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DISPERSION CHARACTERISTICS OF PARTIAL H-PLANE WAVEGUIDES

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Abstract—In this paper, dispersion characteristics of the partial H-plane waveguides are theoretically investigated by applying Galerkin's method in Fourier domain. By extracting the dyadic Green's functions of the structure and satisfying the boundary conditions along the longitudinal slit, propagation constant and consequently, the fields in the structure are obtained. It is seen than propagation constant not only depends on the waveguide dimensions, but also on the location and dimension of the slit. A significant feature of the structure is that its first and second propagation modes can be separately controlled which is very useful in designing singlemode and multimode filters. Two examples are given which in the first one, the parameters of the structure are assigned in such a way the first and second cut off frequencies are at $f = 3.1\,\text{GHz}$ and $f = 6.2\,\text{GHz}$ respectively but in the second example, first and second modes are degenerate. The validity of the method is confirmed by comparing our results with ones from others.

1. INTRODUCTION

Waveguide circuits, as low loss structures with high power handling capability, have been widely used in various microwave systems since the birth of microwave technology and are still very useful in designing microwave components. However, conventional waveguides have a limitation of size since a rectangular waveguide is bulky especially at relatively low microwave frequencies. Recently, to overcome this problem, the partial H-plane waveguide as a folded waveguide having a one-quarter crosssection compared to that of the conventional

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waveguide was presented [1]. Afterwards, many structures such as filters [2–4], and antennas [5, 6] were designed based on partial H-plane waveguides. Nowadays, these structures are being extensively used in substrate integrated waveguide topology [7]. Also, they are very compatible with LTCC technology. In [8], this structure has been used in designing multilayer partial H-plane filters in LTCC technology. Also, these low profile structures are good candidates for designing frequency scanning leaky wave antennas [9].

In [10], propagation constant of the partial H-plane waveguide has been derived for fundamental mode based on the theoretical equivalence between the cross sections of a wave guide and a 2D transmission line of TEM propagation. But their formula is only valid for the case in which the slit has been inserted at the end of vane. Owing to wide range of applications mentioned above, the analysis of the structure based on field theory is useful. The spectral domain approach is a kind of full wave analysis which uses hybrid modes including both the transverse electric (TE) mode and the transverse magnetic (TM) mode along the longitudinal direction [11, 12]. In the last step, Galerkin's method is employed to obtain a group of linear equations. Propagation constants of the structure are the roots of the coefficient matrix's determinant. It is seen that dispersion characteristics of the fundamental mode of the structure depends strongly on the location and the width of the slit, while the dispersion characteristic of the second mode doesn't depend on the H-plane vane. It can be controlled by waveguide width. Hence, the dispersion characteristics of first and second modes can separately be engineered for different applications.

In this paper, first the method of analysis is presented and then the effects of various parameters of the structure on the dispersion characteristics are investigated.

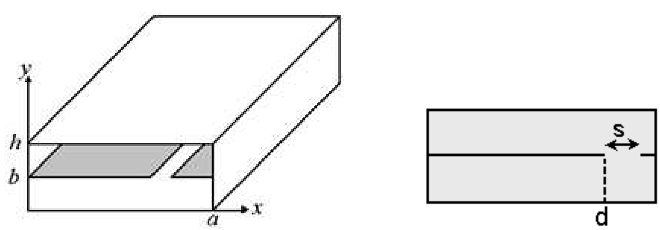

Figure 1. Configuration of the H-plane waveguide.

2. METHOD OF ANALYSIS

The schematic view of the structure under consideration is shown in Figure 1. It consists of a waveguide with an H-plane vane in it. Considering zero thickness of the vane, according to Figure 1, the scalar potential functions in the structure can be expanded in terms of TE and TM modes as follows

$$\begin{aligned}\varphi_1(x,y,z) &= \sum_{n=1}^{N} A_1 \cos\left(\frac{n\pi}{a}x\right)\cosh(\lambda y)\, e^{-j\beta z}\\ &\quad + \sum_{n=1}^{N} B_1 \sin\left(\frac{n\pi}{a}x\right)\sinh(\lambda y)\, e^{-j\beta z}\end{aligned}\tag{1a}$$

For $0 < y < b$, and

$$\begin{aligned}\varphi_2(x,y,z) &= \sum_{n=1}^{N} A_2 \cos\left(\frac{n\pi}{a}x\right)\cosh(\lambda(y-h))\, e^{-j\beta z}\\ &\quad + \sum_{n=1}^{N} B_2 \sin\left(\frac{n\pi}{a}x\right)\sinh(\lambda(y-h)) e^{-j\beta z}\end{aligned}\tag{1b}$$

For $b < y < h$ and

$$\lambda^2 = \left(\frac{n\pi}{a}\right)^2 + \beta^2 - \omega^2\mu\varepsilon$$

In above scalar potential functions, first term includes TE modes and second term includes TM modes. Applying the continuity of the tangential electric fields at $y = h$ led to the following relations:

$$A_1 \sin(\lambda b) = A_2 \sin(\lambda(b-h))\tag{2a}$$

$$B_1 \sin(\lambda b) = B_2 \sin(\lambda(b-h))\tag{2b}$$

Considering the following boundary conditions

$$J_z = H_1^x - H_2^x\tag{3a}$$

$$J_x = H_2^z - H_1^z\tag{3b}$$

And expanding the currents at $y = h$ as follows

$$J_x = \sum_{n=1}^{N} \tilde{j}_n^x \cos\left(\frac{n\pi}{a}x\right) e^{-j\beta z}\tag{4a}$$

$$J_z = \sum_{n=1}^{N} \tilde{j}_n^z \sin\left(\frac{n\pi}{a}x\right) e^{-j\beta z}\tag{4b}$$

We have

$$\tilde{J}_n^z = \left[\left(\frac{-n\pi\beta}{\omega\mu\varepsilon a}\right)A_1 - \left(\frac{\lambda}{\mu}\right)B_1\right]\cdot[\coth(\lambda b) - \coth(\lambda(b-h))]\cdot\sinh(\lambda b) \quad (5a)$$

$$\tilde{J}_n^x = \left(\frac{-j\left(\omega^2\mu\varepsilon - \beta^2\right)}{\omega\mu\varepsilon}\right)\cdot[\coth(\lambda b) - \coth(\lambda(b-h))]\cdot\sinh(\lambda b) \quad (5b)$$

Using relations (2), (4), (5) and considering Maxwell equations, the currents at $y = b$ can be written in terms of tangential electric fields in Fourier domain as following

$$\begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} \\ \tilde{X}_{21} & \tilde{X}_{22} \end{bmatrix} \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} \quad (6)$$

in which

$$\tilde{X}_{11} = \frac{j(\omega^2\mu\varepsilon - \beta^2)}{\omega\mu\lambda}\cdot[\coth(\lambda b) - \coth(\lambda(b-h))]\cdot\sinh(\lambda b)$$

$$\tilde{X}_{12} = \tilde{X}_{21} = \frac{n\pi\beta}{\omega\mu\lambda}\cdot[\coth(\lambda b) - \coth(\lambda(b-h))]\cdot\sinh(\lambda b)$$

$$\tilde{X}_{22} = \left[\frac{-j\beta^2(np/a)^2}{(\omega^2\mu\varepsilon - \beta^2)\lambda} + \frac{j\omega\varepsilon\lambda}{\omega^2\mu\varepsilon - \beta^2}\right]\cdot[\coth(\lambda b) - \coth(\lambda(b-h))]\cdot\sinh(\lambda b)$$

$\sim$ indicates Fourier domain. In order to apply Galerkin's testing procedure to (6), first the tangential electric fields on the slit surface are expanded in terms of known basis functions with unknown coefficients as below [11, 12]

$$E_{ax} = \sum_{r=1}^{R} a_r P_r(x) \quad (7a)$$

$$E_{az} = \sum_{r=1}^{R} b_r Q_r(x) \quad (7b)$$

P_n and Q_n are pulse basis functions. By multiplying two sides of (6) with basis functions defined in (7) and using Parseval's theorem, the following equations are obtained:

$$\sum_{s=1}^{R} a_s \sum_{n=1}^{N} \tilde{P}_r(n)\cdot\tilde{X}_{11}\tilde{P}_s(n) + \sum_{s=1}^{R} b_s \sum_{n=1}^{N} \tilde{P}_r(n)\cdot\tilde{X}_{12}\cdot\tilde{Q}_s(n) = 0 \quad (8a)$$

$$\sum_{s=1}^{R} a_s \sum_{n=1}^{N} \tilde{Q}_r(n)\cdot\tilde{X}_{21}\cdot\tilde{Q}_s(n) + \sum_{s=1}^{R} b_s \sum_{n=1}^{N} \tilde{Q}_r(n)\cdot\tilde{X}_{21}\cdot\tilde{Q}_s(n) = 0 \quad (8b)$$

in which R is the number of basis functions. By nulling the determinant of the above equation at each frequency, the propagation constants of the partial H-plane waveguide are obtained [13].

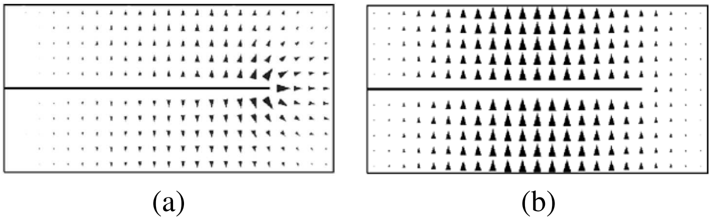

Figure 2. (a) Electric field of the dominant and (b) second mode.

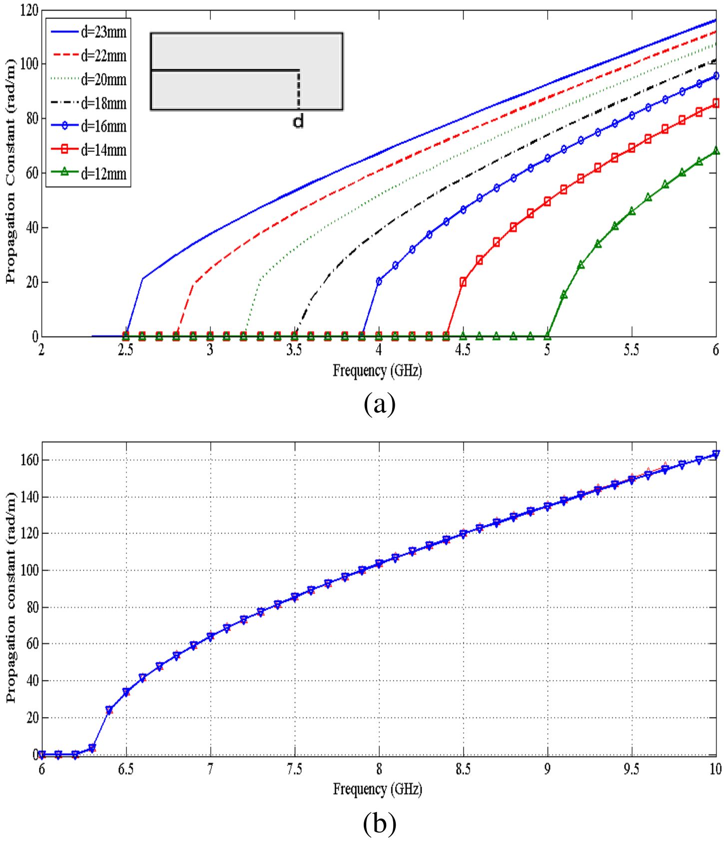

Figure 3. (a) Dispersion characteristics of the dominant mode. (b) Dispersion characteristics of the second mode versus vane's width for $a = 23.8\,\text{mm}$, $b = 6\,\text{mm}$, $h = 12\,\text{mm}$, and $s = a - d$.

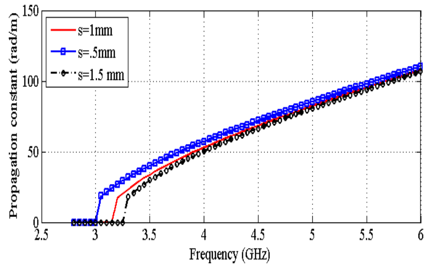

Figure 4. Dispersion characterisAcs of the waveguide versus slit width for $d = 18.8\,\mathrm{mm}$.

3. RESULTS AND DISCUSSION

In this section, the proposed method is used to analyze the structure shown in Figure 1. The parameters of a, b and h are chosen as 23.8 mm, 6 mm, and 12 mm respectively. Without H-plane vane, the first propagation mode of the waveguide is TE_{10} which its cut-off frequency is 6.25 GHz with above mentioned dimensions. But inserting an H-plane vane longitudinally along the waveguide (in Figure 1) allows another mode with lower propagation constant flows along the waveguide. In other words, by adding the H-plane vane, the same waveguide acts at lower frequencies. In Figure 2, transversal electric field of the dominant and second modes is shown. In Figure 3, dispersion characteristics of the structure are shown in terms of vane's width. As figures show, by decreasing the vane's width, since the effective length of the first mode is decreased, its cut-off frequency shifts up. But, the dispersion characteristic of the second mode doesn't depend on the vane. Now consider Figure 1 in which $d = 18.8\,\mathrm{mm}$ and only the width of slit (s) is changed. In this case, the slit's width determines the coupling between two metallic planes. Hence, slit acts as a capacitor [10]. According to Figure 4, as the width of slit is decreased, by increasing the equivalent capacitor between two sections, the effective length of the vane for first mode is increased and its cut-off frequency shifts down. Hence, the propagation constant of the first mode can be controlled by slit's width and location, and the dispersion characteristics of the second mode can be adjusted by waveguide's width. This degree of freedom is very useful in designing singlemode and multimode filters. In the former case, it's desirable to separate these modes as far as possible to improve the out of band response of the filter. But in the later case, it is the coupling between degenerate resonant modes that makes filter response. In Table 1, the

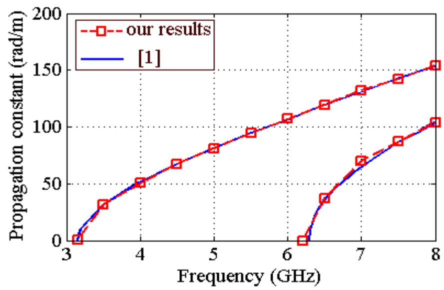

Figure 5. Propagation constant of the structure resulted from our method in comparison with ones from [1] for $d = 20.2\,\mathrm{mm}$, $s = a - d$.

Table 1. Dimensions of the structure appropriate for single ode and multi mode applications.

	a (mm)	S (mm)	d (mm)	First mode's cut-off (GHz)	Second mode's cut-off (GHz)	Single mode bandwidth (GHz)
1	23.8	1	22.8	3.1	6.2	3.1
2	41.5	30	11.5	3.6	3.6	0

dimensions of the structure appropriate for these two cases are shown. In Figure 5, our results are compared with ones from [1].

4. CONCLUSION

The method of moments in spectral domain was applied to analyze the dispersion characteristics of the partial H-plane waveguides. The effects of the various parameters of the structure such as slit's width and location were studied. It was shown that dispersion characteristics of the propagation modes can separately be engineered for different applications. It is instructive to note that above mentioned method can be extended easily for multi-layer waveguides with different dielectric parameters which is very common in SIW and LTCC applications. The method of analysis is very time saving and can easily be implemented on an engineering calculator.

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