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INVESTIGATION ON PROPAGATION CHARACTERISTICS OF SUPER-GAUSSIAN BEAM IN HIGHLY NONLOCAL MEDIUM

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Abstract—We investigate the propagation characteristics of super-Gaussian beam in highly nonlocal nonlinear media. The optical beam propagation has been modeled by well known nonlocal nonlinear Schrödinger equation. The variational method is employed to find the initial beam propagation parameters and then split step Fourier method is used for numerical simulations. A generalized exact analytical solution of the model is obtained and critical power of soliton is determined. The evolution of super-Gaussian beam has shown oscillatory propagation.

1. INTRODUCTION

Accessible solitons were first proposed by Snyder and Mitchell [3]. They are basically the optical spatial solitons in nonlocal nonlinear media and are widely researched theoretically and experimentally [4–27, 32, 42–45]. The modulation instability of accessible solitons along with stabilization [5, 19, 23–25, 32] has been studied theoretically. Nonlocality of optical material can be categorized as local, weakly nonlocal, generally nonlocal, and strongly nonlocal. These nonlocality nomenclature of optical materials are based on the relative length of optical beam width and length of response function [5–8, 12–15, 18–21, 23–26, 32]. The nonlocality is called weak nonlocal, if characteristic length of response function is much narrower than the width of the optical beam and when the characteristic length of the nonlinear response is much broader than the width of the optical

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beam, the nonlocality is called strong nonlocal, whereas the general nonlocality is the case between the weak nonlocality and the strong nonlocality [5–8, 12–15, 18–21, 23–26, 32]. The theoretical research has taken two different type of response function profiles, one is a Gaussian-type response function [5] and the other is an exponential-decay response function [41]. Investigations on weakly nonlocal spatial solitons [6, 7], generally nonlocal spatial solitons [26], and strongly nonlocal spatial solitons [8, 12–15, 18, 20, 21] were performed. Extensive investigations on strongly nonlocal media have given many new phenomena such as large phase shift [14], attraction between out-of-phase solitons [3, 9, 30, 41], attraction between dark solitons [29], and long-range interaction between solitons [22], etc., which are different from local solitons. In strongly nonlocal media, the beam always evolves periodically during propagation [38].

The experimental confirmation of single nonlocal spatial soliton [4, 17], interaction of such a soliton pair [9, 22], and modulation instability [10] have been demonstrated. Nematic liquid crystals (NLC) were found to be an excellent nonlocal media which supports accessible solitons [11, 16]. These solitons are named as Nematicons. Two-color anisotropic nonlocal vector solitons were observed in unbiased nematic liquid crystals [27].

Snyder and Mitchell [3] simplified the nonlocal nonlinear Schrödinger equation NNLSE to a linear model named the Snyder-Mitchell model in the strongly nonlocal case, and they found an exact Gaussian-shaped stationary solution (called accessible soliton). In highly nonlocal media various type of beam shapes have been investigated, namely Hermite-Gaussian (HG) form [33], cosh-Gaussian form [40], Gaussian forms [3, 14], Laguerre-Gaussian and Hermite-Gaussian forms [32, 33], Ince-Gaussian form [34, 35], complex-variable-function Gaussian form [37]. To the best of our knowledge, no one has studied super-Gaussian (SG) shape yet. Our motivation for using SG beam is mainly due to two reasons. First, SG beams are narrow on tail which makes it easy to accommodate more numbers of pulses in less space and it has flat top which make them easy to be identified by detector. Second, the SG beams contain less energy as compare with Gaussian beam of same beam width [2].

This article is organized as follows. Mathematical formulation of NNLSE and implementation of variational method are presented in Section 2. The results of numerical investigation have been presented in Section 3. Finally, our conclusions are given in Section 4.

2. MATHEMATICAL MODEL

The propagation of a optical beam through nonlocal nonlinear medium is modeled by NNLSE of the form

$$i\frac{\partial\psi}{\partial z}+\mu\frac{\partial^2\psi}{\partial x^2}+\rho\psi\int_{-\infty}^{\infty}R(x-x')I(x',z)dx'=0 \tag{1}$$

where $\psi(x,z)$ is a slowly varying envelop, $\mu = 1/2k$, $\rho = k\eta$, k is the wave number, η is the material constant, z is the longitudinal propagation distance, $R(x)$ is the nonlocal response function, and $I(x',z)$ is the intensity of the paraxial beam. In present investigation, we consider a quasi-monochromatic partially incoherent beam propagating in isotropic nonlocal kerr media possessing a Gaussian nonlocal response kernel $R(x)$ such that $\int R(x)dx = 1$, i.e.,

$$R(x)=\frac{1}{\sqrt{\pi}\sigma}\exp\left(-\frac{x^2}{\sigma^2}\right), \tag{2}$$

where σ is the extent of the nonlocality or the length of response function. In strongly nonlocal media, the width of $R(x)$ is much larger than the beam width, hence expansion of response function to the second order by Taylor series gives a linear model of optical beam propagation into highly nonlocal medium, which is suggested by Snyder and Mitchell [3, 14, 28],

$$i\frac{\partial\psi}{\partial z}+\mu\frac{\partial^2\psi}{\partial x^2}+\rho\psi\frac{P_0}{\sqrt{\pi}\sigma}\left(1-\frac{x^2}{\sigma^2}\right)=0. \tag{3}$$

The NNLSE (3) is a nonlinear differential equation which do not have any direct solution, hence we proceed to solve it by an approximation technique, called variational analysis [1], which has been used successfully by various authors [14, 31, 36, 39, 46] for solving nonlinear equations. The required Lagrangian density for Eq. (3) is given as

$$L=\frac{i}{2}\left(\psi^*\frac{\partial\psi}{\partial z}-\psi\frac{\partial\psi^*}{\partial z}\right)-\mu\left|\frac{\partial\psi}{\partial x}\right|^2+\eta\left|\psi\right|^2\left(1-\frac{x^2}{\sigma^2}\right) \tag{4}$$

where $\eta=\rho P_0/2\sqrt{\pi}\sigma$.

The success of variational method lies in proper choice of trial function. In this article we have chosen a SG ansatz,

$$\psi\left(x,z\right)=A(z)\exp\left(-\frac{1}{2}\left(\frac{x}{w(z)}\right)^{2n}+ic(z)x^2+i\theta(z)\right) \tag{5}$$

where $A(z)$ is amplitude, $w(z)$ is width, $c(z)$ is phase front curvature, and $\theta(z)$ is phase of the beam. Here the parameter n controls the

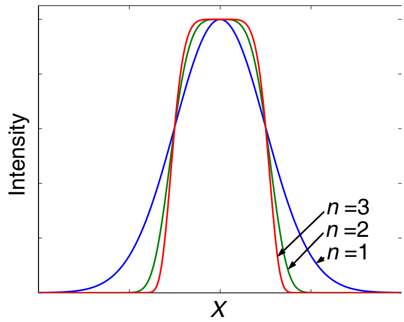

Figure 1. Profile of the super-Gaussian beam for various values of parameter n.

degree of edge sharpness. We plot in Eq. (5) the intensity profile of beam for $n = 1, 2$ and 3 in Fig. 1. For $n = 1$ this function reduced to Gaussian function and for $n > 1$ it is called SG function. It can be clearly seen that as value of n increases the beam's edge becomes steeper and its top becomes flatter. This type of beam output can be obtained from directly modulated lasers [2].

We proceed further by inserting ansatz function Eq. (5) into Eq. (4) to obtain average Lagrangian $\langle L\rangle$ as,

$$\begin{aligned}\langle L\rangle &= \int_{-\infty}^{\infty} L dx \\ &= A^2\left(\frac{w}{n}\left(\frac{c}{z}w^2\Gamma\left(\frac{3}{2n}\right)+\frac{\theta}{z}\Gamma\left(\frac{1}{2n}\right)\right)\right. \\ &\quad -\frac{\mu}{wn}\left(n^2\Gamma\left(\frac{4n-1}{2n}\right)+4w^4c^2\Gamma\left(\frac{3}{2n}\right)\right) \\ &\quad \left.-\frac{\eta w}{\sigma^2 n}\left(-\sigma^2\Gamma\left(\frac{1}{2n}\right)+w^2\Gamma\left(\frac{3}{2n}\right)\right)\right). \qquad (6)\end{aligned}$$

The average Lagrangian results in a set of dynamical equations corresponding to different free pulse parameters after applying the Euler-Lagrange equation

$$\frac{\partial\langle L\rangle}{\partial r_j} - \frac{d}{dz}\left(\frac{\partial\langle L\rangle}{\partial \dot{r}_j}\right), \qquad (7)$$

where $r_j = A(z)$, $c(z)$, $w(z)$, and $\theta(z)$. Now we have substituted Eq. (6) into Eq. (7) to solve it for pulse parameters, which results in several first order ordinary differential equations showing the variation of free pulse parameter along the propagation distance. These equations are shown as,

$$\frac{\partial c}{\partial z} = -\frac{1}{4}\frac{\mu\left(2n-1\right)\Gamma\left(-\frac{1}{2n}\right)}{w^4\Gamma\left(\frac{3}{2n}\right)} - 4c^2\mu - \frac{\eta}{\sigma^2}, \tag{8}$$

$$\frac{\partial \theta}{\partial z} = \frac{1}{2}\frac{\mu\left(2n-1\right)\Gamma\left(-\frac{1}{2n}\right)}{w^2\Gamma\left(\frac{1}{2n}\right)} + \eta, \tag{9}$$

$$\frac{\partial w}{\partial z} = 4\mu c w, \tag{10}$$

where Γ is gamma function. The amplitude $A(z)$ and width $w(z)$ of the beam are related to $\Gamma(\frac{1}{2n})\frac{wA^2}{n} = P_0$, where P_0 is initial beam power.

We differentiate Eq. (10) with respect to z with normalization $y(z) = w/w_0$, where $w_0 = w(0)$, to yield

$$\frac{1}{\mu}\frac{\partial^2 y}{\partial z^2} = \frac{(1-2n)\,\Gamma\left(-\frac{1}{2n}\right)\mu}{{w_0}^3 y^3\Gamma\left(\frac{3}{2n}\right)} - \frac{2w_0\rho P_0 y}{\sqrt{\pi}\sigma^3} \equiv F(y). \tag{11}$$

The physical interpretation of Eq. (11) was discussed in Ref. [14] in details, which is analogous to the Newton's second law of motion in classical mechanics under the force $F(y)$. This force is a balance between diffractive and refractive forces represented by the first and the second terms of the equation, respectively. If both forces are equal and $y = 1$, we can obtain the critical power for soliton propagation as

$$P_c = \frac{(1-2n)\sqrt{\pi}\Gamma\left(-\frac{1}{2n}\right)\mu\sigma^3}{2\Gamma\left(\frac{3}{2n}\right)w_0^4\rho}. \tag{12}$$

As the $F(y)$ is a conservative force, i.e., $F(y) = -dV(y)/dy$, the equivalent potential can be written as,

$$V\left(y\right) = \frac{2k\left(y^2-1\right)\left(y^2-\alpha\right)}{\mu {w_0}^2 y^2}, \tag{13}$$

where $k = \mu\, w_0^3 \rho\, P_0/2\sqrt{\pi}\sigma^3$ and $\alpha = P_c/P_0$. Integration of Eq. (11) with assumption $dw(z)/dz|_{z=0} = 0$ gives the first order differential equation in the form of

$$\frac{1}{2}\left(\frac{\partial y}{\partial z}\right)^2 = \frac{2k\left(y^2-1\right)\left(y^2-\alpha\right)}{{w_0}^2 y^2}, \tag{14}$$

resulting in the solution for the width as

$$w^2 = w_0^2\big(\cos^2(\beta z) + \alpha\sin^2(\beta z)\big), \tag{15}$$

where $\beta = 2\sqrt{k}/w_0$. Substitution of Eq. (15) into Eq. (10) and Eq. (9) gives respectively,

$$c = \frac{(\alpha-1)\,\beta\cos\left(\beta z\right)\sin\left(\beta z\right)}{4\mu\left(\cos\left(\beta z\right)^2 + \alpha\sin\left(\beta z\right)^2\right)}. \tag{16}$$

and

$$\theta = \frac{(2n-1)\,\Gamma\left(-1/2n\right)\mu \arctan\left(\sqrt{\alpha}\tan\left(\beta z\right)\right)}{2w_0^2\Gamma\left(1/2n\right)\beta\sqrt{\alpha}} + \eta z. \tag{17}$$

The exact solution of Eq. (3) can be written by combining Eqs. (15)–(17) with Eq. (5) as,

$$\psi\left(x,z\right) = \sqrt{\frac{nP_0}{w}\Gamma\left(\frac{1}{2n}\right)} \exp\left[-\frac{1}{2}\left(\frac{x}{w(z)}\right)^{2n} + ic(z)x^2 + i\theta(z)\right], \tag{18}$$

where the values of $w(z)$, $c(z)$ and $\theta(z)$ are given by Eqs. (15), (16) and (17), respectively. For $n = 1$, Eq. (18) will become,

$$\psi\left(x,z\right) = \sqrt{\frac{P_0}{\sqrt{\pi}w}} \exp\left[-\frac{1}{2}\left(\frac{x}{w(z)}\right)^{2} + ic(z)x^2 + i\theta(z)\right] \tag{19}$$

which is same as Eq. (13) of Ref. [14].

3. RESULT AND DISCUSSION

The shapes of potential (13) in nonlocal nonlinear media for various initial beam powers are shown in Fig. 2. It is notable from Fig. 2 that the shape of potential corresponding to parameter $n = 1$ is parabolic, which is similar to the case of Snyder and Mitchell's model [3], but for the cases of $n = 2$ and 3 the parabola becomes to be more contracted and the balance point becomes to be more narrower. When incident beam power P_0 equal to critical power P_c of soliton, the particle located at the balance point w_b of potential V and $F = 0$, i.e., both the diffractive and the refractive forces are in balance to each other, in this case, a soliton propagation is stable. On the other hand, when $P_0 < P_c$, the diffraction force is greater then the refraction force, hence the beam width w first start increasing until it reaches to maximum $w_{\max}$, after which it decreases to $w_{\min}$ so that the soliton propagation is unstable since the beam width oscillate periodically. Finally, when $P_0 > P_c$, the diffraction force is less than refractive force, which makes beam width w decrease initially but increase subsequently so that the soliton propagation is unstable with periodic beam width oscillation.

We simulated Eq. (3) to investigate the behavior of the Gaussian and the SG beam in highly nonlocal media for which the typical parameters are fixed as $\sigma = 10$, $w_0 = 1$, $\mu = 1/2$, and $\rho = 1/6$. The evolution of optical beam peak amplitude of Gaussian and SG beam in highly nonlocal media has been shown in Fig. 3 for various input beam powers. The first column is for $P_0 < P_c$, the second column is for $P_0 = P_c$ and the third column is for $P_0 > P_c$. The first row

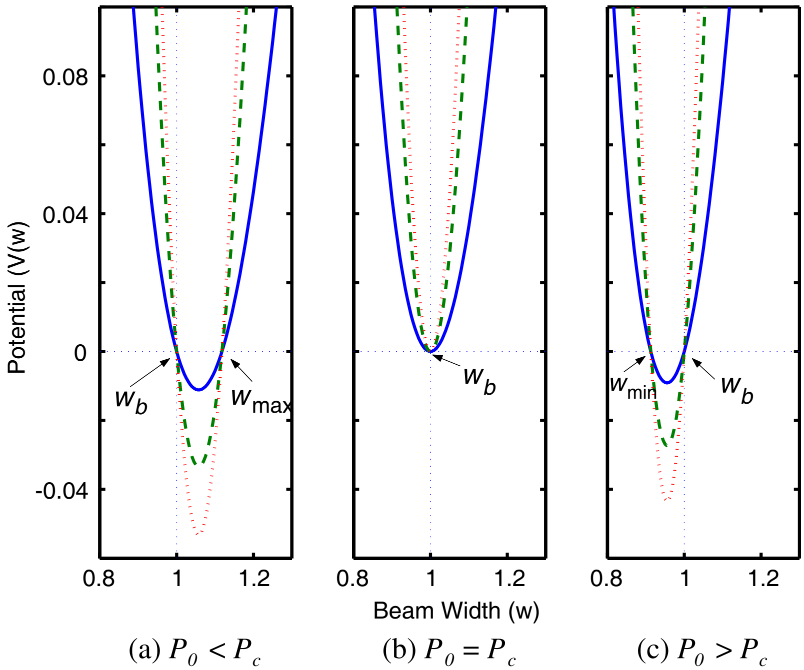

Figure 2. Profile of potential $V(w)$ in nonlocal nonlinear medium for different values of Super-Gaussian beam parameter n. (a) $P_0 < P_c$, (b) $P_0 = P_c$, and (c) $P_0 > P_c$. Solid line is for $n = 1$, dashed line is for $n = 2$, and dotted line is for $n = 3$. The other parameters are $\sigma = 10$, $\mu = 1/2$, $\rho = 1/6$, and $w_0 = 1$.

is for $n = 1$, the second row is for $n = 2$ and the third row is for $n = 3$. The beam peak amplitude variation of Gaussian beam has clearly shown when initial beam power is equal to critical power of soliton, i.e., $P_0 = P_c$ optical beam propagate without changing it's shape (Fig. 3(b)) but when initial beam power is less than to critical power, i.e., $P_0 < P_c$, the beam amplitude decreases initially and then increases (Fig. 3(a)) as in this case diffractive force is greater then the refractive force. Whereas in the case when initial beam power is greater than to critical power, i.e., $P_0 > P_c$, the beam amplitude increases initially and then decreases (Fig. 3(c)). Dynamics of beam peak amplitude for SG beam has been depicted in Figs. 3(d) to 3(i), which has shown non stationary propagation of SG beam for all the cases. In our opinion Gaussian shape behaves like a attractor for other shape, hence when the SG beam is allowed to propagate in media it tries to

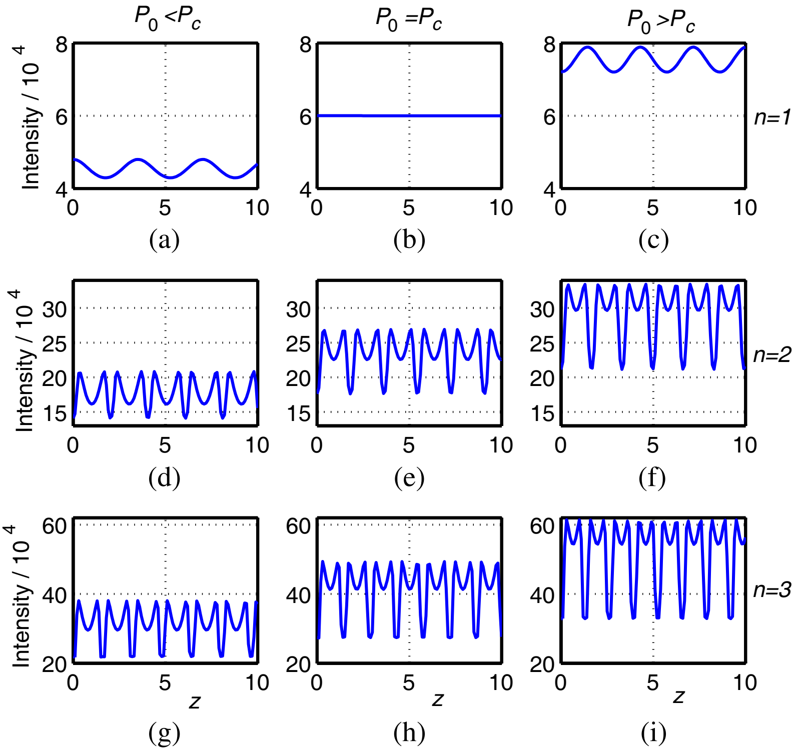

Figure 3. Variation of beam peak vs propagation distance z. The first column is for $P_0 < P_c$, the second column is for $P_0 = P_c$ and the third column is for $P_0 > P_c$. The first row is for $n = 1$, the second row is for $n = 2$ and the third row is for $n = 3$. The other parameters are $\sigma = 10$, $\mu = 1/2$, $\rho = 1/6$, and $w_0 = 1$.

reshape into the Gaussian shape and oscillates between the Gaussian and the SG shapes. Consequently, even when $P_0 = P_c$, no stable propagation is noticed for the SG shape. Three dimensional mesh plots corresponding to Figs. 3(b) and 3(e) have been shown in Figs. 4(a) and 4(b) respectively. It is clear from Fig. 4 that when the Gaussian beam is allowed to propagate through nonlocal nonlinear medium, it propagate without changing the shape while the SG beam has shown oscillatory propagation behavior in highly nonlocal nonlinear media. The SG beam has undergone a periodic variation in shape from SG to Gaussian and again back to SG shape in a periodic manner but non-sinusoidal.

Soliton Interaction: The interaction properties of accessible

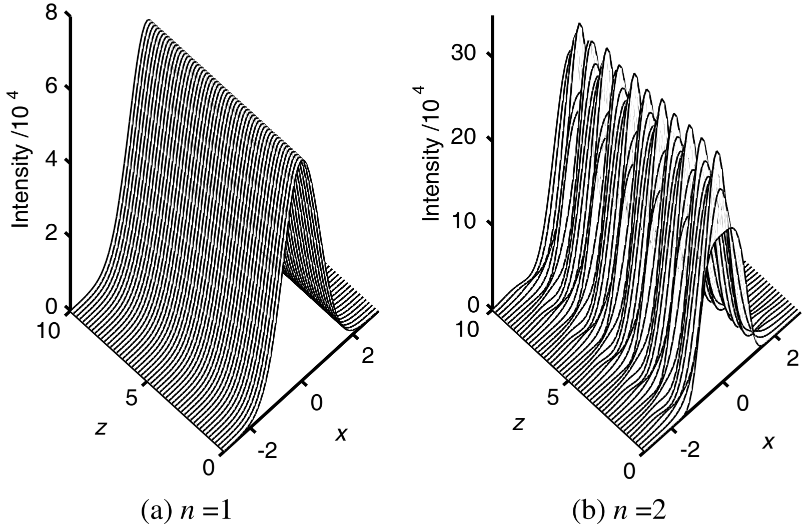

Figure 4. Evolution of the Gaussian and the SG beams for $P_c = P_0$. The parameters are $\sigma = 10$, $\mu = 1/2$, $\rho = 1/6$, and $w_0 = 1$.

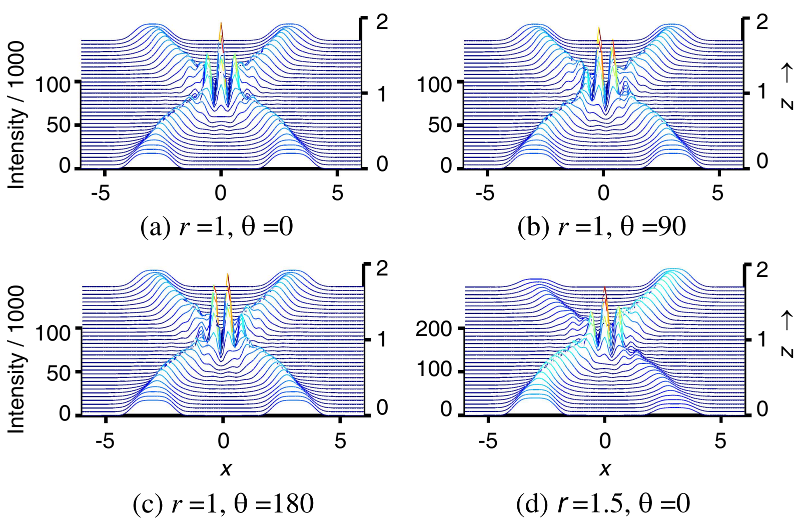

Figure 5. Collision of the two SG beams ($n = 2$). (a) $r = 1$, $\theta = 0°$, (b) $r = 1$, $\theta = 90°$, (c) $r = 1$, $\theta = 180°$, and (d) $r = 1.5$, $\theta = 0°$.

solitons are well known [3, 9, 30, 41]. We further investigated the interaction of SG beams in highly nonlocal nonlinear media by taking co-propagating two beams with initial peak separation of $2d$, which

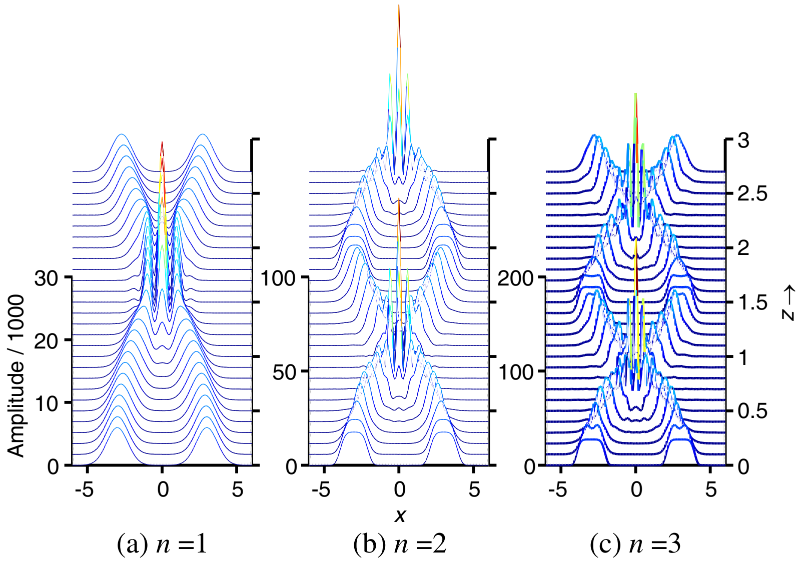

Figure 6. Collision of the two beams. (a) Gaussian beam, (b) super-Gaussian beam ($n = 2$), and (c) super-Gaussian beam ($n = 3$).

can be modeled by equation,

$$\psi(x,z) = \sqrt{\frac{nP_0}{w_0}\Gamma\left(\frac{1}{2n}\right)}\left[\exp\left(-\frac{1}{2}\left(\frac{x-d}{w_0}\right)^{2n}\right) + r\exp(i\theta)\exp\left(-\frac{1}{2}\left(\frac{x+d}{w_0}\right)\right)^{2n}\right] \tag{20}$$

where r is relative amplitude and θ is relative phase difference of both beams. The interaction dynamics of the Gaussian and the SG beams in the nonlocal media are demonstrated in Fig. 6. It is clear from Fig. 6 that SG beams collides earlier than Gaussian beam, which may be due to oscillatory propagation of SG beams. We have also investigated the effect of relative amplitude and phase on interaction of SG beams ($n = 2$), the result is shown in Fig. 5. The figure correlate the results of Ref. [41] that these solitons are always in attractive nature and their relative phase difference have no effect on collision dynamics.

4. CONCLUSION

In this article, we have investigated the propagation of SG beam into a highly nonlocal nonlinear media. The propagation of optical beam is modeled by nonlocal nonlinear Schrödinger equation (NNLSE). The variational method is used to obtain generalized exact analytical solution of the model, which is valid from Gaussian beam ($n = 1$) to flat top beam (higher order SG beam). This result will be much useful for future researchers as the dynamical equations are in generalized form. Further discussion on dynamic of SG beam and its possible applications can be found in Ref. [47]. The numerical simulation has shown stable propagation for Gaussian beam where as propagation of SG beam has been found oscillatory. We conclude, while designing laser for highly nonlocal media, care must be taken, as deviation from Gaussian shape will cause unstable behavior in beam propagation. These results can be applied into fields of Photorefractive Crystals, Nematic liquid crystal and Bose-Einstein condensates and found potential applications in optical switching and all optical devices.

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