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DUAL-FREQUENCY ELECTROMAGNETIC CLOAKS ENABLED BY LC-BASED METAMATERIAL CIRCUITS

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Abstract—A dual-frequency cloak based on lumped LC-circuits is proposed. Multiple LC-resonant tanks are employed to satisfy the specific conditions for dual-frequency operations. In this way, the designed cloak features greatly reduce scattering cross sections at the two working frequencies simultaneously. Besides, explicit design equations are derived for the developed circuit systems. Based on these formulas, the range of the realizable frequency ratio of the presented cloak (the ratio between the two operating frequencies) is discussed. To verify the theoretical predictions, full-wave electromagnetic simulations are implemented. Good consistency between the numerical results and the design theories is achieved.

1. INTRODUCTION

With the development of metamaterials, the concept of invisible cloak as an important topic has drawn much attention. By the aid of the cloaking structures, the waves can propagate through the target with very small perturbations to the total fields. Many methods have been proposed for this purpose. In Ref. [1], anomalous localized resonance is applied to achieving the cloaking effect. Alù and Engheta [2, 3] propose to use plasmonic shell as a cloak to reduce the total scattering cross section of a particle. Especially, the

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spatial coordinate transformation method has been widely employed since its appearance [4–8]. These ideas have also been extended to other applications such as electromagnetic wormholes [9], field concentrator [10], field rotator [11], and beam forming [12, 13]. Meanwhile, recent advance in complex systems has imposed more stringent system requirements including dual-frequency operation [14–27]. As for the cloaking applications, it is noted that most of the proposed cloaking structures can only work at single frequency due to the causality constraints imposed by the metaterials [28]. To relax this limitation, Chen et al. [29] show a cloak design with broader bandwidth, which is realized by the sacrifice of the scattering cross section. Meanwhile, Alù and Engheta [30] propose to use multilayered plasmonic shells to cancel the scattering at different frequencies, leading to multi-frequency cloaks. After that, several multi-frequency cloaks have been achieved by using different methods [31–34]. In this paper, we demonstrate how to construct a dual-frequency cloak based on the coordinate transformation method. Metamaterials based LC-circuits are proposed to realize the desired parameters at different frequencies, where the concept of multiple resonances is applied. Cylindrical cloak with ten layers is studied. Each layer of the cloak is then approximated by the proposed circuit. The simulation results show the desired dual-frequency operation of the whole cloak. The same concept can be also applied for other cloaking structures designed by the spatial coordinate transformation method for multiple-frequency operations.

2. THEORETICAL ANALYSIS

The general schematic of the cloak studied is shown in Fig. 1. It is cylindrical in shape. Applying the coordinate transformation $r = R_1 + r'(R_2 - R_1)/R_2$, the original cylindrical region, $0 \leqslant r' \leqslant R_2$, is compressed into a concentric cylindrical shell, $R_1 \leqslant r \leqslant R_2$, where r' is the coordinate in the original space and r is the coordinate in the transformed space. By doing this, the region $0 \leqslant r \leqslant R_1$ (the dark grey area marked in Fig. 1) is excluded from the transformed space so that anything can be placed into it without affecting the wave propagation in the whole space, resulting in the cloaking of the region. After the transformation, the relative permeability and permittivity in the cloaking shell are expressed as:

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r} \tag{1}$$

$$\varepsilon_\theta = \mu_\theta = \frac{r}{r - R_1} \tag{2}$$

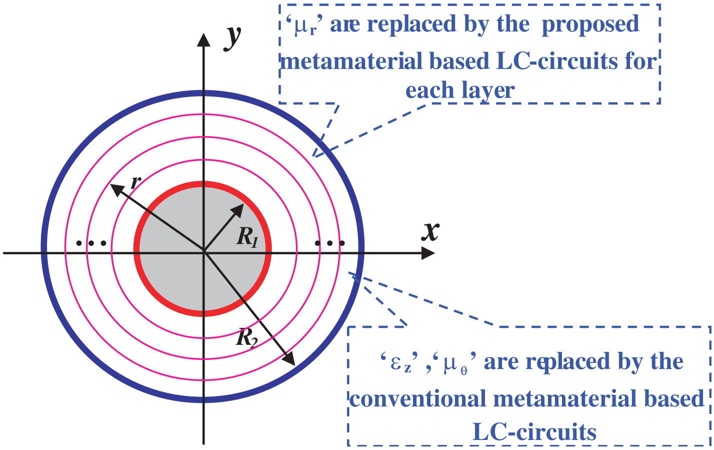

Figure 1. Schematic of the proposed dual-band cloak.

$$\varepsilon_z = \mu_z = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r} \tag{3}$$

To study the performance of the cloaks, time harmonic plane waves with transverse-electric (TE) polarization, transverse-magnetic (TM) polarization or mix of them are excited. Without loss of generality, in the following, we will focus our analysis on the TE case. The same theory can be easily extended to the other cases. It is found that, under the TE illuminations, the wave propagations depend on three parameters, namely, μ_r, μ_θ and ε_z. For single frequency cloak, these parameters are calculated by Eqs. (1)–(3) and can be emulated using the well defined metamaterial structures. In practice, for the ease of implementation, the permittivity and permeability are further reduced to:

$$\varepsilon_z = \left(\frac{R_2}{R_2 - R_1}\right)^2, \quad \mu_\theta = 1, \quad \mu_r = \left(\frac{r - R_1}{r}\right)^2 \tag{4}$$

In Eq. (4), both μ_θ and ε_z are constant and can be realized with the conventional substrate. As for μ_r (the permeability in the r-direction), it is found that, when r is close to R_1, the required permeability is quite small (close to 0), which cannot be realized using conventional material. Therefore, metamaterials are employed to realize this parameter [6]. However, as mentioned before, due to the causality of the metamaterials, the designed parameters can be satisfied only at the single assigned frequency. For dual-frequency or multiple-frequency operations, metamaterials with different structures

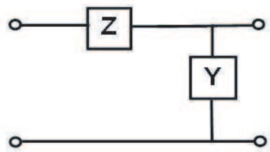

Figure 2. The general circuit model with the analogy to the effective medium.

are required. In general, two types of metamaterials are widely used, one is composed of metal wires and split-ring resonators [35] and the other is based on the LC-loaded transmission line networks [36–40]. In our case, we find that it is possible to employ the second type metamaterial to achieve the dual-frequency operation of the cloak. A typical illustration of this type of metamaterials is shown in Fig. 2, where "Z" is the impedance of the circuit in series and "Y" is the admittance of the circuit in shunt. Due to the analogy between the circuits and the electromagnetic wave propagation models, we have:

$$\varepsilon = \frac{Y}{j\omega\varepsilon_0} \tag{5}$$

$$\mu = \frac{Z}{j\omega\mu_0} \tag{6}$$

Applying Eqs. (5) and (6), the μ_θ and ε_z can be realized by the LC-loaded circuits shown in Fig. 3. And the relations between the lumped components and the permittivity and permeability are:

$$\mu_\theta = \frac{L_\theta}{\mu_0\Delta}, \quad \varepsilon_\theta = \frac{C_\theta}{\varepsilon_0\Delta} \tag{7}$$

$$\varepsilon_z = \frac{C_z}{\varepsilon_0\Delta}, \quad \mu_z = \frac{L_z}{\mu_0\Delta} \tag{8}$$

where Δ is the length of the total LC-circuit and the other parameters are labeled in Fig. 3. From Eqs. (7) and (8), the resulting μ_θ and ε_z are not dependent on the frequency and further calculation show that both of them can be realized by lumped inductor and capacitor with practical values. Therefore, they are suitable for the dual-frequency cloak application. However, for μ_r, due to its very small value at the region close to the inner boundary ($r = R_1$ as shown in Fig. 1), it is impossible to achieve the desired parameter using the conventional LC-loaded circuit as shown in Fig. 3 (e.g., if $\mu_r = 0.002$, the corresponding inductor should be as small as 0.025 nH with $\Delta = 0.01$ m, which is not practical). This motivates us to propose

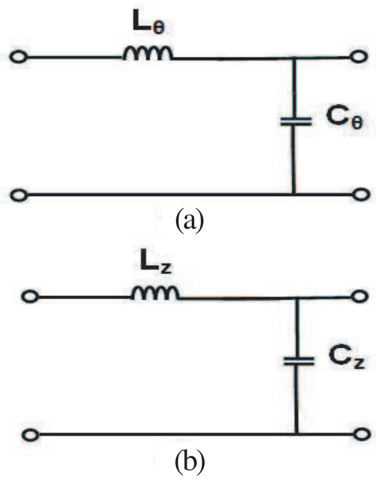

Figure 3. Schematics of the LC-loaded circuits for the medium in (a) θ-direction and (b) z-direction.

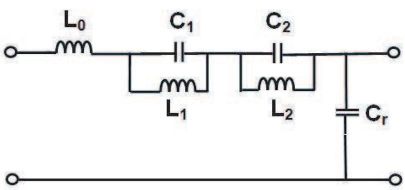

Figure 4. The proposed LC-loaded circuit with multiple resonances to support dual-frequency operation.

the circuit as shown in Fig. 4 to satisfy the assigned conditions for μ_r at two distinct frequencies.

Based on Eqs. (5) and (6) and after some derivations, the effective permittivity and permeability of the proposed circuit systems (as shown in Fig. 4) are expressed as:

$$\mu_r = \frac{1}{\mu_0 \Delta}\left[L_0 + \frac{L_1}{1-\omega^2 L_1 C_1} + \frac{L_2}{1-\omega^2 L_2 C_2}\right] \tag{9}$$

$$\varepsilon_r = \frac{C_r}{\varepsilon_0 \Delta} \tag{10}$$

Two additional LC-resonant tanks are added to the original circuits. Both of these LC-resonators contribute to the effective permeability of the medium. By exploiting the additional freedom introduced by these circuits, it is possible to realize the unconventional permeability at two different frequencies. Physically, this behaves similarly as a two-component system [41] and it is a metamaterial with very flexible properties.

3. SIMULATION RESULTS

To prove the theoretical predictions of the proposed dual-frequency cloak, we have designed a prototype as shown in Fig. 1, where $R_1 = 0.1\,\mathrm{m}$ and $R_2 = 0.2\,\mathrm{m}$. This concentric cloaking shell is further decomposed into ten layers so that each layer has a uniform thickness of $0.01\,\mathrm{m}$. In the θ- and z-directions, the media are constructed by the LC-loaded circuits as shown in Fig. 3. Applying Eq. (4), $\mu_\theta = 1$ and $\varepsilon_z = 4$ are held for all of these ten layers. Plug these values into Eqs. (7) and (8) with $\Delta = 0.01\,\mathrm{m}$, it is found that $L_\theta = 12.6\,\mathrm{nH}$, $C_\theta = 0.35\,\mathrm{pF}$, $L_z = 12.6\,\mathrm{nH}$ and $C_z = 0.35\,\mathrm{pF}$. In the r-direction, the permeability is a function of the position. With Eq. (4), the calculated μ_r are: 0.0041, 0.018, 0.0405, 0.0672, 0.0963, 0.1259, 0.1552, 0.1837, 0.2111, 0.2373, respectively, for the ten layers from inside to outside. Each layer is then approximated by the proposed LC-circuits as shown in Fig. 4 with different LC-component values. For a dual-frequency system, one parameter of particular note is the ratio between the two working frequencies (f_2/f_1). In our case, the range of this ratio is constrained by the practical values of the inductors and capacitors.

Table 1. Calculated LC-loaded circuit parameters for each layer when $f_1 = 3\,\mathrm{GHz}$ and $f_2 = 5\,\mathrm{GHz}$.

Layer	$C_1 = C_2$ (pF)	L_1 (nH)	L_2 (nH)
1	0.5	2.55	8.37
2	0.6	2.03	6.65
3	0.7	1.69	5.5
4	1	1.12	3.55
5	1.1	1	3.18
6	0.8	1.47	4.76
7	0.9	1.29	4.14
8	1	1.14	3.66
9	1.1	1.03	3.28
10	1.2	0.94	2.97

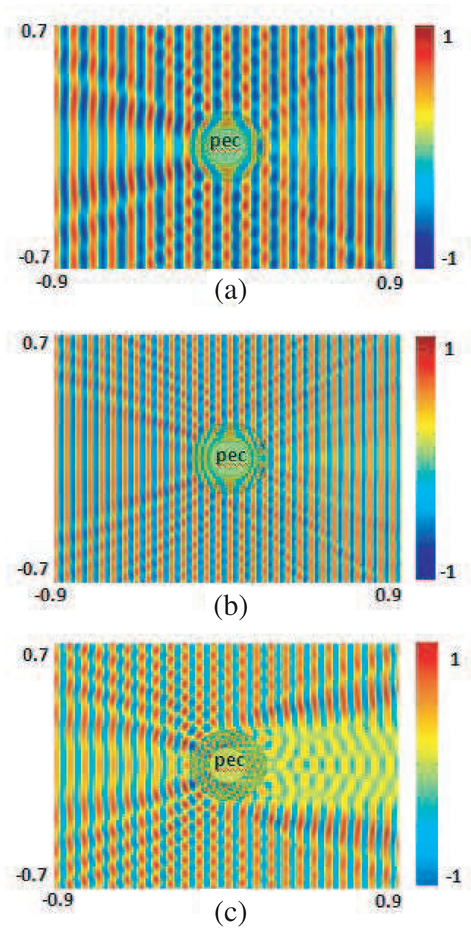

Figure 5. The E-field distributions of the proposed dual-frequency cloak at different frequencies (a) 3 GHz (first working frequency, f_1), (b) 5 GHz (second working frequency, f_2), (c) at 4 GHz.

Based on Eq. (9), a numerical algorithm is applied to calculate the corresponding circuit components values with different frequency ratios. It is found that, for the ratio from $f_2/f_1 = 4\,\mathrm{GHz}/3\,\mathrm{GHz}$ to $f_2/f_1 = 9\,\mathrm{GHz}/3\mathrm{GHz}$ or even higher, the proposed dual-frequency cloaks work well (e.g., for frequency ratio within this range, the dual-frequency cloak can work properly). Some of the typical calculation results are listed in Table 1, for which $L_0 = 12.6\,\mathrm{nH}$ and $C_r = 0.88\,\mathrm{pF}$.

Based on the above calculations, a dual-frequency cloak working at 3 GHz and 5 GHz is designed and its performance is simulated by the full wave simulator COMSOL Multiphysics. In the simulations, the

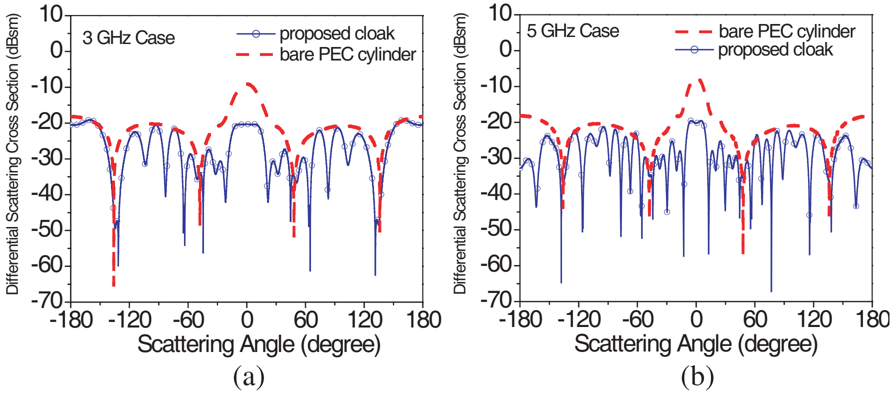

Figure 6. The calculated differential scattering cross sections of the proposed cloak along with the results for the bare PEC, (a) at 3 GHz and (b) at 5 GHz.

plane waves are incident from the left to the right with the frequency range from 2 to 6 GHz. Perfectly matched layers (PML) boundaries and scattering boundaries are placed on all four sides of the simulation domain to remove the undesired reflections. The simulated E-field distributions for this cloak are shown in Fig. 5 at different frequencies. As desired, at 3 and 5 GHz as shown in Figs. 5(a) and (b), the incident waves are bent smoothly around the cloaked area (the perfect electric conductor (PEC) as labeled in Fig. 5) with small perturbations to the far fields (due to singular values of the designed parameters). Meanwhile, at other frequencies (e.g., 4 GHz as shown in Fig. 5(c)), strong scattering is observed because the corresponding permeability and permittivity at those frequencies do not satisfy the suitable cloak parameters.

To further evaluate the performance of the designed dual-frequency cloak, the differential scattering cross section of the cloak is calculated at the two working frequencies. The results are plotted in Fig. 6, where the bistatic scattering is calculated as a function of the scattering angle φ. Also in Fig. 6, the bistatic scatterings with bare PEC cylinder at those two operating frequencies are plotted for the purpose of comparison. It is observed that, at 3 GHz, the scattering cross section at $\varphi = 0°$ for the proposed cloak is 11 dB lower than that of the bare PEC. For most of the other angles (scattering angle), the scattering cross section of the cloaked area is reduced by the aid of the designed cloak. At 5 GHz, similar reductions of the scattering cross section are achieved. Besides, the oscillations of the cloak's differential

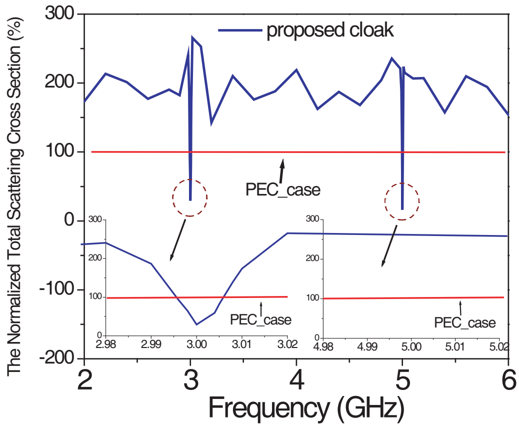

Figure 7. Calculated normalized total scattering cross section of the proposed dual-frequency cloak over the frequency range from 2 GHz to 6 GHz.

scattering cross section in Fig. 6 are faster than that of the bare PEC. This indicates that the zeroth order scattering is reduced by the proposed cloak at the two operating frequencies.

Finally, the normalized total scattering cross section (normalized to the bare PEC) of the proposed cloak is investigated. The results are plotted in Fig. 7 over the frequency range from 2 to 6 GHz. It is observed that, at the working frequencies (3 and 5 GHz), the total scattering cross section of the cloak is only 30% and 16% of those of the bare PEC, respectively, demonstrating great scattering reductions. Deviated from the center frequency, the total scattering cross section increases dramatically so that the bandwidth of the proposed cloak is narrow. This is due to the highly resonant property of the proposed LC-loaded metamaterials.

So far, the performance of the dual-frequency cloak is verified by the results presented above. In addition, it is worth pointing out that, applying the same design concept, it is possible to design cloak for multiple-frequency operations (e.g., triple-frequency, quadruple-frequency, etc.). For these applications, high order LC-based circuits similar with that shown in Fig. 4 but with multiple resonant tanks are needed to satisfy the corresponding requirements. Moreover, for practical implementations, several issues need to considered including the loss of applied components and the physical size of the corresponding components and interconnects.

4. CONCLUSION

We have developed a dual-frequency electromagnetic cloak based on the LC-loaded circuits. The limitations imposed by the causality and the extreme small value of the permittivity/permeability for cloaking applications are relaxed by the proposed LC-circuit with multiple resonances. Full-wave simulations are then carried out to prove the design concept, where a cylindrical cloak working at 3 and 5 GHz is designed and simulated. The simulation results including the E-field distributions and the scattering cross sections show the desired dual-frequency operations of the cloak. Moreover, the proposed structures are suitable for other applications (e.g., field concentrator, beam forming, etc.) featuring dual-frequency operations. Based on the same concept, it is even possible to design cloaking structures with multiple-frequency operations.

REFERENCES

1. Nicorovici, N. A., G. W. Milton, R. C. Mcphedran, and L. C. Botten, "Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance," *Optics Express*, Vol. 15, 6314–6323, May 2007.
2. Alù, A. and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," *Physical Review E*, Vol. 72, 016623, Jul. 2005.
3. Alù, A. and N. Engheta, "Cloaking and transparency for collections of particles with metamaterial and plasmonic covers," *Optics Express*, Vol. 15, 7578–7590, Jun. 2007.
4. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780–1782, May 2006.
5. Leonhardt, U., "Optical conformal mapping," *Science*, Vol. 312, 1777–1780, May 2006.
6. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Material electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, 977–980, Oct. 2006.
7. Chen, H. S., B. I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic wave interactions with a metamaterial cloak," *Physical Review Letters*, Vol. 99, 063903, Oct. 2007.
8. Ruan, Z. C., M. Yan, C. W. Neff, and M. Qiu, "Ideal cylindrical cloak: Perfect but sensitive to tiny perturbations," *Physical Review Letters*, Vol. 99, 113903, Sep. 2007.
9. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann,

"Electromagnetic wormholes and virtual magnetic monopoles from metamaterials," *Physical Review Letters*, Vol. 99, 183901, Oct. 2007.
10. Rahm, M., D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, "Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations," *Photonics and Nanostructures — Fundamentals and Applications*, Vol. 6, 87–95, Apr. 2007.
11. Huanyang, C. and C. T. Chan, "Acoustic cloaking in three dimensions using acoustic metamaterials," *Applied Physics Letters*, Vol. 91, 183518, Nov. 2007.
12. Kong, F., B.-I. Wu, J. A. Kong, J. Huangfu, S. Xi, and H. Chen, "Planar focusing antenna design by using coordinate transformation technology," *Applied Physics Letters*, Vol. 91, 253507, Dec. 2007.
13. Jiang, W. X., T. J. Cui, H. F. Ma, X. Y. Zhou, and Q. Cheng, "Cylindrical-to-plane-wave conversion via embedded optical transformation," *Applied Physics Letters*, Vol. 92, 261903, Jul. 2008.
14. Wu, Y., Y. Liu, and S. Li, "Dual-band modified Wilkinson power divider without transmission line stubs and reactive components," *Progress In Electromagnetics Research*, Vol. 96, 9–20, 2009.
15. Heidari, A. A., M. Heyrani, and M. Nakhkash, "A dual-band circularly polarized stub loaded microstrip patch antenna for GPS applications," *Progress In Electromagnetics Research*, Vol. 92, 195–208, 2009.
16. Wu, Y., Y. Liu, and S. Li, "An unequal dual-frequency Wilkinson power divider with optional isolation structure," *Progress In Electromagnetics Research*, Vol. 91, 393–411, 2009.
17. Wu, Y., Y. Liu, and S. Li, "A new dual-frequency Wilkinson power divider," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 4, 483-492, 2009.
18. Abdalla, M. A. and Z. Hu, "Multi-band functional tunable LH impedance transformer," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 1, 39–47, 2009.
19. Zhou, S., J. Ma, and J. Deng, "A novel dual band-notched ultra-wideband antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 1, 57–63, 2009.
20. Wang, X. H., L. Chen, X. W. Shi, Y. F. Bai, L. Chen, and X. Q. Chen, "Planar dual-frequency power divider using umbrella-shaped resonator," *Journal of Electromagnetic Waves*

and Applications, Vol. 24, No. 5/6, 597–606, 2010.

21. Li, J. C., J. C. Nan, X. Y. Shan, and Q. F. Yan, "A novel modified dual-frequency Wilkinson power divider with open stubs and optional isolation," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 16, 2223–2235, 2010.
22. Abu, M., M. K. A. Rahim, O. Ayop, and F. Zubir, "Triple-band printed Dipole antenna with single-band AMC-HIS," *Progress In Electromagnetics Research B*, Vol. 20, 225–244, 2010.
23. Vegesna S. and M. Saed, "Novel compact dual-band bandpass microstrip filter," *Progress In Electromagnetics Research B*, Vol. 20, 245–262, 2010.
24. Qaroot, A., N. Dib, and A. Gheethan, "Design methodology of multi-frequency un-equal split Wilkinson power divider using transmission line transformers," *Progress In Electromagnetics Research B*, Vol. 22, 1–21, 2010.
25. Wu, Y., Y. Liu, S. Li, C. Yu, and X. Liu, "Closed-form design method of an N-way dual-band Wilkinson hybrid power divider," *Progress In Electromagnetics Research*, Vol. 101, 97–114, 2010.
26. Lin, Z. and Q. X. Chu, "A novel approach to the design of dual-band power divider with variable power dividing ratio based on coupled-lines," *Progress In Electromagnetics Research*, Vol. 103, 271–284, 2010.
27. Yang, R. Y., K. Hon, C. Y. Hung, and C. S. Ye, "Design of dual-band bandpass filters using a dual feeding structure and embedded uniform impedance resonators," *Progress In Electromagnetics Research*, Vol. 105, 93–102, 2010.
28. Wood, B. and J. B. Pendry, "Metamaterials at zero frequency," *Journal of Physics: Condensed Matter*, Vol. 19, 076208, Feb. 2007.
29. Chen, H., Ruan, Z. Liang, P. Yao, X. Jiang, H. Ma, and C. T. Chan, "Extending the bandwidth of electromagnetic cloaks," *Physical Review B*, Vol. 76, 241104, Dec. 2007.
30. Alù, A. and N. Engheta, "Multifrequency optical invisibility cloak with layered plasmonic shells," *Physical Review Letters*, Vol. 100, 113901, May 2008.
31. Wang, H. L. and X. D. Zhang, "Achieving multifrequency transparency with cylindrical plasmonic cloak," *Journal of Applied Physics*, Vol. 106, 053302, 2009.
32. Liu, Y. W. and Y. J. Meng, "Electrically controlled multifrequency ferroelectric cloak," *Optics Express*, Vol. 18, No. 12, 12646–12652, 2010.
33. Serebryannikov, A. E. and E. Ozbay, "Non-ideal multifrequency

cloaking using strongly dispersive materials," *Physica B: Condensed Matter*, Vol. 405, No. 14, 2959–2963, 2010.

34. Li, P. N., Y. W. Liu, Y. J. Meng, and M. J. Zhu, "A multifrequency cloak with a single shell of negative index metamaterials," *Chin. Phys. Lett*, Vol. 28, No. 6, 064206, 2011.
35. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Physical Review Letters*, Vol. 84, 4184–4187, May 2000.
36. Grbic, A. and G. V. Eleftheriades, "Periodic analysis of a 2-D negative refractive index transmission line structure," *IEEE Trans. Antennas Propag.,* Vol. 51, 2604–2611, Oct. 2003.
37. Lin, I.-H., M. DeVincentis, C. Caloz, and T. Itoh, "Arbitrary dual-band components using composite right/left-handed transmission lines," *IEEE Trans. Microwave Theory Tech.,* Vol. 52, 1142–1149, Apr. 2004.
38. Alitalo, P., S. Maslovski, and S. Tretyakov, "Experimental verification of the key properties of a three-dimensional isotropic transmission-line superlens," *Journal of Applied Physics,* Vol. 99, 124910, Jun. 2006.
39. Iyer, A. K. and G. V. Eleftheriades, "A multilayer negative-refractive-index transmission-line (NRI-TL) metamaterial free-space lens at X-band," *IEEE Trans. Antennas Propag.,* Vol. 55, 2746–2753, Oct. 2007.
40. Iyer, A. K. and G. V. Eleftheriades, "A three-dimensional isotropic transmission-line metamaterial topology for free-space excitation," *Applied Physics Letters,* Vol. 92, 261106, Jul. 2008.
41. Chen, A. K., P. Fischer, and F. W. Wise, "Negative refraction at optical frequencies in nonmagnetic two-component molecular media," *Physical Review Letters*, Vol. 95, 067402, Aug. 2005.