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A COMPACT MICROSTRIP RAT-RACE COUPLER WITH MODIFIED LANGE AND T-SHAPED ARMS

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Abstract—A compact microstrip rat-race coupler is proposed with a new phase inverter which is realized by a modified Lange coupling structure with a slotted ground plane and a floating-potential conductor. Based on the even- and odd-mode theory, its parameters are generally synthesized. In order to further miniaturize the rat-race coupler, a T-shaped line is utilized. A prototype operating at 2.0 GHz is designed and fabricated for verification. The circumference of our proposed rat-race coupler is only 0.52λ. Its in-phase and out-of-phase bandwidths are also enhanced, with reasonable agreement obtained between its simulated and measured S-parameters.

1. INTRODUCTION

With the rapid development of modern mobile and wireless communication systems, passive microwave devices, such as filters [1], antennas [2], power dividers [3, 4] and couplers [5, 6], with high performance are being required in large quantities. Rat-race coupler, also referred as hybrid ring coupler, is one type of key passive components for various microwave applications, such as mixers, power amplifiers, phase shifters and antenna feeding networks. This is

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because of its good isolation between two inputs as well as the flat phase difference response between two outputs. The conventional rat-race coupler is composed of one $3\lambda/4$ and three $\lambda/4$ arms, where λ is the guided wavelength at its central frequency, and its total circumference is 1.5λ. Such a coupler often has a large size and a narrow bandwidth [7], while modern communication systems require a large number of rat-race couplers with compactness and wideband high performance.

In order to reduce its size and enhance the bandwidth effectively, a phase inverter of $\lambda/4$ long is usually introduced into a rat-race coupler instead of the $3\lambda/4$ arm. In [8], a $\lambda/4$ arm has been proposed with an additional phase reversal realized by short-ended parallel coupled lines, leading to tight coupling needed between the parallel lines. However, such tightly coupled lines are very difficult to be implemented with conventional microstrip structures. For the purpose of miniaturization, a new microstrip rat-race coupler is developed by incorporating a microstrip-to-coplanar waveguide (CPW) broadside-coupled structure into the circuit [9], which can provide a strong coupling with a phase reversal. In [10], a phase inverter is realized using a transition between microstrip and CPW on the ground, and it is used to construct a compact rat-race hybrid ring. A phase reversal swap, based on double-sided parallel-strip line (DSPSL), is utilized to save about half the area of conventional rat-race coupler [11], which provides a 180° phase shift and introduces small insertion loss within a wide frequency range.

In a rat-race coupler, the $3\lambda/4$ arm can also be replaced by a $\lambda/4$ left-handed arm for the same purpose [12]. However, left-handed transmission lines are usually realized by a periodical structure with complicated configuration and narrow band. So, the performance of left-handed rat-race coupler is not superior to the coupled-line rat-race one [13].

On the other hand, there are many useful methods for area reduction in the development of rat-race couplers, such as multi-folded tracing [14], stepped-impedance [15] and slow-wave structures [16, 17]. The combination of high-impedance transmission lines and shunted lumped capacitors is proposed in [18]. In [19], stepped-impedance stub loading technique is introduced into the design of branch-line and rat-race couplers, which shows the advantages of compactness and dual bands, etc.

In fact, it is not necessary to make three of the four arms in a rat-race coupler have the same impedance and the same length of $\lambda/4$. Some rat-race couplers with $5\lambda/4$, $7\lambda/6$ and even $19\lambda/18$ circumferences are designed [20, 21]. Generalized synthesis of the rat-race coupler, with its two opposite arms having identical characteristic impedances, is proposed in [22]. The derived solutions show that the circuit circumference can be further reduced even shorter than a single wavelength. Using the stepped-impedance technique, a rat-race coupler with its circumference of 0.54λ is developed, which is much compact.

In this paper, based on the previous studies on rat-race couplers, we at first propose a set of generalized equations for their design with an ideal phase inverter according to the even- and odd-mode theory. Then, the phase reversal, with the arm length shorter than $\lambda/4$, is realized by short-ended coupled lines, and the equations given in [8] are extended to a more generalized case. A modified microstrip Lange coupling structure, with a slotted ground plane and a floating-potential conductor, is proposed to meet the specification. A T-shaped line is utilized to further shrink the overall component size. In order to validate our idea, a microstrip rat-race coupler prototype operating at 2.0 GHz is designed and measured. Its circumference is about 0.52λ, which shows its size reduction efficiently. At the same time, its bandwidth is also enhanced, with reasonable agreement obtained between its measured and simulated S-parameters.

2. ANALYSIS AND DESIGN

2.1. Design Equations of the Rat-race Coupler with a Phase Inverter

Figure 1 shows the layout of a rat-race coupler, where the third arm is separated by an ideal phase inverter with frequency-independent 180° phase shifting. The parameters θ_i and Y_i ($i = 1$, 2 and 3) denote the electrical lengths and the normalized characteristic admittances of corresponding arms in the rat-race coupler, respectively.

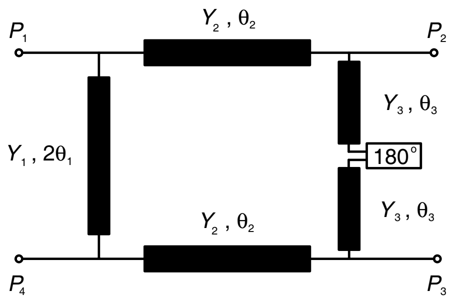


Figure 1. Schematic of the rat-race coupler.

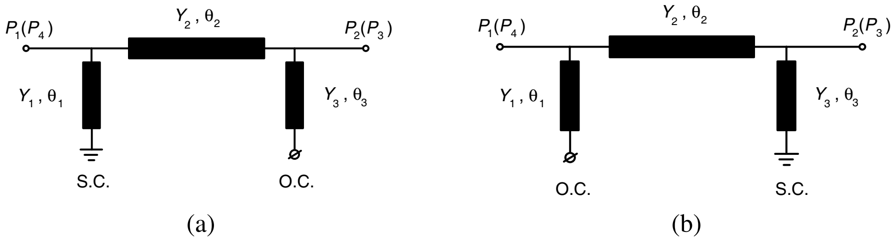


Figure 2. Equivalent circuits of the rat-race coupler for (a) even- and (b) odd-modes.

The even- and odd-mode equivalent circuits of the rat-race coupler are shown in Figures 2(a) and 2(b), respectively. According to [7] and [22], both reflection and transmission coefficients for even- and odd-modes can be derived. Then, the S-parameters of the rat-race coupler are easily calculated. By using the conditions of perfect matching, perfect isolation, in-phase and out-of-phase responses [22], its design equations can be derived as

$$Y_3 = Y_1 \quad (1)$$

$$\theta_3 = \theta_1 \quad (2)$$

$$\sin\theta_2 = R\sin 2\theta_1 \quad (3)$$

$$1/Y_1 = \sqrt{1 + R^2 - 2R\cot 2\theta_1 \cot\theta_2} \quad (4)$$

where $R = Y_2/Y_1$, $\theta_1 \leq 45^\circ$ and $\theta_2 \leq 90^\circ$. Obviously, the two arms on opposite sides should have identical characteristic impedances and identical electrical lengths. These equations are directly used for miniaturizing the rat-race coupler, and the area reduction factor depends on the impedance ratio R and electrical length θ_1 adopted in the design.

2.2. Phase Reversal Implemented with Short Coupled Lines

Short-ended coupled lines with their electrical length of $\lambda/4$ can be used to realize an additional phase reversal [8], as shown in Figure 3. In order to provide an additional phase reversal by the coupled lines with an arbitrary electrical length, the design equations presented in [8] should be extended.

A pair of short-ended coupled lines with their electrical lengths of θ is equivalent to a transmission line with its electrical length of $180^\circ + \theta$, which is a good candidate to build up a phase inverter. Assume that $\theta_e = \theta_o = \theta$, where θ_e and θ_o are the electrical lengths of

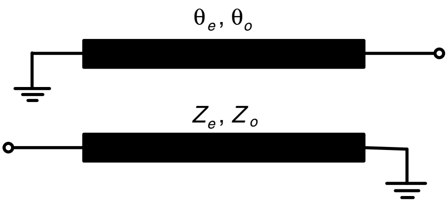


Figure 3. Schematic of two coupled lines with their opposite ends short-circuited.

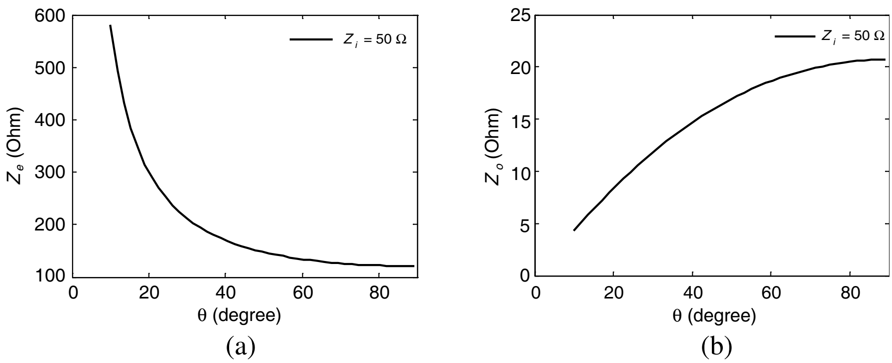


Figure 4. (a) Z_e and (b) Z_o as functions of θ with $Z_i = 50\,\Omega$.

even- and odd-modes, respectively. Then, the equivalent characteristic impedance of the coupled section is given by [8]

$$Z_i = \frac{2Z_eZ_o\sin\theta}{\sqrt{(Z_e - Z_o)^2 - (Z_e + Z_o)^2\cos^2\theta}} \tag{5}$$

where Z_e and Z_o are the even- and odd-mode impedances, respectively, and we also have

$$Z_i = \sqrt{Z_eZ_o} \tag{6}$$

Then, the following equations are derived, i.e.,

$$Z_e = Z_i\left(\sqrt{1+\csc^2\theta} + \csc\theta\right) \tag{7}$$

$$Z_o = Z_i\left(\sqrt{1+\csc^2\theta} - \csc\theta\right) \tag{8}$$

When $\theta = 90°$, both $Z_e = Z_i(\sqrt{2}+1)$ and $Z_o = Z_i(\sqrt{2}-1)$ are identical to those given in [8].

As $Z_i = 50\,\Omega$, Z_e and Z_o against θ are plotted in Figures 4(a) and 4(b), respectively. It is found that Z_e decreases and Z_o increases with θ increased from 0° to 90°. Note that Z_e is quite high while Z_o is quite low when the value of θ is small. In other words, as we want to implement the design values obtained from (7) and (8), very tight coupling and low odd-mode impedance should be realized in those cases.

Considering the short-ended coupled lines together with the rat-race coupler, we set $Z_i = Y_1^{-1}$ and $\theta = 2\theta_1$. Based on (1)–(4), (7) and (8), Z_e and Z_o versus θ_1 for different values of R can be calculated when the port impedance Z_{port} is $50\,\Omega$, which are plotted in Figures 5(a) and 5(b), respectively. For a given R, both Z_e and Z_o increase with increasing θ_1. When $R > 1$, the calculated even-mode impedance Z_e will be very high with small θ_1 given, and at the same time, both Z_e and Z_o change quickly with θ_1. From these design curves, it can be found that $R \leq 1$ is a good choice to design the rat-race coupler using the above method. The variations of Z_e and Z_o are shown in Figure 5(c) for $R = 0.94$.

2.3. Modified Lange Coupling Structure

Parallel coupled lines are widely used for building various passive components [23–25] because of their simple configurations and easy fabrication. However, for edge-coupled microstrip lines, it is difficult to get such low odd-mode impedance as shown in Figure 5(b). On the other hand, conventional coupled microstrip lines have the inherent nature of weak coupling. Thus, four-line parallel-coupled microstrip lines are introduced into the design of typical Lange couplers, and their equivalent even- and odd-mode impedances, denoted by Z_{e4} and Z_{o4}, are approximated by [26]

$$Z_{e4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}} Z_{0e} \tag{9}$$

$$Z_{o4} = \frac{Z_{0o} + Z_{0e}}{3Z_{0e} + Z_{0o}} Z_{0o} \tag{10}$$

where Z_{0e} and Z_{0o} are the even- and odd-mode impedances of a two-conductor pair, respectively.

To further enhance the coupling, some modifications are introduced into the design of Lange structure. In [27], a large aperture is etched under the Lange coupled lines. In [28], a floating-potential conductor is adopted for filter design, together with a slotted ground plane to strengthen the coupling between two-conductor coupled microstrip lines. The floating-potential conductor can also be migrated into the realization of Lange coupling structure, with our proposed modified structure shown in Figure 6. It consists of a set of four-line parallel-coupled microstrip lines, a slotted ground plane and a floating-potential conductor. The four coupled lines have identical widths, and they are short-circuited through four metallic via holes. In Figure 6(b),

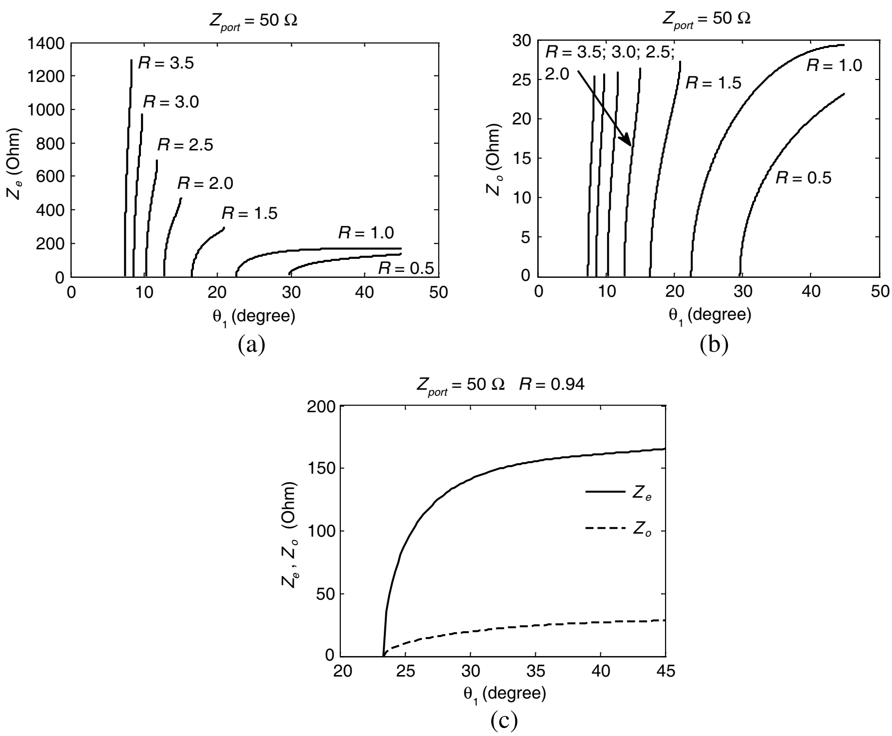


Figure 5. (a) Z_e and (b) Z_o as functions of θ_1 for different values of R, (c) Z_e and Z_o as functions of θ_1 for $R = 0.94$.

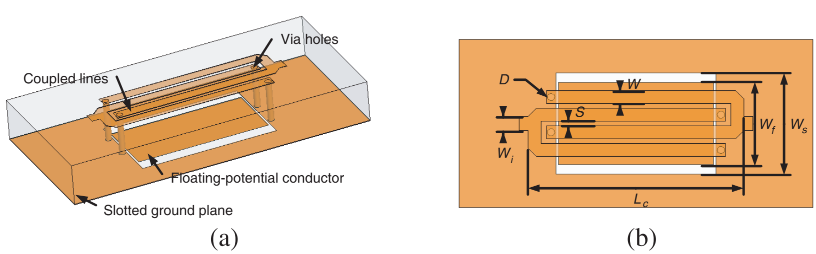


Figure 6. Configuration of the proposed modified Lange coupling structure. (a) 3-D and (b) top views.

W_i is the width of coupled lines whose characteristic impedance is designed to be Y_1^{-1}. L_c, S and W_s are their length, spacing and width, respectively. W_f is the width of the floating-potential conductor, and D is the diameter of via holes. In our design, the gap between the floating-potential conductor and the slotted ground plane has a width of 0.2 mm.

As the slotted ground plane is introduced below the coupled lines, both even- and odd-mode impedances increase simultaneously. However, the even-mode impedance increases much faster than the odd-mode one. Meanwhile, the floating-potential conductor placed under the coupled lines also helps to reduce the odd-mode impedance. Thus, the provided coupling can be enhanced effectively. It is evident that the design of slotted ground and floating-potential conductor is important since both Z_e and Z_o are mainly determined by their dimensions. As Z_e, Z_o and a specific substrate are given, the design parameters of the modified Lange section can be obtained by using a full-wave EM simulator, such as Ansoft HFSS.

It should be pointed out that the total length of signal-path through the coupling structure with electrical length θ is not equal but close to $180^\circ + \theta$ in practice. However, the equivalent electrical length can be easily compensated to $180^\circ + \theta$ by adding two short extra sections with the characteristic impedances of Z_i to the Lange coupling structure.

2.4. T-Shaped Line Section

In order to further miniaturize the structure, a T-shaped line section is used to replace the conventional transmission lines in the rat-race coupler [29]. A shunted stub is tapped to the center of the uniform section, as shown in Figure 7, where Z_T, θ_T, Z_a, θ_a, Z_b and θ_b represent the characteristic impedances and electrical lengths of the uniform section, the series and shunted sections. Using the even- and odd-mode theory, the design equations of T-shaped line section are derived

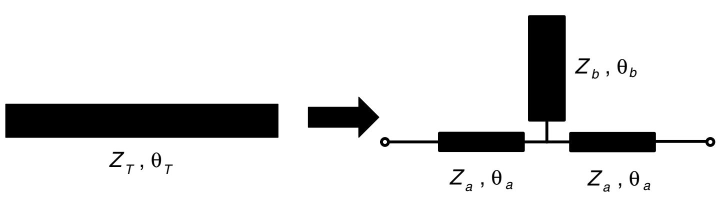


Figure 7. Substitution of a uniform section by a T-shaped line.

as

$$y_1 = -\frac{\tan\theta_T}{2}\left(\frac{R_T\tan\theta_b+\tan\theta_a}{1-R_T\tan\theta_b\tan\theta_a}-\frac{1}{\tan\theta_a}\right)=\frac{Z_a}{Z_T} \tag{11}$$

$$y_2 = \frac{\sin\theta_T}{2}\left(\frac{R_T\tan\theta_b+\tan\theta_a}{1-R_T\tan\theta_b\tan\theta_a}+\frac{1}{\tan\theta_a}\right)=\frac{Z_a}{Z_T} \tag{12}$$

where $R_T = Z_a/(2Z_b)$. For the given Z_T and θ_T, when R_T and θ_a are determined, the values of θ_b and y_1 can be obtained according to (11) and (12). Then, we have $Z_a = y_1 Z_T$ and $Z_b = Z_a/(2R_T)$.

3. RESULTS AND DISCUSSION

In order to validate our method, a microstrip rat-race coupler operating at 2.0 GHz is designed on a F4B-2 substrate, with its relative permittivity of $\varepsilon_r = 2.65$, a loss tangent of 0.003, and a thickness of 0.5 mm. As shown in Figure 8, the Y_1-arm is a conventional microstrip line; the Y_3-arm with a phase inverter is realized using the modified Lange coupling structure; and the two opposite Y_2-arms are substituted by T-shape lines for further miniaturization.

Based on (1)–(4), (7) and (8), the design parameters are obtained with $R = 0.94$, $\theta_1 = \theta_3 = 30°$, $Y_1^{-1} = 52.66\,\Omega$, $\theta_2 = 54.5°$, $Y_2^{-1} = 56.02\,\Omega$, $Z_e = 141.25\,\Omega$ and $Z_o = 19.63\,\Omega$. Some critical parameters of the modified Lange arm of the miniaturized rat-race coupler are listed in Table 1. The dimensions of T-shaped arms

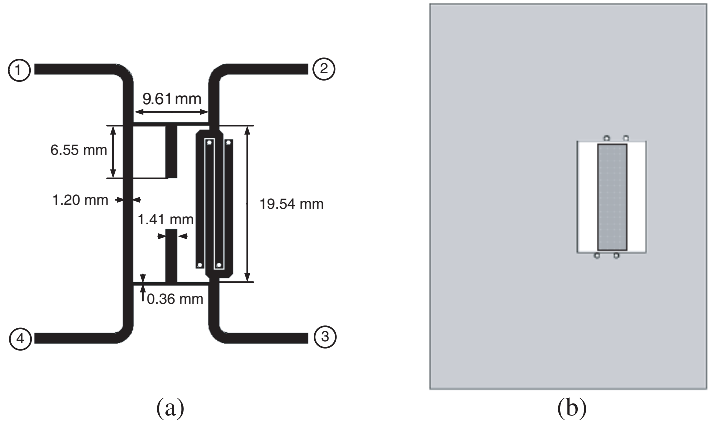
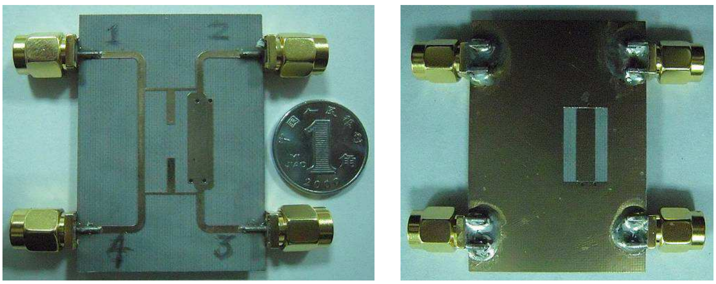


Figure 8. Configuration of the proposed rat-race coupler. (a) Top and (b) bottom views.

Table 1. Critical dimensions of the modified Lange coupling section.

Parameter	W_i	L_c	W	S	D	W_s	W_f
Unit: mm	1.20	18.44	1.00	0.20	0.60	8.43	3.62



(a) (b)

Figure 9. Photo of the rat-race coupler prototype. (a) Top and (b) bottom views.

are further determined by (11) and (12). In above case, $R_T = 1.0$, $Z_a = 94.36\,\Omega$, $\theta_a = 17°$, $Z_b = 47.18\,\Omega$, and $\theta_b = 23.9°$. In other words, the electrical length of Y_2-section is reduced by 20.5°. According to our theoretical derivation, it can be found that the total circumference of the microstrip rat-race coupler is only 0.52λ at $f_0 = 2.0\,\text{GHz}$. So, the size of our realized rat-race coupler has been reduced greatly.

Figure 9 shows the photo of the fabricated prototype. Its size without the four ports is $20.3 \times 15.6\,\text{mm}^2$, while a conventional microstrip rat-race coupler with the same operating frequency and substrate occupies an area of about $1936.6\,\text{mm}^2$. The S-parameters of the prototype are measured by an Agilent 8722ES vector network analyzer, and they are plotted in Figure 10 together with the simulated ones. It is observed that reasonable agreement is obtained between them.

The in-phase and out-of-phase magnitude responses of the rat-race prototype are given in Figures 10(a) and 10(b), respectively. At the central frequency $f_0 = 2.0\,\text{GHz}$, the measured $|S_{21}|$, $|S_{41}|$ (in-phase outputs) and $|S_{11}|$ (reflection) are -3.15, -3.85 and $-15.39\,\text{dB}$, respectively. The measured $|S_{23}|$, $|S_{43}|$ (out-of-phase outputs) and $|S_{33}|$ (reflection) are -3.96, -3.29 and $-15.83\,\text{dB}$, respectively. And the isolation between Ports 1 and 3 is 29.07 dB at 2.0 GHz. Figure 10(c) shows the phase difference responses of the proposed coupler. At

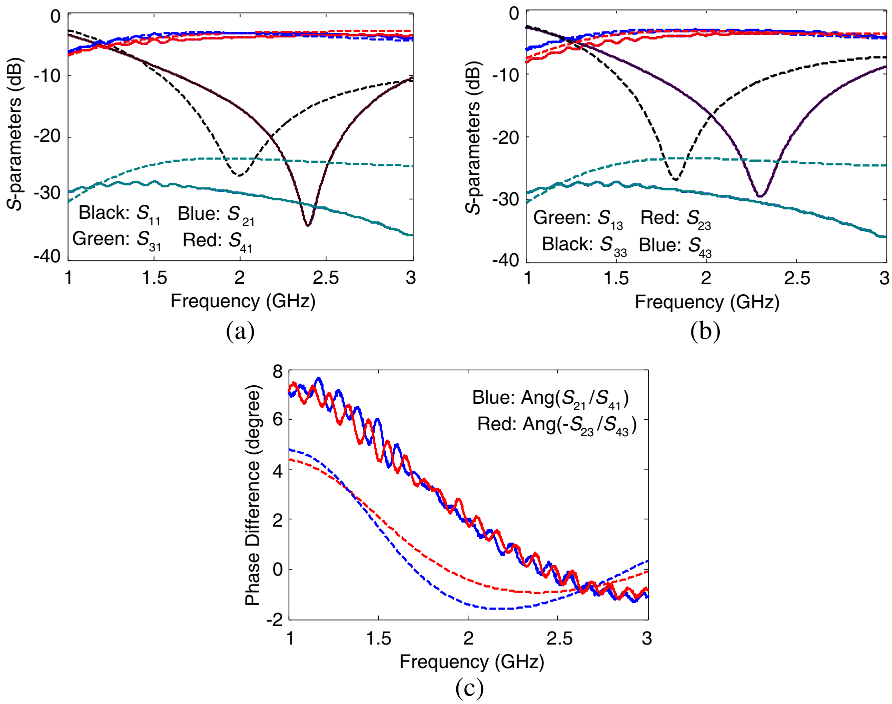


Figure 10. Measured and simulated frequency responses of the rat-race coupler prototype, where solid and dashed lines represent measured and simulated results, respectively. (a) In-phase and (b) out-of-phase magnitude responses. (c) Phase differences.

$f_0 = 2.0\,\text{GHz}$, the phase difference between S_{21} and S_{41} ($\angle S_{21} - \angle S_{41}$) is 2.0°, while the corresponding phase difference between S_{23} and S_{43} ($\angle S_{23} - \angle S_{43}$) is 181.8°. In comparison with the simulated S-parameters, we would like to point out that the measured performance is slightly degenerated by not only the permittivity deviation of substrate but also the fabrication tolerance, in particular of the modified Lange coupling structure.

The measured reflection zeros of S_{11} and S_{33} are located at 2.42 and 2.31 GHz, respectively, with the frequency shift between the measured and simulated ones observed. At 2.41 GHz, the measured results are $|S_{21}| = -3.27\,\text{dB}$, $|S_{41}| = -3.55\,\text{dB}$, $|S_{11}| = -33.79\,\text{dB}$, $|S_{23}| = -3.24\,\text{dB}$, $|S_{43}| = -3.55\,\text{dB}$, $|S_{33}| = -23.55\,\text{dB}$, $|S_{31}| = -31.12\,\text{dB}$, $\angle S_{21} - \angle S_{41} = 0.1^\circ$, and $\angle S_{23} - \angle S_{43} = 180.0^\circ$, which shows a better performance than the measured results at 2.0 GHz.

Furthermore, the measured fractional bandwidths with $|S_{11}| \leq$

$-15\,\text{dB}$ and $|S_{33}| \leq -15\,\text{dB}$ are 31.7% and 27.1%, respectively, while the 0.54λ rat-race coupler in [22] can only provide a small fractional bandwidth of 4.6%.

On the other hand, the circumference of our realized rat-race coupler is about 1/3 of that of the conventional one, with good isolation between two input ports achieved. Also, good phase difference responses are observed over a wide frequency range. Its frequency responses are much flatter than those of other counterparts, together with a wide reflection bandwidth obtained. These characteristics are demonstrated by the simulated as well as measured S-parameters of the fabricated prototype as shown above.

4. CONCLUSION

In this paper, a compact microstrip rat-race coupler is proposed and realized by using standard PCB technology. At first, some design equations for the rat-race coupler with an ideal phase inverter are derived using the even- and odd-mode theory. Then, the phase inverter is implemented by a pair of short-ended coupled lines, whose electrical length is not restricted to 90°. According to our deduced equations, a modified Lange coupling structure is presented to get tight coupling and low odd-mode impedance for the phase inverter, including four-line parallel-coupled microstrip lines, a slotted ground plane and a floating-potential conductor. In order to further reduce the coupler size, two opposite arms are built up by a set of T-shaped lines. Finally, a prototype of the microstrip rat-race coupler is designed at 2.0 GHz and fabricated so as to validate our method. The circumference of our developed rat-race coupler is only 0.52λ, and it offers a good isolation and a wide operating bandwidth.

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