Progress In Electromagnetics Research B, Vol. 32, 107–127, 2011

TROPICAL RAIN CLASSIFICATION AND ESTIMATION OF RAIN FROM Z-R (REFLECTIVITY-RAIN RATE) RELATIONSHIPS

L. S. Kumar[1, *], Y. H. Lee[1], J. X. Yeo[1], and J. T. Ong[2]

[1]Nanyang Technological University, 50, Nanyang Avenue, Singapore

[2]C2N Pte. Ltd., Research and Technology, 41, Cheng Soon Garden, Singapore

Abstract—A Z-R relation is derived using a data set which consists of nine rain events selected from Singapore's drop size distribution. Rain events are separated into convective and stratiform types of rain using two methods: the Gamache-Houze method, a simple threshold technique, and the Atlas-Ulbrich method. In the Atlas-Ulbrich method, the variability of the rain integral parameters R, Z, N_w, D_0 and gamma model parameter μ are used for the classification of rain into convective, stratiform and transition. Z-R relations are derived for each type of rain after classification. The changes in the coefficients of the Z-R relations for different rain events are plotted and analyzed. The Z-R relations of the different methods using the Singapore data are compared and analyzed. It is concluded that the coefficient A of the Z-R relation is higher for the convective stage followed by the stratiform and transition stages. The coefficient b values are higher for the transition stage followed by the stratiform and convective stages. Reflectivities are extracted from RADAR data above NTU site for rain events and compared with the reflectivities derived from the distrometer data. Rain rates retrieved from RADAR data using the proposed relations from Singapore's data set are compared with the distrometer rain rates. The RADAR extracted rain rates are found to be constantly lower than the distrometer derived rain rates but matches well.

Received 4 April 2011, Accepted 30 June 2011, Scheduled 11 July 2011

* Corresponding author: Lakshmi Sutha Kumar (laks0008@ntu.edu.sg).

1. INTRODUCTION

Rain rate estimation from RADAR measurements is based on empirical models such as the reflectivity (Z) and rain rate (R) relation, the Z-R relation, which has been studied for more than 60 years [1]. In RADAR meteorology, the accurate determination of the rain rate from the measured reflectivity is important. The variations in reflectivity-rain rate (Z-R) relationships are strongly dependent on Drop Size Distribution (DSD) variations [2]. Other integral rain parameters such as rainwater content, attenuation, and optical extinction are also functions of DSD.

The Z-R relationships relate the value of the measured reflectivity to the value of the rain rate according to the general formula (1) by Marshall and Palmer [1],

$$Z = AR^b \tag{1}$$

where the RADAR reflectivity factor $Z\,(\mathrm{mm}^{-6}\mathrm{m}^3)$ and rain rate $R\,(\mathrm{mm/hr})$ depend on the DSD [2–5]. Marshall and Palmer [1] published the Z-R relation using the exponential DSD with a set of generic parameters of $A = 200$ and $b = 1.6$. Battan [3] presented a list with 69 different Z-R relationships for different climatic conditions in different parts of the world. There can be dramatic changes in Z-R law parameters within an individual storm as well as between storms [6–10]. These changes are clearly identified with the physical processes acting to form the rain event. In the past, research work was done to improve the accuracy of Z-R relations by classifying the rain into different types [6–10].

Many studies have demonstrated that stratiform rain is characterized by larger raindrop diameters relative to convective type rain for the same liquid water content [6, 7]. In recent years [6–9], variations in the gamma DSD parameters are used for rain type classification. Tokay and Short [6] observed a significant change in the gamma parameter, the intercept parameter, N_0, during the transition from convective to stratiform rain. Later, many researchers reported the existence of a transition region between the convective and stratiform regimes [7–10]. Bringi et al. [11] used a simple scheme to separate stratiform and convective rain types based on the standard deviation of rain rate over 5 consecutive DSD samples. A standard deviation of ≤ 1.5 mm/hr is classified as stratiform type rain, otherwise convective type rain is assumed.

Atlas et al. in [7] and Ulbrich and Atlas in [12] studied the DSDs during the three regimes (stratiform, convective and transition) and determined the Z-R relations for each of these regimes. They pointed out that there is a systematic variation of the Z-R relations for these three types of rain [12]. They identified a rain event consisting of all three regimes. A rain event initially starts off as convective, where the rain rate, R, rises sharply and reaches its peak while the median volume diameter, D_0, does not vary greatly. When D_0 and R decrease simultaneously following the initial convective period, the rain is classified as transition. This is followed by the stratiform rain which is characterized by its approximately steady rain rate of $R \leq 10$ mm/hr and its higher median volume diameter, D_0, values. In [12, 13], they also studied the variations of N_w, the normalizing constant defined as the intercept of an equivalent exponential DSD with the same water content W and variations of μ, the shape parameter for different types of rain. They concluded that the coefficient A is smaller for stratiform rain and increases for increasing convective activity, while b behaves in the opposite manner (smaller for convective rain and larger for stratiform systems) [7, 12].

Motopoli et al. [14] used the D_m and μ variations for rain classification along with the classification used in [11]. The mean diameter D_m revolves around the values of 1.5 mm, and the shape parameter μ is nearly zero (lower values) for their convective samples. For the transition and stratiform samples, both D_m and μ are found to slightly decrease and increase, respectively, relative to their values in the convective zone. On the other hand, in the stratiform time slots, D_m oscillates around 1 mm, whereas μ oscillates around five. Wilson and Tan [10] used Singapore's RADAR (installed by the Rutherford Appleton Laboratory, UK at Nanyang Technological University, Singapore) data and distrometer data to determine the Z-R relationships. They also used the variations in the integral parameters to classify the rain types.

In this paper, the RADAR data from the RADAR installed by the Meteorological Service of Singapore (MSS) at Changi airport, Singapore is used for the determination and classification of rain types. The results are compared with those reported by Wilson and Tan in [10]. This paper intends to study the characteristics of tropical DSDs in terms of bulk rain integral parameters during convective, transition, and stratiform rain, and to determine the best Z-R relations for rain rate retrieval during these three stages. Nine rain events from the nine months data are considered, and different Z-R relationships are derived. The derived Z-R relations are compared with those reported in [10] which makes use of a different database collected also from Singapore. This paper then verifies the results by performing an inter-comparison between the distrometer derived rain rates and the RADAR derived rain rates using the derived Z-R relations.

2. DATA MEASUREMENT AND CALCULATION OF RAIN INTEGRAL PARAMETERS

2.1. Distrometer Data

The data recorded from January 1998 to September 1998, using a Joss-Waldvogel distrometer RD-69, are used in this study. The Joss-Waldvogel Distrometer is capable of measuring drop diameters ranging from 0.3 mm to > 5 mm with an accuracy of $\pm 5\%$. It distinguishes between drops with time interval of about 1 ms. The total number of drops of diameters ranging from 0.3 mm to > 5 mm is divided into 20 different bins with 1 minute integration time [15].

Table 1 presents the list of the 9 rain events involved in this study. As shown in Table 1, there are 1674 minutes of data in total for the 9 rain events. Of the 1674 minutes of data, only DSDs having number of rain drops greater than 100 (1496, one minute samples) are considered. The rain events in Singapore which have all the three rain types exhibit a similar trend; they reach high intensity very rapidly during the leading edge of the convective stage, remain heavy for a few minutes, then decrease slowly during the trailing edge of convective stage and transition period. Transition stage is followed by the steady stratiform rain. One rain event refers to the beginning of the rainfall till the end. The rain events which have all the three rain stages, convective, transition and stratiform, are selected for rain classification. One stratiform rain event is also included for analysis. Most of the selected rain events last for a long duration. The selection of rain events is also dependent on the availability of the RADAR data which are limited to days within the year 1998.

Table 1. Selected rain events from 9 months of year 98 distrometer data for analysis.

Rain event	Date of event	Time (UTC)	No. of samples	Maximum rain rate (mm/hr)
1	09/01/98	1353–1854	127	85.76
2	28/01/98	1400–1513	74	78.57
3	05/04/98	1700–1754	55	114.71
4	09/05/98	500–959	300	123.37
5	12/05/98	1353–1859	307	107.40
6	18/05/98	1825–1959	43	5.92
7	07/06/98	1900–2359	300	38.47
8	10/06/98	135–520	223	64.41
9	25/09/98	515–759	165	90.69

2.2. Measurement of Rain Integral Parameters

The rain rate in mm/hr and reflectivity in $\mathrm{mm}^{-6}\mathrm{m}^3$ can be calculated from the measured DSD data by [15]

$$R = \frac{3600\pi}{6ST} \sum_{i=1}^{20} D_i^3 n_i \tag{2}$$

$$Z = \frac{10^6}{ST} \sum_{i=1}^{20} \frac{D_i^6 n_i}{v(D_i)} \tag{3}$$

where n_i is the number of rain drops in the ith bin; D_i is the mean drop diameter in mm; $S = 5000\,\mathrm{mm}^2$ is the sample area; $T = 60\,\mathrm{sec}$ is the integration time; and $v(D_i)$ is the terminal velocity of rain drop in m/s obtained from Gunn and Kinzer [16]. Reflectivity in dBZ can be calculated as

$$Z\,(\mathrm{dBZ}) = 10 \log_{10}(Z) \tag{4}$$

where Z is found using (3). The relations between reflectivity and rainfall-rate are traditionally derived from a linear regression in which the logarithm of rainfall rate is an independent variable. Campos and Zawadski [17] found that the Z-R relationship depends on the regression technique and is therefore highly method dependent. Atlas et al. [18] found that the Z-R relationship derived from linear regression produces an accurate representation of the relationship at low rain rates but not necessarily at high rain rates. The water content (W) is estimated from radar reflectivity in [19] using the Z-W relation. The relative bias and error are calculated in both linear and logarithm domain fittings between the parameters reflectivity (Z) and water content (W) in [19] because the error analysis in linear domain is highly weighted by heavy rain and that in logarithm domain accounts for more contribution from light rain data points. It is found in [19] that the estimations of microphysical processes are generally improved in the logarithm domain. Therefore, in this paper, a linear regression of $10 * \log(Z)$ versus $\log(R)$ is used.

The DSD is assumed to have the form of the gamma function given by

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \tag{5}$$

where $N(D)$ is the distribution of rain drops per diameter interval D to $D + \Delta D$ (mm); N_0 is the intercept parameter ($\mathrm{mm}^{-1-\mu}\mathrm{m}^{-3}$); Λ is the slope of the exponential (mm^{-1}); and μ is the dimensionless shape parameter. The error effects on DSD moments and moment estimators for DSD parameters were analyzed in [20], and it is verified that there is a tendency for the middle-order moments to have relatively low errors.

Therefore, the 2nd, 3rd and 4th moments are used to model gamma DSD (MM234) in Singapore suggested by Smith et al. [21], and the gamma model parameters N_0, μ and Λ are calculated.

The variations in the rain integral parameters R, Z, D_0, N_w and the gamma model parameter μ are used for classifying the rain into three different types. D_0 is calculated using the following equation

$$D_0 = (3.67 + \mu)/\Lambda \,[\text{mm}] \tag{6}$$

Here D_0 is the median volume diameter [7]. N_w is generalized number concentration of an exponential DSD having the same liquid water content W and mass-weighted diameter D_m as the actual DSD [22], and it is calculated using

$$N_W = \frac{4^4}{\pi \rho_w} \left[\frac{1000W}{D_m^4} \right] \; [\text{mm}^{-1}\text{m}^{-3}] \tag{7}$$

where W is in $g\text{m}^{-3}$, proportional to the third moment of the drop size distribution $N(D)$; $\rho_w = 1$ is the water density in $g\,\text{cm}^{-3}$; D_m is the mass weighted mean diameter and defined as the ratio of the fourth to the third moments of the DSD.

$$D_m = \frac{\int_0^\infty N(D) D^4 dD}{\int_0^\infty N(D) D^3 dD} \,[\text{mm}] \tag{8}$$

2.3. RADAR Data

The RADAR data which is utilized to calculate rain rates is produced by the S-band Meteorological Doppler Weather RADAR (MDWR) system from Meteorological Service of Singapore (MSS). The antenna of the MDWR system is located at 1.3512N, 103.97E, which is adjacent to the Changi airport located on the east coast of Singapore. The location of Nanyang Technological University (NTU) is 1.3423N, 103.68E, located on the west coast of Singapore. The distance between the RADAR and NTU is 32.21 km, and the bearing angle is 268.24°. There are two types of modes in the RADAR data, aerial and airport modes. Both modes take about 250 seconds (about 4 minutes) for one full volume scan. The elevation angles of the rays for the aerial mode are: 0.1°, 1°, 1.5°, 2°, 3°, 5°, 7.5°, 10°, 15° and 20°. The elevation angles of the rays for airport mode are: 1°, 1.5°, 1.7°, 15°, 20°, 30° and 40°. The reflectivity data above NTU can be extracted from the ray of the RADAR using the distance, elevation angle and bearing angle. Z-R relations are proposed using Singapore's DSD data base for different criteria described in Section 3. These Z-R relations are used to calculate rain rate from RADAR data and compared with the rain rates derived from distrometer data at the end of Section 3.

3. RESULTS AND DISCUSSIONS

3.1. Gamache-Houze Method

3.1.1. Rain Classification

Figure 1 shows the Z (dBZ) versus log (R) plot for 1496 DSD minutes, which contains greater than 100 rain drops from the 9 rain events listed in Table 1. Linear regression of $10*\log(Z)$ versus $\log(R)$ is used to find the Z-R relation for the 1496 DSD minutes, and the fitted Z-R relation is named as 'SG'. Reflectivity value of 38 dBZ is used as the threshold as explained by Gamache and Houze [23] to distinguish the precipitation types such that all the DSD minutes having reflectivity values above this threshold are assumed to be convective while those below 38 dBZ are assumed stratiform. After splitting the DSD minutes based on this threshold, Z-R relations are found separately for the convective rain type and stratiform rain type. These Z-R relations are named as 'GH-SG'. Of the total 1496 DSD minutes, the numbers of stratiform and convective points are 1094 and 402, respectively.

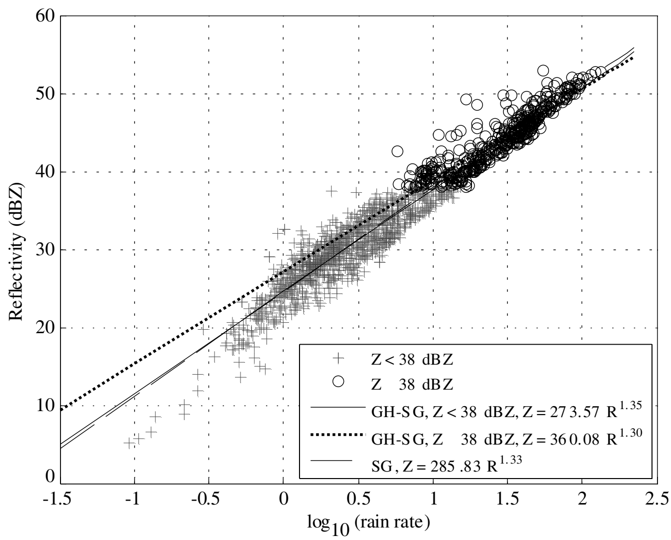


Figure 1. Scatter plot of reflectivity versus $\log_{10}$ (rain rate) (1496 minutes of data from year 98 rain events, DSDs having rain drops greater than 100 only considered) and Z-R fits of SG and GH-SG.

3.1.2. Comparison of Z-R Relations

The Z-R relations are shown in Fig. 1. It is clear from the values of the coefficients derived from (1) that A is larger and b is smaller for convective rain type whereas A value is reduced, and b value is increased for stratiform rain type. The coefficients of the overall Z-R relation, SG, are closer to the coefficients of the stratiform rain type, GH-SH ($Z < 38\,\mathrm{dBZ}$), than the convective rain type, GH-SH ($Z \geq 38\,\mathrm{dBZ}$). This indicates that when fitting the Z-R relation to both the convective and stratiforn rain types, the stratiform rain points dominate. Therefore, the lines of SG and GH-SH ($Z < 38\,\mathrm{dBZ}$) are closer to each other. It might be better to use a separate relation for the different rain types so that more accurate estimate of the rain rates can be obtained from the RADAR reflectivities.

The coefficients of the Z-R relations, A and b, proposed by Marshall and Palmer (MP) [1], from the 1496 DSD minutes of Singapore (SG), from the Gamache and Houze method (GH-SG) and reported by Wilson (W-SG) [10], are tabulated in Table 2. In W-SG, the A coefficient for the convective rain is less than the A coefficient of stratiform rain, and b values are in the reverse order. As explained in the introduction, in general, a transition stage is present in between the convective and stratiform rain stages within each rain event. Wilson and Tan used the method proposed by Atlas-Ulbrich [7] to derive the Z-R relations. This paper also uses the Atlas-Ulbrich method to separate the nine rain events into different rain stages. One of the events that occurred on the 12th May 1998 which had all of the rain types is used in this analysis.

Table 2. Reflectivity-rain rate (Z-R) relationships for individual rain type and for the overall data set derived by linear regression of Z (dBZ) versus $\log_{10}(R)$.

Type	General	C	T	ST
MP	$Z = 200R^{1.6}$	-	-	-
SG	$Z = 285.83R^{1.33}$	-	-	-
W-SG	-	$Z = 139R^{1.5}$	$Z = 271R^{1.25}$	$Z = 330R^{1.35}$
GH-SG	-	$Z = 360.08R^{1.3}$	-	$Z = 273.57R^{1.35}$

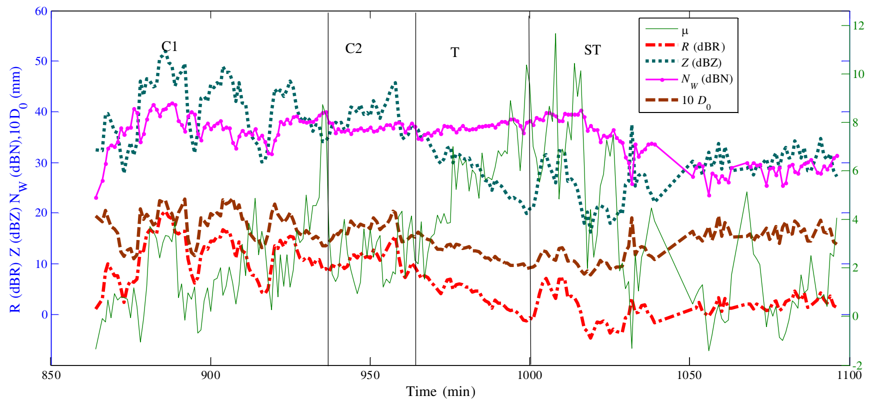
3.2. Atlas-Ulbrich Method

3.2.1. Rain Classification

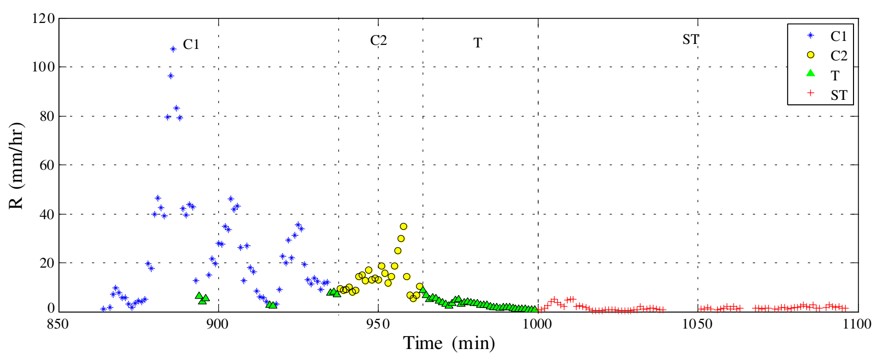
The rain event recorded on 12th May 1998 is selected for the analysis. It was a long convective rain event that lasted for around 4 hours.

This rain event is chosen for analysis because it is one of the long rain events and because it consists of all types of rain. It can be classified into three different rain types, convective, stratiform and transition, using the variations in the integral parameters. The variability of the rain integral parameters rain rate R (dBR), reflectivity Z (dBZ), the parameter N_w (dBN), the median volume diameter D_0 (mm) as $10 * D_0$ and the shape parameter μ for this rain event is plotted in Fig. 2(a). Fig. 2(b) shows the rain rate of the same rain event as a function of time.

The rain event has two convective peaks (C1 and C2) followed by



(a)



(b)

Figure 2. (a) Classification of distrometer data, recorded on 12th May 1998. (a) R (dBR), Z (dBZ), N_w (dBN), $10 * D_0$ (mm) and μ are plotted for around 240 minutes (b) Rain rate in mm/hr as a function of time.

transition (T) and stratiform (ST) stages. Stage C1, from 864 to 937 minutes, 74 minutes of duration, is characterized by DSDs of higher Z and R values, D_0 values above 1.5 mm and μ values less than 5. Some of the DSD minutes in these 73 minutes of duration have drop diameters less than 1.5 mm and rain rates less than 10 mm/hr. These points are added with the transition stage and marked by triangles in Fig. 2(b). The highest rain rate for the whole rain event occurs within stage C1 at 886 minutes and is 107.40 mm/hr with a corresponding reflectivity of 51.87 dBZ.

Stage C2, from 938 to 968 minutes, 31 minutes of duration, has a sharp convective peak. There is a short transition at the end of stage C1, DSD minutes 934 to 937, and afterwards, there is a increase in D_0, Z and R values. This stage also has lower μ values. The maximum rain rate for the C2 stage occurs at 958 minutes with 34.94 mm/hr of rainfall. All the rain integral parameters have stable values with very little variations during the C2 stage. After reaching the highest rain rate, the convective peak starts to decrease. The point at which the rain rate starts to decrease to less than 60% of the maximum rain rate, and $D_0 < 1.5$ mm is identified as the beginning of the transition stage [7, 12]. This stage can be clearly distinguished from the convective stages by the reduction in all the integral parameters.

After 105 minutes of convective rain, three of the integral parameters, R, Z and D_0, decrease continuously and reach the minimum point where $D_0 = 0.90$ mm, $Z = 19.95$ dBZ and $R = 0.73$ mm/hr. The T stage lasts for 31 minutes. During the T stage, the N_w shows a slight decrease, and the gamma parameter μ increases continuously and reaches a higher value of 10.34. From 1000 minutes onwards, R, Z and D_0 values again start to increase. This indicates the beginning of the stratiform (ST) stage. Over the steady stratiform duration of 97 minutes, R has values less than 10 mm/hr, and around 60 minutes of rain have D_0 values less than 1.5 mm. The other two variables Z and D_0 have the same trend as R. N_w, which have almost constant values around 40 dBN throughout the C1, C2 and T stages, have a clear decrease during the stratiform stage and less than 30 dBN for the last 60 minutes of the stratiform stage. The gamma parameter μ has ups and downs in the ST stage.

Table 3 shows the mean and standard deviation values of rain integral parameters in different stages for the rain event recorded on 12th May 1998. As can be seen from Table 3 and Fig. 2(a), the convective rain is dominated by bigger raindrops relative to stratiform rain. The median volume diameter, D_0, of greater than 1.5 mm is observed during the convective stages. The stratiform regime has the next bigger drops followed by the transition stage. Similarly, the other

Table 3. Mean and standard deviation of rain integral parameters during convective (C1, C2), transition (T), and stratiform (ST) stages for the rain event 12th May 1998.

Rain type	R (mm/hr)		Z (dBZ)		D_0 (mm)		N_w (dBN)		μ	
	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
C1	25.23	23.24	41.49	5.82	1.82	0.30	36.67	3.35	1.80	1.55
C2	14.10	7.00	39.47	2.88	1.70	0.16	36.76	0.65	2.46	1.08
T	3.50	2.09	29.64	4.59	1.25	0.18	37.02	1.36	5.15	2.37
ST	1.66	1.08	27.49	4.73	1.35	0.32	32.36	4.70	3.20	3.23

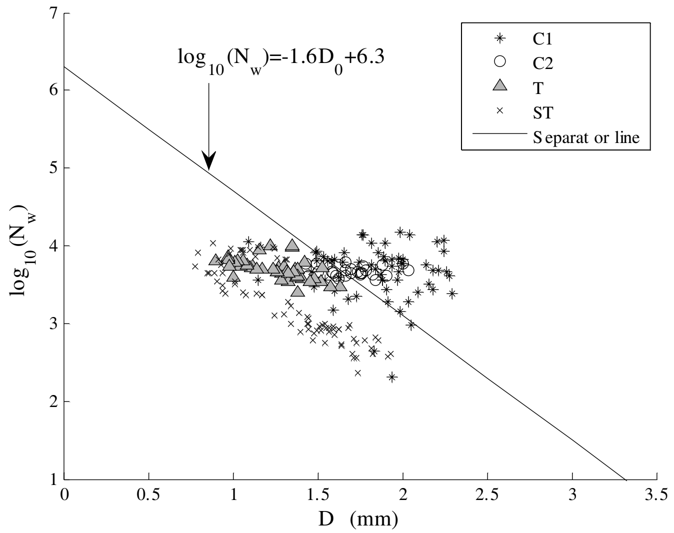


Figure 3. Dependence of $\log_{10}(N_w)$ and D_0 (mm) for the rain event on 12th May 1998 where the unit of N_w is $\mathrm{mm}^{-1}\mathrm{m}^{-3}$. The separator line separates convective and stratiform points.

two rain integral parameters R (mm/hr) and reflectivity Z (dBZ) have higher mean values for the convective stage followed by transition and stratiform stage, and the reverse is true for the rain integral parameter μ. The parameter N_w (dBN) has higher values for convective and transition stages. The standard deviations of parameters are lower for the C2 stage followed by T and ST stages. As the convective rain in C1 stage has ups and downs, the other parameters also deviates more from their mean values.

Figure 3 shows a very clear separation of convective and stratiform

Table 4. Reflectivity-rain rate (Z-R) relationships of the nine rain events for different rain types.

Rain event	Date	C		T	ST	ALL
1	09/01/98	$Z=330.74\,R^{1.25}$		$Z=149.44R^{1.55}$	$Z=182.61\,R^{1.43}$	$Z=192.13\,R^{1.40}$
2	28/01/98	$Z=129.87\,R^{1.58}$		$Z=175.36R^{1.48}$	$Z=295.49\,R^{1.13}$	$Z=179.87\,R^{1.48}$
3	05/04/98	$Z=411.68\,R^{1.27}$		$Z=248.71.03R^{1.34}$	$Z=142.63\,R^{1.59}$	$Z=212.52\,R^{1.42}$
6	18/05/98				$Z=445.89\,R^{1.29}$	
8	10/06/98	$Z=645.09\,R^{1.12}$		$Z=139.83\,R^{1.43}$	$Z=309.47\,R^{1.57}$	$Z=325.63\,R^{1.30}$
9	25/09/98	$Z=318.10\,R^{1.29}$		$Z=232.33\,R^{1.31}$	$Z=271.04\,R^{1.34}$	$Z=256.87\,R^{1.33}$
Rain event	Date	C1	C2	T	ST	
4	09/05/98	$Z=240.83\,R^{1.30}$	$Z=394.10\,\mathrm{R}^{1.26}$	$Z=122.77\,R^{1.54}$	$Z=277.21R^{1.43}$	$Z=285.33\,R^{1.31}$
5	12/05/98	$Z=407.11\,R^{1.27}$	$Z=218.06\,\mathrm{R}^{1.46}$	$Z=172.70\,R^{1.58}$	$Z=352.04\,R^{1.43}$	$Z=311.92\,R^{1.34}$
7	07/06/98	$Z=286.19\,R^{1.24}$	$Z=189.85\,\mathrm{R}^{1.42}$	$Z=174.59\,R^{1.40}$	$Z=321.65\,R^{1.52}$	$Z=380.69\,R^{1.19}$
AU-SG		**$\mathbf{Z=328.64\,R^{1.29}}$**		**$\mathbf{Z=173.24\,R^{1.42}}$**	**$\mathbf{Z=309.20\,R^{1.39}}$**	-

rain types in the N_w-D_0 domain.

The straight line in Fig. 3 represents the separator, defined by Bringi et al. in [24], between the two rain types, given by

$$\log_{10}(N_w) = -1.6D_0 + 6.3 \tag{9}$$

As can be seen from Fig. 3, most of the convective points lie above the separator line with high values of D_0 and $\log_{10}(N_w)$.

The straiform points lie below the separator line with low values of D_0 and $\log_{10}(N_w)$. The transition points lie on both sides of the separator line. But the transition points and a few of the convective points which lie below the line have relatively higher values of $\log_{10}(N_w)$ than the stratiform points. After classifying the rain into different types, reflectivity-rain rate relationships are calculated for each type of rain and also for the entire rain event. The empirical relation of reflectivity (Z) and rain rate (R) has the form of (1), where A is the intercept, and b is the exponent. According to the above procedure, 9 rain events in the year 1998 are classified, and the A and b parameters for the Z-R relation are derived for each stage of the rain event and also for the entire rain event. The Z-R relation parameters are tabulated in Table 4. According to Ulbrich and Atlas [12], the coefficient A increases when the median volume diameter D_0 is large and constant or when the shape parameter μ is small.

It is clear from the Singapore's Z-R relations that the large and constant value of D_0 produces higher A values. For the rain event on the 12th May 1998, A values in descending order is from C1 $\rightarrow$ ST $\rightarrow$ C2 $\rightarrow$ T, and b values are in the reverse order. As shown in Fig. 2 and Table 3, the D_0 is larger as well as constant for most of the time except for the three troughs during the C1 stage, larger during the C2 stage but increases from lower value to higher value slowly and decreases at the end of C2 stage, small for most of the time but less variations at the last 50 minutes of ST stage and smallest during the transition stage. Therefore, the A value of C1 stage is large followed by ST, C2 and T stages. AU-SG Z-R relation is fitted by considering all the C (including C1 and C2), T and ST points of the nine rain events in Table 4 respectively.

After analyzing the nine rain events, it is found that convective stages have higher rain rates (> 10 mm/hr for most of the points) and higher reflectivites (> 37 dBZ for most of the points). A few exceptional points are present during the initial convective period. The N_w values are 40 dBN and above for the convective points and 35 dBN to 40 dBN for the transition points and spreads from less than 30 dBN to 40 dBN for the stratiform points. In the stratiform regime, D_0 is mainly between 0.78 mm and 2.03 mm, which is smaller than the convective center where D_0 is in the range of 0.98 mm to 3.27 mm, for the analyzed rain events. The transition regions have D_0 values from 0.90 mm to 1.96 mm. The gamma parameter, μ, is lower, < 9, for the convective stage followed by transition stage for which μ is less than 11. But the value of μ is larger in the stratiform region, and its spread is also large with abrupt changes in the consecutive minutes.

3.2.2. AU-SG Z-R Relations

Figure 4 shows the coefficients of Z-R relations, A and b, of different rain events for the convective, transition and straitiform stages. As can be seen from Table 4, the coefficient A in the convective rain varies from 129.87 to 645.09, and the coefficient b varies from 1.12 to 1.58. Similarly, for stratiform rain the values of the coefficients A and b are found to vary from 142.63 to 445.89 and 1.13 to 1.59, respectively. Rain rates are higher for the convective rain type. Therefore, if b is large, R^b increases significantly resulting in a small A value. Similarly, if b is small, R^b decreases significantly resulting in a large A value. This can be seen by the large range in A value with the corresponding change in b value. However, during stratiform rain, rain rate is comparatively small, and therefore R^b does not vary too much. The resulting range of A for stratiform rain is small because the rain rate is small. The A coefficients of the Z-R relations obtained for the nine rain events have a mean value of 288.63 with a standard deviation of 88.95 for stratiform rain. But for the convective rain, the A coefficients of the Z-R relations

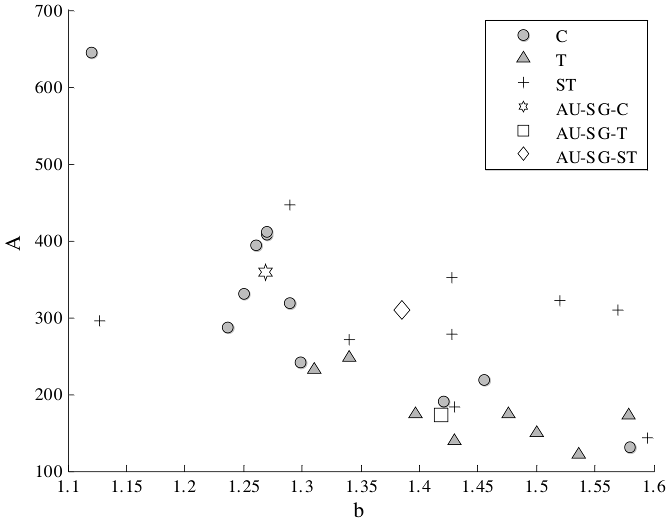


Figure 4. Relations between the coefficient A and the exponent b in the Z-R relations of the nine rain events for different rain types and for the AU-SG Z-R relation.

have a mean value of 324.69 with a large standard deviation of 140.69. Therefore, even small A and large b obtained by Wilson can also work well in the convective stage.

The stability of rain integral parameters during the entire rain stage leads to higher values of coefficient A and lower values of coefficient b. The stratiform rain event recorded on 18th May 1998 has rain rates less than 6 mm/hr, but there are less variations in the rain integral parameters, which will lead to the higher value, 445, of coefficient A and lower value of coefficient b. The transition stages have generally low A values compared to the convective and stratiform stages and higher b values than C and ST stages. The A values for the three rain types in the AU-SG Z-R relation are in the order C $>$ ST $>$ T, and the b values are in the reverse order T $>$ ST $>$ C. Several studies [25, 26] have shown how the DSD, and therefore the Z-R relationship, varies geographically, with rainfall intensity. It is also stated in [25] that the coefficient A is smaller for stratiform rain and increases for increasing convective activity, while b behaves in the opposite manner (smaller for convective rain and larger for stratiform systems). As expected, the coefficient A is larger for the tropical convective rain (Singapore) followed by the stratiform stage because of the presence of larger drops in these stages.

Clearly visible from Table 4, the coefficients of the overall Z-R relation for the 8 rain events (excluding the purely stratiform rain event on 18th May 1998) are closer to the stratiform rain types than the convective or transition rain types as concluded from Fig. 1. According to Wilson and Tan in [10], the A values for the three rain types are in the order C < T < ST, and the b values are in the order T < ST < C which is in a reverse order to that obtained in this paper, due to the analysis of only one rain event for the rain classification in [10]. The Z-R relation, SG, which is derived using all the convective, transition and stratiform rain DSD minutes, has higher A coefficient and lower b coefficient than the MP Z-R relation, and it represents the tropical rain pattern in Singapore. The GH-SG and AU-SG Z-R relations have the same trend that the A values for the two rain types are in the order C > ST, and the b values are in a reverse order ST > C. The Z-R relations proposed in this section are used to derive the rain rates from RADAR data, and the resultant rain rates are compared with the distrometer derived rain rates next.

3.3. Radar and Distrometer Reflectivities

Figure 5 shows the RADAR reflectivities in dBZ derived from the RADAR data at a height of 1.2 km and the distrometer data derived reflectivity at NTU. The minimum elevation angle for the airport

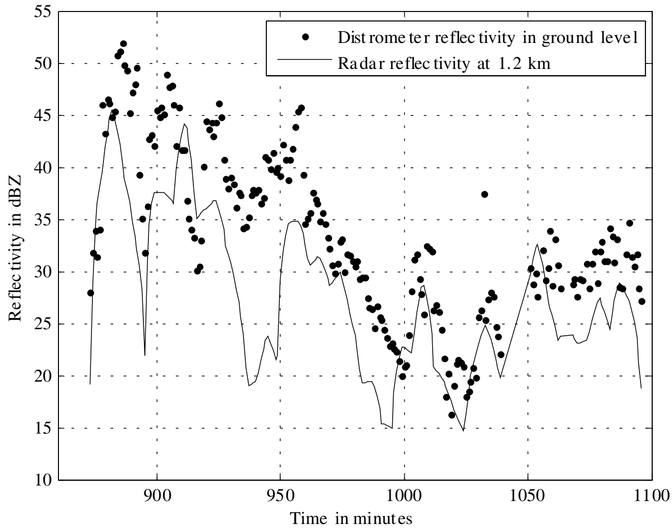


Figure 5. Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 12th May 1998.

mode is 1°; the beam width of the RADAR ray is 1°; therefore, the minimum height of the RADAR ray above NTU is 281 m. The DSD measurements are taken at NTU, 50 m above ground level. Ladd et al. [27] used RADAR reflectivities at a distance of 1 km to compare with the distrometer reflectivities data from Papua New Guinea. The distance 1.2 km is selected in this paper since at this distance RADAR reflectivity is nearer to the distrometer reflectivities.

In Fig. 5, the RADAR reflectivity and distrometer derived reflectivity are in very good qualitative agreement with each other. But the RADAR derived reflectivities are less than the distrometer derived reflectivites for around 86% of time. The deviations are higher at the convective peaks. The differences between the two reflectivity values could arise from a number of reasons; over the integration volume for the RADAR data, precipitation is inhomogeneous which is more obvious during the convective stage of the rain events; and there is a difference in sampling volume between the RADAR data and the distrometer data which also accounts for the difference in reflectivity.

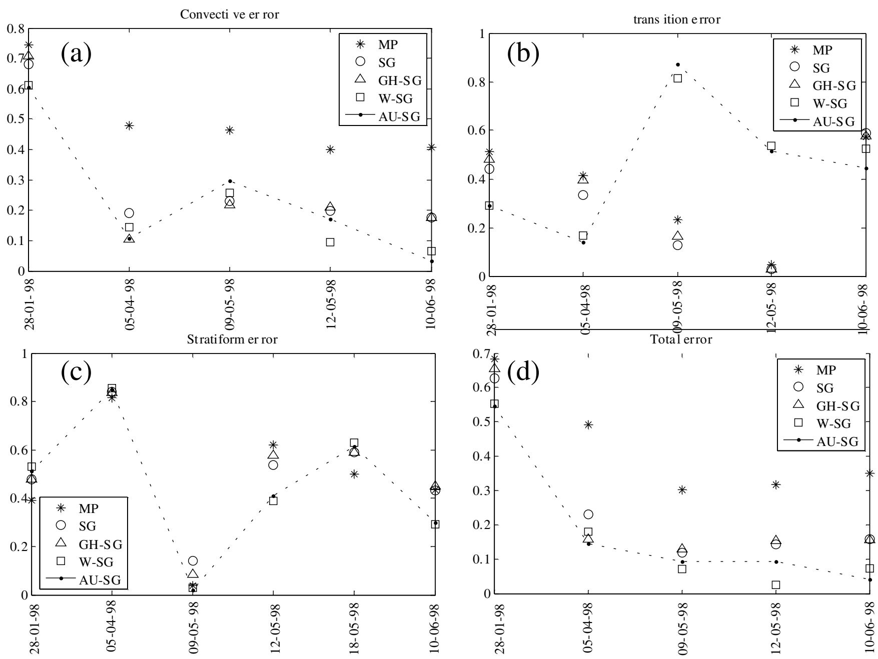


Figure 6. Error between the total accumulated rain rates derived from RADAR data using the MP, SG, GH-SG, W-SG and AU-SG Z-R relations with original rain rates measured by the Joss distrometer.

The distrometer samples the data near the ground over a sampling area of 5000 mm^2 over a period of one minute integration time, whereas the RADAR samples over a comparatively larger volume. The distance between NTU and Changi (where the RADAR is located) is 32.21 km. As the distance, between RADAR and the place where the data is taken increases, the sampled volume of the RADAR increases, and at the higher height of 1.2 km, the sampling volume further increases.

The correlation between the RADAR reflectivities and distrometer reflectivities for the whole rain event is 78%. The difference between the two reflectivities is from −2 dBZ to less than 10 dBZ, for 83% of the minutes. Since the deviations are larger, 5 dBZ of reflectivity is added to the RADAR data for calibration. This calibration reflectivity value of 5 dBZ with the RADAR data is derived through the analysis of the rain events in Table 1.

Figure 6 shows the error between the total accumulated DSD rain rates, $R_{DSD\text{-}T}$, and the total accumulated RADAR rain rates, $R_{RADAR\text{-}T}$, for the 5 rain events. X-axis and Y-axis represent the day of the rain event and the error for the corresponding rain event, respectively. The stratiform rain event recorded on 18th May 1998 is included in the stratiform error plot (subplot 3). The subplots in Fig. 6 show the convective error, transition error, stratiform error and total error for the MP, SG and GH-SG, W-SG and AU-SG Z-R relations respectively. The error is calculated using

$$Error = \frac{(R_{DSD\text{-}T} - R_{RADAR\text{-}T})}{R_{dsd\text{-}T}} \tag{10}$$

The total accumulated RADAR rain rates are calculated from the RADAR rain rates. The RADAR rain rates are calculated from the RADAR reflectivity at a height of 1.2 km using

$$R = \left(\frac{Z}{A}\right)^{(1/b)} \tag{11}$$

where A and b are the derived coefficients of the Z-R relation, and Z is the reflectivity in $\mathrm{mm}^{-6}\mathrm{m}^3$.

The calibrated RADAR data is used in the Z-R relations.

3.4. Comparison of Z-R Relations

As shown in Fig. 6(a), MP Z-R relation has a greater error when used on the convective stages than that of the Singapore Z-R relations, derived from Singapore data. AU-SG shows least convective error for three of the rain events and lower error for the other two rain events. W-SG relation gives lower error for the two of rain events and lower error for the other three events followed by AU-SG Z-R relation. It is clear from Fig. 6(a) that the GH-SG and SG Z-R relations give slightly higher errors than the AU-SG and W-SG Z-R relations. So they also represent well the relation between rain rate and reflectivity compared to MP Z-R relation. The total error in Fig. 6(d) also has a similar trend to the convective error, and AU-SG Z-R relation produces the least error followed by W-SG, GH-SG and SG relations.

AU-SG Z-R relation gives lower error for three of the five rain events in the transition stage. W-SG gives next lower errors similar to the convective stage for these three events. For the other two events, SG and GH-SG give lower error followed by MP-ZR relation. It is clear from Fig. 6(c) that all the Z-R relations produce comparatively lower errors in the stratiform stage. MP Z-R relation also gives lower errors for three of the stratiform stages.

All the stratiform fits work well during the stratiform stage of rain. For the convective stage, as can be seen from Fig. 6(a), the fits with higher A coefficients and lower b coefficients work well for the considered six rain events. As explained in Section 3.2.2, even small A and large b obtained by Wilson and Tan can also work well in the convective stage. The AU-SG Z-R relation produces lower error at all the three rain stages and therefore produces lower error for the entire rain event. Therefore, the AU-SG Z-R relation, derived using the detailed rain classification method, works well for the tropical climate of Singapore.

4. CONCLUSION

Z-R relations are derived using the 1496 minutes of DSD data from the year 1998 rain events. Rain is classified into different types based on the rain integral parameters. In Gamache-House method, one of the rain integral parameters, reflectivity, is used to classify the rain into convective and stratiform types whereas Atlas-Ulbrich method uses the variations in the rain integral parameters, rain rate, reflectivity, the parameter N_w, median volume diameter and the gamma model parameter μ to classify the rain into convective (C1, C2), transition (T) and stratiform (ST) types. It is found that convective stages have higher rain rates ($> 10\,\text{mm/hr}$ for most of the points) and higher reflectivites ($> 37\,\text{dBZ}$ for most of the points). Few exceptional points are present during the initial convective period. The N_w values are higher for the convective stage followed by the transition stage and spread a lot in the stratiform stage with lower values than C and T stages. In the transition regime, D_0 is the smallest followed by stratiform and convective stage. The gamma parameter, μ, is very least in the convective stage and increases continuously in the transition stage, and its spread is large with abrupt changes in the consecutive minutes in the stratiform stage.

Three Z-R relations, SG without classifying the rain, using the 1496 DSD points, GH-SG for C and ST type rains, using a simple threshold technique, AU-SG for C, T and ST type rains, using the variations in the rain integral parameters, are derived from the DSD of Singapore. GH-SG relation has a higher value of A and lower value of b for the convective rain, and the reverse is true for stratiform rain. For the Z-R relations derived using the Atals-Ulbrich, the A and b values spread from lower to higher in the convective and stratiform stages. Even though it is difficult to find the systematic variation in the values of A the b values for the convective and stratiform stages, A values (b values) of C stages are higher (lower) than the A values (b values) of ST stages. But transition stages have clear trend with lower A values and higher b values. The AU-SG Z-R relation is derived using all the convective, transition, and stratiform points, respectively. AU-SG Z-R relation has the trend that the A values for the three rain types are in the order C $>$ ST $>$ T, and the b values are in the reverse order T $>$ ST $>$ C. The rain rates calculated using the Z-R relations from the RADAR data are compared with the distrometer derived rain rates. The DSD derived rain rates are always higher than the RADAR derived rain rates. Therefore, 5 dBZ of reflectivity is added with the RADAR data as calibration. The total rain accumulation for the entire rain event is compared between the distrometer and RADAR rain rates. The derived AU-SG Z-R relation works well for the tropical climate of Singapore.

REFERENCES

1. Marshall, J. S. and W. M. Palmer, "The distribution of raindrops with size," *Journal of Atmos. Sci.*, Vol. 5, 165–166, 1948.
2. Ulbrich, C. W., "Natural variation in the analytical form of the raindrop size distribution," *J. Appl. Meteor.*, Vol. 22, No. 10, 1764–1775, 1983.
3. Battan, L. J., *Radar Observations of the Atmosphere*, Univ. of Chicago Press, 323, 1973.
4. Feingold, G. and Z. Levin, "The lognormal fit to raindrop spectra from frontal convective clouds in Israel," *J. Appl. Meteor.*, Vol. 25, 1346–1363, 1986.
5. Fujiwara, M., "Raindrop-size distribution from individual storms," *J. Atmos. Sci.*, Vol. 22, 585–591, 1965.
6. Tokay, D. and A. Short, "Evidence from tropical raindrop spectra

of the origin of rain from stratiform versus convective clouds," *J. Appl. Meteor.*, Vol. 35, No. 3, 355–371, 1996.

7. Atlas, D., C. W. Ulbrich, F. D. Marks, E. Amitai, and C. R. Williams, "Systematic variation of drop size and radar — Rainfall relations," *J. Geophysical Research*, Vol. 104, 6155–6169, 1999.
8. Tokay, A., D. A. Short, C. R. Williams, W. L. Ecklund, and K. S. Gage, "Tropical rainfall associated with convective and stratiform clouds: Intercomparison of disdrometer and profiler measurements," *J. Appl. Meteor.*, Vol. 38, No. 3, 302–320, 1999.
9. Maki, M., T. D. Keenan, Y. Sasaki, and K. Nakamura, "Characteristics of the raindrop size distribution in tropical continental squall lines observed in Darwin, Australia," *J. Appl. Meteor.*, Vol. 40, 1393–1412, 2001.
10. Wilson, C. L. and J. Tan, "The characteristics of rainfall and melting layer in Singapore: Experimental results from radar and ground instruments," *11th International Conference on Antennas and Propagation*, No. 480, 852–856, Conference Publication, Apr. 17–20, 2001.
11. Bringi, V. N., V. Chandrasekar, J. Hubbert, E. Gorgucci, W. L. Randeu, and M. Schoenhuber, "Raindrop size distribution in different climatic regimes from disdrometer and dual-polarized radar analysis," *J. Appl. Meteorol.*, Vol. 60, No. 2, 354–365, 2003.
12. Ulbrich, C. W. and D. Atlas, "Microphysics of raindrop size spectra: Tropical continental and maritime storms," *J. Appl. Meteor. Climatol.*, Vol. 46, 1777–1791, 2007.
13. Ulbrich, C. W. and D. Atlas, "Radar measurement of rainfall with and without polarimetry," *J. Appl. Meteor. Climatol.*, Vol. 47, 1929–1939, 2008.
14. Montopoli, M., F. S. Marzano, and G. Vulpiani, "Analysis and synthesis of raindrop size distribution time series from disdrometer data," *IEEE Trans. Geosci. Remote Sens.*, Vol. 46, No. 2, 466–478, 2008.
15. Distrome Ltd., Distrometer RD-69 Instruction manual, 1993.
16. Gunn, R. and G. D. Kinzer, "The terminal velocity of fall for water droplets in stagnant air," *J. Atmos. Sci.*, Vol. 6, No. 4, 243–248, 1949.
17. Campos, E. and I. Zawadski, "Instrumental uncertainties in Z-R relations," *J. Appl. Meteor.*, Vol. 39, 1088–1102, 2000.
18. Atlas, D., C. Ulbrich, F. D. Marks, R. A. Black, E. Amitai, P. T. Willis, and C. E. Samsury, "Partitioning tropical oceanic

convective and stratiform rains by draft strength," *J. Geoph. Res.*, Vol. 105, No. D2, 2259–2267, 2000.

19. Zhang, G., J. Sun, and E. Brandes, "Improving parameterization of rain microphysics with disdrometer and radar observations," *J. Atmos. Sci.*, Vol. 63, 1273–1290, 2006.
20. Cao, Q. and G. Zhang, "Errors in estimating raindrop size distribution parameters employing disdrometer and simulated raindrop spectra," *J. Appl. Meteor. Climatol.*, Vol. 48, No. 2, 406–425, Feb. 2009.
21. Smith, P. L., D. V. Kliche, and R. W. Johnson, "The bias and error in moment estimators for parameters of drop size distribution functions: sampling from gamma distributions," *J. Appl. Meteor. Climatol.*, Vol. 48, No. 10, 2118–2126, 2009.
22. Testud, J., S. Oury, R. Black, P. Amayenc, and X. Dou, "The concept of "normalized" distribution to describe raindrop spectra: A tool for cloud physics and cloud remote sensing," *J. Appl. Meteorol.*, Vol. 40, No. 6, 1118–1140, 2000.
23. Gamache, J. F. and A. R. Houze, "Mesoscale air motions associated with a tropical squall line," *Monthly Weather Review*, Vol. 110, 118–135, 1982.
24. Bringi, V. N., C. R. Williams, M. Thurai, and P. T. May, "Using dual-polarized radar and dual-frequency profiler for DSD characterization: A case study from Darwin, Australia," *J. Atmos. Oceanic Technol.*, Vol. 26, 2107–2122, 2009.
25. Sharma, S., M. Konwar, D. K. Sarma, M. C. R. Kalapureddy, and A. R. Jain, "Characteristics of rain integral parameters during tropical convective, transition, and stratiform rain at Gadanki and its application in rain retrieval," *J. Appl. Meteor. Climatol.*, Vol. 48, 1245–1266, 2009.
26. Villarini, G. and W. F. Krajewski, "Review of the different sources of uncertainty in single polarization radar-based estimates of rainfall," *Surveys in Geophysics*, Vol. 31, 107–129, 2009.
27. Ladd, D. N., C. L. Wilson, and M. Thurai, "Radar measurements from papua new guinea and their implications for TRMM PR retrieval algorithms," *Geoscience and Remote Sensing, IGARSS'97, Remote Sensing — A Scientific Vision for Sustainable Development*, Vol. 4, 1648–1650, 1997.