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EFFICIENT PROPER ORTHOGONAL DECOMPOSITION FOR BACKSCATTER PATTERN RECONSTRUCTION

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Abstract—A novel approach is presented for efficiently solving electromagnetic (EM) scattering problems using proper orthogonal decomposition (POD). As a proof of concept and demonstration of how to use the POD to solve EM scattering problems, two ways of implementing the POD procedure have been proposed and realized for calculating EM scattering from PEC targets. Numerical results obtained show that the POD is quite accurate for reconstructing backscatter patterns over wide range of frequencies and angles of interest based on the given snapshots.

1. INTRODUCTION

Efficient modeling of electromagnetic (EM) scattering from large and complex targets for real application often requires generating useful data of responses over wide range of frequencies and angles of interest, which is usually a very expensive computing process using numerical methods. Several methods and algorithms have been proposed for speeding up such a process. Of these, the asymptotic waveform evaluation (AWE) method has been successfully applied to analyze EM problems [1–9] and offers some features of model-order-reduction (MOR) techniques.

Proper orthogonal decomposition (POD) [10–14] is one of the most popular MOR techniques in the field of aerodynamics. In principle, it exploits the correlations in the dynamics of the system states under different excitations to determine dominant modes (POD basis vectors), or coherent structures (eigenfunctions), governing the system behavior. It is essentially based on second-order statistical

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properties that result in a linear invertible transformation such that the eigenfunctions yielded would optimally represent the data, i.e., for any given orthonormal bases, the POD eigenfunction bases ensure it is the best bases and that the two-norm or L2-norm error between the original and reconstructed data is minimum. In addition to being optimal in a least square sense, the POD has the property that it is completely data dependent and does not assume any prior knowledge of the process that generates the data. This property is advantageous in situations where a priori knowledge of the underlying process is insufficient to warrant a certain choice of basis [15, 16].

The POD has been successfully applied to analyze some microwave problems [13, 14]. More recently, it has also been applied to obtain design parameters for multidisciplinary design of S-shaped intake [15–17]. In this paper, the POD method is used to the analysis of EM scattering problem over wide range of frequencies and angles of interest.

2. FUNDAMENTAL OF POD

The procedure for constructing the optimal POD basis vectors Φ_i to reconstruct a function $U(x,t)$ over some domains of interest based on its M given snapshots U_k can be briefly summarized as follows [10–15]:

1) Construct the correlation matrix $\mathbf{R}$ by computing the inner product between every pair of snapshots as shown in Eq. (1).

$$R_{ik} = \frac{1}{M}\left(U_i, U_k\right) \tag{1}$$

2) Solve the eigenvalue problem of Eq. (2) to get the eigenvectors of $\{\beta_{ij}\}$ corresponding to eigenvalues $\lambda_i,\ i = 1, \ldots, M$.

$$R\beta = \Lambda\beta \tag{2}$$

3) Construct the POD basis vector Φ_i using Eq. (3)

$$\Phi_i = \sum\nolimits_{j=1}^{M} \beta_{ij} U_j \tag{3}$$

4) Select the number of modes (POD basis vectors) (P) to be employed in the reconstruction by considering the relative “energy” (measured by the 2-norm) captured by the ith basis vector ($\lambda_i / \sum_{i=1}^{M} \lambda_i$).

The approximate prediction of the function U is then given by a linear combination of the basis vectors using Eq. (4)

$$U \approx \sum\nolimits_{i=1}^{P} \alpha_i \Phi_i \tag{4}$$

where $P \leq M$ is chosen to capture the desired level of energy.

3. IMPLEMENTATION OF POD

Numerical analysis of a given EM problem in frequency domain usually results in a matrix equation as follows:

$$Z(s)I(s) = V(s) \tag{5}$$

where Z is the impedance matrix, I is unknown current vector, V is the known vector related to the source or excitation, and s is a modeling parameter related to frequency or angle. Let $I_k = I(s_k)$ be the solution of Eq. (5) for $V_k = V(s_k)$, $k = 1, \ldots, M$, and Φ_i, $i = 1, \ldots, P$ denote the POD modes constructed from these M current solutions (snapshots). The unknown vector $I(s)$ can be expressed as a linear combination of the POD modes Φ_i

$$I(s) = \sum\nolimits_{i=1}^{P} \alpha_i \Phi_i \tag{6}$$

Substituting (6) into (5) and applying testing procedure yields a matrix equation that can be used to determine coefficients α_i as follows

$$\begin{bmatrix} (Z(s)\Phi_1, Z(s)\Phi_1) & (Z(s)\Phi_1, Z(s)\Phi_2) & \ldots & (Z(s)\Phi_1, Z(s)\Phi_P) \\ (Z(s)\Phi_2, Z(s)\Phi_1) & (Z(s)\Phi_2, Z(s)\Phi_2) & \ldots & (Z(s)\Phi_2, Z(s)\Phi_P) \\ \vdots & \vdots & \vdots & \vdots \\ (Z(s)\Phi_P, Z(s)\Phi_1) & (Z(s)\Phi_P, Z(s)\Phi_2) & \ldots & (Z(s)\Phi_P, Z(s)\Phi_P) \end{bmatrix}$$
$$\cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_P \end{bmatrix} = \begin{bmatrix} (Z(s)\Phi_1, V(s)) \\ (Z(s)\Phi_2, V(s)) \\ \vdots \\ (Z(s)\Phi_P, V(s)) \end{bmatrix} \tag{7}$$

If parameter s represents angle, the coefficients α_i can be simplified as

$$\alpha_i = (Z\Phi_i, V(s))/(Z\Phi_i, Z\Phi_i)$$

in which

$$Z\Phi_i = \sum\nolimits_{j=1}^{M} \beta_{ij} V_j, \quad i = 1, \ldots, P.$$

It is observed that if the impedance matrix Z is a function of parameter s, P matrix-vector multiplications are needed to set up the matrix Equation (7) for solving α_i, which may be computationally expensive for modeling EM problem with electrically large and complex configuration. To reduce the computational cost for reconstructing $I(s)$ using POD basis vectors, $I(s)$ can be alternatively expressed as follows

$$I(s) = \sum\nolimits_{i=1}^{P} \alpha_i(s) \Phi_i \tag{8}$$

$\alpha_i(s)$ can be expanded into a Taylor series or Padè approximation.

Let us expand $\alpha_i(s)$ into a Taylor series

$$\alpha_i(s) = \sum\nolimits_{l=0}^{L} \alpha_{il} s^l,\ i = 1, \ldots, P.$$

Considering the orthonormality of the POD basis vectors Φ_i yields

$$\sum\nolimits_{l=0}^{L} \alpha_{il} s^l = (I(s), \Phi_i),\ i = 1, \ldots, P \tag{9}$$

Choosing $L + 1$ different samples of parameter s, for example, s_{k_l}, $(l = 0, \ldots, L)$, and applying their corresponding snapshots $I_{k_l} = I(s_{k_l})$, $(l = 0, \ldots, L)$ to Eq. (9) results in

$$\begin{bmatrix} 1 & s_1 & \ldots & s_1^L \\ 1 & s_2 & \ldots & s_2^L \\ \vdots & \vdots & \vdots & \vdots \\ 1 & s_{L+1} & \ldots & s_{L+1}^L \end{bmatrix} \begin{bmatrix} \alpha_{i0} \\ \alpha_{i1} \\ \vdots \\ \alpha_{iL} \end{bmatrix} = \begin{bmatrix} (I_{k_0}, \Phi_i) \\ (I_{k_1}, \Phi_i) \\ \vdots \\ (I_{k_L}, \Phi_i) \end{bmatrix},\ i = 1, \ldots, P \tag{10}$$

Let us expand $\alpha_i(s)$ into a rational function or Padè approximation [1, 8, 9] as follows:

$$\alpha_i(s) = \sum\nolimits_{l=0}^{L} \alpha_{il} s^l = \frac{\sum_{m=0}^{L_P} a_{im} s^m}{1 + \sum_{n=1}^{M_p} b_{in} s^n}, \quad i = 1, \ldots, P \tag{11}$$

where $Lp + Mp = L$. Following the standard procedure of the Padè approximation yields

$$\begin{bmatrix} \alpha_{iL_P} & \alpha_{iL_P-1} & \cdots & \alpha_{iL_P-M_P+1} \\ \alpha_{iL_P+1} & \alpha_{iL_P} & \cdots & \alpha_{iL_P-M_P+2} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{iL_P+M_P-1} & \alpha_{iL_P+M_P-2} & \cdots & \alpha_{iL_P} \end{bmatrix} \begin{bmatrix} b_{i1} \\ b_{i2} \\ \vdots \\ b_{iM_P} \end{bmatrix} = -\begin{bmatrix} \alpha_{iL_P+1} \\ \alpha_{iL_P+2} \\ \vdots \\ \alpha_{iL_P+M_P} \end{bmatrix} \tag{12}$$

$$a_{im} = \sum\nolimits_{n=0}^{m} b_{in} \alpha_{im-n}, \quad m = 0, 1, \ldots, L_P \tag{13}$$

Equations (7), (10), (11) and (12) can be easily solved to obtain reconstruction coefficients, as both P and L are much smaller than the dimension of matrix Z.

4. NUMERICAL RESULTS

As a proof of concept and demonstration of how the POD works for EM modeling, two ways of implementation of the POD procedure mentioned above have been realized for calculating EM scattering from 2D PEC targets using electric field integral equation (EFIE) for TE polarization and magnetic field integral equation (MFIE) for TM polarization [18]. The snapshots and benchmarking data are obtained using an in-house method of moments (MoM) code [18].

The first example is the scattering from a 2D circular cylinder of radius 1.0 m. Figure 1 shows the backscatter patterns for TE polarization as function of frequency obtained using MoM and POD. Both results agree very well over the frequency band from 0.03 to 1.0 GHz.

The second example is the scattering from a 2D rectangular cylinder with dimension of 3.0 m by 3.0 m. Figure 2 shows the backscatter patterns for TE polarization at incident angle of 15° as function of frequency obtained using MoM and POD. Both results agree very well over the frequency band from 0.1 to 0.5 GHz.

Compared to MoM, the snapshots used for these two examples for generating POD modes are down sampled by 10 and the speedup achieved is about 10 for reconstructing the backscattering response over the frequency band of interest.

The same rectangular cylinder used in second example is also simulated at 1.2 GHz for both TE and TM polarizations. Figures 3 and 4 show the backscatter patterns as the function of angles. It is observed that the simulation results obtained using POD agree well

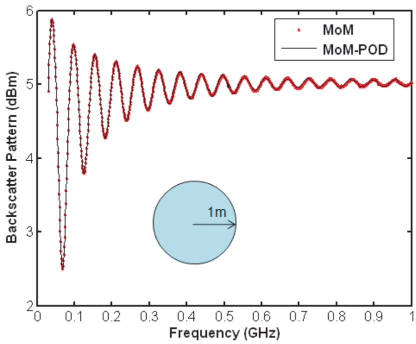

Figure 1. Backscatter pattern of circular cylinder for TE polarization.

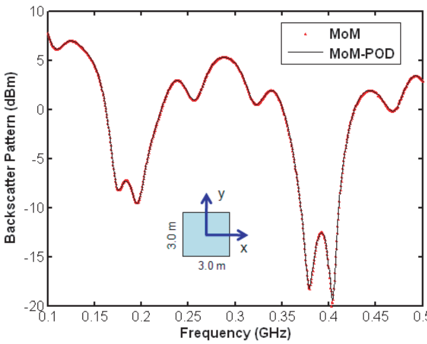

Figure 2. Backscatter pattern of rectangular cylinder at incident angle of 15° for TE polarization.

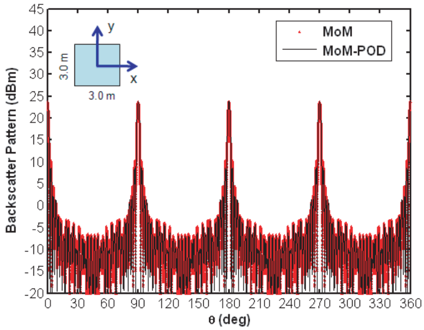

Figure 3. Backscatter pattern of rectangular cylinder at 1.2 GHz for TE polarization.

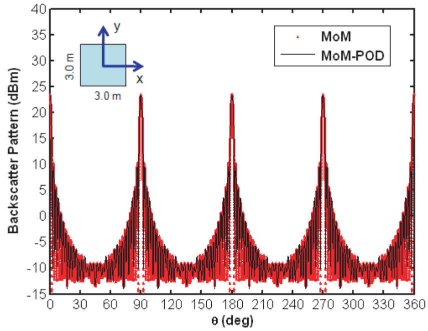

Figure 4. Backscatter pattern of rectangular cylinder at 1.2 GHz for TM polarization.

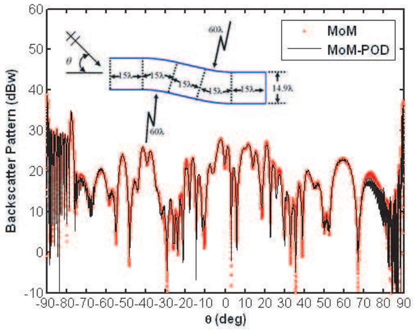

Figure 5. Backscatter pattern of 2D open-ended S-shaped cavity for TE polarization.

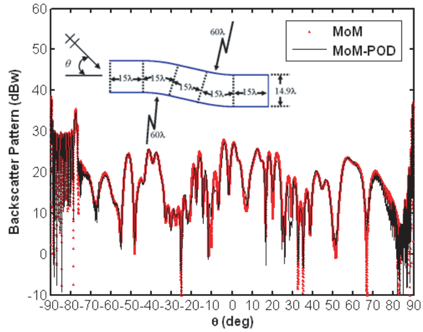

Figure 6. Backscatter pattern of 2D open-ended S-shaped cavity for TM polarization.

with that obtained using MoM. It can be seen these two Figures exactly characterize the scattering behavior of rectangular cylinder. Compared with MoM, the snapshots used in this simulation for generating POD modes are down sampled by 100 and acceleration ratio achieved is about 100.

The last example is the scattering from a large 2D open-ended S-shaped cavity [19, 20]. Figures 5 and 6 show the backscatter patterns as function of observation angles for TE and TM polarizations, respectively. It is observed that the simulation results obtained using POD agree well with that obtained using MoM. Compared with MoM, the snapshots used in this example for generating POD modes are down sampled by 50 for accurately capturing the scattering behavior around -90° and 90°. Acceleration ratio achieved is about 50.

5. DISCUSSION AND CONCLUSION

All the numerical examples described hereinabove show that the proposed implementation of the POD is accurate and efficient for reconstructing EM backscattering response over a wide range of frequencies and angles of interest. The advantage of the POD is it generates high resolution data based on actual experiments or snapshots with much less sampling rate to save computational cost for multiaspect responses. This ensures the POD can be applied to other EM solvers through inputting snapshots. The number of snapshots is also much smaller than the dimension of MoM matrix, so the computational cost of the POD procedure is not much.

It should be noted that the proposed POD with MoM can be consider as an electric current based POD and its POD modes are actually a set of dominant eigen currents that represent the solution space of currents of interest based on given snapshots of currents. So the "energy" of the POD for EM problem is directly related to the EM energy, which is the underlying physics of the POD with MoM for EM problem. Further study on how to automatically determine the optimal number of snapshots and sources of possible errors of the proposed POD method will make it more useful and practical for solving large and complex EM problems.

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