A NOVEL DUAL-BAND Π-SHAPED BRANCH-LINE COUPLER WITH STEPPED-IMPEDANCE STUBS

N. Zheng[1], L. Zhou[1,*], and W.-Y. Yin[2]

[1]Center for Microwave and RF Technologies, Shanghai Jiao Tong University, Shanghai 200240, China

[2]Center for Optics and EM Research, Zhejiang University, Hangzhou 310058, China

Abstract—A novel compact branch-line coupler operating in two arbitrary frequencies is proposed, analyzed and designed. Stepped-impedance stubs are used in the branch-line coupler to achieve dual-band applications. Parameters of the structure are chosen and provided for design guidelines. Broader operating frequency ratios and compactness are achievable. For the purpose of validation, a microstrip coupler operating at 2.4/5.2 GHz is fabricated and measured.

1. INTRODUCTION

Branch-line coupler is one of the most important passive components widely used in modern communication systems. Many researchers investigated it to reduce its size [1–3] and enhance its bandwidth [4] with novel structures, theories and techniques obtained. Recently, to meet the increasing demands of multi-band communication systems, various dual-band branch-line couplers have been investigated and designed [5–8]. A dual-band T-shaped branch-line coupler was designed in [5]. Instead of using T-shape stubs, stepped-impedance stubs coupler was demonstrated to achieve more realizable frequency ratios and compactness [6]. Also, the utilization of Π-shaped branch-lines is realized in [7]. However, the added stubs take a lot of room, and due to the restriction of realizable impedances, the dual-band frequency ratios are also very limited.

In this paper, a novel dual-band branch-line coupler with Π-shaped stepped-impedance stubs is proposed, and the design

equations are provided. Broader operating frequency ratios and more compactness are achievable.

Several design parameters, which are brought in by the stepped-impedance stubs, are studied and analyzed to provide the design guidelines. For demonstration, a quadrature hybrid operating in 2.4/5.2 GHz is simulated, fabricated and measured. The measured results validate the proposed structure.

2. PROPOSED STRUCTURE AND ANALYSIS

2.1. Dual-Band Branch-Line

The proposed dual-band branch-line is shown in Fig. 1. Two stepped-impedance stubs are connected at the two ends of a conventional branch-line. The entire impedance of the stubs Z_b and Z_c is

$$Z_{bc} = jZ_b \frac{Z_b \tan\theta_b - Z_c \cot\theta_c}{Z_b + Z_c \tan\theta_b \cot\theta_c} \tag{1}$$

The $ABCD$ matrix of the novel branch-line can be derived by cascading the matrices of three sections in Fig. 1 as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{bc}} & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_a & jZ_a \sin\theta_a \\ \frac{j\sin\theta_a}{Z_a} & \cos\theta_a \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{bc}} & 1 \end{bmatrix} \tag{2}$$

The proposed structure can act as a dual-band 90° branch-line if

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & \pm jZ_0 \\ \pm j\frac{1}{Z_0} & 0 \end{bmatrix} \tag{3}$$

If we let $\alpha = Z_c/Z_b$ and $\beta = \theta_c/\theta_b$, then by equating (2) and (3) at the frequency f_1 and f_2 $(f_2 > f_1)$, one can obtain

$$Z_a \tan\left((1-\sigma)\theta_{a0}\right) = Z_b \frac{Z_c \cot\left((1-\sigma)\theta_{c0}\right) - Z_b \tan\left((1-\sigma)\theta_{b0}\right)}{Z_b + Z_c \tan\left((1-\sigma)\theta_{b0}\right) \cot\left((1-\sigma)\theta_{c0}\right)} \tag{4}$$
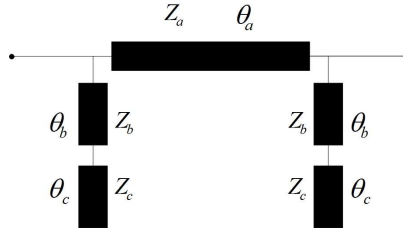

Figure 1. Proposed dual-band branch-line.

$$Z_a = \frac{Z_0}{\sin\left(\frac{1-\sigma}{2}n\pi\right)} \tag{5}$$

$$\theta_{a0} = \frac{n\pi}{2} \tag{6}$$

$$\tan(2\theta_{b0}) = \frac{2\alpha\sin(2\beta\theta_{b0})}{(1-\alpha^2)\cos(2\beta\sigma\theta_{b0}) - (1+\alpha^2)\cos(2\beta\theta_{b0})} \tag{7}$$

where Z_0 is the characteristic impedance of the objective branch-line. $\sigma = (f_2 - f_1)/(f_2 + f_1)$. θ_{a0} and θ_{b0} are electrical lengths of corresponding parts evaluated at the frequency $f_0 = (f_1 + f_2)/2$. Consequently, once the values of σ, α and β are decided, the values of characteristic impedances and electrical lengths for each part in the proposed dual-band branch-line are then determined through Equations (4)–(7).

2.2. Quadrature Hybrid

Based on above analysis, the structure of a dual-band quadrature hybrid is implemented as shown in Fig. 2. Note that every two stepped-impedance stubs of adjacent branch-lines are combined to be an equivalent stub (i.e., $Z'_{b,c} = Z_{b,c}/(1+2^{0.5})$), and the electrical lengths are evaluated at the frequency f_0. Since there are two free parameters α and β, various values of Z'_b, Z'_c, θ_{b0} and θ_{c0} are eligible for the dual-band design. By choosing the right values, the size of the quadrature hybrid can be significantly reduced with the broader frequency range obtained.

For the designed dual-band coupler working at the frequencies of 2.4 GHz and 5.2 GHz, the length of the stepped-impedance stub
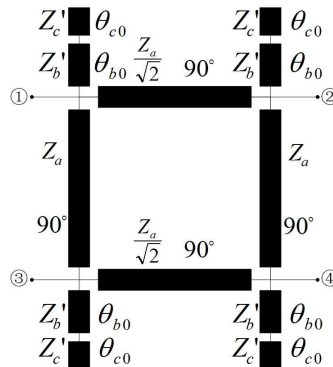

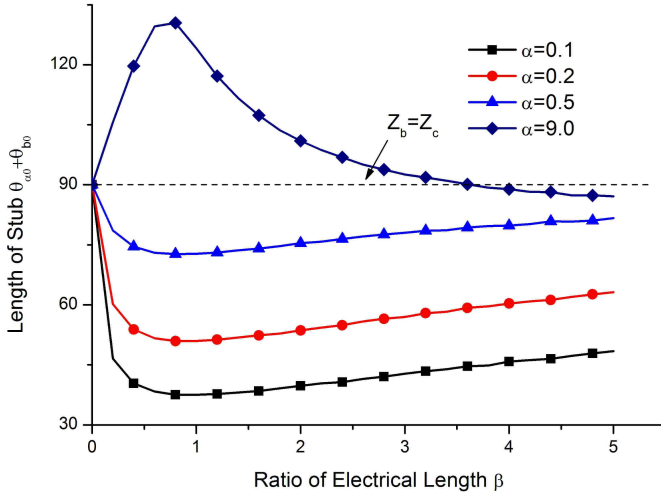
Figure 2. Structure of proposed coupler.

Figure 3. Variation of the electrical length of the stepped-impedance stub versus β.

according to various α and β is shown in Fig. 3. It can be seen that no matter which value α is chosen, various values of β exist. However, a small α and a β around 0.9 are more preferable for the compactness design. Fig. 4 shows the curve of Z_b' and Z_c' under different α and β. To take $20\,\Omega$–$120\,\Omega$ as an achievable impedance range, one can utilize all possible impedance ratios (i.e., $0.2 < \alpha < 6$) to fabricate the proposed coupler. Since the minimum of α is 0.2, couplers aiming to be compact in size should take $\alpha = 0.2$ and β around 0.9 according to the results of Fig. 3 and Fig. 4. Besides, due to the variety of qualified α and β, a broader operating frequencies ratio range of 1.1–6.3 can be achieved, which is much larger than the 1.7–2.7 in [6] and the 1.7–3.8 in [7].

Figure 5 compares the lengths of the stubs between the proposed structure and the traditional Π-shaped branch-line coupler [7]. It is, apparently, that the new structure reduces the stub size remarkably, especially at the lower frequency ratio. For the frequency ratio (i.e., 2.17 for 2.4/5.2 GHz operation) which we are interested in, the electrical length of the stub achieves a reduction from $56.8°$ to $30.4°$.

3. SIMULATION AND MEASURE RESULTS

A proposed coupler operating at the frequencies of 2.4/5.2 GHz is simulated, fabricated and measured. The values of the design parameters in Fig. 2 are chosen as $Z_a = 59.73\,\Omega$, $Z_b' = 107.31\,\Omega$,
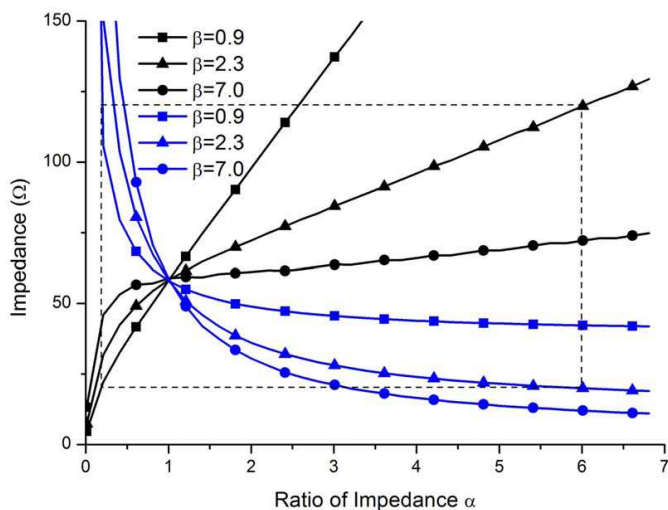
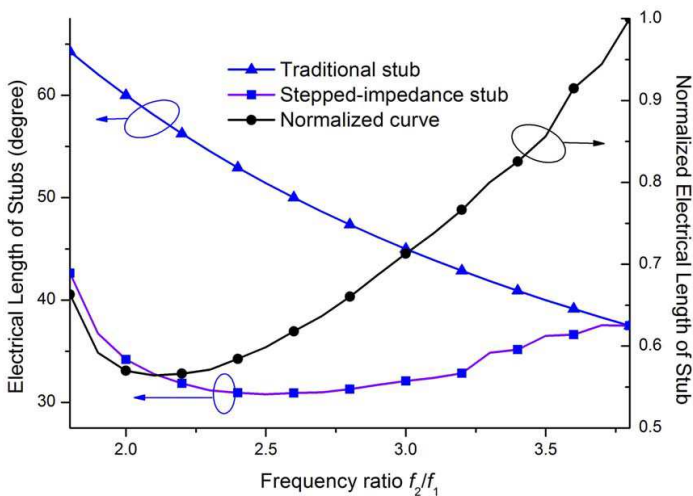**Figure 4.** Variation of Z_b' and Z_c' versus α.

Figure 5. Comparisons between the electrical length of the proposed stub and the conventional stub. The normalized curve is the length of the stepped-impedance stub normalized with the traditional design.

$Z_c' = 21.46\,\Omega$, $\theta_{b0} = 25.35°$, and $\theta_{c0} = 22.82°$.

In order to reduce the unexpected influence brought by the insertion of the ports, these parameters are slightly optimized by full-wave EM simulator HFSS. For the purpose of demonstration,

the coupler is fabricated using Taconic RF35A2 with a thickness of
0.76 mm and a dielectric constant of 3.5. As shown in Fig. 6, the
coupler, in size, is only about 25% of the one in [6], which operates
in similar frequencies 2.4 GHz/5.8 GHz (i.e., the ratio 2.22). Fig. 7
shows the simulation and measurement results with close agreements
obtained. The center frequency of the second band slightly moves
to 5.3 GHz. This deviation is possibly due to the fabricating error.
Table 1 gives the performance of the proposed coupler at 2.4/5.2 GHz.
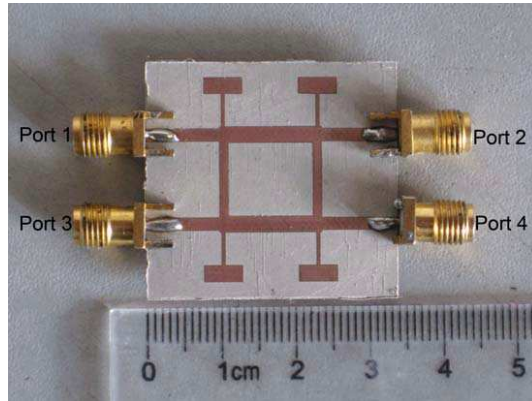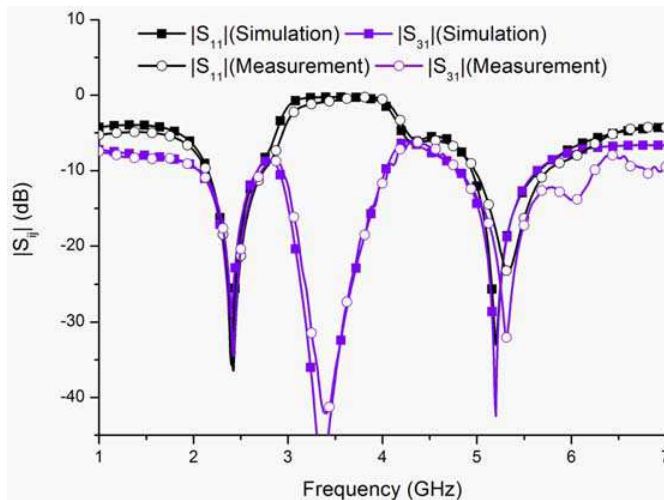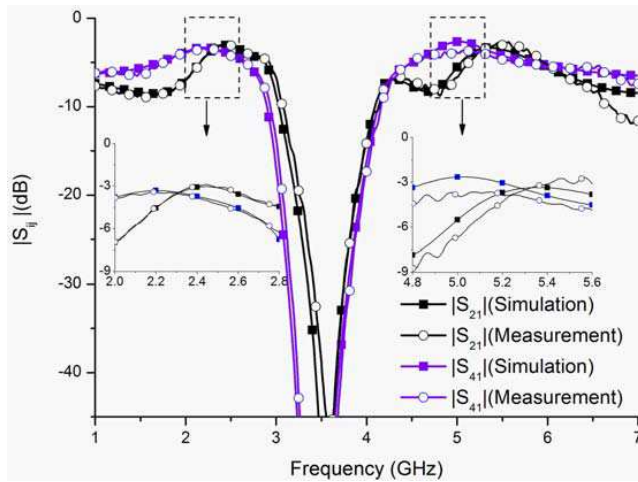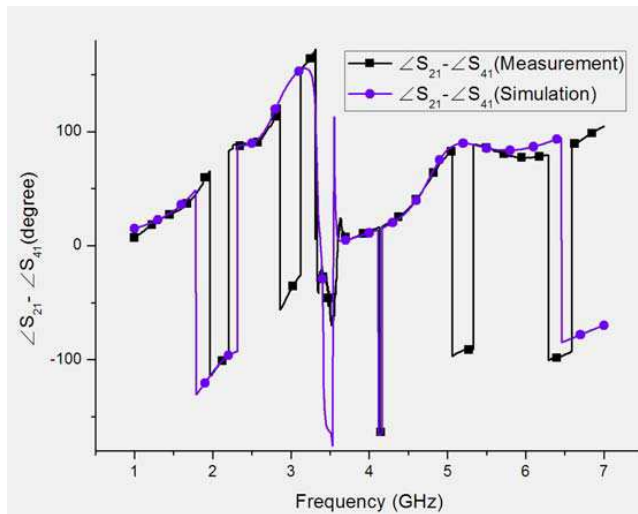The bandwidth is defined with ±0.5 dB amplitude-mismatch.

Figure 6. Fabricated dual-band quadrature hybrid.

(a)

(b)

(c)

Figure 7. Simulation and measurement results of the proposed dual-band coupler (a) The $|S_{11}|$ and $|S_{31}|$. (b) The $|S_{21}|$ and $|S_{41}|$. (c) The $\angle S_{21} - \angle S_{41}$.

Table 2 compares the proposed coupler with those in [5–7]. There are various impedance combinations for the stepped impedance to have the same impedance values with the single stub in [7] at the two frequencies. One can easily utilize stepped impedance which consists
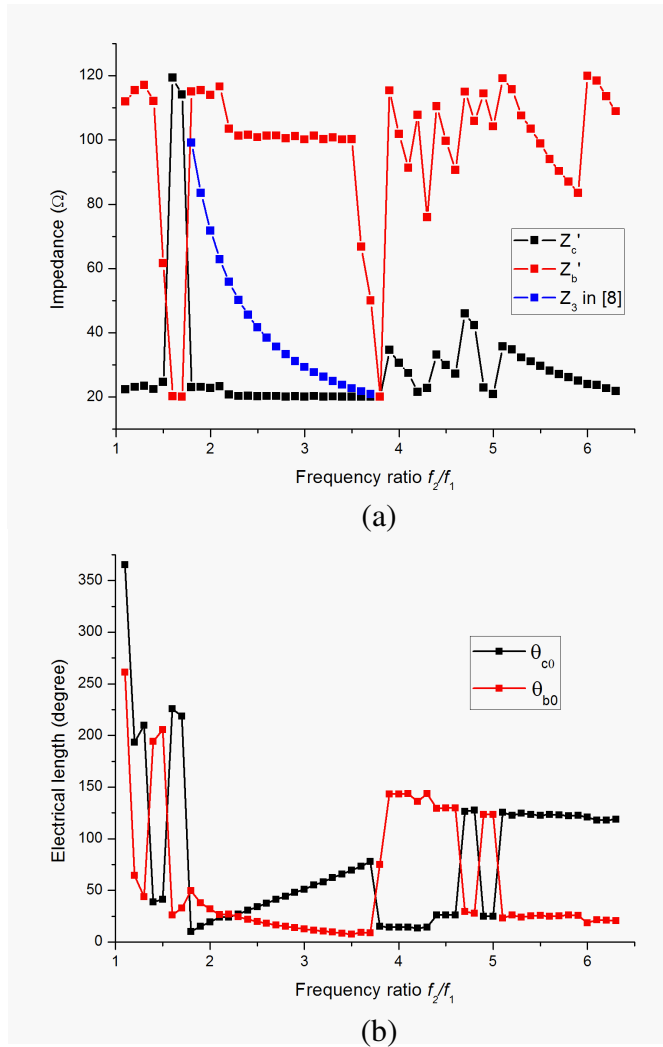
Figure 8. Impedance and electrical length of the stepped impedance stub for various frequency ratios.

of two practical impedance (e.g., $20\,\Omega$–$120\,\Omega$) to replace those single stubs whose impedance values are out of practical range. Consequently, the coupler in this work can achieve a broader frequency ratio range than that in [7]. For a given frequency ratio, one can get the values of the stepped impedance parameters α and β by solving transcendental equation (6) and then bring the results into Equation (4). Afterwards, a transversal can be done on α and β to find those combinations that

Table 1. Measurement results.

Frequency	2.4 GHz	5.3 GHz
Input Return Loss ($|S_{11}|$)	-30.5 dB	-22.0 dB
Isolation factor ($|S_{31}|$)	-29.5 dB	-31.6 dB
Insertion Loss ($|S_{21}|$)	-3.0 dB	-3.5 dB
Insertion Loss ($|S_{41}|$)	-3.9 dB	-4.0 dB
$\angle S_{21}-\angle S_{41}$	87.7 degree	88.8 degree
Relative Bandwidth (Port 2)	12.5%	3.8%
Relative Bandwidth (Port 4)	26.9%	15.9%

Table 2. Comparison results.

Coupler	Frequency ratio range	Operating frequencies	Size
Coupler in [5]	1.25–2.85	2.4/5.2 GHz	340.8 degree × 340.8 degree
Coupler in [6]	1.7–2.7	2.4/5.8 GHz	272.4 degree × 282.4 degree
Coupler in [7]	1.8–3.7	2.4/5.2 GHz	56.8 degree × 170.5 degree
This work	1.1–6.3	2.4/5.2 GHz	56.8 degree × 117.6 degree

can satisfy specific purposes. A numerical searching program has been developed to find the shortest stepped impedances with impedance values falling into the practical range $20\,\Omega$–$120\,\Omega$. Fig. 8(a) shows the values for Z_b', Z_c' and impedance value of the single stub Z_3 in [7] under different frequency ratios, and Fig. 8(b) shows the results for θ_{b0} and θ_{c0}. For example, if the stepped impedance stub for the frequency ratio is 1.5, the parameters can be realized as: $Z_b' = 61.55\,\Omega$, $Z_c' = 24.62\,\Omega$, $\theta_{b0} = 205.41°$, and $\theta_{c0} = 41.08°$. Besides the frequency ratio, another comparison is made on sizes. All sizes in Table 2 are calculated with design equations in each reference, and the size is described by the product of the length and the width's electrical lengths, which are estimated at $2.4\,\mathrm{GHz}$.

4. CONCLUSION

A novel $2.4/5.2\,\mathrm{GHz}$ compact dual-band branch-line coupler using stepped-impedance stub is demonstrated and analyzed with closed form formulas obtained. The parameters are given to provide design guidelines which show more flexibility and diversities. The performance of the proposed coupler is evaluated and measured. The simulation and measurement results show close agreements.

ACKNOWLEDGMENT

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