Progress In Electromagnetics Research B, Vol. 31, 357–379, 2011

SMART ELECTROMAGNETIC SIMULATIONS: GUIDE-LINES FOR DESIGN OF EXPERIMENTS TECHNIQUE

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Abstract—Electromagnetic design problems usually involve a large number of varying parameters. A designer can use different kinds of models in order to achieve optimum design. Some models, e.g., finite-element model, can be very precise: however, it requires large computational costs (i.e., CPU time). Therefore, the designer should use a screening process to reduce the number of parameters in order to reduce the required computational time. In this paper, using the Design of Experiments (DOE) approach to reduce the number of parameters is explored. The benefits of this technique are tremendous. For example, once researchers realize how much insight and information can be obtained in a relatively short amount of time from a well-designed experiment, DOE would become a regular part of the way they approach their simulation projects. The main objective of this paper is to apply the DOE technique to electromagnetic simulations of different systems and to explore its effectiveness on a new field, namely the magnetic refrigeration systems. The methodology of the DOE is presented to assess the effects of the different variables and their interaction involved in electromagnetic simulations design and optimization processes.

1. INTRODUCTION

Designing a system in engineering is a cooperative and iterative process. It is usually a multidisciplinary task involving skilled engineers specialized in different areas. The aim of the designing process is to generate a system (a product) with a predefined set of requirements and

Received 21 May 2011, Accepted 27 June 2011, Scheduled 5 July 2011
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constraints. Traditionally, this process has involved the development of prototypes (physical models) to test. This is a time consuming and expensive approach for developing new systems and can represent a large percentage of the total system. Reducing time and costs has always been, and still is, a key issue [1].

The first and most obvious place to reduce both costs and time is by simulating the physical models rather than building actual prototypes. Computer-aided analysis, also known as simulation, has revolutionized engineering design. In the span of about 35 years, simulation software has evolved into a variety of complex packages equipped with sophisticated layout and visualization modules [2]. The numerical engines employ computational methods whose efficiency and versatility have improved greatly since their time of inception, mostly in the 1970s. Such packages, which are now commercially available, have allowed designers to leave behind the drawing board for the most part and to shortcut through several stages of expensive prototyping and manual cut-and-try tuning [2].

The process of building, verifying, and validating a simulation model can be arduous, but once completed, it can be used to explore different aspects of the modeled system. One extremely effective way for accomplishing this is to use experimental designs, also called Design of Experiments (DOE), to help explore the simulation models [3]. According to [4], many simulation practitioners could obtain more information from their analysis if they used statistical theories, especially with the use of DOE developed specifically for computer models. If the input variables to the process are varied, the outputs will vary, even though the variation may only be due to random effects or noise. The question is which input variables (factors) are causing the majority of the variability in the output (responses)? In other words, which factors are the significant "drivers"? It is desirable to determine where the variability is coming from (also known as "sensitivities") with an optimum expenditure of resources [5]. Like most statistical methods, DOE has the primary objective of obtaining maximum information at minimum cost: its goal is to find the best ratio between benefits of information and information costs [5]. More specifically, in the case of DOE, the desired information is the quantification of the influence of several factors on a given phenomenon. Due to this quantification, it is possible to predict the behavior of the system studied in different possible configurations and, consequently, optimize the operation of such systems. To achieve this, the DOE methodology offers a testing strategy, where one of its main characteristics leads to minimizing the number of tests to be performed.

Recently, the DOE technique has been adopted in the design and testing of various applications including automotive assembly [6], computational intelligence [7], bioassay robustness studies [8] and many others.

The objective of this paper is to apply the DOE technique as a screening tool for electromagnetic simulations. The effect of the input variables on the output variables is intended to be evaluated. In particular, the input variables — which are also called factors or parameters — will include (for example) the shape parameters of an electric machine, where the output variables — which are the objective functions to be evaluated — will include (for example) the torque produced by an electric machine.

The remaining sections of the paper are organized as follows. Section 2 describes the main concepts and typical methods used in the design process of any system comprising several factors. Section 3 discusses a practical implementation of the DOE technique using a tool built on Matlab and Java for efficient use of the technique. Section 4 presents three different applications to which the DOE is applied in order to facilitate their design and optimization processes. These applications are magnetizer, Problem 25 of Team Workshop and a magnetic refrigeration system. Results from such systems are also presented in Section 4. Finally, the overall paper conclusions is drawn in Section 5.

2. METHODOLOGY

Designing an electromagnetic device or system, is usually a study of a phenomenon depending on different factors. The intuitive (classical) method is not always the best choice. The classical method consists generally of fixing the level of all variables except one and then measuring the response for several values of the variable factor.

Let's assume for example that an electric machine is aimed to be optimized. To simplify the problem we assume that only the global dimensions (the length and the external radius) are the varying factors. Thus the length is the first factor and is denoted by x_1 and the external radius is the second factor and is denoted by x_2. Each factor can take several values between $\{x_{1_{\min}}, x_{1_{\max}}\}$ and $\{x_{2_{\min}}, x_{2_{\max}}\}$. We desire to study the influence of each of these factors on the system response or output (torque) called Y. The classical or traditional approach is to study the two factors x_1 and x_2, separately.

First we fix x_2 at the average level $x_{2_{average}}$ and study the response of the system when x_1 varies from $x_{1_{\min}}$ to $x_{1_{\max}}$ following, for example, 4 steps (experiments or simulations) as shown in Figure 1. We then repeat the same experience to study the influence of x_2. The total

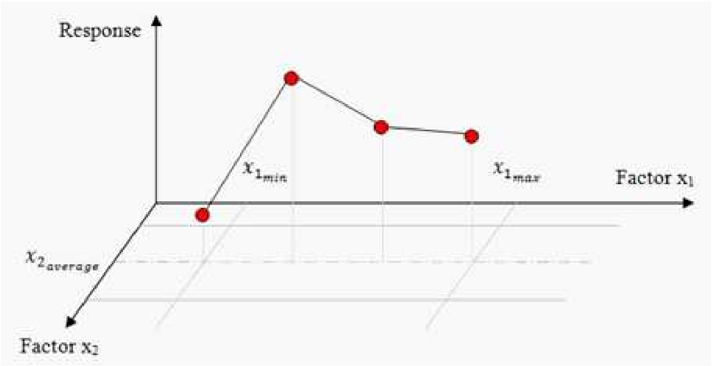


Figure 1. Traditional method of experiments.

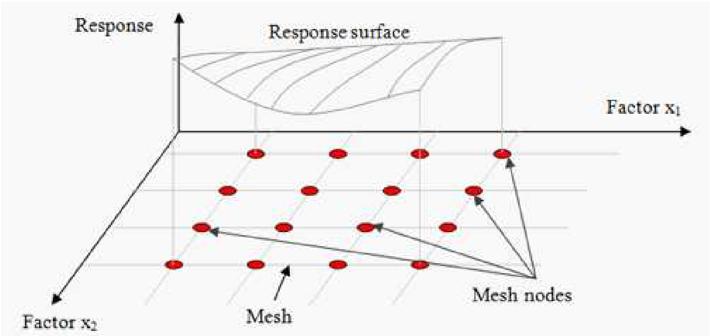


Figure 2. One experience at each node of the mesh.

number of tests is 8. Nevertheless, one can ask if we have a good knowledge about the system with these 8 experiences. It is obvious that the answer of this question is no.

To get a better knowledge about the system, we have to mesh the validity domain of the two factors and test each node of this mesh as shown in Figure 2. In this case, we have to achieve $4 \times 4 = 4^2 = 16$ experiences.

However, in this example only two factors are taken into account. If the number of factors increases to 7 for example, the number of tests to be performed will rise to $4^7 = 16384$ experiences, which is a time- and cost-consuming process. Knowing that it is impossible to reduce the number of values that one factor can take less than 2, the designer often reduces the number of factors, which leads to incertitude of results. To reduce both cost and time, the DOE is used where it aims to establish a design experiment with less number of tests. The DOE, for example, allows identifying the influence of 7 factors with 2

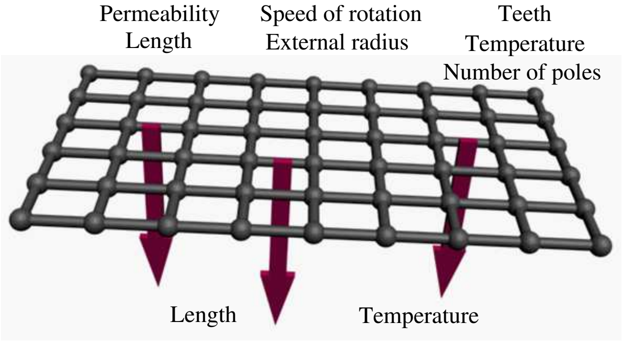


Figure 3. Schematic of a screening experimental design, where many factors are reduced to a significant few.

points per variable with only 8 or 12 tests rather than 128 tests with the traditional method [5]. When the number of factors increases a screening process using the DOE technique should first be adopted to reduce this number. This screening process is schematically shown in Figure 3. It shows an example of parameters screening where the initial factors are: permeability, length, speed of rotation, external radius, teeth, temperature and number of poles. Then, after the screening process, only two parameters are influent on the output (temperature and length).

2.1. Mathematical Concept

Assume that y is the response (or output) of an experiment (or a simulation) and $\{x_1, x_2, x_3, \ldots, x_k\}$ are k factors acting on this experiment where each factor has two levels of variation x_{i-} and x_{i+}. To predict the value of y, it is approximated by an algebraic model given by the following equation:

$$y = a_0 + a_1x_1 + a_2x_2 + \ldots + a_kx_k + \ldots + a_1x_1x_2 + \ldots + a_1x_1x_k + a_{1\ldots k}x_{1\ldots k} \qquad (1)$$

where a_j are coefficients which represent the effect of factors and their interactions on the response of the experiment.

2.2. Full Factorial Design

The study of full factorial design consists of exploring all possible combinations of the factors considered in the experiment [9]. Note that the design X^k means that this experiment concerns a system with k factors with X levels.

Usually, two values of the X's (called levels) are used. The use of only two levels implies that the effects are monotonic on the response variable, but not necessarily linear [5]. For each factor, the two levels are denoted using the "rating Yates" notation (named after its author) by: -1 the low level of each factor, $+1$ the high level of each factor (Figure 4). Thus, the number of experiments carried out by a full factorial design with 2 levels is given by:

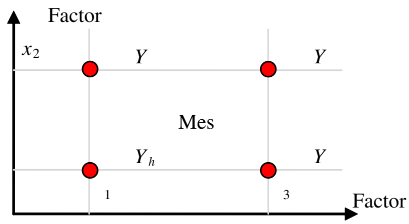
$$n = 2^k \tag{2}$$

where k is the number of factors to be considered.

Figure 4 shows the design matrix of a full factorial design for 2 factors and the mesh of the experimental field where points correspond to nodes.

Test number	Factor x_1	Factor x_2	Response y
1	-1	-1	Y_1
2	-1	+1	Y_2
3	+1	-1	Y_3
4	+1	+1	Y_4

(a)



(b)

Figure 4. Full factorial design for 2 factors and 2 levels. (a) Design matrix. (b) Strategy of experimentation; points corresponding to nodes in the mesh of the experimental field.

The advantage of full factorial designs, is the ability to estimate not only the main effects of factors, but also all their interactions in two by two, three by three, etc., up to the interaction involving the k factors. However, when the number of factors increases, the use of such a design leads to a prohibitive number of experiments or simulations to perform.

The question to be asked is then: is it necessary to perform all experiments of the full factorial design to estimate the system's model? In other words, is it necessary to conduct a test at each node of the mesh?

2.3. Fractional Factorial Design

It is not necessary to identify the effect of all interactions of the analytical model given by Equation (2), because the interactions of order ≥ 2 (like $x_1x_2x_3$) are usually negligible. To illustrate this phenomenon, an analogy can be made with a Taylor series approximation where the information given by each term decreases if the order of this one increases. So, fractional factorial designs can be used to estimate factors effect and interactions that act more on the experiments with a reduced number of experiments [10]. Taguchi tables [9], or G. Box generators [11], give the fractional factorial design matrix of experiments [12].

To illustrate fractional factorial designs let's take an example, if the number of factors is $k = 3$, the design matrix of these three factors is given by G. Box generators in a way that the third factor is the product of the two other factors. It is said that factor x_3 and interaction x_1x_2 are confused, or x_3 and x_1x_2 are aliased and there is a confusion of these aliases because only their sums are reachable [9, 10, 12].

2.4. Estimation of Model Coefficients

The value of the coefficient a_0 is estimated from the arithmetic average of all observed responses and it is given by:

$$a_0=\bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i \tag{3}$$

where y_i is the response observed for the experiment i and n is the number of experiments.

The effect of a factor x_j at the level x_{j+} can be calculated thus, the coefficient associated with this effect can be identified by using the following equations:

$$a_j = e_j = y^+_{x_j} - a_0 \tag{4}$$

and

$$y^+_{x_j}=\frac{1}{n^+}\sum_{i=1}^{n} y^+_i \tag{5}$$

where $y^+_{x_j}$ is the response observed for experiment i when x_j is at level x_{j+}, n^+ is the number of experiments where x_j is at the level x_{j+} and e_{a_j} is the effect of coefficient a_j.

Once the method of how to calculate the coefficients of the model and how to identify the existing confusion between these factors has been presented, we can evaluate the contributions of contrasts (the sum of confusions) and therefore the most significant factors (affecting the response). In [11] the identification of the significant factors has been proposed by evaluating the coefficients contribution (or contrasts, for fractional designs) on the model response from the normalization of their values compared to the sum of squared responses, such as given in the following equations:

$$C_{a_j} = \frac{SCE\left(a_j\right)}{SCE\left(y\right)} \ [\%] \tag{6}$$

with

$$SCE\left(y\right) = \sum_{i=1}^{n} \left(y_i - \overline{y}\right)^2 \tag{7}$$

$$SCE\left(a_j\right) = \frac{n}{s} \sum_{j=1}^{s} \left(e_{a_j}\right)^2 \tag{8}$$

where S is the number of levels (equals to 2 in this case), e_{a_j} is the effect of coefficient a_j, and C_{aj} is the contribution of the contrast associated with the coefficient a_j.

According to [13]:

- The contribution given by (6) is deemed significant if $C_{a_j} \leq 5\%$.
- The interactions of order higher than two are negligible.
- If a contrast is negligible, all effects composing this contrast are negligible also.
- Two significant factors can generate a significant interaction. On the other side, two insignificant factors do not generate a significant interaction.

3. IMPLEMENTATION

For an efficient use of the DOE methodology, it has been implemented in the form of interactive tool called Design of experiments Tool (DOET) using a combination of Matlab and Java. Matlab is an efficient software which puts the powerful calculation function, visual and program designing together in an easily used development environment. Java is a cross-platform program development language which is created by Sun. It is the most advanced program language which also has the richest characteristic and has the most powerful function. In DOET, Matlab is used for all calculation functions, and java is used to generate interactive User Interfaces.

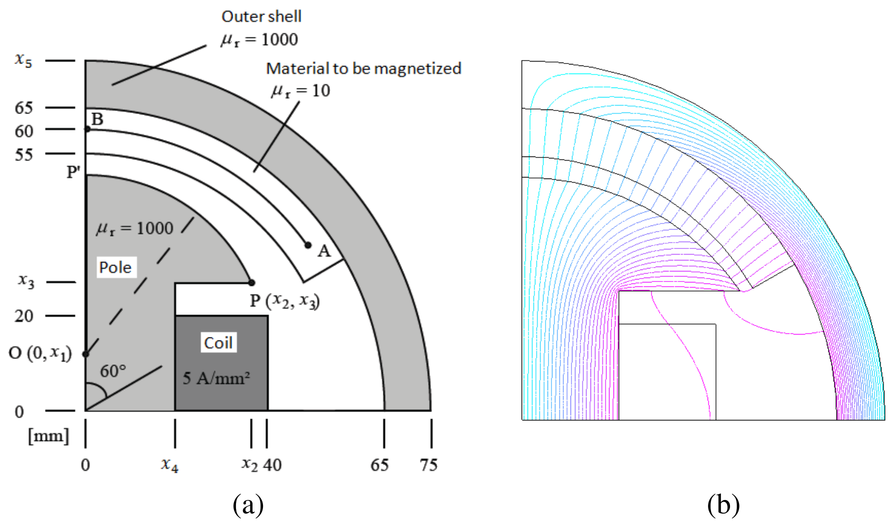


Figure 5. Magnetizer to be studied; (a) geometry, (b) contour of the solution.

4. APPLICATIONS

4.1. Application to Magnetizer

4.1.1. Description

The application of the DOE technique is first demonstrated on a magnetizer where the geometry is shown in Figure 5. The study and optimization of this magnetizer is given in [9, 10]. The shape of the pole face is to be optimized. In the finite element model the object to be magnetized is treated as if it were made of nonmagnetic material. A permeability value very close to that of air is assigned to that region ($\mu_r = 10$). A high current is applied to the coil. The linear magnetostatic field analysis is carried out in 2-D using "finite element method" (FEM).

4.1.2. Objective Function and Constraints

The goal of the optimization is to achieve a constant magnetic flux density distribution along chord A-B positioned halfway through the width of the magnetized piece and subtending an angle of $\pi/3$.

The constraints of x_1, x_2, x_3, x_4 and x_5 are represented in Table 1.

To avoid intersection of the pole with the material to magnetize and to ensure an air-gap at least equal to 2 mm, the parameters are

Table 1. Parameters and their constraints.

Parameter	Minimum Value [mm]	Maximum Value [mm]
x_1	0	15
x_2	35	45.9
x_3	25	26.5
x_4	15	20
x_5	70	80

subject to an additional geometric constraint. This constraint is given by

$$x_1 + \sqrt{x_2^2 + (x_3 - x_1)^2} \leq 53 \tag{9}$$

The optimization problem can be formulated as follows:

$$F_{obj} = \frac{1}{1 + f_{obj}} \tag{10}$$

$$f_{obj} = \frac{\sqrt{n \sum_{i=1}^{n} B_i^2 - \left(\sum_{i=1}^{n} B_i\right)^2}}{\sum_{i=1}^{n} B_i} \tag{11}$$

where B_i is the value of magnetic flux density at a point i of the arc A-B, and $n = 50$ is the total number of points on the arc.

When the geometric constraint is violated, f_{obj} is expressed by a penalty term given by:

$$f_{obj}=\lambda_1+\lambda_2\left(x_1+\sqrt{x_2^2+(x_3-x_1)^2}-53\right) \quad (\lambda_1=\lambda_2=10) \tag{12}$$

4.1.3. Identification of Significant Parameters Using the DOE

The aim of this section is to identify the most influential factors on the objective function using the DOE. As explained earlier the use of two-level full factorial design needs $2^5 = 32$ runs (simulation) to evaluate the objective function. Using Generators of G.Box, the possible designs are $2^{5-2} = 8$ runs and $2^{5-1} = 16$ runs.

Design 1: the choice of a 2^{5-2} design means that we have a 2 levels design with 5 factors where 2 of these factors are generated using the other 3 factors. In other words; the factor (4) will be generated using the product of factors (1) and (2). The factor (5) will be generated using the product of factors (1) and (3) (see Table 2 and Figure 6).

Table 2. Generators of G. Box for 5 factors.

Resolution	Design name	Number of tests	Generators
3	2^{5-2}	8	$4 = \pm 1, 2 \quad 5 = \pm 1, 3$
5	2^{5-1}	16	$5 = \pm 1, 2, 3, 4$

Table 3. Design matrix of generated by the 2^{5-2} fractional factorial design.

x_1 [mm]	x_2 [mm]	x_3 [mm]	x_4 [mm]	x_5 [mm]	F
0	35	25	2	8	0.6102
0	35	26	2	7	0.4973
0	46	25	15	8	0.6085
0	46	26	15	7	0.4969
15	35	25	15	7	0.4903
15	35	26	15	8	0.5884
15	46	25	2	7	−0.0019
15	46	26	2	8	−0.0019

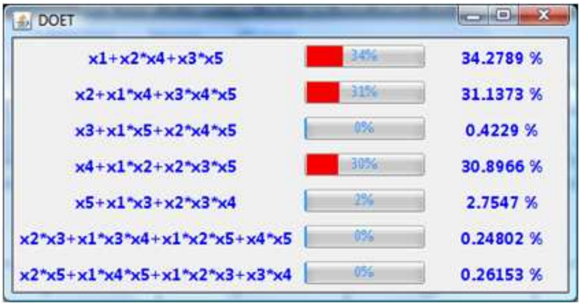


Figure 6. Contributions obtained with a 2^{5-2} fractional factorial design: The contrasts (left side), and the influence on the objective function (right side)

Design 2: the choice of a 2^{5-1} design means that we have a 2 levels design with 5 factors where one of these factors is generated using the other 4 factors. This means that the factor (5) will be generated using the product of factors (1), (2), (3) and (4) (Table 2 and Figure 7).

So, using a fractional design will effectively reduce the number of runs from 32 to 16 or 8 depending on the chosen design. Thus the time of simulation using Design 1, and Design 2 is reduced to 50% and 75% of the initial time, respectively.

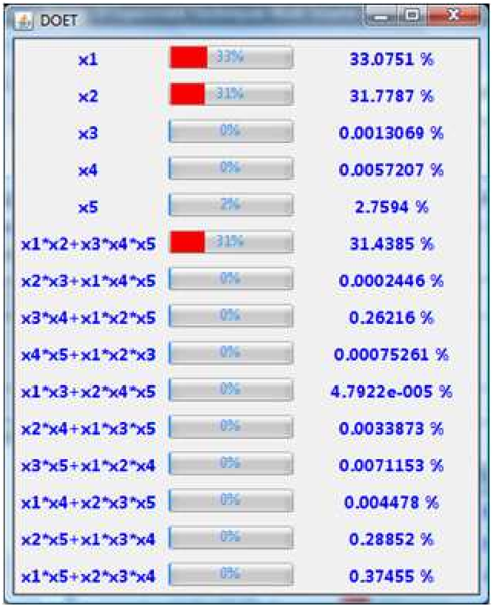


Figure 7. Contributions obtained with a 2^{5-1} fractional factorial design: The contrasts (left side), and the influence on the objective function (right side).

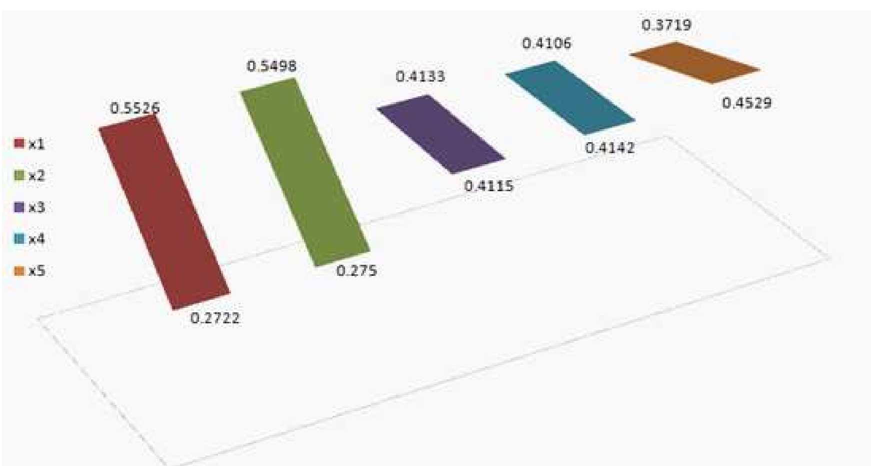


Figure 8. Graph of the main effects.

Table 4. Design matrix of generated by the 2^{5-1} fractional factorial design.

x_1 [mm]	x_2 [mm]	x_3 [mm]	x_4 [mm]	x_5 [mm]	F
0	35	25	15	8	0.6102
0	35	25	2	7	0.4973
0	35	26	15	7	0.4973
0	35	26	2	8	0.6085
0	46	25	15	7	0.4972
0	46	25	2	8	0.6085
0	46	26	15	8	0.6048
0	46	26	2	7	0.4969
15	35	25	15	7	0.4903
15	35	25	2	8	0.6065
15	35	26	15	8	0.5884
15	35	26	2	7	0.5
15	46	25	15	8	−0.0019
15	46	25	2	7	0.0019
15	46	26	15	7	−0.0019
15	46	26	2	8	−0.0019

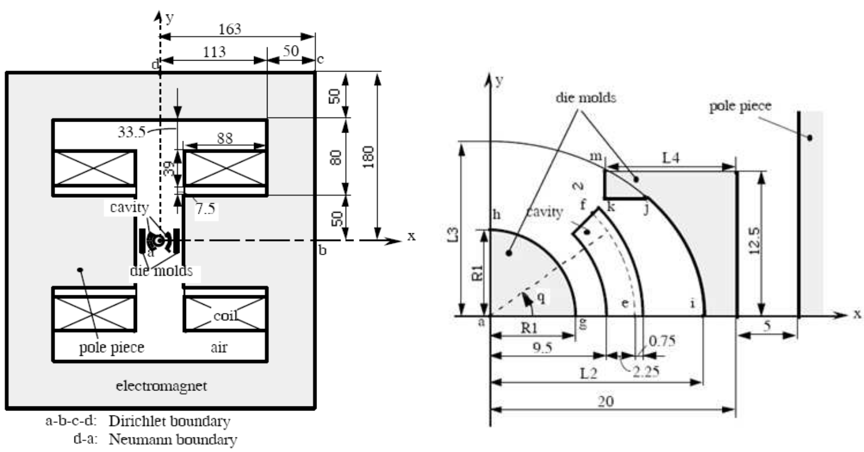
The DOET allows running this problem. The design matrix (values of the parameters used in each experience, as well as the values of the objective function for each configuration), is shown for Design 1 in Table 3 and for design 2 in Table 4.

The contributions of contrasts on the objective function of this case study are given for designs 1 and 2 in Figure 6 and Figure 7, respectively. Each figure illustrates the contrasts and their influence on the objective function. The results of Figures 9 and 10 are similar; this shows the accuracy of the DOE technique.

Figure 11 depicts the main effects of each factor. Based on the results shown in Figure 6, Figure 7 and Figure 8 it can be concluded that among the factors analyzed in this study, factor x_1 and x_2 are the ones that influence most significantly the response variable. Moreover, the interaction x_1x_2 also influences significantly the response variable. It is important to highlight that the number of factors to be considered in this optimization process is significantly reduced from 5 to 2 factors only.

Table 5. Parameters and their constraints.

Parameter	Minimum Value [mm]	Maximum Value [mm]
R_1	5.0	9.4
L_2	12.6	18
L_3	14.0	45.0
L_4	4.0	19,0
A_1	170.0	190.0
A_2	70.0	90.0
A_3	86.0	88.0
A_4	9.5	11.0



(a) Whole view (b) Enlarged view

Figure 9. Model of die press with electromagnet.

4.2. Application to the Problem 25 of Team Workshop

4.2.1. Description

The aim of this problem is to obtain the shape of a die molds used for producing anisotropic permanent magnet by using the optimization method. The model can be assumed as two-dimensional. The die mold is described by an internal circle of radius R_1 and by an external ellipse represented by L_2, L_3 and L_4. It is required to find the values of R_1, L_2, L_3 and L_4 such that a constant radial magnetic induction on ten different points defined on the arc ef, as shown in Figure 9, can be obtained.

To show the efficiency of the DOE Method in the identification of significant parameters, 4 new parameters (A_1, A_2, A_3 and A_4) have been added to the original problem [5]. We can identify these parameters in Figure 9, and their respective constraint in Table 5.

4.2.2. Objective Function and Constraints

The objective function describing this problem W is given by

$$W = \sum_{i-1}^{n} \left\{ (B_{xip} - B_{xio})^2 + (B_{yip} - B_{yio})^2 \right\} \tag{13}$$

where $n = 10$ is the number of specified points. The subscripts p and o refer to the calculated and specified values respectively. B_{xo} and B_{yo} along the line e-f are specified as follows:

$$\begin{cases} B_{xo} = 0.35\cos\theta \ \ (\mathrm{T}) \\ B_{yo} = 0.35\sin\theta \ \ (\mathrm{T}) \end{cases} \tag{14}$$

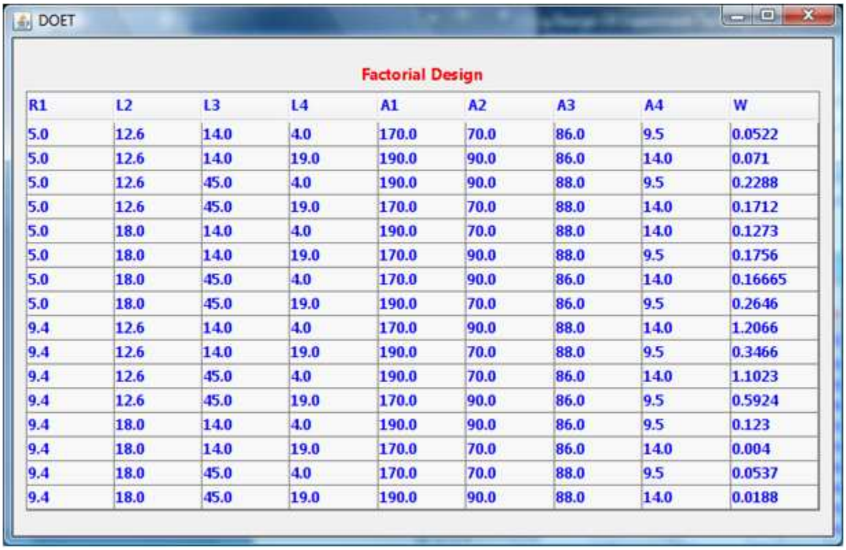
The constraints on R_1, L_2, L_3, L_4, A_1, A_2, A_3 and A_4 are given in Table 5.

4.2.3. Identification of Significant Parameters Using the DOE

The aim of this section is to identify the most influential factors on the objective function using the DOE. As explained earlier, the use of two-level full factorial design needs $2^8 = 256$ runs (simulations) to evaluate the objective function. Using a 2^{8-4} fractional factorial design will reduce effectively the number of runs from 256 to 16. Thus the time is reduced to around 93% of the initial time. The 2^{8-4} design is given using Generators of G. Box as shown in Table 6 The choice of a 2^{8-4} means that we have a 2 levels design with 8 factors where 4 of these factors are generated using the other 4 factors. So we can write that:

Table 6. Generators of G. Box for 8 factors.

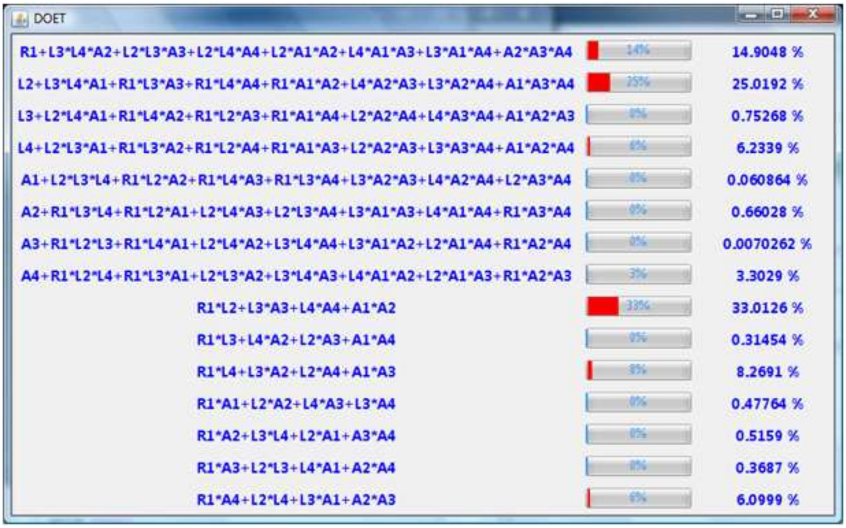
Resolution	Design name	Number of tests	Generators
4	2^{8-4}	16	$5 = \pm 2,3,4$ $6 = \pm 1,3,4$ $7 = \pm 1,2,3$ $8 = \pm 1,2,4$
4	2^{8-3}	32	$6 = \pm 1,2,3$ $7 = \pm 1,2,4$ $8 = \pm 2,3,4,5$
5	2^{8-2}	64	$7 = \pm 1,2,3,4$ $8 = \pm 1,2,5,6$



Factorial Design

R1	L2	L3	L4	A1	A2	A3	A4	W
5.0	12.6	14.0	4.0	170.0	70.0	86.0	9.5	0.0522
5.0	12.6	14.0	19.0	190.0	90.0	86.0	14.0	0.071
5.0	12.6	45.0	4.0	190.0	90.0	88.0	9.5	0.2288
5.0	12.6	45.0	19.0	170.0	70.0	88.0	14.0	0.1712
5.0	18.0	14.0	4.0	190.0	70.0	88.0	14.0	0.1273
5.0	18.0	14.0	19.0	170.0	90.0	88.0	9.5	0.1756
5.0	18.0	45.0	4.0	170.0	90.0	86.0	14.0	0.16665
5.0	18.0	45.0	19.0	190.0	70.0	86.0	9.5	0.2646
9.4	12.6	14.0	4.0	170.0	90.0	88.0	14.0	1.2066
9.4	12.6	14.0	19.0	190.0	70.0	88.0	9.5	0.3466
9.4	12.6	45.0	4.0	190.0	70.0	86.0	14.0	1.1023
9.4	12.6	45.0	19.0	170.0	90.0	86.0	9.5	0.5924
9.4	18.0	14.0	4.0	190.0	90.0	86.0	9.5	0.123
9.4	18.0	14.0	19.0	170.0	70.0	86.0	14.0	0.004
9.4	18.0	45.0	4.0	170.0	70.0	88.0	9.5	0.0537
9.4	18.0	45.0	19.0	190.0	90.0	88.0	14.0	0.0188

Figure 10. Design matrix generated by the 2^{8-4} fractional factorial design.



R1+L3*L4*A2+L2*L3*A3+L2*L4*A4+L2*A1*A2+L4*A1*A3+L3*A1*A4+A2*A3*A4	14.9048 %
L2+L3*L4*A1+R1*L3*A3+R1*L4*A4+R1*A1*A2+L4*A2*A3+L3*A2*A4+A1*A3*A4	25.0192 %
L3+L2*L4*A1+R1*L4*A2+R1*L2*A3+R1*A1*A4+L2*A2*A4+L4*A3*A4+A1*A2*A3	0.75268 %
L4+L2*L3*A1+R1*L3*A2+R1*L2*A4+R1*A1*A3+L2*A2*A3+L3*A3*A4+A1*A2*A4	6.2339 %
A1+L2*L3*L4+R1*L2*A2+R1*L4*A3+R1*L3*A4+L3*A2*A3+L4*A2*A4+L2*A3*A4	0.060864 %
A2+R1*L3*L4+R1*L2*A1+L2*L4*A3+L2*L3*A4+L3*A1*A3+L4*A1*A4+R1*A3*A4	0.66028 %
A3+R1*L2*L3+R1*L4*A1+L2*L4*A2+L3*L4*A4+L3*A1*A2+L2*A1*A4+R1*A2*A4	0.0070262 %
A4+R1*L2*L4+R1*L3*A1+L2*L3*A2+L3*L4*A3+L4*A1*A2+L2*A1*A3+R1*A2*A3	3.3029 %
R1*L2+L3*A3+L4*A4+A1*A2	33.0126 %
R1*L3+L4*A2+L2*A3+A1*A4	0.31454 %
R1*L4+L3*A2+L2*A4+A1*A3	8.2691 %
R1*A1+L2*A2+L4*A3+L3*A4	0.47764 %
R1*A2+L3*L4+L2*A1+A3*A4	0.5159 %
R1*A3+L2*L3+L4*A1+A2*A4	0.3687 %
R1*A4+L2*L4+L3*A1+A2*A3	6.0999 %

Figure 11. Contributions obtained: the contrasts (left side), and the influence on the objective function (right side).

- The factor (5) will be generated using the product of factors (2), (3), and (4).
- The factor (6) will be generated using the product of factors (1), (3), and (4).
- The factor (7) will be generated using the product of factors (1), (2), and (3).
- The factor (8) will be generated using the product of factors (1), (2), and (4).

The design matrix (values of the parameters used in each simulation, as well as the values of the objective function for each configuration), is shown in Figure 10.

The contributions of contrasts obtained are given in Figure 11. As set earlier a contribution is significant if it is higher than 5%. High order interactions (higher than 2) are considered negligible and only interactions of significant parameters are also significant. Thus, factors $R_1(14,9\%)$, $L_2(25\%)$, $L_4(6.2\%)$ and interactions $R_1L_2(33\%)$, $R_1L_4(8.3\%)$ and $L_2L_4(6.1\%)$ are the only significant factors for the objective function value. So, for an optimization process only R_1, L_2 and L_4 are considered. This represents a reduction of 93% of the number of factors and consequently causes a significant time saving.

4.3. Application to a Magnetic Refrigeration Prototype

Magnetic refrigeration (MR) becomes a promising competitive technology to the conventional gas-compression/expansion [14]. MR is a compact, reliable and efficient technology since it does not require compressor (which is the most inefficient part in conventional refrigeration). The MR is based on the magneto-caloric effect (MCE) [14–16]. The MCE was first discovered in iron compound by Warburg 1881 [14]. The MCE is defined as the response of a solid material to an applied magnetic field (generated with permanent magnet for instance), which appears as a change in its temperature. When such solid materials are placed in a magnetic field, their temperature increases and when the materials are removed from the magnetic field, they cool down.

4.3.1. Description

The MR structure studied here is quite similar to a rotating machine as shown in Figure 12. The stator of this machine consists of a cylindrical yoke and two pairs of containers called beds (b_1, b_2, b_3 and b_4) filled with MCE materials (for instance Gadolinium). The beds are placed around a rotating magnet (rotor). The yoke has two major roles: the

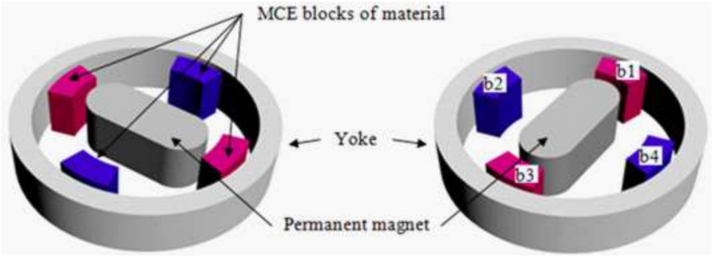


Figure 12. Configuration and operating principle of the PM cooling machine.

first one is to canalize the magnetic flux and the second one is to support the containers. The rotation of the permanent magnet (PM) will generate cycles of varying magnetic field between B_{high} and B_{low}.

An electromagnetic study is undertaken to compute the profiles of the variation of magnetic field, torque and forces. The electromagnetic study is achieved by simulating the system using FEM. Due to symmetrical geometry of the chosen configuration, the investigations were performed in two dimensions. The problem has been solved in magnetostatic formulation. In these studies, the remanent magnetization and the relative permeability of the magnet are fixed respectively to 1.46 T and 1.046 (NdFeB magnets). The stator is described by a constant permeability equals to 1000 and the MCE material by an isotropic bulk gadolinium material with a relative permeability of 5.

4.3.2. Objective Function and Constraints

Since the magneto-caloric effect depends on the magnitude of the magnetic field, the performance of this machine are directly related to the magnetic field range ΔB. Thus, the ΔB should be the first criterion to be taken into account in the design and the optimization process. The objective for this criterion is to maximize the ΔB. The second criterion is the minimization of the magnetic efforts (Forces and Torque). The objective function for the optimization process is given by:

$$F_{\text{objective}} = \{f_1, f_2, f_3\} \tag{15}$$

where

$$f_1 = \max(\Delta B) \tag{16}$$

$$f_2 = \min(\text{Forces}) \tag{17}$$

$$f_3 = \min(\text{Torque}) \tag{18}$$

4.3.3. Identification of Significant Parameters Using the DOE

Since eight parameters define the shape of the machine, it is advisable to determine the effect of each parameter on the objective function. Thus, it is very important to provide proper parameter ranges. The considered parameters (names, definitions, ranges and types) are listed in Table 7. There are two types of parameters: continuous parameters (which can take any value inside the defined range like the length of the machine, the radius of the rotor, etc.) and discrete parameters (which can take only the limits of the defined range like the number of blocks; since a 2 level design is used). Using two-level full factorial design needs $2^8 = 256$ runs (simulations) to evaluate the objective function. Using a 2^{8-4} fractional factorial design will significantly reduce the number of runs from 256 to 16. The 2^{8-4} design is given using Generators of G. Box as shown in Table 8. In this application, the output function is a multi-objective function rather than a single-objective function as in application 1 and 2. The results obtained when the procedure described above is run are given in Table 8 and

Table 7. Considered parameters.

Name	Description	Initial Value	Minimum Value	Maximum Value	Type
L	Length of the machine	100 mm	90 mm	110 mm	Continuous
R_e	External radius of the yoke	88.5 mm	80.5 mm	99.5 mm	Continuous
R_i	Internal radius of the yoke	73.5 mm	73.5 mm	77.5 mm	Continuous
R_r	Radius of the rotor	53.5 mm	47.5 mm	53.5 mm	Continuous
a_r	Angular size of the rotor	60°	45°	60°	Continuous
a_b	Angular size of the magnetocaloric material Blocks	45°	30°	45°	Continuous
w_b	Width of the magnetocaloric material Blocks	17 mm	10 mm	17 mm	Continuous
N_b	Number of the magnetocaloric material Blocks	4	2	6	Discreet

Table 8. Design matrix generated by the 2^{8-4} Box-Wilson fractional factorial design and the simulation results.

N^o	L [mm]	R_e [mm]	R_i [mm]	R_r [mm]	a_r [o]	a_b [o]	w_b [mm]	N_b	ΔB [T]	Torque [N.m]	Forces [N]
1	90	85.5	73.5	47.5	45.3	0	10	2	0.56579	8.3397	-124.95
2	90	85.5	73.5	53.5	60.4	5	10	6	0.81441	4.0532	-532.66
3	90	85.5	77.5	47.5	60.4	5	17	2	0.61438	19.4440	-354.97
4	90	85.5	77.5	53.5	45.3	0	17	6	0.85772	16.5120	-290.60
5	90	99.5	73.5	47.5	60.3	0	17	6	0.85831	6.1528	-266.31
6	90	99.5	73.5	53.5	45.4	5	17	2	0.91797	40.4441	-645.45
7	90	99.5	77.5	47.5	45.4	5	10	6	0.44317	2.0692	-154.36
8	90	99.5	77.5	53.5	60.3	0	10	2	0.77045	16.5523	-253.22
9	110	85.5	73.5	47.5	45.4	5	17	6	0.63619	6.1388	-375.38
10	110	85.5	73.5	53.5	60.3	0	17	2	1.17272	55.1549	-617.04
11	110	85.5	77.5	47.5	60.3	0	10	6	0.55053	3.1761	-163.76
12	110	85.5	77.5	53.5	45.4	5	10	2	0.61404	16.9514	-363.43
13	110	99.5	73.5	47.5	60.4	5	10	2	0.60799	14.7476	-372.99
14	110	99.5	73.5	53.5	45.3	0	10	6	0.80641	14.1823	-293.53
15	110	99.5	77.5	47.5	45.3	0	17	2	0.61751	16.7636	-181.47
16	110	99.5	77.5	53.5	60.4	5	17	6	0.85549	5.8050	-749.81

Table 9. Contrasts and contribution obtained.

Contrasts	ΔB	Significant?	Torque	Significant?	Forces	Significant?
L	0	No	1	No	3	No
Re	0	No	0	No	0	No
Ri	14	Yes	6	Yes	6	Yes
Rr	46	Yes	18	Yes	36	Yes
ar	8	Yes	0	No	9	Yes
ab	6	Yes	2	No	22	Yes
wb	23	Yes	16	Yes	18	Yes
Nb	0	No	35	Yes	0	No
*L*Re+Ri*wb+Rr*Nb+ar*ab*	1	No	5	Yes	0	No
*L*Ri+Rr*ab+Re*wb+ar*Nb*	0	No	4	No	1	No
*L*Rr+Ri*ab+Re*Nb+ar*wb*	0	No	0	No	0	No
*L*ar+Re*ab+Rr*wb+Ri*Nb*	1	No	4	No	3	No
*L*ab+Ri*Rr+Re*ar+wb*Nb*	0	No	9	Yes	0	No
*L*wb+Re*Ri+Rr*ar+ab*Nb*	0	No	1	No	1	No
*L*Nb+Re*Rr+Ri*ar+ab*wb*	1	No	1	No	0	No

Table 9. The significance of each parameter on the output functions are summarized in Table 9.

5. CONCLUSION

Traditional design methodologies — based on physical prototyping of a system — are often seen expensive and time-consuming processes. In order to optimize a design process while reducing design costs, the Design of Experiments (DOE) approach is a widely advisable solution. The use of DOE technique, in conjunction with computer simulation, allows for more efficient analysis of the simulated models. In this paper, the DOE technique was introduced and evaluated. In particular, the effect of significant factors and their interaction with the objective functions of electromagnetic simulations were investigated. The use of this technique was illustrated using three case studies. The results presented in the paper demonstrated that the DOE approach has a great potential to reduce the required simulation time in all cases. In more detail, the number of factors was reduced from 5 to 2 and from 8 to 3 in the first and second cases, respectively. In the third case, the DOE was applied to a multi-objective design and optimization process related to a magnetic refrigeration system. The study showed that the application of DOE can reduce the number of simulations from 256 to 16, thereby reducing the computational cost significantly (by approximately 94%). From the results of this case study, one can analyze the influence of each parameter on each output of the multi-objective function. Since magnetic refrigeration systems involve many design parameters, it was demonstrated that the DOE technique would be very helpful in the design and optimization of such systems.

The approach suggested in this paper is aimed at trying to quantify the influence of parameters on an objective function. This approach is very interesting and it could be a first step into a long design optimization process. Moreover, the effectiveness of the presented approach can be further explored through applying it to a broad range of design applications where the design problem depends on a large set of varying parameters. Areas of interest may include antenna design, electronic circuit design and automatic control design.

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