

## A SECOND-ORDER CONE PROGRAMMING APPROACH FOR ROBUST DOWNLINK BEAMFORMING WITH POWER CONTROL IN COGNITIVE RADIO NETWORKS

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**Abstract**—The downlink beamforming technology plays a key role in a cognitive radio network (CR-Net). It can be used to reduce transmission power and interference to other users, etc. This paper presents a robust downlink beamforming method with power control for a multiuser multiple-input-single-output (MISO) CR-Net. In this proposed approach, the beamforming optimization problem is formulated as the second-order cone programming (SOCP). The presented method can not only minimize the transmitted power but also guarantee that the received signal-to-interference-plus-noise ratio (SINR) is strictly above the prescribed quality-of-service (QoS)-constrained threshold at each secondary user (SU) and the the interference power (IP) is strictly below the prescribed threshold at the primary user (PU). Simulation results are presented to verify the efficiency of the proposed method.

### 1. INTRODUCTION

Fixed spectrum allocation is the predominant spectrum allocation methodology for traditional wireless communications. Specifically, in order to avoid interference, different wireless communication systems are allocated to different licensed bands. With the popularity of various wireless technologies, fixed spectrum allocation strategy has resulted in

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a scarcity of radio spectrum, due to the fact that most of the available spectrum has been assigned. Therefore, one of the fundamental challenges faced by the wireless communication industry is how to meet rapidly growing demands for wireless services and applications with limited radio spectrum. Cognitive radio (CR) technology has been proposed as a promising solution to tackle such a challenge [1–3]. The idea of CR is initially introduced by Mitolo and Maguire [1]. CR is an intelligent wireless communication system that is aware of its surrounding environment, learns from the environment and adapts its internal states to statistical variations in the incoming radio frequency (RF) stimuli by making corresponding changes in certain operating parameters in real time. The cognitive radio approach can be extended to cognitive networks [3]. A cognitive radio network (CR-Net) is an intelligent multiuser wireless communication system that perceives the radio-scene, adapts to variations in the environment, facilitates communication between users by cooperation, and controls the communication through proper allocation of resources. In a CR-Net, the secondary users (SUs) are allowed to operate within the service range of the primary users (PUs), although the PUs have higher priority in utilizing the spectrum. In other words, the key feature of the CR-Net is to allow a cognitive secondary user to simultaneously share a licensed spectrum as long as the secondary transmission does not interfere with the primary link. As a result, two major challenges are imposed on the CR-Net: (1) to maximize its own transmit throughput, and (2) to ensure the quality-of-service (QoS) of the primary links [4].

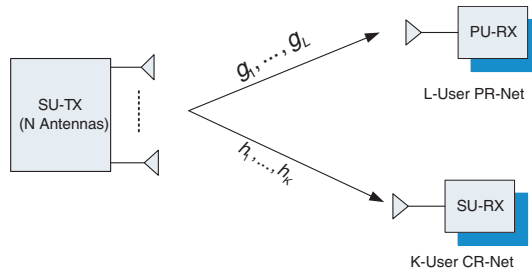
Recently, joint beamforming and power control has been widely studied for a CR-Net [5–7]. However, most works on joint beamforming and power control are studied under the assumption of perfect channel knowledge. The perfect channel knowledge could be difficult to obtain due to limited feedback, lack of cooperation among the nodes, or when the channel reciprocity does not hold. Several robust algorithms [8–10] are presented by making use of the channel uncertainty models which are summarized in [11]. A robust downlink beamforming design [8] is considered for a MISO CR-Net. It is assumed that the channel-state information (CSI) for all relevant channels is imperfectly known, and the imperfectness of the CSI is modelled using a Euclidean ball-shaped uncertainty set. The beamforming optimization problem is translated to a relaxed semidefinite programming. The ellipsoid model is used to describe uncertainty channel, and worst case equivalent channel expressions are derived by use of Lagrangian function and dual method [9]. With the derivation, the uncertainty optimization problem is transformed into a certainty problem. The worst-case robust beamforming with rank relaxation [10] is addressed for a

downlink multiuser multi-antenna CR system coexisted with a number of PUs.

In this paper, we consider the robust downlink beamforming with power control problem for a CR-Net with multiple SU-Rxs coexisting with multiple PU-Rxs, whose relevant CSI is imperfectly known. The design objective is to minimize the transmit power of the SU-Tx while simultaneously targeting a lower bound on the received signal-to-interference-plus-noise ratio (SINR) for the SUs and imposing an upper limit on the interference power (IP) at the PUs. This paper is organized as follows. Section 2 briefly introduces the system model. The proposed method based on SOCP is addressed in Section 3. In Section 4, simulation results are presented to verify the performance of the proposed approach. Section 5 concludes the paper.

## 2. SYSTEM MODEL

Consider the downlink scenario of a multiuser MISO CR-Net coexisting with a primary radio network (PR-Net) having  $L$  PUs each equipped with a single antenna. The SU-Tx, which is equipped with  $N$  antennas, transmits independent symbols  $s_k$  to  $K$  different single-antenna SU-RXs, as shown in Fig. 1. The channel from SU-Tx to each SU-RX and PU-RX are determined using the complex-valued vector  $\{\mathbf{h}_k \in \mathbb{C}^{N \times 1}\}_{k=1}^K$  and  $\{\mathbf{g}_l \in \mathbb{C}^{N \times 1}\}_{l=1}^L$ , respectively.



**Figure 1.** MISO CR-Net system.

In beamforming design, the transmit signal is multiplied by the beamformers, which lead to the following form

$$\mathbf{x} = \sum_{i=1}^K \mu_i \mathbf{w}_i = \mathbf{W}\mathbf{s} \tag{1}$$

where  $\mathbf{s} = [\mu_1, \dots, \mu_K]^T \in \mathbb{C}^{K \times 1}$  contains the transmitted symbols. The precoding matrix  $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{N \times K}$  with  $\mathbf{w}_i$  denoting

the beamforming vector for use  $i$ . In the following part of this paper, we assume that  $E\{|\mu_i|\} = 1$  for simplicity of analysis.

For the system depicted in Fig. 1, the total transmitted power TxP is given by

$$\text{TxP} \triangleq E\{\|\mathbf{x}\|^2\} = \sum_{k=1}^K \|\mathbf{w}_k\|^2 = \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k. \quad (2)$$

where  $\|\cdot\|$  denotes the Euclidean norm.

The received signal at the  $k$ th SU-Rx is expressed as

$$y_k = \mathbf{h}_k^H \mathbf{w}_k \mu_k + \sum_{i=1, i \neq k}^K \mathbf{h}_k^H \mathbf{w}_i \mu_i + n_k \quad (3)$$

where  $(\cdot)^H$  stands for the Hermitian operation. The right-hand side of (3) has three terms. The first term is the received signal of the intended message, whereas the second and third terms show the interference from other messages and noise. Assume that the noise  $n_k$  is white and Gaussian, i.e.,  $n_k \sim \mathcal{CN}(0, \sigma_n^2)$ . The SINR at the  $k$ th SU-Rx  $\text{SINR}_k$  is given by

$$\text{SINR}_k \triangleq \frac{|\mathbf{w}_k^H \mathbf{h}_k|^2}{\sigma_n^2 + \sum_{i=1, i \neq k}^K |\mathbf{w}_i^H \mathbf{h}_k|^2} \quad \forall k \in [1, \dots, K]. \quad (4)$$

Similarly, the received signal at the  $l$ th PU can be written as

$$z_l = \sum_{k=1}^K \mathbf{g}_l \mathbf{w}_k \mu_k + \nu_l \quad (5)$$

where  $\nu_l$  is the received noise. The IP $_l$  to this PU-Rx is defined as

$$\text{IP}_l \triangleq \sum_{k=1}^K |\mathbf{w}_k^H \mathbf{g}_l|^2 = \sum_{k=1}^K \mathbf{w}_k^H \mathbf{g}_l \mathbf{g}_l^H \mathbf{w}_k \quad \forall l \in [1, \dots, L]. \quad (6)$$

For realistic reasons, i.e., to maximize the expiration of the battery, we must minimize the transmitted power TxP while guaranteeing that the SINR at each SU-Rx for all the channel realizations is higher than the QoS-constrained threshold  $\{\text{SINR}_k \geq$

$\gamma_k\}_{k=1}^K$ , and simultaneously, the IP at each PU-Rx is less than PR-Net-imposed threshold  $\{IP_l \leq \xi_l\}_{l=1}^L$ , respectively. Mathematically, this problem can be described as

$$\begin{aligned} & \min \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k \\ & \text{s.t.} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{g}_l \mathbf{g}_l^H \mathbf{w}_i \leq \xi_l \quad (l = 1, \dots, L) \\ & \frac{|\mathbf{w}_k^H \mathbf{h}_k|^2}{\sigma_n^2 + \sum_{i=1, i \neq k}^K |\mathbf{w}_i^H \mathbf{h}_k|^2} \geq \gamma_k \quad (k = 1, \dots, K). \end{aligned} \quad (7)$$

### 3. ALGORITHM FORMULATION

#### 3.1. Channel Uncertainty Model

In practice, the CSI available to the SU transmitter is destined to be imperfect, due to some kind of uncertainty such as estimation errors etc. This uncertainty is described using an uncertainty set  $\mathcal{H}_k$ , which is defined as a Euclidean ball as

$$\mathcal{H}_k = \left\{ \mathbf{h} \mid \left\| \mathbf{h} - \tilde{\mathbf{h}}_k \right\| \leq \delta_k \right\} \quad (8)$$

In the above definition, the ball is centered around the actual value of the channel vector  $\tilde{\mathbf{h}}_k$ , and the radius of the ball is determined by  $\delta_k$ , which is a positive constant. Using this notion, the channel is modelled as

$$\mathbf{h}_k = \tilde{\mathbf{h}}_k + \mathbf{a}_k \quad (9)$$

where  $\mathbf{a}_k$  is a norm-bounded uncertainty vector, namely,  $\|\mathbf{a}_k\| \leq \delta_k$ .

Similarly, the uncertainty of the channel from SU-Tx to PU-Rx is defined using a set  $\mathcal{G}_l$ , which is

$$\mathcal{G}_l = \left\{ \mathbf{g} \mid \left\| \mathbf{g} - \tilde{\mathbf{g}}_l \right\| \leq \eta_l \right\} \quad (10)$$

Equivalently, we may write

$$\mathbf{g}_l = \tilde{\mathbf{g}}_l + \mathbf{b}_l \quad (11)$$

where  $\tilde{\mathbf{g}}_l$  is the actual value of the channel and  $\mathbf{b}_l$  is a norm-bounded uncertain vector, namely,  $\|\mathbf{b}_l\| \leq \eta_l$ .

From (9), we have the following equation

$$\begin{aligned} \left| \mathbf{w}_k^H \mathbf{h}_k \right|^2 &= \mathbf{w}_k^H (\tilde{\mathbf{h}}_k + \mathbf{a}_k) (\tilde{\mathbf{h}}_k + \mathbf{a}_k)^H \mathbf{w}_k \\ &= \mathbf{w}_k^H (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{w}_k \end{aligned} \quad (12)$$

where  $\tilde{\mathbf{H}}_k = \tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^H$  is the constant covariance matrix of the nominal CSI, and  $\Delta_k$  is given by

$$\Delta_k = \tilde{\mathbf{h}}_k \mathbf{a}_k^H + \mathbf{a}_k \tilde{\mathbf{h}}_k^H + \mathbf{a}_k \mathbf{a}_k^H \quad (13)$$

Note that  $\Delta_k$  is norm-bounded matrix,  $\|\Delta_k\| \leq \varepsilon_k$ . It is straightforward to find the following relation

$$\begin{aligned} \varepsilon_k &\geq \|\Delta_k\| = \left\| \tilde{\mathbf{h}}_k \mathbf{a}_k^H + \mathbf{a}_k \tilde{\mathbf{h}}_k^H + \mathbf{a}_k \mathbf{a}_k^H \right\| \\ &\leq \left\| \tilde{\mathbf{h}}_k \right\| \|\mathbf{a}_k\| + \|\mathbf{a}_k\| \left\| \tilde{\mathbf{h}}_k \right\| + \|\mathbf{a}_k\|^2 = \delta_k^2 + 2\delta_k \left\| \tilde{\mathbf{h}}_k \right\|. \end{aligned} \quad (14)$$

Therefore, it is possible to choose  $\varepsilon_k = \delta_k^2 + 2\delta_k \|\tilde{\mathbf{h}}_k\|$ .

Similarly, we can get the following equation

$$\begin{aligned} \mathbf{w}_k^H \mathbf{g}_l \mathbf{g}_l^H \mathbf{w}_k &= \mathbf{w}_k^H (\tilde{\mathbf{g}}_l + \mathbf{b}_l) (\tilde{\mathbf{g}}_l + \mathbf{b}_l)^H \mathbf{w}_k \\ &= \mathbf{w}_k^H (\tilde{\mathbf{G}}_l + \Lambda_l) \mathbf{w}_k \end{aligned} \quad (15)$$

where  $\tilde{\mathbf{G}}_l$  is the constant matrix,  $\tilde{\mathbf{G}}_l = \tilde{\mathbf{g}}_l \tilde{\mathbf{g}}_l^H$  and  $\Lambda_l$  is the norm-bounded uncertainty matrix  $\|\Lambda_l\| \leq \zeta_l$ . Similarly, we know that  $\zeta_l = \eta_l^2 + 2\eta_l \|\tilde{\mathbf{g}}_l\|$ .

Adopting the preceding notations, the problem (7) with bounded channel uncertainties can be rewritten as

$$\begin{aligned} &\min \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k \\ &\text{s.t.} \sum_{i=1}^K \mathbf{w}_i^H (\tilde{\mathbf{G}}_l + \Lambda_l) \mathbf{w}_i \leq \xi_l \quad (l = 1, \dots, L) \\ &\frac{\mathbf{w}_k^H (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1, i \neq k}^K \mathbf{w}_i^H (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{w}_i} \geq \gamma_k \quad (k = 1, \dots, K). \end{aligned} \quad (16)$$

### 3.2. Worst-case Constraints

The worst-case beamformer for the problem(16) can be rewritten as

$$\begin{aligned} &\min \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k \\ &\text{s.t.} \max_{\|\Lambda_l\| \leq \zeta_l} \sum_{i=1}^K \mathbf{w}_i^H (\tilde{\mathbf{G}}_l + \Lambda_l) \mathbf{w}_i \leq \xi_l \quad (l = 1, \dots, L) \\ &\min_{\|\Delta_k\| \leq \varepsilon_k} \frac{\mathbf{w}_k^H (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1, i \neq k}^K \mathbf{w}_i^H (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{w}_i} \geq \gamma_k \quad (k = 1, \dots, K). \end{aligned} \quad (17)$$

To solve the above problem, the worst-case can be obtained as follows

$$\begin{aligned}
 \max_{\|\Lambda_l\| \leq \zeta_l} \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{G}}_l + \Lambda_l \right) \mathbf{w}_i &= \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{G}}_l + \zeta_l \mathbf{I} \right) \mathbf{w}_i \quad (18) \\
 \min_{\|\Delta_k\| \leq \varepsilon_k} \frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1, i \neq k}^K \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{w}_k} \\
 &= \frac{\min_{\|\Delta_k\| \leq \varepsilon_k} \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{w}_k}{\max_{\|\Delta_k\| \leq \varepsilon_k} \sigma_n^2 + \sum_{i=1, i \neq k}^K \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{w}_k} \\
 &= \frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1, i \neq k}^K \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I} \right) \mathbf{w}_k} \\
 &\geq \frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1}^K \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}. \quad (19)
 \end{aligned}$$

Using (18) and (19), the problem (17) can be formulated as

$$\begin{aligned}
 \min \quad & \sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k \\
 \text{s.t.} \quad & \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{G}}_l + \zeta_l \mathbf{I} \right) \mathbf{w}_i \leq \xi_l \quad (l = 1, \dots, L) \\
 & \frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1}^K \mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I} \right) \mathbf{w}_k} \geq \gamma_k \quad (k = 1, \dots, K). \quad (20)
 \end{aligned}$$

The above optimization problem (20) can be converted to a convex one and solved by the second order cone programming method.

### 3.3. Second-order Cone Formulation

The standard form of convex conic optimization problem is defined as

$$\max_{\mathbf{y}} \mathbf{b}^T \mathbf{y} \quad \text{subject to} \quad \mathbf{c} - \mathbf{A}^T \mathbf{y} \in \mathcal{K} \quad (21)$$

where  $\mathbf{y}$  is a vector containing the designed variables.  $\mathbf{A}$  is an arbitrary matrix.  $\mathbf{b}$  and  $\mathbf{c}$  are arbitrary vectors.  $\mathcal{K}$  is a symmetric cone consisting

of Cartesian products of elementary cones (each corresponding to a constraint). Note that  $\mathbf{A}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  can be complex-valued and must have matching dimensions.

For our problem, the elementary cones are SOCs. The  $q$ -dimensional SOC is defined as

$$\text{SOC}^{q+1} \triangleq \left\{ (x_1, \mathbf{x}_2) \in \mathcal{R} \times \mathcal{C}^q \mid x_1 \geq \|\mathbf{x}_2\| \right\} \quad (22)$$

where  $x_1$  is a real scalar.  $\mathbf{x}_2$  is a complex  $q$ -dimensional vector.

First of all, we convert the quadratic objective function of (20) to a linear one. Notice that  $\sum_{k=1}^K \mathbf{w}_k^H \mathbf{w}_k = \mathbf{W}^H \mathbf{W}$ , where  $\mathbf{W} = [\mathbf{w}_1^T, \dots, \mathbf{w}_K^T]^T$ . Clearly, minimizing  $\mathbf{W}^H \mathbf{W}$  is equivalent to minimizing  $\|\mathbf{W}\|^2$ . Introducing a new scalar nonnegative variable  $\tau$  and a new constraint  $\|\mathbf{W}\|^2 \leq \tau$ , we can convert (20) into the following form

$$\begin{aligned} & \min_{\mathbf{w}, \tau} \tau \\ & \text{subject to} \\ & \text{C1: } \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{G}}_l + \zeta_l \mathbf{I} \right) \mathbf{w}_i \leq \xi_l \quad (l = 1, \dots, L) \\ & \text{C2: } \frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I} \right) \mathbf{w}_i} \geq \gamma_k \quad (k = 1, \dots, K) \\ & \text{C3: } \|\mathbf{W}\|^2 \leq \tau. \end{aligned} \quad (23)$$

Since  $\sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{G}}_l + \zeta_l \mathbf{I} \right) \mathbf{w}_i = \mathbf{W}^H \mathbf{L}_l^H \mathbf{L}_l \mathbf{W}$ , where  $\mathbf{L}_l = \text{diag}\{[\mathbf{L}_{Gl}, \dots, \mathbf{L}_{Gl}]\}$  with  $\mathbf{L}_{Gl}$  being the Cholesky factor of  $\mathbf{G}_l + \zeta_l \mathbf{I}$ , i.e.,  $\mathbf{L}_{Gl}^H \mathbf{L}_{Gl} = \mathbf{G}_l + \zeta_l \mathbf{I}$ . The constraint C1 can be expressed as

$$\|\mathbf{L}_l \mathbf{W}\|^2 \leq \xi_l. \quad (24)$$

Let  $\mathbf{C}_{k1}$  and  $\mathbf{C}_{k2}$  be the Cholesky factors of  $\tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I}$  and  $\tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I}$ , respectively.

The left hand side of constraint C2 has the following form

$$\frac{\mathbf{w}_k^H \left( \tilde{\mathbf{H}}_k - \varepsilon_k \mathbf{I} \right) \mathbf{w}_k}{\sigma_n^2 + \sum_{i=1}^K \mathbf{w}_i^H \left( \tilde{\mathbf{H}}_k + \varepsilon_k \mathbf{I} \right) \mathbf{w}_i} \geq \frac{\|\mathbf{C}_{k1} \mathbf{w}_k\|^2}{\sigma^2 + \|\Gamma_k \mathbf{W}\|^2} \quad (25)$$

where  $\Gamma_k = \text{diag}\{[\mathbf{C}_{k2}, \dots, \mathbf{C}_{k2}]\}$ . According to [12], the constraint C2 can be rewritten as

$$\left\| \begin{array}{c} \sigma \\ \Gamma_k \mathbf{W} \end{array} \right\|^2 \leq \frac{1}{\gamma_k} \left( \mathbf{C}_{k1} \mathbf{w}_k \right)^2$$



Making use of (24) and (26), the optimization problem (23) can be translated to the following optimization problem

$$\begin{aligned}
 & \min_{\mathbf{w}, \tau} \tau \\
 & \text{subject to} \\
 & \text{C1: } \|\mathbf{L}_l \mathbf{W}\|^2 \leq \xi_l \quad (l = 1, \dots, L) \\
 & \text{C2: } \left\| \begin{matrix} \sigma \\ \Gamma_k \mathbf{W} \end{matrix} \right\|^2 \leq \frac{1}{\gamma_k} \left( \mathbf{C}_{k1} \mathbf{w}_k \right)^2 \quad (k = 1, \dots, K) \\
 & \text{C3: } \|\mathbf{W}\|^2 \leq \tau.
 \end{aligned} \tag{26}$$

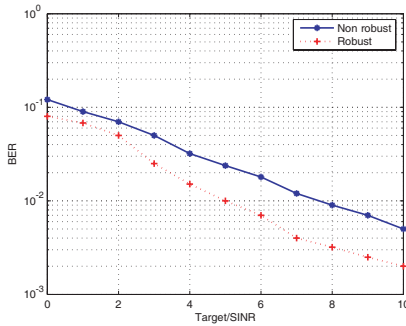
If the above convex optimization problem (27) is feasible, the proposed robust beamformer can be obtained from the optimal solution. However, the problem may be infeasible, depending on the chosen norm constrained parameters  $\xi_l$  ( $l = 1, \dots, L$ ) and  $\gamma_k$  ( $k = 1, \dots, K$ ). Fortunately, the infeasibility can be easily detected by the convex optimization software [13].

#### 4. SIMULATION RESULTS

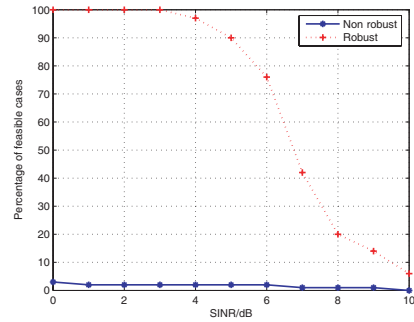
In this section, we consider the base station (BS) is equipped with a seven-element uniform linear array (ULA) with baseline separation of  $\Delta = 0.5$ . Assume that three SU-RXs ( $K = 3$ ) are served and the CR-Net should protect two PU-RXs ( $L = 2$ ). The SU-Rxs are located in the directions of  $\theta_1 = -10^\circ$ ,  $\theta_2 = 20^\circ$ , and  $\theta_3 = 45^\circ$ , respectively. The PU-Rxs are located at the directions of  $-20^\circ$ ,  $60^\circ$ , respectively. It is assumed that the change in direction of arrival (DOA) of the input waves to the SU-Tx may be up to  $\pm 1.5^\circ$  arbitrarily. The noise power is assumed to be  $\sigma_n^2 = 0.01$  and identical for all of the users. In addition, a constant SINR level of 10 dB is targeted for all the SUs, whereas the interference threshold of 0.01 is used to protect the PUs. The uncertainty sets are characterized with  $\varepsilon_k = \zeta_l = 0.04$ .

Figure 2 depicts the BER performance of the robust scheme and non robust scheme for one of the secondary users as a function of target SINR values. The robust scheme outperforms the non robust scheme for different target SINR values.

A beamforming solution is defined as feasible solution, if it satisfies all the constraints in problem (7). The percentage of feasible cases is defined as  $r = r_f/r_t$  where  $r_f$  denotes the number of the feasible solution and  $r_t$  stands for the number of total tests. Fig. 3 plots the percentage of feasible cases of all the schemes. It can be seen that the non-robust approach yields infeasible solutions in almost all the cases. In the robust approaches the percentage of feasible runs decreases monotonously as the target SINR increases.



**Figure 2.** The BER curves of secondary user for robust scheme and non robust scheme.



**Figure 3.** Feasibility percentage of different schemes.

## 5. CONCLUSION

In this paper, a robust downlink beamforming is presented in CR-Net. The proposed beamformer has been optimized for worst possible errors in the CSI. The optimization problem is efficiently formulated in terms of SOCP problem. Simulation results show the proposed method has superior performance, such as better robustness to channel change etc..

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