al. worked on coordinate transformation to make perfect invisibility cloak with arbitrary shape [15]. Yan et al. discussed the influence of geometrical perturbation at inner boundaries of invisibility cloaks [16]. An extensive research has been made on cloaks by many researchers [17–21].

Up to now, most of the discussions on invisibility cloaks have focussed on the cylindrical and spherical cloaks produced by a coordinate transformation only in the radial direction. The invisibility performance of such cylindrical and spherical cloaks are further confirmed by obtaining the exact fields in the cloak medium directly from the Maxwell equations. In practice, it is sometimes desirable to have invisibility cloaks whose shapes are according to the shape of objects. Thus, one needs to understand well the properties of invisibility cloaks with arbitrary shaped produced by general coordinate transformation. Such coordinates transformation technique is discussed in [5,18]. Using this technique, it is observed that the

Maxwell equations in a general curved space have same form as in cartesian space but with different permittivity, permeability, current density and electric charge density [5, 18]. It is also shown in [5] that when the fields \mathbf{E}^i and \mathbf{H}^i are incident upon the cloak. The fields in the cloaked medium can be obtained using the coordinate transformation as

$$\begin{aligned}\hat{E}_r(r, \theta, z) &= f'(r)E_r^i(f(r), \theta, z) \\ \hat{H}_r(r, \theta, z) &= f'(r)H_r^i(f(r), \theta, z) \\ \hat{E}_\theta(r, \theta, z) &= \frac{f(r)}{r}E_\theta^i(f(r), \theta, z) \\ \hat{H}_\theta(r, \theta, z) &= \frac{f(r)}{r}H_\theta^i(f(r), \theta, z) \\ \hat{E}_z(r, \theta, z) &= E_z^i(f(r), \theta, z) \\ \hat{H}_z(r, \theta, z) &= H_z^i(f(r), \theta, z)\end{aligned}$$

where $[E_r^i, E_\theta^i, E_z^i]$ and $[H_r^i, H_\theta^i, H_z^i]$ are the components of incident fields expressed in cylindrical coordinate form.

PEMC material introduced by Lindell and Shivola [22] attracted the attention of many researchers [22–41]. Ruppin developed an analytic theory for the electromagnetic scattering from PEMC cylinder [26]. Ahmed and Naqvi studied the behavior of the buried perfect electromagnetic conductor (PEMC), coated PEMC, and coated nihility circular cylinder for the plane wave or a line source excitation [31–35]. Many other scientists explored the important properties of PEMC material in [36–41]. PEMC material is the generalization of PEC and PMC materials, and boundary conditions are

$$\vec{n} \times (\vec{H} + M\vec{E}) = 0 \quad (1)$$

$$\vec{n} \cdot (\vec{D} - M\vec{B}) = 0 \quad (2)$$

where M is the admittance parameter of PEMC material.

PEMC cylinder has many applications including geophysical prospecting and remote sensing. Due to the particular property of short-circuiting, the PEMC material offers a medium which can be exploited in microwave engineering applications. Potential examples of such are, e.g., ground planes for profile antennas, radar reflectors, field pattern purifiers for aperture antennas, polarization transformer and generalized high-impedance surfaces.

In this work, an invisibility cloak termed as cylindrical invisibility cloak incorporating perfect electromagnetic conductor (PEMC) at perturbed void region is discussed. Our main focus is to explore the scattering characteristics of the cylindrical invisibility cloak and to

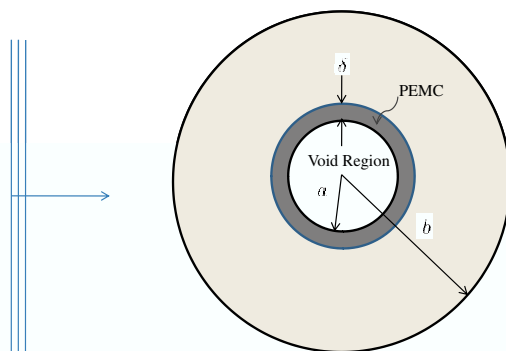


Figure 1. A schematic of a cylindrical invisibility cloak with PEMC thin perturbation.

improve the convergence rate of the scattered fields from the cylindrical invisibility cloak.

The next sections deal with the formulation of problem. Based on the formulation, numerical results are presented. We have used $e^{-j(\omega t)}$ time dependence which is suppressed through out the analysis.

2. ANALYTICAL FORMULATION

A two-dimension (2-D) cylindrical invisibility cloak incorporating PEMC layer at tiny perturbation δ can be constructed by compressing electromagnetic fields in a cylindrical region $r' < b$ into concentric cylindrical shell $a < r < b$ as shown in Figure 1. At the inner boundary of cylindrical cloak some components of medium parameters may reach infinity. To avoid this situation, a thin layer of width δ has been removed from the inner boundary. The region outside the shell $r > b$ is termed as free space with $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ as wave number and η_0 as impedance. Region between $a + \delta < r < b$ is termed as region 1.

Region inside the cloak is composed of unknown medium. Consider a coordinate transformation in cylindrical coordinates such that

$$f(r) = r' = \frac{b}{(b-a)}(r-a)$$

with characteristics $f(a) = 0$ and $f(b) = b$, while θ and z are kept unchanged [5]. Following the coordinate transformation method, the

permittivity and permeability of the cloak region may be obtained as

$$\begin{aligned}\epsilon_r &= \mu_r = \frac{f(r)}{r f'(r)} \\ \epsilon_\theta &= \mu_\theta = \frac{f(r) f'(r)}{f(r)} \\ \epsilon_z &= \mu_z = \frac{f(r) f'(r)}{r}\end{aligned}$$

where the superscript $'$ denotes differentiation.

2.1. Electromagnetic Fields in Free Space ($r > b$)

Consider the case when a H -polarization wave is incident on the cloak from free space and a geometrical perturbation δ is introduced on the inner boundary of the cloak. It is assumed that PEMC is placed at the inner boundary of cloak to avoid the situation when some parameters become infinity.

The incident electromagnetic field in terms of cylindrical coordinates (r, θ) , can be written as

$$H_{0z}^i = \sum_{n=-\infty}^{\infty} J_n(k_0 r) e^{in(\theta)} \quad (3)$$

$J_n(\cdot)$ is the Bessel function of first kind. Using Maxwell equation, the corresponding θ component of incident electric field may be obtained as

$$E_{0\theta}^i = -i\eta_0 \sum_{n=-\infty}^{\infty} J'_n(k_0 r) e^{in(\theta)} \quad (4)$$

where η_0 is the impedance of the free space.

In response to this incident field, the corresponding scattered field from the outer boundary of cylindrical invisibility cloak may be written in terms of unknown coefficient as

$$H_{0z}^s = \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(k_0 r) e^{in(\theta)} \quad (5)$$

The θ component of electric field is written as

$$E_{0\theta}^s = -i\eta_0 \sum_{n=-\infty}^{\infty} a_n H_n^{(1)'}(k_0 r) e^{in(\theta)} \quad (6)$$

where $H_n^{(1)}(\cdot)$ is the Hankel function of first kind.

2.2. Electromagnetic Fields in Region 1 ($a + \delta < r < b$)

The transmitted field in region 1 can be written in terms of unknown transmission coefficient as

$$H_{1z}^i = \sum_{n=-\infty}^{\infty} b_n J_n(k_0 f(r)) e^{in(\theta)} \quad (7)$$

The corresponding electric field can be written by using Maxwell equation as

$$E_{1\theta}^i = -i\eta_0 f'(r) \sum_{n=-\infty}^{\infty} b_n J'_n(k_0 f(r)) e^{in(\theta)} \quad (8)$$

The co-polarized component of the scattered field by cloak in region 1 is

$$H_{1z}^s = \sum_{n=-\infty}^{\infty} c_n H_n^1(k_0 f(r)) e^{in(\theta)} \quad (9)$$

The corresponding electric field is

$$E_{1\theta}^s = -i\eta_0 f'(r) \sum_{n=-\infty}^{\infty} c_n H_n^{(1)'}(k_0 f(r)) e^{in(\theta)} \quad (10)$$

The field scattered from a PEMC boundary contains E -polarized fields in addition to the H -polarized fields for E -polarized excitation [24]. Hence, the cross-polarized component of the scattered field in region 1 may be expressed as

$$E_{1z}^s = -i\eta_0 \sum_{n=-\infty}^{\infty} d_n H_n^{(1)}(k_0 f(r)) e^{in(\theta)} \quad (11)$$

Corresponding θ component of scattered magnetic field is

$$H_{1\theta}^s = f'(r) \sum_{n=-\infty}^{\infty} d_n H_n^{(1)'}(k_0 f(r)) e^{in(\theta)} \quad (12)$$

In all the above expressions, a_n , b_n , c_n , and d_n are unknown coefficients, which are to be determined by using appropriate boundary conditions at interfaces.

The boundary conditions at the interface $r = b$ are

$$H_{0z} = H_{1z} \quad r = b, \quad 0 \leq \theta \leq 2\pi \quad (13)$$

$$E_{0\theta} = E_{1\theta} \quad r = b, \quad 0 \leq \theta \leq 2\pi \quad (14)$$

which means that the tangential components of total electric and magnetic fields are continues.

The boundary conditions at the interface $r = a + \delta$ are

$$H_{1z} + ME_{1z} = 0, \quad r = a + \delta, \quad 0 \leq \theta \leq 2\pi \quad (15)$$

$$H_{1\theta} + ME_{1\theta} = 0, \quad r = a + \delta, \quad 0 \leq \theta \leq 2\pi \quad (16)$$

where M is the admittance parameter of the PEMC material. By applying these boundary conditions at $r = a + \delta$ and $r = b$, the following set of equations are obtained

$$J_n(k_0b) + a_n H_n^{(1)}(k_0b) = b_n J_n(k_0f(b)) + c_n H_n^{(1)}(k_0f(b))$$

$$J'_n(k_0b) + a_n H_n^{(1)'}(k_0b) = b_n f'(b) J'_n(k_0f(b)) + c_n f'(b) H_n^{(1)'}(k_0f(b))$$

and

$$b_n J_n(k_0f(a + \delta)) + c_n H_n^{(1)}(k_0f(a + \delta))$$

$$-Mi\eta_0 d_n H_n^{(1)}(k_0f(a + \delta)) = 0$$

$$d_n H_n^{(1)'}(k_0f(a + \delta)) - Mi\eta_0 b_n J'_n(k_0f(a + \delta))$$

$$-Mi\eta_0 c_n H_n^{(1)'}(k_0f(a + \delta)) = 0$$

After simplifications, the unknown scattering coefficients are

$$a_n = D'_n B'_n - C'_n \quad (17)$$

$$b_n = D'_n \quad (18)$$

$$c_n = D'_n A'_n \quad (19)$$

$$d_n = \frac{D'_n I_n + D'_n A'_n N_n}{\beta N_n} \quad (20)$$

where

$$A'_n = \frac{\beta^2 N_n L_n - I_n K_n}{K_n N_n - \beta^2 K_n N_n}$$

$$B'_n = \frac{C_n + A'_n H_n}{I_n}$$

$$C'_n = \frac{E_n}{I_n}$$

$$D'_n = \frac{C'_n B_n - A_n}{B_n B'_n - C_n - A'_n D_n}$$

$$A_n = J_n(k_0b)$$

$$B_n = H_n^{(1)}(k_0b)$$

$$C_n = J_n(k_0f(b))$$

$$D_n = H_n^{(1)}(k_0f(b))$$

$$E_n = J'_n(k_0b)$$

$$F_n = H_n^{(1)'}(k_0b)$$

$$\begin{aligned}
 G_n &= f'(b)J'_n(k_0f(b)) \\
 H_n &= f'(b)H_n^{(1)'}(k_0f(b)) \\
 I_n &= J_n(k_0f(a + \delta)) \\
 K_n &= H_n^{(1)'}(k_0f(a + \delta)) \\
 L_n &= J'_n(k_0f(a + \delta)) \\
 N_n &= H_n^{(1)}(k_0f(a + \delta)) \\
 \beta &= iM\eta_0
 \end{aligned}$$

Under matched condition as discussed by Yan et al. in [16], it is seen that the cloaked medium is free space. By considering this fact, it is clear from the above mentioned fields that $a_n = c_n$ and $b_n = 1$. Now the above coefficients become

$$\begin{aligned}
 c_n &= \frac{\beta^2 N_n L_n - I_n K_n}{K_n N_n (1 - \beta^2)} \\
 d_n &= \frac{\beta N_n L_n - \beta K_n I_n}{K_n N_n (1 - \beta^2)}
 \end{aligned}$$

As PEC and PMC are the limiting cases of PEMC [24], one can find the scattering coefficients for the case of PEC or PMC. These coefficients are

$$M \rightarrow 0 \text{ (PMC)} : \quad c_n = \frac{-I_n}{N_n} = \frac{-J_n(k_0f(a + \delta))}{H_n^{(1)}(k_0f(a + \delta))} \quad (21)$$

$$M \rightarrow \pm\infty \text{ (PEC)} : \quad c_n = \frac{L_n}{K_n} = \frac{J'_n(k_0f(a + \delta))}{H_n^{(1)'}(k_0f(a + \delta))} \quad (22)$$

These coefficients are the same as those discussed in [16], and the cross-polarized coefficient d_n becomes zero for PEC or PMC case.

3. NUMERICAL RESULTS AND DISCUSSION

In this section, some numerical results based on the proposed analytical formulations are presented for cylindrical invisibility cloak with PEMC layer at δ . Plots deal with the zeroth-, first-, and, second-order scattering co- and cross-polarized coefficients for the case of H -polarization. The radius of the inner boundary is taken as $a = 0.1$, and that of the outer boundary is $b = 0.2$, while the range of tiny perturbation δ is taken as $10^{-8}a < \delta < 10^{-2}a$. The frequency of incident wave is 2GHz. Throughout the plots, blue line shows the co-polarized coefficient and red line the cross-polarized scattering coefficient.

Zeroth-order co- and cross- scattering coefficients are plotted to explain the worth of cylindrical invisibility cloak. Figure 2 shows the co- and cross-polarized scattering coefficients when $M = 0$, i.e., the PMC case. This result is compared with that given in [16] and found in good agreement. In this case, the cross-polarized coefficient disappears, while the co-polarized coefficient also decreases with decrease in δ . In Figure 3, the co- and cross-polarized coefficients are plotted for $M = \pm 1$, and it is observed that the convergence of these coefficients is better than the case when $M = 0$. Comparison shows that this plot gives better convergence rate than that given in [16]. With decreasing value of δ from $10^{-5}a$ to $10^{-8}a$, the scattering coefficients decrease from 0.0815 to 0.0476. Figure 4 presents the co- and cross-polarized coefficients when $M \rightarrow \pm\infty$ (PEC case) and overlaps the result given in [16]. It is observed that the cross-polarized coefficient disappears, and convergence rate of the scattering coefficients is much better.

Figure 5 to Figure 7 represent first-order co- and cross-polarized scattering coefficients for different values of M . Figure 5 shows the co- and cross-polarized coefficients for $M = 0$, i.e., the PMC case. It is clear from the plots that with decreasing value of δ , co-polarized coefficient also decreases while the cross-polarized coefficient is zero. In Figure 6 co- and cross-polarized coefficients are plotted for the case of $M = \pm 1$. In this case, the cross-polarized is in good agreement with the plots given in [16], while the convergence rate of co-polarized is much better than the results given in [16]. With decreasing value of δ from $10^{-5}a$ to $10^{-8}a$, the scattering coefficients decrease from 1.81×10^{-16} to 3.12×10^{-28} and 5.50×10^{-9} to 5.50×10^{-15} for co- and

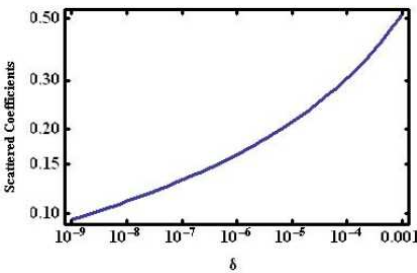


Figure 2. Co- and cross-pol. zeroth-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = 0$, and the medium inside the cloak is assumed to be air.

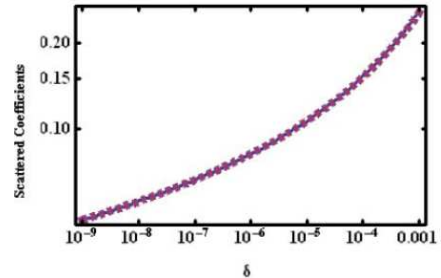


Figure 3. Co- and cross-pol. zeroth-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = \pm 1$, and the medium inside the cloak is assumed to be air.

cross-polarized respectively. Figure 7 shows the co- and cross-polarized scattering coefficients when $M \rightarrow \pm\infty$, i.e., the PEC case, which is in excellent agreement with the published literature.

Figure 8 to Figure 10 represent second-order co- and cross-polarized scattering coefficients for different values of M . Figure 8 shows the co- and cross-polarized coefficients for $M = 0$, i.e., the PMC case. It is clear from the plots that with decreasing value of δ , co-polarized coefficient also decreases while the cross-polarized coefficient is zero. In Figure 9, co- and cross-polarized coefficients are plotted for the case of $M = \pm 1$. In this case, the cross-polarized is in good comparison with the plots given in [16], while the convergence rate of co-polarized is much better than the results given in [16]. With decreasing value of δ from $10^{-5}a$ to $10^{-8}a$, the scattering coefficients decrease from 2.81×10^{-27} to 2.95×10^{-45} and 4.82×10^{-15} to 4.82×10^{-30} for co- and cross-polarized respectively. Figure 10 shows the co- and cross-polarized scattering coefficients when $M \rightarrow \pm\infty$, i.e., the PEC case, which is in excellent comparison with the published literature.

For the E -polarization case, all the plots for PEMC material are exactly the same as presented above.

In this section, we focus on the convergence rate of the co- and cross-polarized scattered fields. For an ideal cloak, these fields should be zero, but this situation is not possible up to now. The convergence rate observed in this work is better than that in the previous literature when PEMC material is used instead of PEC or PMC. In the available literature either PEC or PMC is used as core to achieve good convergence rate depending upon the type of incident field. But in this work PEMC material core is used which is the

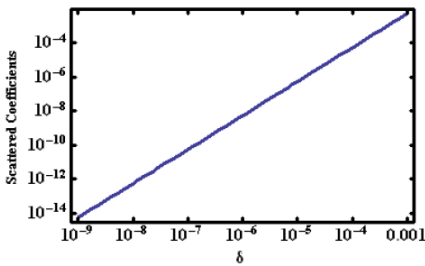


Figure 4. Co- and cross-pol. zeroth-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M \rightarrow \pm\infty$, and the medium inside the cloak is assumed to be air.

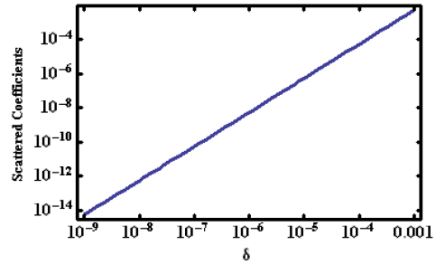


Figure 5. Co- and cross-pol. first-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = 0$, and the medium inside the cloak is assumed to be air.

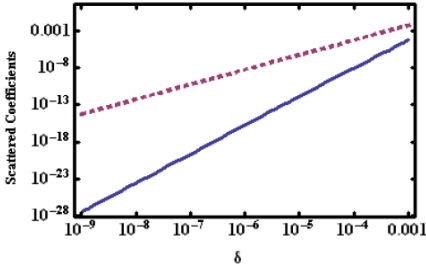


Figure 6. Co- and cross-pol. first-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = \pm 1$, and the medium inside the clock is assumed to be air.

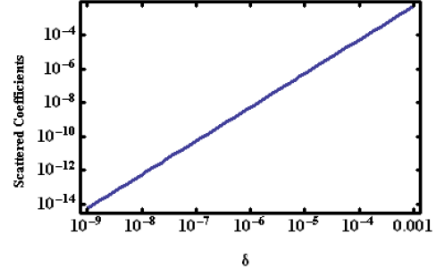


Figure 7. Co- and cross-pol. first-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M \rightarrow \pm\infty$, and the medium inside the clock is assumed to be air.

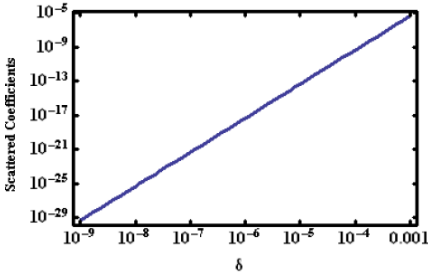


Figure 8. Co- and cross-pol. second-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = 0$, and the medium inside the clock is assumed to be air.

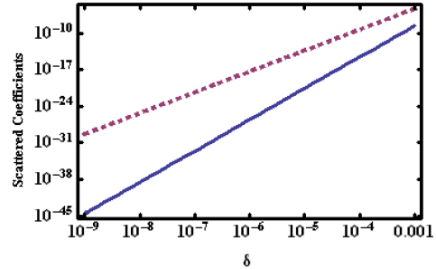


Figure 9. Co- and cross-pol. second-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M = \pm 1$, and the medium inside the clock is assumed to be air.

generalization of both the PEC and PMC. Due to the dual nature of PEMC, better convergence rate is observed. Also it is seen that the convergence rate is independent of the type of incident field for PEMC material.

The better convergence rate observed in this paper may be due to the dual nature of PEMC and its admittance parameter M . The best results can be obtained when we set $M = \pm 1$ which leads to a better convergence rate of the coefficients. Also by changing the parameter, i.e., M , we can improve the convergence rate of the scattered coefficients.

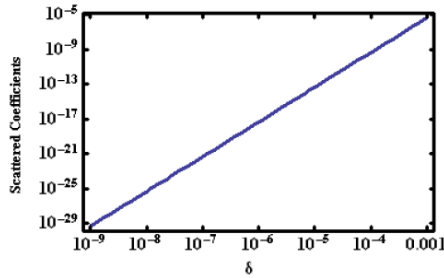


Figure 10. Co- and cross-pol. second-order scattering coefficients with $a = 0.1$, $b = 0.2$, $M \rightarrow \pm\infty$, and the medium inside the cloak is assumed to be air.

4. CONCLUSIONS

By observing the behavior of the co- and cross-polarized scattering coefficients, it is concluded that cylindrical invisibility cloak incorporating PEMC at δ can be constructed, which is the generalized case of PEC and PMC. Based on the numerical results, the following conclusions have been drawn.

1. For the zeroth-order scattering, PEMC gives better convergence rate than PMC, while the other limiting case (PEC) of PEMC is exactly the same as those in [9, 16].
2. For the first- and second-order scattering, the convergence rate in case of PEMC is much better than both PEC and PMC especially the co-polarized scattering.
3. The convergence rate is independent of the type of incident field when PEMC material is used instead of PEC or PMC.

REFERENCES

1. Smith, D. R., J. B. Pendry, and M. C. K. Wiltshire, "Metamaterials and negative reflection index," *Science*, Vol. 305, No. 5685, 788–792, 2004.
2. Shalaev, V. M., "Optical negative-index metamaterials," *Nature Photonics*, Vol. 1, 41–48, 2006.
3. Shelby, R. A., D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, No. 5514, 77–79, 2001.
4. Pendry, J. B., "Negative refraction makes a perfect lens," *Physical Review Letters*, Vol. 85, No. 18, 3966–3969, 2000.

5. Pendry, J. B. and D. R. Schuring, "Controlling electromagnetic fields," *Science*, Vol. 312, No. 5781, 1780–1782, 2006.
6. Cummer, S. A., B. I. Popa, D. Schuring, D. R. Smith, and J. B. Pendry, "Full-wave simulations of electromagnetic cloaking structures," *Physical Review E*, Vol. 74, No. 3, 036621, 2006.
7. Schuring, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, No. 5801, 977–980, 2006.
8. Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shaleaev, "Optical cloaking with metamaterials," *Nature Photonics*, Vol. 1, 224–227, 2007.
9. Ruan, Z., M. Yan, C. W. Neff, and M. Qiu, "Ideal cylindrical cloak: Perfect but sensitive to tiny perturbations," *Physical Review Letters*, Vol. 99, No. 11, 113903, 2007.
10. Yan, M., Z. Ruan, and M. Qiu, "Cylindrical invisibility cloak with simplified material parameters is inherently visible," *Physical Review Letters*, Vol. 99, No. 23, 233901, 2007.
11. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Improvement of cylindrical cloaking with the SHS lining," *Optics Express*, Vol. 15, No. 20, 12717–12734, 2008.
12. Yan, M., Z. Ruan, and M. Qiu, "Scattering characteristics of simplified cylindrical invisibility cloaks," *Optics Express*, Vol. 15, No. 26, 17770–17782, 2007.
13. Zhang, B., H. Chen, B. I. Wu, Y. Luo, L. Ran, and J. A. Kong, "Response of a cylindrical invisibility cloak to electromagnetic waves," *Physical Review B (Condensed Matter and Materials Physics)*, Vol. 76, No. 12, 121101, 2007.
14. Cai, W., U. K. Chettiar, A. V. Kildishev, V. M. Shaleaev, and G. W. Milton, "Nonmagnetic cloak with minimized scattering," *Applied Physics Letters*, Vol. 91, No. 11, 111105, 2007.
15. Yan, W., M. Yan, Z. Ruan, and M. Qiu, "Coordinate transformation make perfect invisibility cloaks with arbitrary shape," *New Journal of Physics*, Vol. 10, No. 4, 043040, 13, 2008.
16. Yan, W., M. Yan, Z. Ruan, and M. Qiu, "Influence of geometrical perturbation at inner boundaries of invisibility cloaks," *Journal of the Optical Society of America, Series A*, Vol. 25, No. 4, 968–973, 2008.
17. Zolla, F., S. Guenneau, A. Nicolet, and J. B. Pendry, "Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect," *Optics Letters*, Vol. 32, No. 9, 1069–1071, 2007.

18. Leonhardt, U. and T. G. Philbin, "General relativity in electrical engineering," *New Journal of Physics*, Vol. 8, 247, 2006.
19. Yan, W., M. Yan, and M. Qiu, "Necessary and sufficient conditions for reflectionless transformation media in an isotropic and homogenous background," arxiv: 0806. 3231 v1, 2008.
20. Chen, H., B. I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic waves interactions with a metamaterials cloak," *Physical Review Letters*, Vol. 99, No. 6, 063903, 2007.
21. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Full-wave invisibility of active devices at all frequencies," *Communications in Mathematical Physics*, Vol. 275, 749–789, 2008.
22. Lindell, I. V. and A. H. Sihvola, "Perfect electromagnetic conductor," *Journal of Electromagnetic Waves and Applications*, Vol. 19, No. 7, 861–869, 2005.
23. Lindell, I. V., *Differential Forms in Electromagnetics*, Wiley and IEEE Press, New York, 2004.
24. Lindell, I. V. and A. H. Sihvola, "Realization of the PEMC boundary," *IEEE Trans. Antennas Propagat.*, Vol. 53, No. 9, 3012–3018, 2005.
25. Lindell, I. V. and A. H. Sihvola, "Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures," *IEEE Trans. Antennas Propagat.*, Vol. 53, No. 9, 3005–3011, 2005.
26. Ruppin, R., "Scattering of electromagnetic radiation by a perfect electromagnetic conductor cylinder," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 13, 1853–1860, 2006.
27. Lindell, I. V. and A. H. Sihvola, "Losses in PEMC boundary," *IEEE Trans. Antennas Propagat.*, Vol. 54, No. 9, 2553–2558, 2006.
28. Sihvola, A. and I. V. Lindell, "Possible applications of perfect electromagnetic conductor (PEMC) media," *Proc. EuCAP 2006*, Nice, France,, Nov. 6–10, 2006 (ESA SP-626, Oct. 2006).
29. Lindell, I. V. and A. H. Sihvola, "The PEMC resonator," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 7, 849–859, 2006.
30. Jancewicz, B., "Plane electromagnetic wave in PEMC," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 5, 647–659, 2006.
31. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from a perfect electromagnetic conductor cylinder buried in a dielectric half-space," *Progress In Electromagnetics Research*, Vol. 78, 25–

- 38, 2008.
32. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from parallel perfect electromagnetic conductor cylinders of circular cross-sections using iterative procedure," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 7, 987–1003, 2008.
 33. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from a two dimensional perfect electromagnetic conductor (PEMC) strip and PEMC strip grating simulated by circular cylinders," *Optics Communications*, Vol. 281, 4211–4218, 2008.
 34. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from a perfect electromagnetic conductor cylinder coated with a metamaterial having negative permittivity and/or permeability," *Optics Communications*, Vol. 281, 5664–5670, 2008.
 35. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering of two or more incident plane waves by a perfect electromagnetic conductor cylinder coated with a metamaterial," *Progress In Electromagnetics Research B*, Vol. 10, 75–90, 2008.
 36. Fiaz, M. A., A. Ghaffar, and Q. A. Naqvi, "High frequency expressions for the field in the caustic region of a PEMC cylindrical reflector using Maslov's method," *Journal of Electromagnetic Waves and Applications*, Vol. 22, Nos. 2–3, 358–397, 2008.
 37. Fiaz, M. A., A. Aziz, A. Ghaffar, and Q. A. Naqvi, "High frequency expression for the field in the caustic region of a PEMC Gregorian system using Maslov's method," *Progress In Electromagnetics Research*, Vol. 81, 135–148, 2008.
 38. Illahi, A. and Q. A. Naqvi, "Scattering of an arbitrarily oriented dipole field by an infinite and a finite length PEMC circular cylinder," *CEJP*, in print, 2009.
 39. Illahi, A., M. Afzaal, and Q. A. Naqvi, "Scattering of dipole field by a perfect electromagnetic conductor cylinder," *Progress In Electromagnetics Research Letters*, Vol. 4, 43–53, 2008.
 40. Shahzad, A., S. Ahmed, and Q. A. Naqvi, "Analysis of electromagnetic field due to a buried coated PEMC circular cylinder," *Optics Communications*, Vol. 283, 4563–4571, 2010.
 41. Ahmed, S., F. Mannan, A. Shahzad, and Q. A. Naqvi, "Electromagnetic scattering from a chiral-coated PEMC cylinder," *Progress In Electromagnetics Research M*, Vol. 19, 239–250, 2011.