DESIGN AND OPTIMIZATION OF EQUAL SPLIT BROADBAND MICROSTRIP WILKINSON POWER DIVIDER USING ENHANCED PARTICLE SWARM OPTIMIZATION ALGORITHM

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Abstract—An enhanced particle swarm optimization (EPSO) algorithm is proposed. To improve convergence accuracy and velocity, we introduce a quadratic interpolation method and perturbation to personal best particles in EPSO. Then, a design procedure based on the EPSO is proposed for the design and optimization of equal split broadband microstrip Wilkinson power dividers (MWPDs). A set of numerical examples and fabricated samples are presented to validate the improvement of the proposed EPSO. Even-odd mode analysis is incorporated in the design procedure to calculate the scattering matrix of the MWPD on the basis of the dispersion and dissipation microstrip line model. A fitness function is then constructed according to the scattering parameters. The optimized widths and lengths of microstrip lines and values of isolation resistors are directly obtained by minimizing the fitness function. EPSO is also compared with the genetic algorithm (GA), standard particle swarm optimization (PSO) and improved particle swarm optimization (IPSO).

1. INTRODUCTION

The Wilkinson power divider (WPD) proposed in 1960 [1] has good match at all ports as well as excellent isolation between two output ports at the central frequency. It has been extensively applied to microwave circuits and antenna arrays, but suffers from its narrow bandwidth. Several design methods [2–4] have been studied to increase bandwidth and high isolation between output ports. Moreover, dual-band [5–12] and multiband [13] WPDs have been studied. Even-odd

mode analysis method [14] and circuit theory are usually used to analyze WPDs. The evolutional optimization algorithm has also been adopted in designing WPDs in [15].

Despite the volume of research devoted to WPDs, the dispersion and dissipation effects of practical transmission lines were not considered in the design methods mentioned above. To generate excellent practical performance, researchers should consider these effects in the design procedure. The least squares method [16] was applied to optimize this complicated problem when the practical microstrip line model was adopted. However, an extra procedure was developed there to endow specified initial values to the optimization procedure. To the best of our knowledge, the design and optimization of broadband MWPDs based on the dispersion and dissipation microstrip line model with randomly initialized values have not been presented. Realizing these objectives necessitates a powerful optimization algorithm.

To this end, we propose an enhanced particle swarm optimization (EPSO) algorithm. The design and optimization of equal split broadband MWPDs are achieved using the EPSO design procedure with randomly initialized values. The proposed EPSO is tested and compared with the GA, PSO and IPSO [17]. A set of numerical examples are presented, and two MWPDs are optimized, fabricated and measured. The measured results validate the effectiveness of the proposed EPSO design procedure.

2. DESIGN PROCEDURE

The scattering matrix of an MWPD is calculated by even-odd mode analysis method based on the dispersion and dissipation microstrip line model [18–20]. As shown in Figure 1, the equal split broadband MWPD consists of N sections of microstrip lines in cascade, with characteristic impedances Z_i, widths W_i and lengths L_i, i =
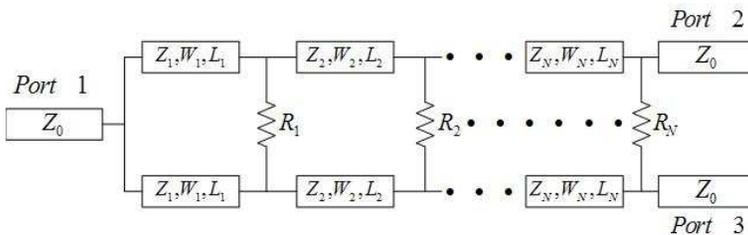

Figure 1. General topology of an equal split broadband MWPD.

$1, 2, 3, \ldots, N$. The isolation between the output ports is achieved by N isolation resistors. The input and output ports impedances are represented by $Z_0 = 50\,\Omega$.

2.1. Dispersion and Dissipation Microstrip Line Model

In the EPSO design procedure, strip widths W_i and lengths L_i are parameters to be optimized. The formulas given in [18] can be used to calculate the effective constant $\varepsilon_{i,e}(0)$ and characteristic impedance $Z_i(0)$ for a given width. Here (0) indicates that dispersion and dissipation effects are disregarded.

On the basis of effective constant $\varepsilon_{i,e}(0)$ computed above, a dispersion formula for effective constant $\varepsilon_{i,e}(f)$ was proposed in [19], where f is the operating frequency. The closed-form expression describing the effect of frequency on characteristic impedance has been proposed in [20] and is expressed as

$$Z_i(f) = Z_i(0)\frac{\varepsilon_{i,e}(f) - 1}{\varepsilon_{i,e}(0) - 1}\sqrt{\frac{\varepsilon_{i,e}(0)}{\varepsilon_{i,e}(f)}} \tag{1}$$

Then propagation constant can be written as

$$\beta_i(f) = k_0\sqrt{\varepsilon_{i,e}(f)} \tag{2}$$

where k_0 is the wavenumber in vacuum.

Moreover, the dissipation caused by dielectric and conductor losses according to [18] is given below

$$\alpha_{i,d}(f) = \frac{k_0 \varepsilon_r (\varepsilon_{i,e}(f) - 1)\tan\delta}{2\sqrt{\varepsilon_{i,e}(f)}(\varepsilon_r - 1)} \tag{3}$$

$$\alpha_{i,c}(f) = \frac{1}{Z_i(f)W_i}\sqrt{\frac{\pi f \mu_0}{\sigma}} \tag{4}$$

where ε_r and $\tan\delta$ are the relative permittivity and loss tangent of the dielectric respectively. σ is the conductivity of strip metal. The complex propagation constant based on the dispersion and dissipation microstrip line model can be expressed as

$$\gamma_i(f) = \alpha_{i,d}(f) + \alpha_{i,c}(f) + j\beta_i(f) \tag{5}$$

2.2. Even Mode Analysis

In the even mode, there is no current flow through the isolation resistors, so the isolation resistors can be neglected. This way, the power divider can be split into two symmetric two-port networks. Only
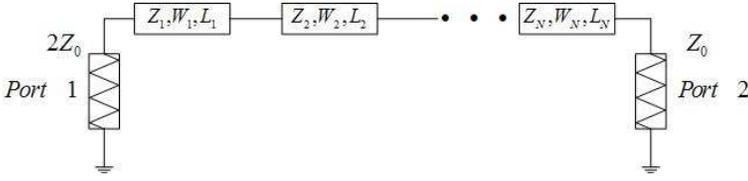
Figure 2. Even mode equivalent circuit for the MWPD.

the upper half equivalent circuit is shown in Figure 2 due to symmetry. The transmission matrix of the i-th microstrip line can be written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TL,i} = \begin{bmatrix} \cosh \gamma_i(f)L_i & jZ_i(f)\sinh \gamma_i(f)L_i \\ j\sinh \gamma_i(f)L_i/Z_i(f) & \cosh \gamma_i(f)L_i \end{bmatrix},$$
$$i = 1, 2, 3, \ldots, N \quad (6)$$

The transmission matrix of the two-port network can be easily expressed as the product of the N cascade transmission matrices thus:

$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TL,i} \quad (7)$$

According to the relationships [18] between the transmission matrix and scattering matrix for a two-port network, the even mode scattering matrix can be expressed as

$$\begin{bmatrix} S_{11e} & S_{12e} \\ S_{21e} & S_{22e} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{A_e Z_{02}+B_e-C_e Z_{01}Z_{02}-D_e Z_{01}}{A_e Z_{02}+B_e+C_e Z_{01}Z_{02}+D_e Z_{01}} & \frac{2\sqrt{Z_{01}Z_{02}}(A_e D_e-B_e C_e)}{A_e Z_{02}+B_e+C_e Z_{01}Z_{02}+D_e Z_{01}} \\ \frac{2\sqrt{Z_{01}Z_{02}}}{A_e Z_{02}+B_e+C_e Z_{01}Z_{02}+D_e Z_{01}} & \frac{-A_e Z_{02}+B_e-C_e Z_{01}Z_{02}+D_e Z_{01}}{A_e Z_{02}+B_e+C_e Z_{01}Z_{02}+D_e Z_{01}} \end{bmatrix} \quad (8)$$

where Z_{01} and Z_{02} indicate the terminated load impedances at ports 1 and 2 respectively. In the even mode circuit Z_{01} and Z_{02} are $2Z_0$ and Z_0 respectively as shown in Figure 2.

2.3. Odd Mode Analysis

In the odd mode, the power divider can also be split into two identical two-port networks. The upper half equivalent circuit is shown in Figure 3. The transmission matrix of the i-th shunt isolation resistor can be expressed as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{R,i} = \begin{bmatrix} 1 & 0 \\ 1/(R_i/2) & 1 \end{bmatrix}, \quad i = 1, 2, 3, \ldots, N \quad (9)$$
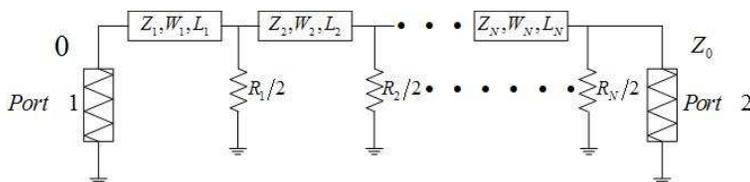
Figure 3. Odd mode equivalent circuit for the MWPD.

The odd mode transmission matrix of the two-port network can be written as the product of the $2N$ cascade transmission matrices thus:

$$\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{TL,i} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{R,i} \tag{10}$$

Then the odd mode scattering matrix is

$$\begin{bmatrix} S_{11o} & S_{12o} \\ S_{21o} & S_{22o} \end{bmatrix} = \begin{bmatrix} \frac{A_o Z_{02}+B_o}{A_o Z_{02}+B_o} & 0 \\ 0 & \frac{-A_o Z_{02}+B_o}{A_o Z_{02}+B_o} \end{bmatrix} \tag{11}$$

This result can be obtained according to Equation (8) by substituting Z_{01} with 0 because the source impedance is zero.

Finally, the scattering matrix of an MWPD [21] can be expressed as

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} S_{11e} & \frac{S_{12e}}{\sqrt{2}} & \frac{S_{12e}}{\sqrt{2}} \\ \frac{S_{12e}}{\sqrt{2}} & \frac{S_{22e}+S_{22o}}{2} & \frac{S_{22e}-S_{22o}}{2} \\ \frac{S_{12e}}{\sqrt{2}} & \frac{S_{22e}-S_{22o}}{2} & \frac{S_{22e}+S_{22o}}{2} \end{bmatrix} \tag{12}$$

2.4. Fitness Function Construction

Given that the scattering matrix of MWPD has been given, the fitness function can be defined as

$$Fitness = |\max S_{11} - a| + |\max S_{23} - b| \tag{13}$$

where $\max S_{11}$ and $\max S_{23}$ are the maximum magnitudes of S_{11} and S_{23} in the desired frequency domain. In this paper, a and b are 0.04 and 0.03 respectively.

3. PARTICLE SWARM OPTIMIZATION

3.1. Standard PSO

Kennedy and Eberhart [22] proposed PSO, which is an evolutionary computation technique based on the movement and intelligence of

particle swarm. Each particle in the swarm represents a possible solution (position in N dimensional space) to the optimization problem. The particle at iteration T is usually expressed as a vector $\mathbf{X}_i = [X_{i,1}(T), X_{i,2}(T), \ldots, X_{i,N}(T)]$ $(1 \leq i \leq M)$, where M is the size of the swarm. A particle adjusts its position with velocity $\mathbf{V}_i = [V_{i,1}(T), V_{i,2}(T), \ldots, V_{i,N}(T)]$ through the solution space. The velocity is also associated with the personal best particle $\mathbf{P}_i = [P_{i,1}(T), P_{i,2}(T), \ldots, P_{i,N}(T)]$ and global best particle $\mathbf{G}(T) = [G_i(T), G_2(T), \ldots, G_N(T)]$ in the swarm. The personal best particles and global best particle are selected by evaluating the fitness function. The PSO is mainly based on the update of the velocity and position:

$$\mathbf{V}_i(T+1) = \omega \mathbf{V}_i(T) + c_1(\mathbf{P}_i(T) - \mathbf{X}_i(T)) + c_2(\mathbf{G}(T) - \mathbf{X}_i(T)) \quad (14)$$
$$\mathbf{X}_i(T+1) = \mathbf{X}_i(T) + \mathbf{V}_i(T+1) \quad (15)$$

where ω (i.e., inertia weight factor) decreases linearly from 0.9 to 0.4. c_1 and c_2 are uniformly distributed random numbers between 0 and 1. The velocity of the i-th dimension is confined in the range $[-V_{i,\max}, V_{i,\max}]$, where $V_{i,\max}$ is the maximum velocity of the i-th dimension given by users.

3.2. Enhanced PSO

To improve convergence accuracy and velocity, we introduce a quadratic interpolation method and perturbation to the personal best particles in the proposed EPSO. The steps involved in the proposed EPSO are described as follows:

Step 1: Randomly initialize M particles with velocity in a given range. Calculate the fitness of each particle according to Equation (13). Record the personal best particles and global best particle.

Step 2: Update the velocity and position of each particle according to Equations (14) and (15).

Step 3: Calculate the fitness of each particle again. Update the personal best particles according to fitness.

Step 4: Apply quadratic interpolation method. The quadratic interpolation method in [17] is applied to the first three best particles to improve the local search ability. However, compared to the particles arbitrarily moving in the space, the personal best particles are closer to the global minimum. Thus, the quadratic interpolation method is more effective when it is directly applied to the personal best particle solutions; this approach can improve the convergence velocity of the EPSO. Pick the first three best particles among the personal best particles. The chosen particles are denoted as $f(\mathbf{P}_1) < f(\mathbf{P}_2) < f(\mathbf{P}_3)$, where $f(\mathbf{P}_i)$ is the fitness function value of the i-th personal best

particle. The quadratic interpolation method is defined as follows:

$$\alpha_k = (P_{1,k} - P_{3,k})f(\mathbf{P}_2) + (P_{3,k} - P_{2,k})f(\mathbf{P}_1) + (P_{2,k} - P_{1,k})f(\mathbf{P}_3),$$
$$k = 1, 2, \ldots, N \quad (16)$$

For each k, if $|\alpha_k| > 10^{-5}$ then

$$p'_k = \frac{1}{2} \frac{(P_{1,k}^2 - P_{3,k}^2)f(\mathbf{P}_2) + (P_{3,k}^2 - P_{2,k}^2)f(\mathbf{P}_1) + (P_{2,k}^2 - P_{1,k}^2)f(\mathbf{P}_3)}{\alpha_k},$$
$$k = 1, 2, \ldots, N \quad (17)$$

If $|\alpha_k| < 10^{-5}$ then

$$p'_k = p_{1,k}, \quad k = 1, 2, \ldots, N \quad (18)$$

Replace the worst particle in the personal best particles with the better one between $\mathbf{P}'$ and $\mathbf{P}_1$.

Step 5: Perturb the personal best particles. GN_i is used to represent the successive iterations in which the i-th personal best particle remains unchanged. The perturbation equation adopted in this study is the same as that used to perturb the global best particle solution in [17], where the global best particle is perturbed unconditionally. However, in this paper the perturbation equation is applied only when GN_i exceeds 20, so that the EPSO can better maintain the merit based on the movement and intelligence of the particles. When the perturbation of the i-th personal best particle is applied, GN_i is initialized with 1. The diversity of the algorithm is enhanced because the size of personal best particles is larger than that of the global best particle.

Step 6: Update global best particle.

Step 7: Repeat steps 2, 3, 4, 5 and 6 until the maximum iteration number is reached.

4. NUMERICAL RESULTS

The EPSO, IPSO, PSO and GA are employed to optimize two-, three- and four-section MWPDs. All the algorithms are implemented using MATLAB. The swarm size of EPSO, IPSO and PSO (or population size for GA) is 40, and the maximum iteration number is 500. The crossover probability and mutation probability of GA are 0.8 and 0.1 respectively. The substrate of all the MWPDs is F4B-2 with a thickness of 1.0 mm, relative permittivity of 2.65 and loss tangent of 0.0009. The strip widths range from 0.72 to 2.72 mm with respect to the characteristic impedances of 100 and 50 Ω. The lower and upper bounds of strip lengths are 10 and 50 mm respectively. The isolation resistors vary from 50 to 1000 Ω.
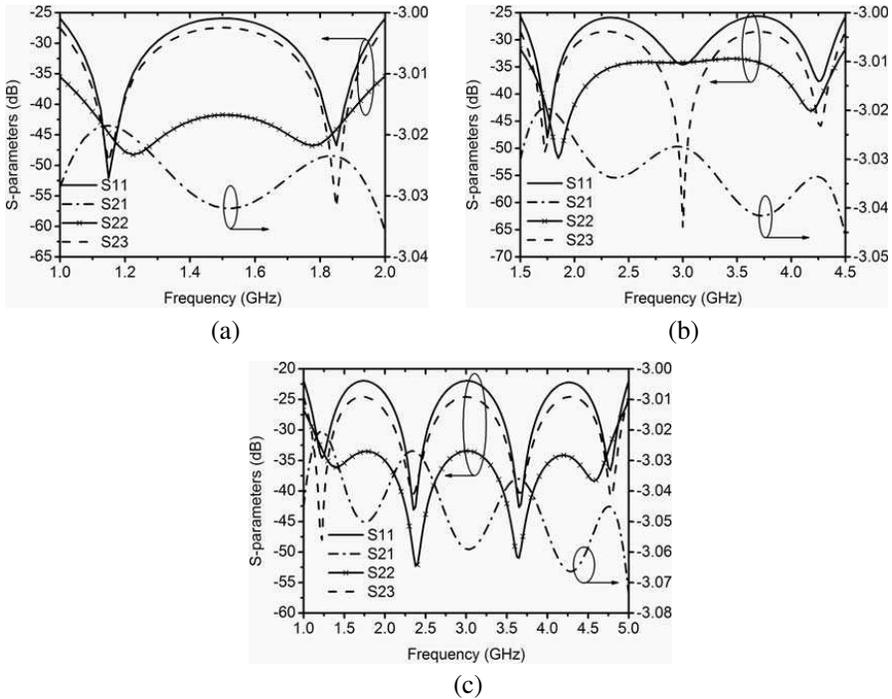
Figure 4. S-parameters computed by the EPSO design procedure for: (a) Two-section MWPD. (b) Three-section MWPD. (c) Four-section MWPD.

A two-section MWPD is designed to operate in the 1.0–2.0 GHz band. Figure 4(a) shows the optimized scattering parameters computed using the EPSO design procedure. This MWPD has a usable $2:1$ bandwidth with $S_{11} < -26.0$ dB and $S_{23} < -27.4$ dB. A similar performance of a two-section hybrid can be found in [2] through extensive mathematical computation. In the present work, the EPSO design procedure automatically provides the optimized parameters without the need for extensive mathematical computation.

For the second example, a three-section MWPD is obtained which covers the 1.5–4.5 GHz with $S_{11} < -25.6$ dB and $S_{23} < -28.4$ dB. A plot of the computed results is shown in Figure 4(b). In comparison, a three-section hybrid [2] has a usable $3:1$ bandwidth with $S_{11} < -26.0$ dB and $S_{23} < -27.9$ dB. Similar performance verifies the validity of the EPSO design procedure again.

To verify the flexibility of EPSO design procedure, we consider a four-section MWPD operating in the 1.0–5.0 GHz frequency band.

The simulated scattering parameters for this MWPD are shown in Figure 4(c). The isolation between output ports is better than $-24.6\,$dB. Input port matching parameter S_{11} is lower than $-22.0\,$dB.

Table 1 shows the best fitness and number of fitness function evaluations for the different algorithms. The IPSO and EPSO have better convergence accuracy and velocity than do the PSO and GA. Moreover, EPSO exhibits the best performance in terms of speed and

Table 1. Performance comparisons for different algorithms.

Algorithm	Example 1		Example 2		Example 3	
	Best Fitness	Fitness function evaluations	Best Fitness	Fitness function evaluations	Best Fitness	Fitness function evaluations
GA	0.0244	20040	0.0341	17040	0.0892	19890
PSO	0.0247	19880	0.0330	18120	0.0810	19800
IPSO	0.0235	7641	0.0231	8412	0.0752	14435
EPSO	0.0231	6207	0.0200	6654	0.0687	13509

Table 2. Parameters optimized using the EPSO design procedure.

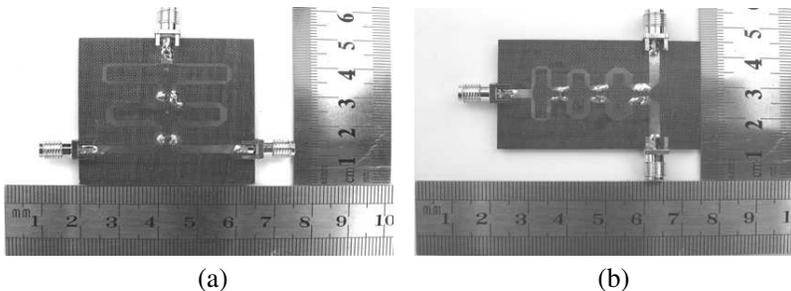| Example 1	Example 2	Example 3
$W1 = 1.128\,$mm	$W1 = 0.985\,$mm $\quad R1 = 117.3\,\Omega$	$W1 = 0.954\,$mm $\quad L3 = 17.24\,$mm
$W2 = 1.937\,$mm	$W2 = 1.489\,$mm $\quad R2 = 223.7\,\Omega$	$W2 = 1.238\,$mm $\quad L4 = 16.86\,$mm
$L1 = 34.7895\,$mm	$W3 = 2.122\,$mm $\quad R3 = 432.6\,\Omega$	$W3 = 1.633\,$mm $\quad R1 = 144.2\,\Omega$
$L2 = 34.1617\,$mm	$L1 = 17.36\,$mm	$W4 = 2.062\,$mm $\quad R2 = 235.6\,\Omega$
$R1 = 100.7\,\Omega$	$L2 = 17.20\,$mm	$L1 = 17.45\,$mm $\quad R3 = 349.8\,\Omega$
$R2 = 242.4\,\Omega$	$L3 = 17.01\,$mm	$L2 = 17.21\,$mm $\quad R4 = 478.1\,\Omega$

Figure 5. Photograph of the fabricated samples. (a) Two-section MWPD. (b) Three-section MWPD.

precision. The strip widths, lengths and values of isolation resistors optimized using the EPSO design procedure are listed in Table 2.

5. FABRICATION AND MEASUREMENTS

The first two samples optimized in Section 4 were fabricated and measured. The microstrip line widths for the all samples are specified in Table 2. The isolation resistors optimized in the table are replaced
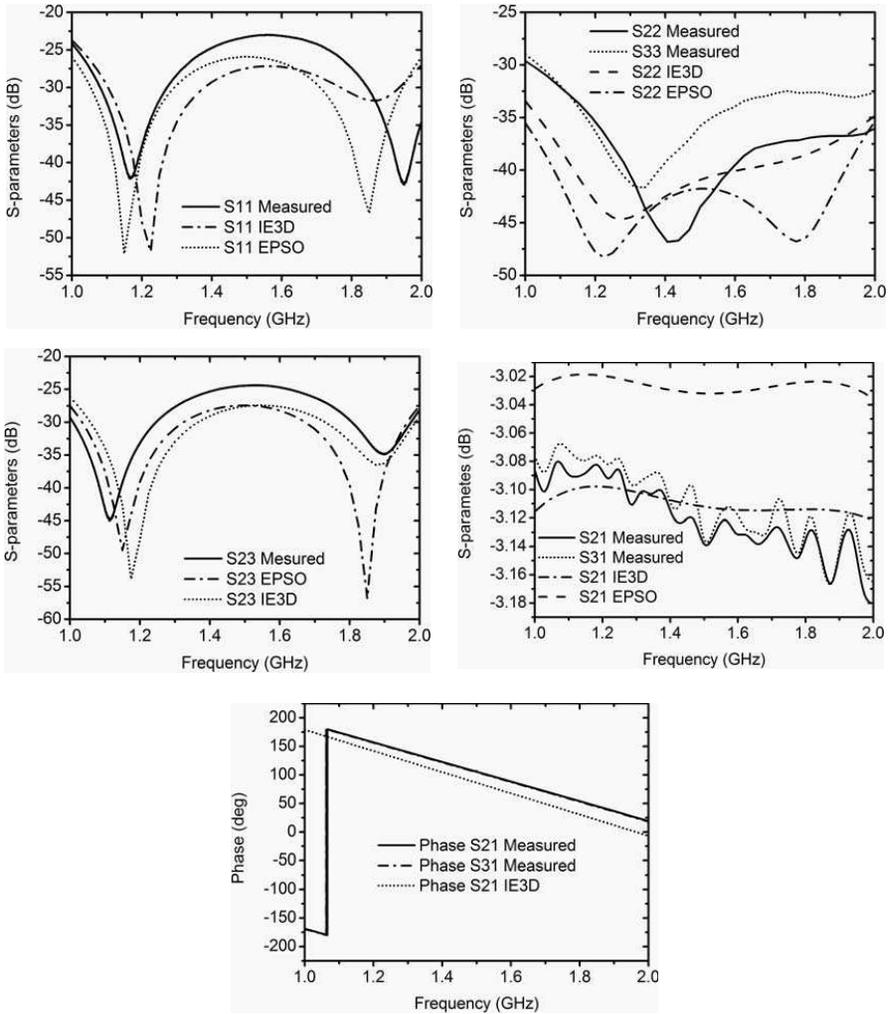

Figure 6. Measured, simulated and computed results for sample 1.

by their nearest standard resistors. The microstrip lines are curved to reduce the longitudinal size of these MWPDs. However, the optimized lengths in Table 2 are applicable to straight microstrip lines. Subsequently, the full-wave simulator IE3D is employed to guarantee the electrical lengths between curved and straight microstrip lines. Figure 5 shows the fabricated power dividers.

For the two-section MWPD, the following standard resistors are selected: $R1 = 100.0\,\Omega$ and $R2 = 240.0\,\Omega$. The measured
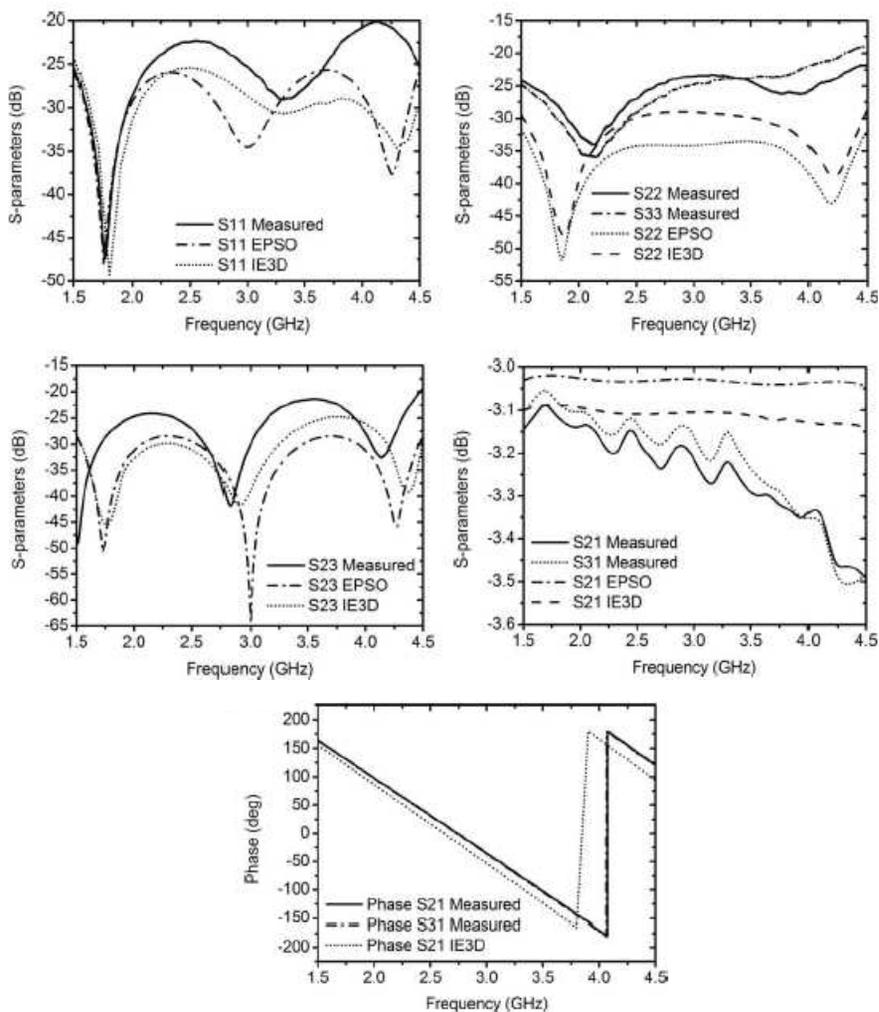

Figure 7. Measured, simulated and computed results for sample 2.

scattering parameters and phase are shown in Figure 6. The simulated and computed scattering parameters are also plotted. The measured S_{11} is lower than $-23.0\,\mathrm{dB}$ in the designed frequency band. The isolation between two output ports is lower than $-25.0\,\mathrm{dB}$. Output port matching parameters S_{22} and S_{33} are nearly better than $-29.0\,\mathrm{dB}$. Ports 2 and 3 exhibit phase balance in the entire frequency band. Good agreement is observed among the measured, simulated and computed scattering parameters. In comparison, a two-section stripline experimental model was constructed in [2]; this model also operates in the 1.0–2.0 GHz band with $S_{11} < -20.8\,\mathrm{dB}$ and $S_{23} < -22.0\,\mathrm{dB}$.

The following standard resistors are selected for the second sample: $R1 = 120.0\,\Omega$, $R2 = 220.0\,\Omega$ and $R3 = 430.0\,\Omega$. As shown in Figure 7, the measured S_{11}, S_{22}, S_{33} and S_{23} are nearly lower than $-20.0\,\mathrm{dB}$. Transmission parameters S_{21} and S_{31} decrease to $-3.5\,\mathrm{dB}$ as the frequency increase to 4.5 GHz. Moreover, excellent phase balance between ports 2 and 3 is observed. The lower frequency band shows good agreement, whereas the upper frequency band shows obvious errors. This may be caused by the fabrication. Step discontinuities on the metal strips also affect the errors between the computed and measured results. Moreover, the chip resistors used in the MWPD are probably no longer ideal resistors [23], which can also result in errors. Generally speaking, the fabricated MWPDs and measured experimental results validate the proposed EPSO design procedure.

6. CONCLUSION

A design procedure for equal split broadband MWPD based on the EPSO has been presented. A dispersion and dissipation microstrip line model has also been incorporated into the design procedure. By applying the quadratic interpolation method and perturbation equation to the personal best particles, the EPSO achieves better performance in terms of convergence accuracy and velocity than do the GA, PSO and IPSO. MWPDs with good performance can be optimized with randomly initialized values in desired frequency band because of the powerful EPSO. Several MWPDs have been optimized, simulated, fabricated and tested. The measured results validate the proposed EPSO design procedure and confirm the improvement of the proposed EPSO.

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